A New Non-linear Control of the Four-Leg Inverter with Decoupled Model and Fast Dynamic Response for PV Generation Systems

M. Pichan* (C.A.), E. Heydari** and H. Rastegar*

Abstract: Distributed generation (DG) will play an important role in future power generation systems, especially in stand-alone applications. Three phase four-leg inverter is a well-known topology which can be used as an interface power converter for DGs. Thanks to the fourth leg to provide the neutral path, the four-leg inverter is able to supply balanced loads as well as unbalanced loads. In this paper, the model of a three phase four-leg inverter with the fourth leg inductor in the αβγ reference frame is investigated thoroughly. Afterward, a decoupled model of the four-leg inverter is adopted to establish the proposed control method. Among non-linear control methods, pole-placement method is a famous solution to ensure fast transient response. Hence, in this paper, a pole-placement method via state feedback is proposed to control the output voltage of the four-leg inverter. Using this method, the transient performance of the system can be adjusted well. On the other hand, to guarantee good performance of the control system under steady state condition, a lead compensator is proposed to be used with the pole-placement method. Therefore, the proposed control system not only can provide fast dynamic response but also, it ensures very low steady state error. To validate the superior performance of the proposed control method, simulation and experimental results under various loading condition are provided based on a DSP-based digital control system.

Keywords: Four-Leg Inverter, Pole-Placement Control Method, State Feedback.

1 Introduction

NOWADAYS, distributed generation (DG) have gained a high significance due to many advantages especially no CO2 emission. There are two kinds of DG systems, stand-alone and grid-connected ones where stand-alone system has attracted much attention for some applications like powering remote loads. Among these systems, the usage of photovoltaic (PV) system has increased due to its capability of generating electricity in a very clean, quiet, and reliable way [1-3]. An interface power converter between PV system and AC loads must supply loads with sinusoidal voltages with fixed amplitude and frequency. According to nature of loads in standalone applications, unbalanced loading condition is inevitable. As a result, for unbalanced loads, the inverter must provide a path for the neutral current. For this purpose, three phase four-leg inverter can be used in which the load neutral is connected to the fourth leg instead of dc-link capacitor midpoint. Compared to other solution for providing neutral current path, the four-leg inverter provides enhanced dc bus utilization, lower ripple on the dc-link voltage and smaller size for dc-link capacitors [4-7]. The schematic of stand-alone photovoltaic system is shown in Fig. 1 where, the four-leg inverter is equipped with an LC filter.

![Fig. 1 Stand-alone photovoltaic system.](image_url)
Since three phase four-leg inverter is a suitable solution for unbalanced/nonlinear loads, the control system play an important task to supply sinusoidal voltage with minimum harmonic distortions. To regulate the load voltages of the four-leg inverter, different research use classic control methods [8-11]. However, most of these methods use frequency domain factors that does not have a precise response in time domain to ensure good transient response [12]. On the other hand, other methods such as PI controller shows weak performance in disturbances rejection even while it was properly tuned [13, 14]. Hence, modern control method is essential to adjust the transient response of control system. In addition, it is important to use control method which is robust to against model disturbances and parameters uncertainties. Due to several advantages of the pole placement strategy [15-19], it can be used to control the load voltages of the four-leg inverter. Although this methods was proposed in [19] for the three phase inverter but, the control system is implemented in dq0 reference frame which suffers from different transformations among reference frames and also, high coupling among different d and q components. It is worth to be mentioned that this coupling makes the controller design and performance analysis somewhat complicated. Moreover, in[19], this coupling is completely ignored that decrease the precise of controller design and performance considerably. Furthermore, this method did not present experimental results.

With respect to several benefits of the pole-placement method, this paper proposes a pole-placement strategy via state feedback to control the output voltage of four-leg inverter without any coupling among state variables. By applying this controller, the transient response of control system can be adjusted precisely which is a significant achievement. Furthermore, to guarantee good performance of the control system under steady state condition, a lead compensator is also designed to be used in parallel with the pole-placement method. It is worth to be mentioned that zero steady state error is a very important issue from voltage stability point of view in standalone applications. Moreover, some devices cannot work properly under voltage decrement or increment so, all of these verify the importance of the low steady state error of the control system. Therefore, the proposed control system not only can provide fast dynamic response but also, it ensures very low steady state error. In addition, the proposed method is highly robust against model parameters mismatches. This paper is organized as follow. The model of the four-leg inverter with a simple decoupling technique is presented in section 2. In section 3, the proposed pole placement control strategy and resonant controller is proposed based on the decoupled model. In section 4 some simulation and experimental results are presented and discussed. Section 5 concludes the paper.

