Design a Guidance Law Considering Approximation of Missile Control Loop Dynamics Using Adaptive Back-Stepping Theory

V. Behnamgol* (C.A.), A. R. Vali* and A. Mohammadi*

Abstract: In this paper, a new guidance law is designed to improve the performance of a homing missiles guidance system in terminal phase. For this purpose first of all, the two dimensions equations of motion are formulated, then the approximation dynamic of missile control loop is added to these equations which are nonlinear with unmatched uncertainty. Then, a new adaptive back-stepping method is developed in order to control this system. An adaptive term is used in the control law that is converged to the uncertainty. This convergence is proved based on Lyapunov stability theorem. Therefore using this adaptive term in the control law can be eliminated the uncertainty. Based on this algorithm, a new guidance law is designed. Then its performance is compared with common guidance laws in a guidance loop simulation in the presence of control loop dynamics.

Keywords: Guidance Law, Control Loop Dynamics, Adaptive Back-Stepping Control, Unmatched Uncertainty.

1 Introduction

Homing missiles guidance system consists of two guidance and missile control loops. The outer is guidance loop and the inner is missile control loop. In the guidance loop, the proper guidance commands are produced to modify the missile trajectory and reach it to the target. The guidance commands (missile lateral acceleration) are generated by guidance law. The guidance law is designed using mathematical rules or control theories considering relative kinematics between the missile and the target. The guidance commands are implemented by the missile control loop [1-4]. So it can be said that the missile control loop acts as an actuator in the homing missiles guidance loop.

In tactical missiles, the proportional navigation (PN) family are widely applicable laws implemented in terminal phase of many missiles. This guidance law is based on nullifying the line of sight (LOS) rate. To intercept maneuvering targets, the augmented proportional navigation (APN) is proposed which needs the target maneuvers measurement or estimation. Therefore it leads to a significant increase in cost and of course complicated calculations [1-4]. PN family is designed based on mathematical theories. In recent years, the guidance law is considered as guidance loop controller and designed using control theories. From this point of view, the relative kinematic between missile and target is a process must be controlled. The outputs of this process are some variables such as LOS rate and closing velocity. These variables are measured by seeker and are sent to the guidance section to generate guidance commands.

Regarding the nonlinear kinematic relations in the terminal phase, nonlinear control theories and in especial case Lyapunov stability theory have been used to design the guidance law [5,6]. In the presence of uncertainties such as target maneuvers, sliding mode control theory has been used to design nonlinear and robust guidance laws. In this case, the guidance law only used the target maneuvers bound. Hence target maneuvers measurement or estimation is not required.
LOS rate is commonly used to introduce sliding variables in sliding mode guidance laws. Then, missile acceleration has been designed such that the sliding variable converges to zero [7-9]. In [10,11] the nonlinear guidance laws have been designed by using sliding mode and partial control theories. The chattering phenomenon has occurred in the sliding mode guidance laws and so the implementation of these guidance laws is impossible. For overcome this problem, an approximation of these guidance laws is used that leads to reducing the accuracy.

In the design procedure of these guidance laws, the dynamics of missile control loop is neglected. This means it is assumed the guidance commands are implemented immediately and with no dynamics. While the dynamics of missile control loop exist and in many conditions, guidance commands are not implemented immediately. So, in reality the performance of these guidance laws are decreased and the stability margins are not guaranteed. Also the line of sight rate may diverge that leads to error in guidance loop. Therefore, consideration of missile control loop dynamics in guidance law designing can improve the performance of missile. Considering relative kinematics and perfect dynamics of missile, an integrated guidance and control system can be designed. This kind of system was designed in [12-18] using different control theories. The aerodynamic surface angle is determined by these integrated guidance and control systems directly for guaranteeing interception. This complicated systems acts as both autopilot and guidance law. An approximation of missile control loop dynamics can be assumed in guidance law designing. Otherwise, major changes in guidance and control loops are not required. Furthermore, the designing procedure will be simpler than integrated algorithms. This task was performed in [19] by using adaptive sliding mode control theory. Also in [20] the Lyapunov control theory, in [21] the conventional sliding mode control, and finally in [22] nonsingular terminal sliding mode control is used for designing guidance law considering approximation of control loop dynamics. In presented algorithms [19-22], the normalization procedure is required. In normalization process, the derivation of target lateral acceleration appears. Whereas this variable is considered as uncertainty, the derivation was not available. Also in some references for calculation simplicity, the commanded acceleration was designed normal to line of sight. For implementation of these guidance laws, calculating perpendicular vector normal to the line of sight is required every time. Moreover, in these references for preventing the chattering the controller approximation was used. Note that this approximation leads to precision decrease in control and the stability is not guaranteed.

