Optimal Placement and Sizing of Distributed Generations in Unbalanced Distribution Networks Considering Load Models and Uncertainties

F. Amini and R. Kazemzadeh

Abstract: Development of distributed generations’ technology, trends in the use of these sources to improve some of the problems such as high losses, low reliability, low power quality and high costs in distributed networks. Choose the correct location to install and proper capacity of these sources, such as important things that must be considered in their use. Since distribution networks are actually unbalanced and asymmetric consumption loads are different, so in this paper with optimal placement and sizing of distributed generation sources that dependent on the load model and type of load connection and the uncertainties which caused by the generated power of wind turbines and solar panels, the positive effects of these sources have been examined on unbalanced distribution network. Hence with linear three-phase unbalanced load flow method and IPSO algorithm, allocation of distributed generation sources in IEEE standard of 37 bus unbalanced network have been done. Obtained results show improvement of voltage profile in each phase and reduction of network power losses and buses’ voltage unbalance factor.

Keywords: Distributed Generations’ Uncertainty, Linear Three Phase Unbalanced Load Flow, Load Model, Unbalanced Distribution Network.
costs. Also the power losses are reduced [8]. The use of these sources in convenient size and location, will have other benefits for distribution systems like improving voltage profile, active and reactive line loss reduction, increasing network reliability, improving electrical efficiency, improving voltage stability and etc. [9]. The important thing about solar and wind generation sources is that, power generation of these sources dependent on external factors such as wind speed and intensity of sunlight. So the generated power of these units are constantly changing, and modeling them with constant speed and constant output power is incorrect and it will create a lot of errors in simulations. Hence, the methods of uncertainty analysis should use to achieve a model for generated power’s uncertainty of these sources [10]. Several studies have been conducted on the above topics. In [11] has been reminded that the usual methods such as Newton-Raphson load flow is not suitable for unbalanced distribution networks, and used ZBus method to investigate load flow in unbalanced network. In [12] with backward-forward load flow and genetic algorithm, reconfiguration and change arrangement of feeders has been done, ultimately this method lead to reducing losses and improving voltage profile in two unbalanced distribution networks. In [13] using the new method, 4-wire unbalanced load flow networks have been investigated.

So far, DGs allocation have been discussed in balanced networks. In this paper, DGs allocation have been done on IEEE standard of 37 bus unbalanced network, which has unequal loads on the phases and mutual inductance between phases of lines. Therefore appropriate size of sources has been obtained according to the uncertainty of generated power and convenient location of sources has been chosen by IPSO algorithm. Also, linear three phase unbalanced load flow method with considering load model and type of loads connection has been used in calculations. According to the obtained results voltage profile of each phase has been improved and both network power losses and voltage unbalance factor have been reduced.

This paper is structured as follows.

Loads are modeled in section 2. Type of loads connection are expressed in section 3. Voltage unbalance is expressed in section 4. Multi objective function that include the reduction of network active power losses, voltage profile improvement and reduction of the voltage unbalance factor is described in section 5. Network constraints are given in section 6. The power of distributed generations (wind turbine and solar panels) are obtained considering their uncertainty in section 7. Linear three phase unbalanced load flow is described in section 8. IPSO algorithm and its implementation are described respectively in section 9 and section 10. Case study and simulation results are elaborated in section 11 and conclusions are expressed in section 12.

2. Load Modeling

In most studies, power of network loads has been assumed constant. In the event that, actually loads are dependent on the voltage and powers will not be constant. Generally static load model is as shown below.

\[
P = P_0 \left( \frac{V}{V_0} \right)^\alpha, \quad Q = Q_0 \left( \frac{V}{V_0} \right)^\beta
\]  

That \( P_0 \) and \( Q_0 \) are rated power at rated voltage and loads are divided into three categories based on the amounts of \( \alpha \) and \( \beta \), as below [14].

- constant power load \( \alpha = \beta = 0 \)
- constant current load \( \alpha = \beta = 1 \)
- constant impedance load \( \alpha = \beta = 2 \)

3. Type of Load Connection

Loads can be connected to the respective bus as star or delta. Voltage of star connected load is considered to be phase to neutral and voltage of delta connected load is considered to be line to line. Connection of unbalanced loads as star and delta are shown respectively in Fig. 1 and Fig. 2.