![Fig. 2 Four-leg voltage source inverter.](image)

2 System modeling

The structure of the four-leg inverter following with the output LC filter and the neutral inductor is shown in Fig. 2. As shown in Fig. 2, the load neutral point is connected to the midpoint of the fourth leg through an inductor ($L_o$) which this connection provides a path for the zero-component of the load current.

For the inverter of Fig.2, each phase voltage with respect to the fourth leg can be defined as:

$$
\begin{bmatrix}
v_{af} \\
v_{bf} \\
v_{cf}
\end{bmatrix}
= V_{DC} \begin{bmatrix}
d_{af} - d_N \\
d_{bf} - d_N \\
d_{cf} - d_N
\end{bmatrix}
$$

(1)

where $d_{af}$, $d_{bf}$, $d_{cf}$ and $d_N$ are the duty cycles of each phase according to the negative point of the DC-Link. Now, applying KVL and KCL at the output of the inverter and after some calculations, the state equations of the system are derived as (2) and (3) [20].

$$
\begin{bmatrix}
\frac{d}{dt} i_{La} \\
\frac{d}{dt} i_{Lb} \\
\frac{d}{dt} i_{Lc}
\end{bmatrix}
= \frac{1}{L + 3L_N} \begin{bmatrix}
1 + \frac{2L_N}{L} & -\frac{L_N}{L} & -\frac{L_N}{L} \\
-\frac{L_N}{L} & 1 + \frac{2L_N}{L} & -\frac{L_N}{L} \\
-\frac{L_N}{L} & -\frac{L_N}{L} & 1 + \frac{2L_N}{L}
\end{bmatrix}
\begin{bmatrix}
v_{af} - v_{an} \\
v_{bf} - v_{bn} \\
v_{cf} - v_{cn}
\end{bmatrix}
$$

(2)

$$
\begin{bmatrix}
\frac{d}{dt} v_{an} \\
\frac{d}{dt} v_{bn} \\
\frac{d}{dt} v_{cn}
\end{bmatrix}
= \frac{1}{C} \begin{bmatrix}
i_{La} - i_{oa} \\
i_{Lb} - i_{ob} \\
i_{Lc} - i_{oc}
\end{bmatrix}
$$

(3)

According to (2), it is clear that there is a coupling between currents of different phases which results in the coupled voltage equations. This coupling makes the analysis and design of the controller a difficult task. To solve the problem, the state equations can be transformed from the abc reference frame to the stationary ($\alpha\beta\gamma$) reference frame [20]. For this purpose, each three phase variable can be transformed from the abc reference frame to the $\alpha\beta\gamma$ reference frame by using the transformation matrix $T$ as defined in (4).
By multiplying both sides of (2) and (3) by (4), the state equations in $a\beta\gamma$ reference frame are obtained according to (5), (6):

$$\frac{dv_{a}}{dt} = V_{DC} \left[ \begin{array}{c} \frac{1}{L} - \frac{1}{L + 3L_{n}} \\ 0 \\ 0 \end{array} \right] \left[ \begin{array}{c} i_{La} - i_{\alpha n} \\ i_{L\beta} - i_{\alpha \beta} \\ i_{L\gamma} - i_{\alpha \gamma} \end{array} \right]$$

where $v_{a}$, $v_{\beta}$, $v_{\gamma}$ and $v_{\alpha n}$, $v_{\beta n}$, $v_{\gamma n}$ denote the inverter and the load voltages in the $a\beta\gamma$ reference frame, respectively. According to (5), (6), it is clear that all the state variables are decoupled from each other. Hence, the currents and voltages in the $a\beta\gamma$ reference frame can be controlled independently with no effect on each other [20].

