Regulatory Impacts on Distributed Generation and Upstream Transmission Substation Expansion Planning: A Novel Stochastic Bi-level Model

F. Misaghi*, T. Barforoushi* (C.A.) and M. Jafari-Nokandi*

Abstract: In this paper, a novel framework is proposed to study impacts of regulatory incentive on distributed generation (DG) investment in sub-transmission substations, as well as upgrading of upstream transmission substations. Both conventional and wind power technologies are considered here. Investment incentives are fuel cost, firm contracts, capacity payment and investment subsidy relating to wind power. The problem is modelled as a bi-level stochastic optimization problem, where the upper level consists of investor's decisions maximizing its own profit. Both market clearing and decision on upgrading of transmission substation aiming at minimizing the total cost are considered in the lower level. Due to non-convexity of the lower level and impossibility of converting to single level problem (i.e. mathematical programming with equilibrium constraints (MPEC)), an algorithm combining enumeration and mathematical optimization is used to tackle with the non-convexity. For each upgrading strategy of substations, a stochastic MPEC, converted to a mixed integer linear programming (MILP) is solved. The proposed model is examined on a six-bus and an actual network. Numerical studies confirm that the proposed model can be used for analysing investment behaviour of DGs and substation expansion.

Keywords: Capacity Payment, Firm Contract, Mathematical Programing with Equilibrium Constraints (MPEC), Mixed-Integer Linear Programming (MILP), Wind Power.

1 Introduction

POWER industry has experienced drastic changes in structures, markets, regulations and managements in the past decades. One of these changes has been entering to new competitive setting, which has made new challenges and opportunities for electric utilities. Electric companies which are responsible to meet consumers’ demands on acceptable reliability levels and optimal costs use various operational and investment options in the short and long-term. In order to make small investors enter into competitive market, DGs were considered as investment option. The objective of independent system operator (ISO) is minimizing the total operation and expansion cost. Also, medium voltage levels are suitable locations for installing DG units [1-2]. In this respect, upgrading of upstream substation in the presence of DGs is an important issue for system operator, because expansion of DGs and network has interaction with each other. Furthermore, investment in DG is an irreversible task which may impose a substantial cost to investors. So, investment return must be guaranteed in the long-term since investors are unwilling under energy only markets mechanisms. In this way, various mechanisms are set to encourage and control investment [3].

Another reason for widely usage of DGs is the environmental concerns. Among renewable DG technologies, applying wind power for its low operating cost and environmental impact is widespread. However, intermittent nature of wind power has significant impacts on the operation and expansion of the power system. Therefore, this uncertainty should be appropriately considered. In this regard, appropriate models should be developed to solve combined conventional and wind power expansion problem and also to investigate impacts of investment incentives on renewable and conventional DGs expansion [4-5]. Several methods have been presented in literatures for expansion of DGs. These methods can be classified into numerical, heuristic and analytical methods [6]. Some of the most common numerical methods include gradient search, linear programming (LP), nonlinear programming (NLP) and dynamic programming (DP). Heuristic methods are based on artificial intelligence advanced techniques such as genetic algorithm (GA), particle swarm optimization (PSO), and Tabu Search (TS). The main feature of these methods is their computational robustness. An overview to methods and
models is presented in [6] to classify research trends in this area. In [7], a method is presented for allocation and sizing of renewable DGs to minimize total losses in radial distribution systems. The problem is formulated as a single objective mixed integer nonlinear programming and is solved by means of the Augmented Lagrangian Genetic Algorithm (ALGA). Authors in [8] presented an algorithm for modelling stochastically dependent renewable energy resources of unbalanced networks, where diagonal band copula and sequential Monte Carlo method have been used to consider multivariate stochastic dependency among wind power, photovoltaic power and demand. In [9], optimal location and capacity of DGs are determined in distribution network in the absence of feeder changes and substation upgrades. Particle swarm optimization with inertia weight (PSO-w) is used to solve the model, while a boundary mutation strategy is given to improve global searching ability. Optimal placement of wind-based DGs in sub-transmission network with the objective of minimizing transmission losses is presented in [10]. An improved analytical (IA) method was presented in [11] for optimal planning of DGs in distribution system. In the absence of DGs, traditional power systems need to expand network by either upgrading the existing or substations installing new ones. Since DG investment and network expansion affect each other, simultaneous consideration of substation upgrading and DGs penetration can lead to better network expansion plans [12-13]. Thus, an appropriate model is required for planning of DGs and network. A dynamic model was presented in [14] where DGs together with substation upgrading and feeders' reinforcement are considered simultaneously, where capital, maintenance, operating and losses costs are taken into account as objectives. DGs installation in sub-transmission substations was studied in [15]. In [16] optimal location of sub-transmission substations is considered under demand uncertainty through a single level problem which is solved by GA. The expansion planning of DGs and sub-transmission substations is presented in [17] where the cost function consists of investment, construction, and installation costs of sub-transmission substations as well as connection costs of substations to the upper network, new medium-voltage feeders, and DGs. However, upgrading of upstream transformers was neglected. In [18], a stochastic bi-level model is proposed for transmission and wind power investment in market environment. It minimizes the total cost in the upper level and maximizes the social welfare in the lower level while taking into account investment subsidy relating to wind power. In [19], wind power investment problem was considered in an investor point of view, where in the upper level, the investor maximizes his profit, while market clearing is performed in the lower level. In [20] a bi-level model is presented to identify the private owner's offer about volume and contract pricing of wind power.