4. Define and Measure Voltage Unbalance

In a three-phase system, the difference in size or angle of the phase or line voltage will cause the creation of voltage imbalance. An unbalanced three-phase system is using symmetrical component method in the form of Eq. (2) to turns in to three components of balanced positive, negative and zero sequence [15].
That $V_a, V_b$ and $V_c$ are respectively phase voltages of phase a, b and c and $V_0, V_1$ and $V_2$ are respectively voltage components of zero, positive and negative and $a = e^{j2\pi}$.

So far, due to the importance of voltage unbalance, different definitions of voltage imbalance is provided. According to IEEE standard, VUF is as follows [16].

\[
\text{VUF} = \frac{V_{ij}}{V_{ij}} \times 100
\]  

(3)

5. Multi Objective Function
The multi objective function in this paper include the reduction of network active power losses, voltage profile improvement and reduction of the voltage unbalance factor.

5.1. Reducing of Network Active Power Losses
One of the important parameters of network is reduce losses and it is possible with the help of the DGs. Power losses of unbalanced distribution network is obtained as follows [17]:

\[
F_i = \sum_{P \in \text{network}} P_{\text{loss}} = \sum_{i=1}^{n} \sum_{j=1}^{n} V_i^{(P)} V_j^{(P)} \cos(\theta_i^{(P)} - \theta_j^{(P)} - \delta_{ij}^{(P)})
\]  

(4)

That $P$ is representing phases A, B and C, $n$ is the number of network buses, $V_i^{(P)}$ and $\theta_i^{(P)}$ are respectively voltage size and angle of phase P at bus i, $V_j^{(P)}$ and $\theta_j^{(P)}$ are respectively voltage size and angle of phase P at bus j, $\delta_{ij}^{(P)}$ are respectively admittance size and angle of phase P between i and j buses.

5.2. Improving of Voltage Profile
Optimal DG placement can improve voltage profile of phases. In fact, the purpose of improving voltage profile is that the voltage of each phase is close to 1 perunit [18].

\[
VD_a = V_{i_A} - V_{i_A}
\]

\[
VD_b = V_{i_B} - V_{i_B}
\]

\[
VD_c = V_{i_C} - V_{i_C}
\]  

(5)

That $V_{i_A}$, $V_{i_B}$ and $V_{i_C}$ are voltage size of bus i in phases A, B and C respectively, and $V_{i_{ref}}$ is the desired voltage value, which is usually considered 1 perunit. The aim of this section is that Eq. (6) is minimized.

\[
F_2 = \max(VD_a, VD_b, VD_c)
\]  

(6)

5.3. Reducing of Voltage Unbalance Factor
The aim of this section is that Eq. (7) is minimized. That was defined in Eq. (3).

\[
F_3 = \max(V_{ij})
\]  

(7)

To improve efficiency and achieve better results, the objective function is defined as a combination of the three above objectives and weighted sum of these objectives is considered as Eq. (8).

\[
F = k_1 \times F_1 + k_2 \times F_2 + k_3 \times F_3
\]  

(8)

\[k_1 + k_2 + k_3 = 1 \quad k_1, k_2, k_3 \in [0,1]\]

Values of the coefficients are selected based on the importance of objectives in such a way that the best results are achieved. The amount of phase voltages and power can obtain as perunit. Also, perunit expressed as a percentage. So all the terms are in perunit.

6. Network Constraints
Network constraints are as follows:

- Voltage of each phase, should be between the minimum and maximum voltage. So standard limit of voltage profile is considered to be 0.9-1.1 (perunit) or close to it [9].
- Losses with presence of DG must be less than the losses without DG.
- Allowable voltage unbalance factor is considered 2 percent [19].

7. Determining Distributed Generation’s Power Considering Uncertainty
Generated power of wind turbines and solar panels are respectively dependent on wind speed and sun intensity. So generated power of these sources are uncertain. In this section, generated power uncertainty modeling of these sources have been studied by probabilistic methods.

7.1. Wind Turbine Power Probabilistic Modeling
Wind speed profile follows the Weibull distribution function [20]. Wind speed probability density distribution function divided into several sections and the probability of each section is attributed to midpoint of each section.

So the probability of wind speed can be obtained as follows [20].

\[
P_{GF} = \int_{v_{ref}}^{v_{ij}} f(v)dv
\]  

(9)
That \( f_r(V) \) is probability density function of wind speed, \( V_{wl1} \) and \( V_{wl2} \) are respectively beginning and end speed of the desired section. Linear relation between active output power and wind speed is shown as follows [20].

\[
P_w = \sum_{i=1}^{s} P_w(i) \times P_i(G_i)
\]

That \( V_{ci} \), \( V_{r} \) and \( V_{co} \) are respectively wind turbine start speed, maximum speed and cut out speed of each section’s speed and \( P_{rated} \) is wind turbine rated power. Finally, wind turbine average power can be obtained from Eq. (11).