### 3 Proposed Controller

In this paper, the pole placement strategy with state feedback method as one of the modern control method is used to control the system under test. Using this method, the transient performance of the control system can be adjusted as well as designer requirements. To implement the proposed controller, the state equations in the $a\beta\gamma$ reference frame are used since as previously discussed, due to complete decoupling, they can be controlled independently with no effect on each other.

Firstly, to consider the general loading condition, an unbalanced resistive load is considered and the state variables is assumed as $x = [v_{\alpha n} i_{La} v_{\beta n} i_{L\beta} v_{\gamma n} i_{L\gamma}]^{T}$. According to (5) and (6), a signal flow graph of the four-inverter is shown in Fig. 3. Therefore, the state equations of the system can be concluded as (7).

$$\dot{x}(t) = Ax(t) + Bu(t) \quad , \quad y = Cx(t)$$

To achieve the desired transient response, the poles of the system can be placed at the appropriate position by proper determination of the feedback matrix $K$. On the other hand, a lead compensator is also used to track the input reference to guarantee achieving this purpose. The
structure of the proposed control method with the state feedback and lead compensator is shown in Fig. 4.

3.1 Pole placement via state feedback strategy

By considering the state feedback \( u = -Kx \), the closed-loop system equation can be expressed as follow:

\[
\dot{x} = [A - BK]x(t) \tag{8}
\]

Now, the goal is to find the appropriate matrix \( K \) such that the poles of the closed-loop system place on the desired value to suitably improve the transient performance. Applying this method, not only the behavior of the open-loop system can be improved significantly but also, the transient response can be adjusted precisely. Assume the \((\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)\) are the eigenvalues of the matrix \( A \) of the open-loop system and the \((\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_4, \hat{\lambda}_5)\) are the desired eigenvalues of the matrix \( A - BK \) of the closed-loop system. The characteristic equations of the open-loop and closed-loop system are shown in (9) and (10), respectively:

\[
p(s) = \prod_{i=1}^{5} (s - \lambda_i) = |sI - A| = s^5 + a_4s^4 + a_3s^3 + ... + a_1
\]

\[
\hat{p}(s) = \prod_{i=1}^{5} (s - \hat{\lambda}_i) = |sI - A + BK| = s^5 + a_4s^4 + a_3s^3 + ... + a_1
\]

By locating the eigenvalues of the closed-loop system at \((\hat{\lambda}_1, \hat{\lambda}_2, ..., \hat{\lambda}_5)\) and considering the eigenvectors of the system as \((\varphi_1, \varphi_2, ..., \varphi_5)\), it is concluded that:

\[
[A - BK] \varphi_i = \hat{\lambda}_i \varphi_i \tag{11}
\]

and consequently:

\[
\begin{bmatrix} A - \hat{\lambda}_i I & B \end{bmatrix} \begin{bmatrix} \varphi_i \\ -K \varphi_i \end{bmatrix} = 0 \tag{12}
\]

Now, by definition \( q_i = -K \varphi_i \) for \( i = 1, 2, ..., n \), the following equations are obtained:

\[
\begin{bmatrix} A - \hat{\lambda}_i I & B \end{bmatrix} \begin{bmatrix} \varphi_i \\ q_i \end{bmatrix} = 0 \tag{13}
\]

According to (13), the matrix \([\varphi_1, q_2, ..., q_n] \begin{bmatrix} \varphi_1 \\ q_2 \\ ... \\ q_n \end{bmatrix}^{-1}\) must be placed in the null space of the matrix \([A - \hat{\lambda}_i I, B]\). Hence, the selection of the eigenvectors is limited by (13) while eigenvalues can be placed at anywhere of the plane. It is worth to be mentioned that due to large variation of the loading condition of the four-leg inverter, the control method should not affected under any single/three phase linear/non-linear loads. In addition, with respect to the variation of the inductors and capacitors during the time, the control method should be robust against model parameter mismatches. As a result, in this paper, eigenvalues placement is done based on a proposed algorithm in [21] in which, it is suitable for multi-input systems and optimizes the choice of eigenvectors for robust operation of the control method.