According to the literature, expansion planning of DGs in sub-transmission systems and upgrading of transmission substations under incentives has not been investigated. So in this paper, a novel stochastic bi-level framework is proposed to study the impacts of incentive policies on investment behaviour of conventional and wind DGs in sub-transmission networks, as well as upgrading of transmission substation in a target year. Thus, wind power and conventional technologies along with substations upgrading are considered as expansion options. Stochastic nature of wind power is modelled by means of scenarios. The upper level problem contains investment decisions on DGs aiming at maximizing of investor's profit, while minimizing the total operator payments, including substation upgrading and the operation cost is considered as objective in the lower level. Incentives are firm contracts, capacity payments, subsidy to wind power investment, and fuel cost. To tackle the non-convexity of the lower level problem and impossibility of converting into a single level MPEC, an algorithm combining the enumeration and mathematical optimization is developed. For each strategy of substations upgrading, the bi-level problem is converted to a single-level MPEC and solved as a MILP after linearization. The main contributions of this paper are as the below:

- Developing a novel bi-level framework to solve expansion planning of DGs and upgrading of upstream substations in a contractual and wholesale energy market.
- Considering both conventional and wind power technologies in the proposed model.
- Investigating the impacts of investment incentives such as firm contract, capacity payment, investment subsidy and fuel cost on the expansion of the system.
- Proposing a combined algorithm based on mathematical optimization and enumeration to solve the non-convex bi-level problem.

The rest of this paper is organized as follows. Mathematical formulation of the proposed model is presented in section 2. In section 3 we analyse the model on a 6-bus test system and a real power network. The conclusion of the numerical study is organized in the last section.

2 Mathematical formulation

2.1 Introducing indices, variables and parameters

In order to define the model, definition of the decision variables, indices, sets, and parameters is presented in the Table 1.

2.2 The bi-level model

The stochastic investment problem is formulated as the following bi-level model which comprises an upper (12)-(18)). In the upper level problem investment
Table 1  Decision variables, indices, sets, and parameters.

Indices and sets:
- $i$: 63 KV buses.
- $j / k$: 20 KV buses.
- $n / m$: Index for conventional / wind units.
- $l$: Demand blocks.
- $d$: Index for demand.
- $b / h$: Size of investment options.
- $w$: Index for wind scenarios.
- $\phi^j$: Set of busses connected to bus $j$.
- $\phi^w_{63}$/ $\phi^w_{63}$: Set of 20/63 KV buses.
- $\phi^d$: Set of indices for demand blocks.
- $\phi^w$: Set of indices for wind scenarios.
- $\phi^b$: Set for investment options.

Constants:
- $B_{jk}^{SU}$: Susceptance of line $j-k$ (pu).
- $F_{jk}^{max}$: Capacity of line $j-k$ (MW).
- $FCV_H$ / $FCV_w$: Contract volume for DGs.
- $CP_{ij}^H$ / $CP_{ij}^{wind}$: Contractual price ($/MWh$).
- $CPR_{ij}^{HFC}$ / $CPR_{ij}^{HP}$: Capacity payment rate ($/MWh$).
- $CPR_{ij}^{wind}$ / $CPR_{ij}^{wind}$: Capacity payment rate ($/MWh$).
- $S_{wind}$: Subsidy for wind power investment.
- $MC_n$: Marginal cost of DGs ($/MWh$).
- $\rho_{ij}$ / $\rho_{ij}^{wind}$: Probability of needing DGs by system operator.
- $\sigma^d$: Weight of demand blocks $i$ (h).
- $\sigma^w$: Weight of scenario $w$.
- $AIC^H$ / $AIC^{wind}$: Annualized investment cost of DGs ($/MW$).
- $AIC^5$: Annualized upgrading cost of substations ($$).
- $IC_{max}$: Available investment budget ($$).
- $P_{m}^{min}$ / $P_{wind}^{max}$: Maximum capacity of DGs that can be installed.
- $X_{ij}^{ES}$: Substation capacity at bus $i$ (MW).
- $X_{ij}^S$: Candidate transformer to be installed at bus $i$ (MW).
- $P_{ij}^H$: Power consumed by demand $d$ (MW).
- $FOR^H$ / $FOR^{wind}$: Force outage rate of DGs.
- $K_{wind}$: Wind intensity at bus $m$ in demand block $l$ and scenario $w$.

Decision variables:
- $P_{n}^{HFC}$ / $P_{n}^{HP}$: Power produced by conventional DGs (MW).
- $P_{n}^{wind}$ / $P_{n}^{wind}$: Power produced by wind DGs (MW).
- $P_{n}^S$: Power produced by substation (MW).
- $X_n^H$ / $X_{wind}$: Investment capacity of DGs (MW).
- $U_{m}^{H}$ / $U_{m}^{wind}$: Binary variable for DGs investment.
- $U_{m}^S$: Binary variable for transmission substation upgrading.
- $\lambda_{m}$: Locational Marginal price ($/MWh$).
- $\theta_{j}$: Voltage angle in radian.

decisions are made by maximizing the expected total profit. The lower level problem represents market clearing and substation upgrading where the total operating and expansion costs are minimized. The Objective function shown by (1) includes 10 terms. The first term is the revenue from selling energy of the conventional DGs via firm contract. The second term represents the revenue from capacity payment to conventional DGs in contractual market. This payment is considered when DGs are available for producing energy; even if they are not called by system operator. In contractual market the fuel costs of DGs are compensated by system operator. Thus, the operation cost of DGs is not considered. The third term represents revenue of the conventional units in wholesale market. According to the wholesale market rules, fuel cost of DGs participating in the market is paid by investor. The fourth term represents the capacity payment to the conventional DGs in the wholesale market. The fifth and the sixth terms represent the revenue of wind units for selling energy and capacity payment in contractual market. The seventh and the eighth terms represent the revenue of wind units for selling energy and capacity payment in wholesale market. Two last terms of (1) represent conventional and wind DGs investment costs, respectively. A subsidy scheme [21] is considered here for wind units. Note that $n \in \phi^f$ and $m \in \phi^w$ identifies respectively the conventional unit $n$ and wind unit $m$ located at bus $j$.