7.2. Solar Panel Power Probabilistic Modeling

Beta probability distribution function follows the probability distribution function for each interval is calculated middle of each interval as representative, the solar irradiance is divided into 0.1 (kw/m\(^2\)) intervals and for the each section's speed and \( P_{rated} \) is wind turbine rated power. Therefore, compared with other methods of distribution and model and type of connection loads are considered. In this method distribution network unbalanced structure is suitable. So in this paper the new method of load flow, named linear three-phase unbalanced load flow is used. In this method distribution network unbalanced structure and model of connection loads are considered. Therefore, compared with other methods of distribution network load flow like backward-forward load flow is more accurate and faster [22].

In this method there is a relation between buses voltage and current as follows.

\[
\begin{bmatrix}
I_s \\
I_n
\end{bmatrix} = \begin{bmatrix}
Y_{ss} & Y_{sn} \\
Y_{ns} & Y_{nn}
\end{bmatrix} \begin{bmatrix}
V_s \\
V_n
\end{bmatrix}
\]

In which the S index represents the slack bus and the N index represents other buses, \( Y_{ss} \) and \( Y_{nn} \) are respectively insider admittance of nodes with S and N index, \( Y_{sn} \) and \( Y_{ns} \) are mutual admittance between nodes with S and N index, \( I_s \) and \( V_s \) are respectively current and voltage of bus with S index, \( I_N \) and \( V_N \) are respectively current and voltage of bus with N index. The N index represents three nodes that is related to each phase. So admittance matrix size will be as \((N_{bus}+N_{bus})^2\) that \( N_{bus} \) shows the number of network buses. There is a relation between current and voltage of each bus as combined load model as follows.

\[
I_s = \frac{S'_s}{V_s} + h \cdot S'_m + h^2 \cdot S'_z \cdot V_s
\]

That \( h=1/V_{nom} \) and \( V_{nom} \) is network nominal voltage. Phrases \( S'_p / V'_K \), \( h \cdot S'_k \) and \( h \cdot S'_z \cdot V \) are respectively current of constant power load model, constant current...
load model and constant impedance load model that $S_{P_k}, S_{I_k}, S_{Z_k}$ are respectively network powers in constant power load model, constant current load model and constant impedance load model and $V_k$ is voltage of bus k. This relation is nonlinear toward current of constant power load model section that is approximated to obtain linear load flow. Therefore, with the help of the Taylor series $1 - AV = \sum_{n=0}^{\infty} \frac{(-AV)^n}{n!}$ and with remove the great exponents of this series and define $[V = 1 - AV]$, linear form can be obtained as follows.

$$\frac{1}{V} = \frac{1}{1 - AV} = 1 + AV = 2 - V$$  \hspace{1cm} (21)

In unbalanced network for loads with delta connection, line voltage can be obtained from phase voltage as follows.

$$V_{N\text{Line}} = M \cdot V_{N\text{Phase}}$$  \hspace{1cm} (22)

M matrix is defined as follows:

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (23)

That $V_{N\text{Line}}$ is line voltage and $V_{N\text{Phase}}$ is phase voltage. After applying the linear approximation in Eq. (20) and using Eq. (22), linear form is arranged as the following phrase.

$$A + BV_A^* + CV_C^* = 0$$  \hspace{1cm} (24)

In this relation A, B and C are as follows.

$$A = Y_{ss} \cdot V_A^2 - 2h \cdot M^T \cdot S_{P_s} \cdot T - h \cdot M^T \cdot S_{C_s} \cdot T$$  \hspace{1cm} (25)

$$B = h^3 \cdot M^T \cdot \text{diag}(S_{P_s} \cdot T^2) \cdot M$$  \hspace{1cm} (26)

$$C = Y_{ss} - h^3 \cdot M^T \cdot \text{diag}(S_{C_s} \cdot T^2) \cdot M$$  \hspace{1cm} (27)

That $T^{\text{as}} = \omega$ and $\varphi = \{0, -2\pi/3, 2\pi/3\}$. In Eq. (24) the real and imaginary values of voltage (respectively with r and i index) can be obtained from the following equation.

$$\begin{bmatrix} -A \\
-A \end{bmatrix} = \begin{bmatrix} B + C & B - C \\
B + C & -B + C \end{bmatrix} \begin{bmatrix} V_r \\
V_i \end{bmatrix}$$  \hspace{1cm} (28)

After obtaining the voltage of all buses, load flow calculations has ended and network power losses is calculated.