3.2 Design of the lead compensator

By using the pole placement method that is discussed in previous section, the dynamic performance of the system will be tuned precisely. However, the good steady state performance of the system may not satisfied completely. As a result, the lead compensator is adopted to better input reference tracking of the proposed method. According to Fig.4, the input of the system can be expressed as (15).

\[
u = Gr - Kx \tag{15}
\]

Now, substituting (15) in (7) will results in (16):

\[
\dot{x} = Ax + Bu \tag{16}
\]

To ensure zero steady state error, the \( \lim x(t) = 0 \), so:

\[
x = -(A - BK)^{-1}BGr \tag{17}
\]

\[
y = Cx = -C(A - BK)^{-1}BGr
\]

Furthermore, for input reference tracking, \( \lim y(t) = r \), hence, the lead compensator can be designed properly as follow:

\[
G = -C^{-1}(A - BK)B^{-1} \tag{18}
\]

Applying the complete control system, not only the transient response is adjusted properly but also, the steady state performance will be adjusted to have a zero steady state error in the load voltages. It is two main achievements of the proposed control system which candidate it as one of the best solutions to control the load voltages of the four-leg inverters.
control system, several simulations are done in Matlab/Simulink software. Also, a 3kW test bench is provided to validate the simulation results through experimental results. The parameters of the system is shown in Table 1.

The photograph of the test bench is shown in Fig. 5. Due to several benefits of the digital signal processors (DSPs) such as several peripherals, six enhanced PWM channels, sixteen high frequency analog to digital (A/D) converters and high speed calculations ability, the control system is implemented by prototype control board based on the TMS320F28335 DSP from TI company. Also, the SKM100FB12T4 IGBTs are used for the power devices which their gate signals are prepared by the prototype board with HCPL316J ICs.

The low pass LC filter connected at the inverter output is responsible to remove or decrease the switching ripples. It is appreciate assumption to set the cut-off frequency of the LC filter at 10% of the switching frequency [20]. The current ripple of the inductor at the worst case is given in (19) [22]:

$$\text{Inductor Current Ripple} = \frac{V_{DC}}{2 * L * f_{sw}} |D| * (1 - |D|)$$  \hspace{1cm} (19)

where \(D\) is defined as the line to neutral duty ratio. According to the maximum allowable current ripple, the \(L\) is selected from (19). Afterward, based on the cut-off frequency, the proper value for \(C\) is calculated. To achieve the best performance from Load voltage THD point of view [23], the neutral inductor \((L_n)\) is selected as half of the three phase inductors \((L)\).

Due to increased number of switching vectors of the four-leg inverter \((2^4 = 16)\) compared to three leg inverters, the common SVM PWM method is somewhat complex for digital implementation [24-26]. On the other hand, the sinusoidal PWM method not only provides good harmonic profile but also, it provides considerably simple structure [27]. Therefore, the SPWM method is adopted in this paper to generate the switching gate signals.

Substituting the parameters of the system in (9), the eigenvalues of the open-loop system are obtained as \(\lambda_{1,2} = -1893 \pm 5699j\) , \(\lambda_{3,4} = -1515 \pm 5812j\) , \(\lambda_{5,6} = -1262 \pm 3526j\) . The desired eigenvalues of the closed-loop system are selected as \(\hat{\lambda}_{1,2} = -49836 \pm 3913j\) ,

Table 1 The parameters of the system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_o)</td>
<td>Nominal output voltage</td>
<td>100v (rms)</td>
</tr>
<tr>
<td>(f_o)</td>
<td>Nominal output voltage frequency</td>
<td>60Hz</td>
</tr>
<tr>
<td>(f_{sw})</td>
<td>Switching frequency</td>
<td>12kHz</td>
</tr>
<tr>
<td>(V_{DC})</td>
<td>DC-Link voltage</td>
<td>390v</td>
</tr>
<tr>
<td>(L_f)</td>
<td>Filter inductance</td>
<td>880 (\mu)H</td>
</tr>
<tr>
<td>(C_f)</td>
<td>Filter capacitance</td>
<td>33 (\mu)F</td>
</tr>
<tr>
<td>(L_n)</td>
<td>Neutral filter inductance</td>
<td>440 (\mu)H</td>
</tr>
</tbody>
</table>

Fig. 6 Simulation and experimental results under no load condition; simulation results: a) phase voltages, b) THD% of the phase voltages; experimental results: c) phase voltages, d) THD% of the phase voltages.
\[ \hat{\lambda}_1 = -30000, \hat{\lambda}_2 = -50000, \hat{\lambda}_3 = -80000, \hat{\lambda}_4 = -80000. \]

By locating the eigenvalues of closed-loop system at \( (\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_4, \hat{\lambda}_5) \), the transient performance of the control system can be adjusted well. On the other hand, a lead compensator is also designed for better input reference tracking by (18).