In this paper, the guidance law is designed assuming the approximation of control loop dynamics. Without normalization procedure, the back-stepping method is used. Due to the unmatched relation between target lateral acceleration as uncertainty and commanded acceleration as control input, an adaptive back-stepping algorithm is proposed.

In next section, the relative kinematic equations and first order dynamics for control loop is formulated. In Section 3 the new adaptive back-stepping method is proposed. In Section 4, the guidance law is designed. The simulation results are presented in Section 5 and finally the conclusion is presented in Section 6.

2 Modeling

In this section, relative kinematics and approximation of control loop dynamics is formulated. First of all, the equations of two dimension motion are derived.

As shown in Fig. 1, $R$ is the relative range and $\sigma$ is line of sight angle. Also $\gamma_m$ and $\gamma_t$ are missile and target velocity vector angles and $A_m$ and $A_t$ are missile and target lateral acceleration, respectively. Closing velocity is achieved as follow:

$$\dot{R} = V_t \cos(\sigma - \gamma_t) - V_m \cos(\sigma - \gamma_m)$$

(1)

Also the relative lateral velocity is

$$R\dot{\sigma} = -V_t \sin(\sigma - \gamma_t) + V_m \sin(\sigma - \gamma_m)$$

(2)

where $\dot{\sigma}$ is the line of sight rate. The relations of missile and target lateral acceleration are as follow:

$$A_m = V_m \dot{\gamma}_m$$

(3)

$$A_t = V_t \dot{\gamma}_t$$

(4)

where $\dot{\gamma}_m$ and $\dot{\gamma}_t$ are the rates of missile and target velocity vectors. By derivation from (1) and (2) and constant velocities assumption we have:

$$\frac{d}{dt}(\dot{R}) = R \ddot{\sigma}^2 + A_t \sin(\sigma - \gamma_t) - A_m \sin(\sigma - \gamma_m)$$

(5)

$$\frac{d}{dt}(R \dot{\sigma}) = -R \ddot{\sigma} + A_t \cos(\sigma - \gamma_t) - A_m \cos(\sigma - \gamma_m)$$

(6)
For guaranteeing the interception with target, noticing (6) the missile lateral acceleration should be in such form that stabilizes the relative lateral velocity \([6,23]\).

In this paper, the stabilized control loop dynamics is approximated as follow:

\[
\begin{align*}
A_{21} &= \frac{1}{rS+1} \\
A_{21} &= -\frac{1}{r}A_{21} + \frac{1}{r}A_{e}
\end{align*}
\]  

(7)

(8)

By adding (8) to relative kinematics (3)-(6), the system relative degree is increased. Therefore, the missile lateral acceleration is considered as a state variable and commanded acceleration is designed such that the relative lateral velocity goes to zero [20-22].

3 Adaptive Back-Stepping Control

In the back-stepping algorithm the Lyapunov theory is applied for high order un-normal nonlinear systems. Consider a second degree nonlinear system as follow:

\[
\begin{align*}
x_1' &= f_1(x) + g_1(x)x_2 \\
x_2' &= f_2(x) + g_2(x)u \\
y' &= x_1
\end{align*}
\]  

(9)

In back-stepping algorithm, considering first equation of (9), \(x_2\) is assumed as virtual control signal. Then, this control signal is designed such that the first state variable \(x_1\) goes to desired state. Assume the virtual control for controlling \(x_1\) exists as follow:

\[
x_2 = \phi(x)
\]  

(10)

Now in second step, the control input \(u\) is designed such that \(x_2\) reaches to \(\phi(x)\). Back stepping algorithm is applied for certain nonlinear systems. The sliding mode theory is used for controlling normal and uncertain nonlinear systems. In un-normal systems, this theory is applied after normalization. While an unmatched uncertainty exists, normalization may leads to uncertainty derivations which are inappropriate. Later a new back-stepping algorithm is proposed for nonlinear systems with unmatched uncertainty as shown in (11).

\[
\begin{align*}
x_1' &= f_1(x) + g_1(x)x_2 + w(t) \\
x_2' &= f_2(x) + g_2(x)u \\
y' &= x_1
\end{align*}
\]  

(11)

where \(w(t)\) is unmatched uncertainty with condition \(|w(t)| \leq L_w, \ |g(t)| = \tilde{w}(t)| \leq L_w\) and \(L_w, L_u\) are positive constants. The Theorem (1) is proposed for controlling these kinds of systems.