Constraints (2) state that conventional units are available in discrete blocks. Constraints (3) impose that only one block for each conventional unit is selected. For example, if conventional unit is available in 5-MW blocks, we will have $P_{n}^{max} = 0$, $P_{n}^{max} = 5$, $P_{n}^{max} = 10$, etc. Thus, If the optimal capacity is 10 MW, we will have $U_{m}^S = 0$, $\forall n \neq 3$. Constraints (4) and (5) represent that wind units are available in discrete blocks. Constraints (6) and (7) represent relation between production of conventional and wind DGs in contractual. Constraint (8) denotes the limitations of power produced by wind units participating in the
Upper Level:
Maximize
\[
\sum_{i \in \delta^+} \left[ \sum_{n \in \delta^+} P_n^{HFC} C_{pi}^{H} + \sum_{n \in \delta^+} P_n^{HFC} P_{CR}^{HFC} \left( 1 - P_{\omega}^{H} \right) \left( 1 - \text{FOR}^{H} \right) + \sum_{w \in \delta^+} \sum_{j \in \delta^+} P_{w}^{HFC} \left( \lambda_{jw} - MC_n \right) \right] + \mathcal{P}_{n}^{HFC} MC_n + \left( X - P_n^{HFC} \right) \text{FOR}_n^{H} \left( 1 - \text{FOR}^{H} \right) \\
+ \sum_{w \in \delta^+} \sum_{j \in \delta^+} P_{w}^{HFC} C_{pi}^{H} \left( 1 - \text{FOR}^{H} \right) + \sum_{w \in \delta^+} \sum_{m \in \delta^+} P_{w}^{HFC} C_{pm}^{H} \left( 1 - \text{FOR}^{H} \right) + \sum_{w \in \delta^+} \sum_{m \in \delta^+} P_{w}^{HFC} \lambda_{jw}
\]
\[
- \sum_{m \in \delta^+} \sum_{w \in \delta^+} \left( k_{w} \text{wind} X_{m} - P_{w}^{HFC} \right) C_{pi}^{H} \left( 1 - \text{FOR}^{H} \right)
\]

Subject to:
\[
X_{n} = \sum_{j \in \delta^+} P_{n}^{HFC} \text{MAX} : \forall n
\]
\[
\sum_{j \in \delta^+} U_{nj}^{H} = 1 , U_{nj}^{H} \in \{0,1\} : \forall n
\]
\[
\sum_{j \in \delta^+} U_{nj}^{H} = 1 , U_{nj}^{H} \in \{0,1\} : \forall m
\]
\[
P_{w}^{HFC} = FCV X_{n}^{H} : \forall n
\]
\[
P_{w}^{HFC} = FCV X_{m}^{H} \sum_{m} P_{m}^{HFC} \left( 1 - \text{FOR}^{H} \right) : \forall m, l, w
\]
\[
0 \leq P_{w}^{HFC} \leq k_{w} \text{wind} X_{m}^{H} - P_{w}^{HFC} : \forall m, l, w
\]
\[
\sum_{m \in \delta^+} X_{n}^{H} \text{AIC}_{n}^{H} + \sum_{m \in \delta^+} X_{n}^{H} \text{AIC}_{w}^{H} : \forall n
\]
\[
\sum_{m \in \delta^+} X_{m}^{H} \left( \text{AIC}_{w}^{H} \left( 1 - S_{w} \right) \right) \leq IC_{\text{MAX}}
\]
\[
D_{d}^{\text{peak}} = \max \left\{ D_{d}^{\text{peak}}, \forall l \right\} : \forall d, l \in \delta^+
\]
\[
\sum_{m \in \delta^+} X_{n}^{H} + \sum_{m \in \delta^+} X_{m}^{H} \leq p f \sum_{d \in \delta^+} D_{d}^{\text{peak}}
\]

Lower Level:
\[
\lambda_{jw} \in \mathcal{P}_{n}^{HFC}, P_{w}^{HFC}, P_{jw}^{HFC} \in \mathcal{P}_{n}^{HFC}, P_{jw}^{HFC} \in \mathcal{P}_{n}^{HFC}, \lambda_{jw} \in \mathcal{P}_{n}^{HFC}, P_{w}^{HFC}, P_{jw}^{HFC} \in \mathcal{P}_{n}^{HFC}, P_{jw}^{HFC} \in \mathcal{P}_{n}^{HFC}
\]
\[
\sum_{n \in \delta^+} \left[ P_{n}^{HFC} C_{pi}^{H} \left( 1 - P_{\omega}^{H} \right) + P_{n}^{HFC} \text{MC}_{n} \left( 1 - \text{FOR}^{H} \right) \right] + \mathcal{P}_{n}^{HFC} \text{MC}_{n} + \left( X - P_n^{HFC} \right) \text{FOR}_n^{H} \left( 1 - \text{FOR}^{H} \right)
\]
\[
- \sum_{m \in \delta^+} \sum_{w \in \delta^+} \left( k_{w} \text{wind} X_{m} - P_{w}^{HFC} \right) C_{pi}^{H} \left( 1 - \text{FOR}^{H} \right)
\]