In the first study, the four-leg inverter is tested under no load condition and both simulation and experimental results are shown in Fig. 6. As evidently appears, the control system tracks the reference values precisely. The total harmonic distortion (THD%) of the load voltages under simulation results is 1.13% which shows that the simulation and experimental results are in good accordance with each other.

Non-linear loads have been used widely in both high power industrial applications such as adjustable AC drives and low power domestic applications such as electronic ballasts and switching power supplies. Since these loads absorb high harmonic polluted currents, the inverter should maintain its normal operation under any non-linear loading condition. To evaluate the performance of the four-leg inverter with proposed control method, three phase non-linear load as depicted in Fig. 7 is connected at inverter outputs and the results are shown in Fig. 8.

Based on Fig. 8 (a), it is evident that even with large harmonic polluted and sharp current, the load voltage does not affected much. At this condition, the maximum THD% of the load voltages is 3.6% which is much lower than 8% so, the performance under non-linear load satisfies the IEC62040-3 international standard. The voltage reference tracking error (\( e_v [%] = (V_{v,reference} - V_{v,measured}) / V_{v,reference} \times 100 \)) is below 1.4%.

To better evaluate the performance of the proposed control system, a hybrid load involving both linear and non-linear loads are connected at inverter outputs and the results are shown in Fig. 9. The THD% of the load voltages are 2.8% under this condition with \( e_v [%] \) equal to 3%. Since the amount of non-linear loads are decreased, the THD% of the phase voltages are improved while, due to increase of loads, the reference tracking error experienced a bit increment.

![Fig. 7 The structure of the non-linear load.](image)

![Fig. 8 The experimental results under non-linear loads: a) voltage (50 v/div) and current (13 A/div) of phase 'a', b) load voltages, c) THD% of the load voltages.](image)

![Fig. 9 Experimental results under hybrid linear/non-linear loads: a) phase voltages; ch1-ch3 (50 v/div), b) phase (ch1-ch3) and neutral currents (ch4) (13 A/div).](image)
As one of the achievements of the proposed control system, the transient response can be adjusted precisely. To examine this characteristic, the transient waveforms in response to the no-load to nominal resistive load step change (rising) and also falling step change are shown in Fig. 10. With respect to this figure, the control system retrieves the load voltage in a time interval lower than 400 μs. Since the load step change is performed near the peak value of the phase 'c' (ch3 with green color) voltage, it experiences the most severe condition from voltage drop point of view. However, the control system compensates the voltage error with a small undershoot of about 12% of the nominal voltage.

In another study, the effect of parameter mismatches on the control performance is considered. According to Fig.3 and (7), the filter inductors and capacitors, \( L \) and \( C \), are the most usable parameter in the control system. Therefore, the performance of the proposed controller in response to mismatches in these parameters is studied in terms of the THD% and the \( e_v \%). The results are shown in Fig.11.

Based on Fig. 11, it can be concluded that the control system maintains its normal operation even under large model parameter mismatches.

5 Conclusion

In power converter interface between DG systems and AC loads, to supply unbalanced/non-linear loads, three phase four-leg inverter with the neutral inductor seems to be the best solution. To achieve the benefits of the four-leg inverter as well as providing clean sinusoidal balanced voltages, control system plays an important task. In this paper, to provide adjustable transient response precisely, a pole-placement strategy via state feedback with no coupling between the state variables is proposed to control the output voltages of the four-leg inverter. In addition, a lead compensator is also designed for precise tracking of the reference input. Several simulation and experimental results were performed to investigate the performance of the proposed control methods. All of these verify the effectiveness of the proposed method where the transient response under 100% step load changing is below 2ms. Also, the THD% value of the load voltages at severe non-linear loading condition is below 3.6% which satisfies the IEC62040-3 international standard. Furthermore, it is confirmed that the proposed control method is highly robust against model parameter mismatches.

References


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