Theorem 1:

Given system (11), by using the control law

\[
\begin{align*}
f_2(x) + \dot{x}_1(x) - k_1 \frac{\tilde{z}}{|\tilde{z}|} &
\end{align*}
\]  

(12)

\[
\begin{align*}
z &= x_2 - \phi(x) \\
\phi(x) &= \frac{1}{g_1(x)}[-f_1(x) - k_1x_1 - \xi_1] \\
\xi_1 &= k_2x_1 + k_3 \frac{x_1 - \xi_2}{|x_1 - \xi_2|} \\
\xi_2 &= -k_4x_1 + k_5 \text{sgn}(x_1 - \xi_2)
\end{align*}
\]  

The stability of uncertain and un-normal nonlinear system (11) is guaranteed with selecting \(k_1, k_2, k_3 > 0, k_4 = (L_w + |\tilde{z}|)L_w + \eta / k_3\) and \(\eta > 0\), where \(\xi_1\) and \(\xi_2\) are adaptive variables.

Proof. For proving stability, in the first step consider the first equation in (11). In this equation, \(x_2 = \phi(x)\) is considered as virtual control input and then it is designed such that stabilizes the first state variable in system (11). By using the virtual control input \(x_2 = \phi(x)\) which is proposed in (12) in the first equation of (11) we have:

\[
\begin{align*}
x_1' &= -k_1x_1 - \xi_1 + w(t) \\
\xi_1 &= k_2x_1 + k_3 \text{sgn}(x_1 - \xi_2) \\
\xi_2 &= -k_4x_1 + k_5 \text{sgn}(x_1 - \xi_2)
\end{align*}
\]  

(13)

For surveying stabilization of (13), the error dynamics are introduced as follow:

\[
\begin{align*}
e' &= \begin{bmatrix} x_1 \\ e_1 \\ e_2 \end{bmatrix}, \\
e_1 &= w(t) - \xi_1, \\
e_1 &= g(t) - k_2x_1 - k_3 \frac{e_2}{|e_2|}, \\
e_2 &= e_1 - k_4 \frac{e_2}{|e_2|}
\end{align*}
\]  

(14)

where \(g(t) = \tilde{w}(t)\). Now, a Lyapunov candidate is selected as

\[
V_1 = \frac{k_2}{2} x_1^2 + \frac{1}{2} e_1^2 + k_3 e_2^2
\]  

(15)

This Lyapunov function is positive definite. The derivative of this Lyapunov function is:
\[ V_k = k_2 x_1 x_1 + e_1 e_1 + k_3 \frac{e_z}{|e_z|} e_z = k_2 x_1 (-k_1 x_1 + e_1) + e_1 \left( g(t) - k_2 x_1 - k_3 \frac{e_z}{|e_z|} \right) + k_3 \frac{e_z}{|e_z|} \left( e_1 - k_3 \frac{e_z}{|e_z|} \right) \]

\[ = k_2 k_1 x_1^2 + e_1 g(t) - k_2 x_1 e_1 - k_3 e_1 + k_3 \frac{e_z}{|e_z|} - k_3 k_4 = -k_1 k_2 x_1^2 + e_1 g(t) - k_2 x_1 \leq e_1 g(t) - k_2 x_1 \] (16)

By selecting \( k_1 = (L_w + \frac{\xi_1}{\xi_1}) L_\theta + \eta / k_3 \), where \( |w(t)| \leq L_w \), \( |g| \leq L_\theta \) and \( \eta > 0 \) yields:

\[ V_k = (w(t) - \xi_1) g(t) - (L_w + \frac{\xi_1}{\xi_1}) L_\theta - \eta \leq -\eta \] (17)

The condition (17) implies

\[
\int_0^0 dV \leq \int_0^{\eta} -\eta dt \Rightarrow -V(0) \leq -\eta \Rightarrow t \leq \frac{V(0)}{\eta}
\] (18)

Therefore condition (17) guarantees the convergence of \( V \) from \( V(0) \) to zero in finite time (18). Therefore the finite time stabilization of error dynamics (13) is in the presence of \( w(t) \) as uncertainty is guaranteed.