wholesale market. Investment budget limitation is represented by (9). Constraint (10) is related to the maximum demand at each bus and (11) is the limitation of penetration factor (pf) of DGs. Equation (12) represents the objective function of the lower level problem. The first term considers payments for energy purchase, while the second and third terms consider payments for operational cost and capacity to
conventional units at contractual market, respectively. The fourth and fifth terms represent payment for purchase and capacity to conventional units in wholesale market. Accordingly, the sixth and seventh terms represent power purchased and capacity payment of wind units at contractual market. The eighth term is the capacity payment to wind units at wholesale market. Payment for energy purchased from substations is considered in the ninth term. The last term is the cost related to upgrading of substations. Constraints (13) represent the energy balance at each bus, being the associated dual variables (LMPs) or nodal prices. Constraints (14) and (15) impose power bounds for generation constraints at transmission substation and wind units at wholesale market, respectively. Constraints (16) define the power flow of line using DC power flow model. Constraints (17) and (18) enforce voltage angle bounds and fix the voltage angle at the reference bus, respectively. $k \in \varphi^0$ identifies that the bus $k$ is connected to bus $j$. Dual variables are indicated at the relevant constraints following a colon. The term $u'_{nh}, X^j$ in (14) is related to the decision on substation upgrading.

2.3 MPEC
The upper-level problem (1)-(11) and the collection of lower-level problems (12)-(18) need to be solved jointly. Thus, it is necessary to convert the bi-level problem into a single-level problem, i.e., MPEC. Regarding the fixed values for variables of upper-level and variables for substation upgrading, the lower-level problems are continuous and linear. Therefore, there are two options to obtain the MPEC:

1) KKT formulation: to replace each lower-level problem by its Karush-Kuhn-Tucker (KKT) conditions.

2) Primal-dual formulation: to replace each lower-level problem by its primal constraints, its dual constraints and the strong duality theorem (SDT) equality.

Both options are equally valid. Using the KKT conditions involves solving a large number of complementarity constraints, one for each inequality constraint of lower level problem. These complementarity constraints are nonlinear, and should be linearized to solve the optimization problem.

The bi-level problem (1)-(18) can be converted to the MPEC by enforcing KKT conditions to the lower level problems [22, 23], represented by (19)-(31).

\[
\sum_{k \in \varphi^0} B^k_{i,j} (\lambda_{i,j} - \lambda_{i,j}^k) + \sum_{k \in \varphi^0} B^k_{i,j} (\nu_{j,k}^{\text{min}} - \nu_{j,k}^{\text{min}}) + \sum_{k \in \varphi^0} B^k_{i,j} (\nu_{j,k}^{\text{max}} - \nu_{j,k}^{\text{max}})
\]

\[
+ \sum_{k \in \varphi^0} B^k_{i,j} (\nu_{j,k}^{\text{min}} - \nu_{j,k}^{\text{min}}) + \xi_k^1 = 0 \quad \forall j, k \in \varphi^0, l, w
\]

(13) \-(18)

0 \leq P^i_{s_{nh}} - \mu^\text{min}_{s_{nh}} \geq 0 \quad \forall i, l, w
\]

(23)

0 \leq B^H_{s_{nh}} \perp \mu^\text{min}_{s_{nh}} \geq 0 \quad \forall n, l, w
\]

(24)

0 \leq X^i_{s_{nh}} + u^i_{s_{nh}} X^j_{s_{nh}} - P^i_{s_{nh}} \perp \mu^\text{max}_{s_{nh}} \geq 0 \quad \forall i, l, w
\]

(26)

0 \leq X^i_{s_{nh}} - P^H_{s_{nh}} \perp \mu^\text{min}_{s_{nh}} \geq 0
\]

(27)

0 \leq F^\text{max}_{j,k} - B^{f_j}_{j,k} (\theta_{j,k} - \theta_{j,k}) \perp \nu_{j,k}^{\text{max}} \geq 0
\]

\forall j, k \in \varphi^0, l, w
\]

(28)

0 \leq F^\text{max}_{j,k} + B^{f_j}_{j,k} (\theta_{j,k} - \theta_{j,k}) \perp \nu_{j,k}^{\text{max}} \geq 0
\]

\forall j, k \in \varphi^0, l, w
\]

(29)

0 \leq \pi - \theta_{j,k} \perp \xi_k^0 \geq 0 \quad \forall j, l, w
\]

(30)

0 \leq \pi + \theta_{j,k} \perp \xi_k^0 \geq 0 \quad \forall j, l, w
\]

(31)

2.4 Linearization
The MPEC problem of (19)-(31) is nonlinear because of the term $\sum_{n} P^i_{s_{nh}} \lambda_{j,w} + \sum_{n} P^i_{s_{nh}} \lambda_{j,w}$ in the objective function and (24)-(31). Each nonlinear equation is linearized according to their nature, as follows:

1) Equations (24)-(31) is linearized by (32) and (33), in which M is a large enough constant [24]:

\[
0 \leq a \perp b \geq 0
\]

(32)

\[
a \geq 0,
b \geq 0,
a \leq \tau M ,
b \leq (1-\tau) M ,
\]

\[
\tau \in \{0,1\}.
\]

(33)