9. IPSO Algorithm

IPSO algorithm is improved PSO algorithm. PSO, first time was presented by Eberhart and Kennedy that it has been inspired from the mass movement of birds that they are looking for food [23]. This algorithm works with a group of particles and search with update generations to find the optimal solution in the problem space. Each particle has two values, position($X_i$) and speed ($V_i$) that update at each stage of the population with the better of two values. The first value is $P_{\text{best}}$ that is the best answer in terms of fitness which has been obtained for each particle and the second value is $G_{\text{best}}$ that is the best value which has been achieved by all particles among the total population so far. In each iteration after finding values of two best, speed and new place of each particle update from Eq. (29) and Eq. (30).

$$V_{i+1} = w \cdot V_i + c_1 r_1 \cdot (P_{\text{best}} - X_i) + c_2 r_2 \cdot (G_{\text{best}} - X_i)$$  \hspace{1cm} (29)

$$X_{i+1} = X_i + V_{i+1}$$  \hspace{1cm} (30)

That $V_{i+1}$ is speed of the particle i in k iteration, $c_1$ and $c_2$ are acceleration coefficient in 1-2 interval, $r_1$ and $r_2$ are random numbers between 0-1, $X_i$ is position of the particle in the k iteration, $P_{\text{best}}$ is the best position of the particle in the k iteration, $G_{\text{best}}$ is the best position of the group in the k iteration and w is stiffness factor parameter that generally defined as follows.

$$w = w_{\text{min}} - \frac{w_{\text{max}} - w_{\text{min}}}{\text{Iter}_{\text{max}}} \times \text{Iter}$$  \hspace{1cm} (31)

That $w_{\text{min}}$ and $w_{\text{max}}$ are respectively lowest and highest weight, Iter$_{\text{max}}$ is the highest number of iteration and Iter is iteration number.

In IPSO algorithm a repeater is used that is a dynamic system and shows irregular behavior. The following equation shows how the behavior of these repeaters [24].

$$D_i = \mu \cdot D_{i-1} \cdot (1 - D_{i-1})$$  \hspace{1cm} (32)

where $D_k$ is an irregular parameter that is sensitive to $\mu$ changes, which is a control parameter and its amount is changed between 0-4. The amount of $\mu$ determines that the amount of $D_k$ remain constant or swing between limited quantities or behaves randomly. When $\mu=4$ and $D_0=\{0.0, 0.25, 0.5, 0.75, 1\}$ the relation shows irregular be-
behavior and the behavior of the system is very sensitive to the initial state of $D_k$.

One of the major weaknesses of PSO is premature convergence and particles do not discover responses better local. So in IPSO, weight parameter is determined as Eq. (33) which enhances the search for PSO in the problem space.

$$w_{\text{new}} = w \cdot D$$  
(33)

That $w_{\text{new}}$ and $w$ are respectively IPSO and PSO weight parameter. $w_{\text{new}}$ is reduced as oscillatory and with the total number of iterations. Whereas $w$ is reduced constantly from $w_{\text{max}}$ to $w_{\text{min}}$.

10. IPSO Algorithm Implementation

In this section by using the IPSO algorithm, optimal placement of DGs would be reviewed. So the algorithm is implemented as follows.

1. The amount of DGs generated power that is obtained due to the methods of uncertainty, is considered as input parameters of the algorithm.
2. The first generation of particles is produced in random order (first place candidates) and $P_{\text{best}}$ and $G_{\text{best}}$ are recorded.
3. Linear three-phase unbalanced load flow run and network constraints are examined.
4. The next generation is produced, if the number of iterations reach to the specified value and network constraints are not violated, $G_{\text{best}}$ is selected and location of DG is introduced and the multi objective function is calculated. Otherwise, the algorithm is repeated so as to reach the maximum number of iterations.

11. Case study

Performance of unbalanced load flow method and proposed algorithm has been applied to IEEE standard of 37 bus unbalanced network. The studied network is shown in Fig. 3. The network is unbalanced due to phases unequal loads and mutual impedances between line phases. Network nominal voltage is 4.8 (kV) and bus with index 701 is slack bus. A variety of load models are shown as constant power (PQ), constant current (I) and constant impedance (Z) and type of loads connection are shown as star (Y) and delta (D) [25].

Simulations have been performed regardless of network transformer and regulator in MATLAB R2010a computing environment with Core i5, 2.20 GHz computer with 8.00 GB RAM. So buses with 799 and 775 index are neglected and the number of buses reduced to 35.

