A Game Theory Approach to Distribution System Reliability Improvement Based on Customer Requests

R. Mohammadi* and H. Rajabi Mashhadi*(C.A.)

Abstract: Distribution system reliability programs are usually based on improvement of average reliability indices. They have weakness in terms of distinguishing between reliability of different customers that may prefer different level of reliability. This paper proposes a new framework based on game theory to accommodate customers’ reliability requests in distribution system reliability provision. To do this, distribution reliability equations are developed so that it is recognized how game theory is suitable for this purpose and why conventional methods could not provide customer reliability requirements appropriately. It would be shown that customer participation in distribution system reliability provision can make conflict of interest and leads to a competition between customers. So, in this paper a game theoretic approach is designed to model possible strategic behavior of customers in distribution system reliability provision. The results show that by implementing the proposed model, distribution utilities would have the capability to respond to customers’ reliability requirements, such that it is beneficial for both utility and customers.

Keywords: Distribution System, Game Theory, Reliability, Smart Grid.

1 Introduction

Providing desired reliability for different customers is changing to a public need in distribution systems. It is because of growing electrical loads with sensitive process in different areas of electrical power consumption including industrial, commercial and even domestic sections.

Introducing concept of smart electrical distribution systems, by which higher levels of reliability can be provided for each customer, is a response to this requirement of customers. In such a system by utilizing smart technologies, the capability of utility to enhance the system reliability improves [1]. Smart distribution systems provide new capabilities which can be vastly different from conventional distribution systems. The differences are not just about hardware in the system but also include the studies related to operation and planning of distribution systems. In current distribution systems which are not fully equipped with smart tools, distribution utilities should find solutions to respond to reliability preferences of customers in distribution system and also, such effort should be done optimally. However, assessing the companies’ reliability improvement plans over time indicates that distribution system reliability planning has not been the priority of the utilities. In addition to this, the system modifications implemented for reliability improvement have not been optimal [2].

Conventional programs for distribution system reliability improvement have some weaknesses that make them inappropriate for providing desired reliability of customers at different load points. These weaknesses are because of the reliability indices they use and the methods they employ for reliability improvement.

About the indices, distribution system reliability is generally divided into two categories: the first type is customer-based reliability indices which are introduced by IEEE Std.1366. The most important average indices that are widely used by distribution companies include SAIFI, SAIDI, CAIDI and ASAI. Average reliability indices give equal importance to different loads, do not...
reflect customer satisfactions and have the inherent weakness in expressing local status of reliability [3]. In some cases, value of the index can be confusing and when reliability improvement planning is implemented, it is possible to produce conflicting results [2, 4]. Consequently, the usage of average indices, which is common in conventional reliability improvement, causes some limitation in allocation of required reliability levels to load points, regardless of employed improvement method.

The second type is load-point reliability indices, which evaluate the quality of service at each load point. According to the IEEE Std. 493, these indices determine the average failure rate, the average outage duration, and the average annual outage duration at load points. To improve average outage duration at load points based on customer preferences, reliability options under utility liability [5] and a market based platform [6] are applied to distribution system reliability provision. But, conventional reliability improvement methods, in distribution system, are usually based on Utility-Based Reliability Planning (UBRP), Value-Based Planning (VBP), Performance-Based Rates (PBR). Reliability Insurance Scheme (RIS) is another reliability provision method based on customer preferences that has attracted attention recently.

In UBRP, the utility considers the reliability as a system performance index and try to improve it in minimum cost for utility. Improvement of EENS and other average reliability indices is prevalent in this approach [7]. VBP minimizes the sum of the customers’ outage costs and the cost of the utility (social cost) in system’ reliability problem [8, 9]. Implementation of VBP may be problematic as the customers’ tendency to pay for reliability improvement can be different from their reliability values [10]. Difficulty in determination of real customers’ reliability values and possibility of making a kind of subsidy between different areas are other weaknesses of VBP [2].

In regulatory structures like PBR, distribution utilities are forced to improve their reliability in order to avoid penalties [11]. PBRs do not completely reflect social cost due to the possibility of creating multiple optimal points and make new financial risks for companies [12]. Furthermore, regulators set targets based on the system average indices and so additional control on load point reliability indices should be performed [13]. Different utilities sometimes use different methods for calculation of reliability indices. This is not good for regulators that want to set out reliability targets in PBR based on these reliability indices. Moreover, by using PBRs, determining an acceptable level of reliability is a questionable task [14]. The above mentioned weaknesses in terms of indices and methods may cause the provided reliability not to be completely corresponding to the customer needs.

Reliability insurance scheme (RIS) is introduced in [15]. In this framework it is allowed to customers to select their desired reliability levels. In RIS, customers select their own reliability levels, pay expenses accordingly, and if utility cannot provide the agreed reliability levels, penalties are paid to the insured customers. With received premiums, distribution company can perform optimal and effective modifications on system to improve reliability so that it becomes profitable for the company [16, 17]. Also, DG owners have the chance of making profitable contracts with adjacent customers in their zones [18]. Therefore, it is expected that both sides have enough tendency and motivation to actively participate in the program. However, since electricity is a network good and a distribution company does not have sufficient control over the system to provide desired reliability at all load points, partial implementation of RIS would be possible. This can mitigate customers’ satisfaction and their real selection rights for desired reliability levels. These limitations occur due to the lack of enough flexibility in restoration process and can cause phenomena like strategic game and free riding [18]. By considering this challenge in conventional RIS, utility has to act in a way that discover and control such phenomena and prevent these behaviors by regulatory policies as penalizing free riders [19]. While in a fully upgraded system of smart grid this problem should be solved technically and so, the insurance scheme has some disadvantages again.