For instance, (24) is linearized based on duality gap and usage of the complementarity conditions. Each of the non-linear relationship is linearized by replacing $P^i_{s_{nh}}$ and $\mu^\text{min}_{s_{nh}}$ instead of a and b in (33).

\[
a = P^i_{s_{nh}}, \quad b = \mu^\text{min}_{s_{nh}}
\]

(34)

\[
0 \leq P^i_{s_{nh}} \perp \mu^\text{min}_{s_{nh}} \geq 0
\]

(35)
2) To find a linear expression for \( \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} \) + 
\[ \sum_{n \in [j, m]} P_{\text{wind}}^{\text{fur}} \lambda_{\text{hub}} \), the strong duality theorem and some KKT equalities are used. Applying the strong duality theorem to each lower-level problem (12)-(18) yields:
\[ \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} [MC_n - CPR_{H} (1 - \text{FOR})] + y = - \left[ \sum_{n \in [j, m]} \mu_{\text{hub}}^{\text{Max}} \left(X_H^n - P_{\text{hub}}^{\text{fur}}\right) + \sum_{m \in [j, m]} P_{\text{wind}}^{\text{fur}} \lambda_{\text{hub}} \right] + \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} (X_H^n - P_{\text{hub}}^{\text{fur}}) + \sum_{m \in [j, m]} P_{\text{wind}}^{\text{fur}} \lambda_{\text{hub}} 
+ \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} + \sum_{m \in [j, m]} P_{\text{wind}}^{\text{fur}} \lambda_{\text{hub}} \right] \] (36)
\[ y = \sum_{i \in [j, m]} P_i^{\text{hub}} E P_i^2 + \sum_{j \in [j, m]} (\sigma_{\text{min}} - \sigma_{\text{max}}) \] 
\[ + \sum_{k \in [k, m]} \sigma_{\text{max}} \left( V_{\text{min}}^{k} / V_{\text{max}}^{k} \right) + \sum_{i \in [j, m]} E \mu_{\text{hub}}^{\text{Max}} \] 
\[ - \sum_{l \in [l, m]} P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} \] (37)

From (27):
\[ \sum_{n \in [j, m]} \mu_{\text{hub}}^{\text{Max}} (X_H^n - P_{\text{hub}}^{\text{fur}}) = \sum_{n \in [j, m]} \mu_{\text{hub}}^{\text{Max}} (P_{\text{hub}}^{\text{fur}}) \] (38)
Substituting (38) in (36) renders:
\[ \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} (MC_n - CPR_{H} (1 - \text{FOR}) + \mu_{\text{hub}}^{\text{Max}}) + 
\[ y = - \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} + \sum_{m \in [j, m]} P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} + \sum_{m \in [j, m]} P_{\text{wind}}^{\text{fur}} \lambda_{\text{hub}} \right] \] (39)

On the other hand, from (20):
\[ \lambda_{\text{hub}} (j, n, \text{cap}) = MC_n - CPR_{H} (1 - \text{FOR}) + \mu_{\text{hub}}^{\text{Max}} \] (40)
Thus:
\[ \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} [\lambda_{\text{hub}} (j, n, \text{cap}) + \mu_{\text{hub}}^{\text{Max}}] = 
\sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} [MC_n - CPR_{H} (1 - \text{FOR}) + \mu_{\text{hub}}^{\text{Max}}] \] (41)

Substituting (41) in (39) renders:
\[ \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} [\lambda_{\text{hub}} (j, n, \text{cap}) + \mu_{\text{hub}}^{\text{Max}}] + y = 
- \left[ \sum_{m \in [j, m]} P_{\text{wind}}^{\text{fur}} \lambda_{\text{hub}} + \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} + 
\sum_{m \in [j, m]} P_{\text{wind}}^{\text{fur}} \lambda_{\text{hub}} \right] \] (42)

Additionally, from (21):
\[ \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} \mu_{\text{hub}}^{\text{Max}} = 0 \] (43)
Substituting (43) in (42) renders:
\[ \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} + \sum_{m \in [j, m]} P_{\text{wind}}^{\text{fur}} \lambda_{\text{hub}} = 
- \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} - \sum_{m \in [j, m]} P_{\text{wind}}^{\text{fur}} \lambda_{\text{hub}} \] (44)
The term \( \sum_{n \in [j, m]} P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} \) of nonlinear term of the equation (43) is as follows:
\[ P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} = FCV_{H} X_{H}^{n} \lambda_{\text{hub}} \] (45)

\[ FCV_{H} \left( \sum_{b \in [b, m]} P_{\text{hub}}^{\text{fur}} \right) \lambda_{\text{hub}} = FCV_{H} \lambda_{\text{hub}} \] (46)
where, \( P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} = Z_{\text{hub}} \) and each of terms \( Z_{\text{hub}} \) can be replaced by the following set of exact mixed-integer linear expressions:
\[ Z_{\text{hub}} = P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} (j, n, \text{cap}) - \xi_{\text{hub}} \] (47)
\[ \sum_{b \in [b, m]} P_{\text{hub}}^{\text{fur}} \lambda_{\text{hub}} (j, n, \text{cap}) \leq Z_{\text{hub}} \] (48)