Now in second step, the virtual variable is introduced as \( z = z_2 - \phi(x) \). For addressing stability of this variable the second Lyapunov function candidate is introduced as follow:

\[ V_2 = \frac{1}{2} z^2 \] (19)

The derivation of this Lyapunov function is

\[ \dot{V}_2 = z \dot{z} = z \left( \dot{z}_2 - \dot{\phi}(x) \right) \] (20)

Replacing the second relation of (11) in (20) we have:

\[ \dot{V}_2 = z \dot{z} = z \left( f_2(x) + g_2(x) u - \dot{\phi}(x) \right) \] (21)

Replacing controller (12) in (21) yields:

\[ \dot{V}_2 = z \dot{z} = z \left( f_2(x) + g_2(x) \frac{1}{g_2(x)} [-f_2(x)] + \dot{\phi}(x) - k_1 \frac{z}{|z|} \right) = -k_1 |z|^2 \] (22)

For \( c = (\theta - \beta^2 / \theta) \) we have \( \dot{V}_2 = -c V_2^2 \), therefore the finite time stabilization of virtual variable \( z = z_2 - \phi(x) \) is guaranteed [6].

4 Guidance Law Designing

In this section the guidance law is designed using the new back-stepping control algorithm that is proposed in previous section.

The Kinematic equations and first order control dynamics are as follow:

\[ \frac{d}{dt} (R \sigma) = -\dot{R} \sigma - \cos(\gamma_{\text{m}} - \sigma) A_z + A_{m \sigma} \] (23)

In these equations the commanded acceleration \( A_z \) should be designed such that controls the relative lateral velocity \( R \dot{\sigma} \). Also, \( A_{m \sigma} \) is the target lateral acceleration normal to line of sight that is considered as uncertainty and has unmatched relation with control signal. By using back-stepping algorithm that is proposed in previous section, by comparing (11) and (21) we have:

\[ f_1(x) = -\dot{R} \sigma \]

\[ g_1(x) = \cos(\gamma_{\text{m}} - \sigma) \]

\[ x_1 = A_{m \sigma} \]

\[ w(t) = A_{\text{m \sigma}} \]

\[ f_2(x) = \frac{1}{\tau} A_z \]

\[ g_2(x) = \frac{1}{\tau} \]

\[ u = A_z \]

Therefore for controlling (23) by using Theorem 1, the guidance law is designed as follow:

\[ A_z = \tau \left( A_{m \sigma} + \frac{\phi(x) - k}{|z|} \right) \]

\[ z = A_{m \sigma} - \phi(x) \]

\[ \phi(x) = \frac{1}{-\cos(\gamma_{\text{m}} - \sigma)} \left( \dot{R} \sigma - k, R \sigma - \xi_2 \right) \]

\[ \dot{\xi}_2 = k_1 R \dot{\sigma} + k_1 \text{sgn} (R \sigma - \xi_2) \]

\[ \dot{\xi}_2 = -k_1 R \dot{\sigma} + k_1 \text{sgn} (R \sigma - \xi_2) \] (25)

5 Simulation Results

In this section, the proposed guidance law is simulated. In all simulation scenarios, the interception
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condition is reaching to 5 meters from target and a 150 m/s² saturation is considered for autopilot. Also, the initial velocities for missile and target are 800 and 700 m/s, respectively. The proposed back-stepping guidance law is compared to approximated sliding mode guidance law and true proportional navigation (TPN) in two different scenarios. These guidance laws are as follow:

\[ A_{SMG} = \frac{1}{\cos(\gamma_w - \sigma)} \left[ -\dot{\gamma} + (\mu + \eta) \text{Sat}_\epsilon (R \sigma) \right] \quad (26) \]

\[ A_{TPN} = \frac{1}{\cos(\gamma_w - \sigma)} [-N R \sigma] \quad (27) \]

5.1 Scenario I

In first scenario, the missile and target are flown with 30 and 150 degree initial angles in pitch plan and target has -3g maneuver. Therefore, target is coming and initial range is 40 km. The other parameters are as listed in Table 1.