Coefficients of multi-objective function are selected as $K_1=K_2=0.3$ and $K_3=0.4$ that gives the best results. The maximum number of iterations is considered 100.

Necessary parameters for modeling generated power uncertainty of wind turbines and solar panels are respectively shown in Table 1 and Table 2.

In wind turbine $k=3$ and $c=8$ and in solar panel $\alpha, \beta=3.5$ and $N=500$ have been considered on the basis of regional conditions.

According to the above information, the appropriate amount of DGs power obtained as shown in Table 3.
Because the system is unbalanced, single phase DG sources must be installed on one phase of elected bus to achieve optimum results.

So by the proposed algorithm and according to the size of DGs obtained in Table 3, elected phases and buses to install DGs are shown in Table 4.

The obtained results for network power losses and maximum voltage unbalance factor, with and without DG installation are shown in Table 5.

Obtained results from the placement of DGs, according to the obtained powers from probabilistic methods and consider the model and type of loads connection, in Table 5 show that network losses and maximum percentage of voltage unbalance factor are reduced after installing DGs. Because the system is unbalanced, the size and angle of the phases’ voltage are different. Therefore, to better illustrate the voltage changes, phases’ voltage profile before and after DG installation are shown in Fig. 4. The percentage of voltage unbalance factor of each bus before and after DG placement has been compared in Fig. 5.

As shown in Fig. 4 voltage profile of each phase after placement of DGs improved. It is observed that the index decreased after placement in all buses by comparing buses’ voltage unbalance factor before and after placement of DGs as shown in Fig. 5.

To demonstrate the effectiveness of the methods used in this paper, a comparison has been done according to Table 6 between the losses that obtained from genetic algorithm in [26] and this article after DG placement. In [26] DGs power is considered 2500 KW for each bus.

<table>
<thead>
<tr>
<th>Number of DGs</th>
<th>Bus Number</th>
<th>Bus Phase</th>
<th>Type of DG</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>725</td>
<td>B</td>
<td>PV</td>
</tr>
<tr>
<td></td>
<td>727</td>
<td>C</td>
<td>wind</td>
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<tr>
<th>P_{max} (KW)</th>
<th>VUF_{max} (%)</th>
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<tr>
<td>Without DG</td>
<td>With DGs</td>
</tr>
<tr>
<td>33.1192</td>
<td>23.3648</td>
</tr>
<tr>
<td>1.3816</td>
<td>1.0490</td>
</tr>
</tbody>
</table>

![Fig. 4. Voltage profile with and without DGs (a) in phase A, (b) in phase B, (c) in phase C.](image)

![Fig. 5. Comparison VUF with and without DGs.](image)
According to Table 6, the losses derived from calculations in this article is lower than that derived from [26]. In [26] allocation has been done regardless of DGs power uncertainty, load model and the type of loads connection. So placement done in this paper is more accurate than [26].

In Table 7, voltage angle results for this paper are observed before and after DG placement.

12. Conclusion

Because of the imbalance loads and mutual inductance between different phases, distributed systems are actually unbalanced systems. An imbalance has various negative effects on distribution networks such as reducing of voltage instability, increasing of network losses and etc. So we should consider all details of the network to reduced imbalance of network by appropriate methods. In this paper first, load models and type of loads connection are described using the corresponding relations. Then since the generated power of wind turbines and solar panels sources depend on external factors such as, wind speed and sun intensity, modeling generated power uncertainty of these sources is done. Then linear three-phase unbalanced load flow method is presented and reviewed that it is a new load flow method with low computation time, for unbalanced distribution networks. And for optimal DG placement, IPSO algorithm has been suggested. To eval-

<table>
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<th>Table 6. Comparison of simulation results</th>
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<tr>
<td><strong>Bus number</strong></td>
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<tr>
<td>----------------</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<table>
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<tr>
<th>Table 7. Comparison angle (degree) of voltage different phases with and without DGs</th>
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<tr>
<td><strong>Bus no.</strong></td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td><strong>θA</strong></td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>3</td>
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**Note:** IPSO algorithm has been suggested.
ulate the effectiveness of three-phase unbalanced load flow method and IPSO algorithm, IEEE standard of 37 bus unbalanced network is studied. Simulations are done in Matlab software with considering buses load models and type of loads connection according to IEEE standard and generated power uncertainty of solar and wind sources. The results of the simulations, represent improving of each phase voltage profile, and reduction both network losses and percentage of buses voltage unbalance factor.

References


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