And linearizing the second part \( \sum_{m \in [j, m]} P_{\text{wind}}^{\text{fur}} \lambda_{\text{hub}} \) of nonlinear term of the equation (43):
\[ P_{\text{wind}} \cdot \lambda_{\text{dis}} = FCV \cdot \text{wind} \cdot \sum_{b,c} \text{wind} \cdot P_{\text{wind}} \cdot \lambda_{\text{dis}} \]

\[ FCV \cdot \text{wind} \cdot \sum_{b,c} \text{wind} \cdot P_{\text{wind}} \cdot \lambda_{\text{dis}} = FCV \cdot \text{wind} \cdot \sum_{b,c} \text{wind} \cdot P_{\text{wind}} \cdot \lambda_{\text{dis}} \]

where \( u_{\text{wind}} \) and each of the terms \( ZE_{\text{mlw}} \) can be replaced by the following set of exact mixed-integer linear expressions:

\[ ZE_{\text{mlw}} = P_{\text{mlw}} \cdot \lambda_{\text{mlw}} \cdot \left( \text{wind} \cdot \left( j, \text{peak} \right) \right) - \zeta \cdot E_{\text{mlw}} \]

\[ u_{\text{wind}} \cdot P_{\text{wind}} \cdot \lambda_{\text{wind}} \cdot \left( j, \text{peak} \right) \leq ZE_{\text{mlw}} \]

\[ \left( 1 - u_{\text{wind}} \right) \cdot P_{\text{wind}} \cdot \lambda_{\text{wind}} \cdot \left( j, \text{peak} \right) \leq \zeta \cdot E_{\text{mlw}} \]

In the above equation, \( \lambda_{\text{mlw}} \) and \( \lambda_{\text{wind}} \) are lower and upper bounds of \( \lambda_{\text{dis}} \), respectively, and \( \zeta \cdot E_{\text{mlw}} \) are auxiliary continuous variables for accomplishing the linearization process.

3 Numerical studies

In this section, effectiveness of the proposed framework is examined using two sample networks including a six-bus test system and a real sub-transmission network.

3.1 Case study 1

The six-bus sample network is shown in Fig. 1. This network consists of two (230/63 kV) substations with capacities of 70 and 50 MW at buses 1 and 2, respectively. Capacity of 63 kV transformers is 40 MVA. The 70 and 50 MW transformers are chosen as candidates for installation at buses 1 and 2, where the investment costs are 0.7 and 0.5 SM, respectively. The discount rate is 12% and the lifetime is 30 years. The 20-kV buses are candidate for installation of conventional units and bus 6 is candidate for wind power units. Maximum capacity of installed DGs at each bus is considered to be 25 MW with blocks of 5 MW. Thus, it is possible to install 15 MW of conventional units and 10 MW of wind units with 2 MW blocks. Investment cost of conventional and wind units are considered to be 0.9 and 1.3 (SM/MW). Moreover, 30% of the annual investment cost is supported by the government for wind power as subsidy. The number of wind generation scenarios and the amount of wind intensity (K mw\(^2\)wind) in each scenario are taken from [19]. Maximum penetration factor of DGs is 30%. Capacity of 63 kV lines is considered as 45 MW. In the first case study, capacity payment is neglected and only the firm contract and subsidy are considered as incentives. The duration of demand blocks for peak, medium and off-peak are 30%, 60% and 25%, respectively. Peak demand at buses 5 and 6 are 75 and 60 MW, respectively. Available investment budget is considered as 150 SM. Table 2 shows some data for simulation. The model is solved using the solver CPLEX [25] under GAMS software [26], Parameter M is set to 10,000. Different scenarios are defined according to Table 3. In order to validate the results obtained with the model, it was firstly implemented using a scenario found in [27], with the same results. Table 4 shows the simulation results.

**Scenario 1:** In this scenario, we deal with network without the presence of DGs and investment incentives. In the absence of sub-transmission lines limitations, optimal solution is upgrading transmission substation at bus 2, which coincides with the minimum cost of the lower level problem (i.e. 72.0 SM). The cost of the lower level problem considering upgrading of substation at bus 1 and both substation has been obtained as 72.04 and 72.15 SM.

**Scenario 2:** Since in this scenario 40 MW conventional DG are installed, there is no need to...
Table 3  6-bus network case study scenarios.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Investment option</th>
<th>Incentives</th>
<th>Wholesale market</th>
<th>Substation upgrading</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NO DG</td>
<td>--</td>
<td>--</td>
<td>✓</td>
</tr>
<tr>
<td>2</td>
<td>Conventional</td>
<td>✓</td>
<td>--</td>
<td>✓</td>
</tr>
<tr>
<td>3</td>
<td>Wind</td>
<td>✓</td>
<td>--</td>
<td>✓</td>
</tr>
<tr>
<td>4</td>
<td>Wind &amp; Conventional</td>
<td>✓</td>
<td>--</td>
<td>✓</td>
</tr>
<tr>
<td>5</td>
<td>Wind &amp; Conventional</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 4  Simulation results for 6-bus network.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line constraints</td>
<td>NO</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Investment subsidy</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Capacity and location of conventional units (MW)</td>
<td>-</td>
<td>25(5) &amp; 15(6)</td>
<td>-</td>
<td>15(5) &amp; 10(6)</td>
<td>-</td>
</tr>
<tr>
<td>Capacity and location of wind units (MW)</td>
<td>-</td>
<td>-</td>
<td>24(5) &amp; 16(6)</td>
<td>-</td>
<td>10(5) &amp; 6(6)</td>
</tr>
<tr>
<td>Investor’s profit ($M)</td>
<td>-</td>
<td>14.47</td>
<td>14.01</td>
<td>14.85</td>
<td>18.7</td>
</tr>
<tr>
<td>Substation upgrading</td>
<td>Bus 2</td>
<td>-</td>
<td>Bus 2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Operator payment ($M)</td>
<td>72</td>
<td>75.16</td>
<td>72.45</td>
<td>72.27</td>
<td>56.07</td>
</tr>
</tbody>
</table>

Fig. 2  Results of participation in wholesale market for scenario 5 in the 6-bus network.