In this scenario, commanded and missile lateral acceleration, line of sight rate and relative lateral velocity is plotted in Figs. 2 and 3. As shown in figures, the missile acceleration is smooth and implementable in all guidance laws. The precision of controlling line of sight rate and relative lateral velocity are higher than two other guidance laws. As shown in Figs.4 and 5, the interception time and velocity losses are lower than other guidance laws. In this scenario, it can be said; due to high relative range the missile has enough time for reactions and the control loop dynamics does not have very bad influences on guidance loop.

**Table 1** The values of parameters in scenario I.

<table>
<thead>
<tr>
<th>Guidance Law</th>
<th>( \eta )</th>
<th>( \mu )</th>
<th>( N )</th>
<th>( \epsilon )</th>
<th>( A_t )</th>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>( k_3 )</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
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<tbody>
<tr>
<td>Robust Back-Stepping</td>
<td>70</td>
<td>30</td>
<td>-</td>
<td>-</td>
<td>0.0008</td>
<td>30</td>
<td>-</td>
<td>-</td>
<td>0.8</td>
<td>0.4</td>
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<tr>
<td>Approximated SMG</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TPN</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</tr>
</tbody>
</table>

**Fig. 2** a) Commanded acceleration and b) Missile acceleration in scenario I.

**Fig. 3** a) Line of sight rate and b) Relative lateral velocity in scenario I.
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Fig. 4 Missile and target trajectories in scenario I.

Fig. 5 Closing velocity in scenario I.

5.2 Scenario II

In second scenario missile and target is flying with 0 and 150 degrees angles in pitch plan, respectively. Target has 2g maneuver. Therefore, target is going and initial relative range is 10 km. the values of parameters in this scenario are as listed in Table 2.

In this scenario, commanded and missile lateral acceleration, line of sight rate and relative lateral acceleration are plotted in Figs. 6 and 7. As illustrated in these figures, the missile accelerations are smooth and implementable. By applying proposed guidance law, the line of sight rate and relative lateral velocity are converging to zero and these variables are not controlled using two other guidance laws. As shown in Figs.8 and 9 the missile intercept to target by using proposed guidance law, but the closing velocity reaches to zero and guidance loop is instable by using other guidance laws.

In this case, the miss distance is 46 and 21 m by using approximated sliding mode and proportional guidance laws, respectively. Therefore, in this scenario that the relative range is shorter than first scenario, the control loop dynamics leads to instability by using approximated sliding mode and proportional guidance laws.

6 Conclusions

In this paper, a guidance law considering approximation of control loop dynamic is designed using robust back-stepping theory. In the guidance loop, the missile-target relative kinematics is considered as a control process. The control input is the commanded acceleration and the output is the relative lateral velocity based on parallel navigation idea. In this guidance loop, the missile control loop act as an actuator and implement the guidance commands. The dynamics of this section commonly is not to be considered in designing guidance law and it is neglected. But for increasing precision, the approximation of missile control loop dynamics is added to kinematics equations and then the guidance law is designed. The simulation results show that the missile control loop dynamics can cause instability in older guidance laws such as approximated sliding mode and pure proportional guidance laws; but the proposed robust back-stepping guidance law has appropriate performance in the presence of missile control loop dynamics.
Table 2 The values of parameters in scenario II.

<table>
<thead>
<tr>
<th>Guidance Law</th>
<th>$\eta$</th>
<th>$\mu$</th>
<th>$N$</th>
<th>$\varepsilon$</th>
<th>$A_1$</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robust Back-Stepping</td>
<td>-</td>
<td>20</td>
<td>-</td>
<td>-</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>5</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>Approximated SMG</td>
<td>10</td>
<td>20</td>
<td>-</td>
<td>0.00003</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TPNG</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>-</td>
<td>20</td>
<td>-</td>
<td>-</td>
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</tbody>
</table>

Fig. 6 a) Commanded acceleration and b) Missile acceleration in scenario II.

Fig. 7 a) Line of sight rate and b) Relative lateral velocity in scenario II.

Fig. 8 Missile and target trajectories in scenario II.
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References


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