Fig. 3  Influence of initial contract price in scenario 5; 6-bus network.

upgrade substations and reinforcement of 63 kV lines. However, operator's payment has been increased by 4% in comparison with scenario 1. The operator's payment has also been increased by 4.39% with respect to scenario 1. The investor benefits from the incentives in this scenario.

Scenario 3: in this scenario, only the effect of wind units is studied. Data of wind scenarios and their intensity factor are taken from [19]. We see that 40 MW of capacity has been totally added in the network, which is similar to the second scenario. However, due to the uncertainty of the wind units, upgrading of substation is required at bus 2 unlike the case for scenario 2. Moreover, compared to scenario 2, investor’s profit has been decreased by 3.18% and operator’s payment has been decreased by 3.6%. Despite of the higher contractual price for the wind units (i.e. 120 $/MWh) comparing with the energy price for conventional units, the profit of wind units has been decreased. Wind intensity factor can also influence on the investor and network behaviour. In this respect, results showed that it is not required to upgrade the substation by increasing the wind intensity factor. This emphasizes the importance of finding the optimal location of wind units in the areas with good wind intensity.

Scenario 4: In this scenario, expansion of both conventional and wind units are considered. It is assumed that available capacity of conventional and wind units installed at buses 5 and 6 are 15 and 10 MW. In this case, the total installed capacity of DGs is 41 MW including 25 MW of conventional and 16 MW of wind power. Uncertainty of the wind power has made the investor to install less capacity with respect to conventional units. Moreover, the expected profit of wind units is less than conventional ones. Therefore,
compared to wind generation units, the less contract price but more available energy of conventional units makes them more profitable. Compared to scenarios 2 and 3, the investor’s profit has been increased by 2.6% and 5.9%. Also, operator payment has been decreased by 3.8% in comparison with scenario 2, and it has a little decrease considering scenario 3. Even though more incentives have been assigned to wind units, considering their uncertainty and higher investment costs, made these units less attractive than conventional units. When the wind intensity factor is increased, the installed capacity of wind DGs is increased. As a result, wind units have different influences on the investment behaviour due to their stochastic nature.

**Scenario 5:** In this scenario, DGs participate in both contractual and wholesale markets. Compared to scenario 4, the operator’s payment is decreased by 22.4% and also the investors’ profit is increased by 25.9%. The total invested capacity of conventional units has been increased by 5 MW with respect to scenario 4, because in the absence of contracts, DGs have more opportunities to enter in wholesale market. Also, the contractual price is lower than the wholesale price, and the fuel cost of DGs is not paid by the operator. In addition, it is not necessary to upgrade the transmission substation. If the LMPs are higher than the contractual price, participating in the wholesale market is more profitable. Therefore, the most profit is earned in this case. The investor’s revenue in the wholesale market is obtained considering the LMPs in different time intervals. LMPs are equal to 100, 80, and 60 $/MWh for three time intervals. Moreover, the lowest operator’s payment has been obtained, since the fuel costs of DGs are paid by the investor in the wholesale market.

Fig. 2 shows the impacts of contractual volume on investor’s profit and operator’s payment. If the total installed capacity of DGs participates in the wholesale market, operator’s payment is decreased by 22.4% compared to the case where the whole capacity is sold through contracts. This increases the investor’s profit by 25.9%. Fig. 3 represents the influence of contract price on investor’s profit and operator’s payment. For a fixed contract volume, contract price reduction leads to decrease in investment profit and operator’s payment. Fuel costs of DGs should be considered for setting contract price, because a low price would produce no profit for investor and high price will increase the operator’s payment.

### 3.2 Case study 2

Single-line diagram of the sub-transmission system of the west of MREC is shown in Fig. 4. Annual demand growth rate is assumed to be 7% which leads to a forecasted demand value of 700 MW for the target year. Three demand blocks were considered as in case of six-bus system. Thermal capacity of sub-transmission lines is considered to be 50 MW. Line’s susceptance per unit in base of 100 MVA is represented on the diagram along with some of the characteristics of existing generating units. There are four major 230/63 kV substations namely CHABOKSAR, NOSHAHR, DANIYAL, and ROYAN which are modelled as generation units. Substations ROYAN and DANIYAL are candidates for upgrading at the amount of 125 and 70 MVA, respectively. Investment cost of transformers with the capacity of 125 and 70 MVA are assumed to be 1.475 and 0.737 SM, respectively. The annual investment costs are calculated considering the discount rate of 12% and lifetime of 30 years. Ten 63 kV buses named ROYAN2, MAHMUDABAD, NOOR, BANK, CISANGAN, NASHTARUD, RAMSAR, TONEKABON, CHALUS, NOSHAHR2 are candidates for installing conventional units. Moreover, buses RAMSAR, NASHTARUD, ROYAN2 and CHALUS are candidates for installing wind units. Practically, DG units are installed at 20 kV buses of 63/20 kV substations. However, for simplicity, they are assumed to be installed at 63 kV buses. Annual investment cost of conventional and wind units as well as subsidy are assumed to be the same as the 6-bus case study. Maximum capacity of DGs which can be installed at each bus is assumed to be 25 MW. The conventional and wind units are available in 5 and 2 MW blocks at the candidate buses up to 25 and 10 MW, respectively. In those buses that both conventional and wind units are installed, the capacity of conventional and wind units are 15 and 10 MW, respectively. Fuel costs of conventional units, is assumed to be 7.5 $/MWh. In all demand blocks, contract price for wind power is considered to be 53 $/MWh which is twice as expensive as conventional units. Investment incentives include firm contracts, capacity payment, investment subsidy for wind power and the payment of fuel cost of conventional units by the operator in the contractual market. Capacity payment rate is 83.33% of contracted price of energy purchase in contractual market, while it is assumed to be 5.4 $/MWh in the wholesale market. Energy price of substation at off-load, shoulder, and peak-load hours is considered to be equal to 12, 14.5, 17 $/MWh, respectively. Furthermore, it is assumed that the network needs the power produced by DGs with the probability of 0.4, 0.6, and 0.8 for off-load, shoulder, and peak-load hours, respectively. The model is solved using the Solver CPLEX [25] under GAMS software [26]. Parameter M is set to 10,000. Different scenarios are defined which are represented in Table 5.

In scenario 1, ROYAN substations must be upgraded. In this situation, the operator's payment is 67.70 SM. In the case of upgrading of DANIYAL, the operator’s payment has been obtained as 67.86 SM, while it has been 68.01 SM for upgrading both DANIYAL and ROYAN. In scenario 2, it is assumed that the total invested capacity of DGs is traded through firm contracts. As we can see in the Table 6, conventional
DGs with total capacity of 210 MW are installed, where there is no need to upgrade the substations. Operator's payment has been increased by 30% with respect to scenario 1. The profit of the investor (24.27SM) shows the reasonable contract price. In scenario 3, the impacts of wind and conventional units are investigated. As shown in Table 6, the total installed capacity of DGs is 210 MW. The capacity of conventional and wind DGs are 170 and 40 MW, respectively. The operator's payment decreases by 2.61% with respect to scenario 2. As a result, the combined use of wind and conventional units makes it more attractive for the network operator. In scenario 4, the investor can participate in the wholesale market. According to Table 6, the total invested capacity of DGs is 203 MW which shows a decrease of 3.3% with respect to scenario 3. The installed capacity of conventional and wind DGs are 165 MW and 38 MW, respectively. Fig. 5 shows the changes in the investor's profit and operator's payment due to change of the firm contract volume. We see that a decrease in the firm contract volume leads to the reduction in operator's payment and the investor's profit. For contract volume of 50%, the investor's profit and the operator’s payment decrease by 34% and 16.7%. However, setting contract volume on zero decreases both the investor’s profit and the operator payment by 69% and 33.4%. That’s why LMPs at peak hours are 18 $/MWh, while the contract price for wind and conventional DGs are 53 $/MWh and 26.5 $/MWh, respectively.

4 Conclusion

In this paper, a new stochastic model was proposed to
solve the non-convex and bi-level optimization problem of DGs expansion and substation upgrading. Also, impacts of incentive policies on the expansion schemes are investigated. A combination of enumeration algorithm and mathematical optimization is used to solve the model. Relevant results obtained by the simulations are as follows:

1) Using the combined enumeration and MILP algorithm, non-convexity and disability to linearization of the stochastic bi-level problem in the lower level problem has been solved.

2) Participation of DG in wholesale market lessens the need for substation upgrading. Due to the uncertainty of wind power, the need for substation upgrading increases in the case of expanding only wind power.

3) Investment of wind and conventional units increases the investor’s profit and a decreases operator’s payment. Moreover, it prevents substation upgrading to some extent. Due to the uncertainty in wind generation, these units cannot compete with conventional units even with superior incentives.

4) For a fixed contract volume, contract price influences the DGs expansion and substation upgrading schemes. Contract price must be selected according to fuel cost of DGs.

5) Investment incentives increases investment tendency. Since the investment incentives have a great influence on DGs expansion and substation upgrading, one must consider them accurately for policy making.

In the proposed model, simultaneous upgrading of sub-transmission system has not been considered. Thus, this feature can be added to the proposed model in the future works.

References


**F. Misaghi** received the B.Sc. degree in electrical engineering from the Babol Noshirvani University of Technology, Babol, Iran, in September 2012 and the M.Sc. degree from the Babol Noshirvani University of Technology, Babol, Iran, in March 2016. He is currently working in the Monenco Iran Consulting Engineers Company, Tehran, Iran, as consulting engineer. His research interests include economics of electric energy systems, the applications of equilibrium, as well as investment problems.

**T. Barforoushi** received the B.Sc. degree in electrical engineering from Iran University of Science and Technology, Tehran, Iran, in 1994 and the M.Sc. and Ph.D. degrees from Tarbiat Modares University, Tehran, in 1999 and 2008, respectively. He worked at Mazandaran Regional Electric Company (MREC) from April 1999 to May 2012. He is currently an assistant professor with the faculty of electrical and computer engineering, Babol Noshirvani University of Technology, Babol, Iran. His research interests are power system planning and operation, Electricity markets.

**M. Jafari-Nokandi** received the B.Sc. degree from Sharif University of Technology, Tehran, Iran, in 2003 and the M.Sc. and PhD degrees from the University of Tehran in 2005 and 2011, all in electrical engineering. He has been with the Faculty of Electrical and Computer Engineering, Babol Noshirvani University of Technology, since 2011 where he currently is an Assistant Professor. His research interests are power system reliability, operation, and planning.