This paper aims to enter customers’ preferences in distribution reliability improvement program and focuses on load point reliability indices. By adding new participants to the reliability improvement problem of distribution utility, new decision makers appears which their decision can be in conflict with each other. Therefore, a game theoretic approach is utilized to model interaction between some players’ decisions and their conflicting objectives. To do this, the minimum requirement to implement the proposed approach is identified and then, it is shown that how game theory basics are applied to the customer reliability provision in distribution system and how it influences on customer preferences for reliability. Furthermore, it is analytically shown why game theory is an appropriate framework for reliability provision in distribution system and why conventional reliability improvement policies have weaknesses in terms of customers’ satisfactions. In this paper, the model formulation based on game theory is presented and analyzed in a distribution system. The output of the proposed model shows how changing view to reliability provision can cause different satisfaction levels among customers.

In this context, one important issue is how much the system is able to allocate different levels of reliability to different load points. The insurance structure considers only a few levels for reliability, usually three levels, and therefore reliability and cost allocation process cannot be accurate. In addition, insurance scheme is not able to consider the effect of location of customers in the
A Game Theory Approach to Distribution System Reliability  

... R. Mohammadi and H. Rajabi Mashhadi

In the proposed model, there is no concern about free riding and reliability requests which are not in accordance to customers’ reliability values. As mentioned earlier, these two points are the main weaknesses of other load point based reliability improvement methods. Because, competition and maximizing each agent payoff function is accepted in this model. Accordingly, distribution utility should act in a way that manages the equilibrium point of this competition to a direction that minimize undesired phenomena like free riding.

In the next part, the reliability of the distribution system is explained in more details to show how much flexibility can be provided in the system. First, the reasons for the weaknesses of the previous methods are analyzed and expressed in form of examples and then it is shown how the proposed game method introduces a more appropriate and transparent framework.

2.2 Flexibility of Distribution System in Providing Reliability

One of the most important issues in providing different reliability levels in distribution system is the amount of controllability or flexibility of system to distinguish between different load points. Fig. 2 shows a radial distribution system with 4 buses.

Fig. 2 shows a distribution system with four load points. In basic state, there is no control over the system through sectionalizing switch, tie switch and breaker. This system is equipped with reliability control instruments, namely breakers and switches, and then their effects on improving reliability of distribution load points are illustrated. To do this, the expected outage time of load point \( i \) (\( D_i \)) is studied. Repair time (\( D_r \)), switching time (\( D_s \)), tie switching time (\( D_{ts} \)) and failure rate of section \( i \) (\( \lambda_i \)) are the parameters which affect the expected customers’ outage time. The cases shown in Fig. 2 are explained as follows:

1. Basic state: As seen in Fig. 2(a), there are no switches in this configuration and therefore if any fault occurs, each load point is faced with power outage which takes as long as repair time of the faulty line. Then, outage time of load point 1 and 2 can be written as follows:
A Game Theory Approach to Distribution System Reliability … R. Mohammadi and H. Rajabi Mashhadi

\[ D_1 = D_2 = (\lambda_1 + \lambda_3 + \lambda_2 + \lambda_4)D_r, \]  

2. Effect of breakers: Fig. 2(b) shows that feeder 2 including loads 3 and 4 is equipped with a breaker. By installment of this equipment at the head of the feeder, obviously all faults occurring on the downstream part of the breaker have no effect on reliability of other sections. As a result, the outage time of other feeders of distribution system will be decreased. The following equation indicates this effect:

\[ D_1 = D_2 = (\lambda_1 + \lambda_2)D_r, \]

3. Effect of sectionalizing switches: as Fig. 2(c) shows, sectionalizing switches improve the reliability of upstream parts of the feeder and have no effect on downstream parts. Installation of sectionalizing switch change reliability of load points 1 and 2 as follows:

\[
\begin{align*}
D_1 &= \lambda_1 D_r + \lambda_2 D_s, \\
D_2 &= (\lambda_1 + \lambda_2)D_r
\end{align*}
\]

In this case, adding a breaker on feeder 1, as mentioned in previous case, just affect reliability of load points on feeder 2, as it isolates the upstream loads and other feeders from faulty line.

4. Effect of tie switches: in this case, as shown in Fig. 2(d), tie switching between two feeders is possible. According to Eq. (4), it is clear that the reliability of bus 1 does not change, thus the tie switch improves the reliability of the downstream parts and has no effect on that of the upstream, so we have:

\[
\begin{align*}
D_1 &= \lambda_1 D_s, \\
D_2 &= \lambda_1 (D_r + D_s) + \lambda_2 D_s
\end{align*}
\]

2.3 Implementation of the Proposed Scheme

2.3.1 Minimum Requirement to Control Reliability

Based on what was mentioned in the last section, the effect of breaker on reliability of load points shows that breakers can be utilized to distinguish between the reliability of different feeders. On the other hand, sectionalizing switch and tie switch have complementary role in reliability of the load points and can be appropriately employed to control the reliability of the loads on one feeder. Then, the existing facilities of the distribution system can provide a minimum level of flexibility by which it is possible to assign different amount of reliability to different load points. As the number of sectionalizing and tie switches on a feeder increase, the flexibility of system to respond to reliability requirement for a larger group of customers increases accordingly. In general, the most flexible system is a smart grid by which any reliability assignment to any load point is possible. It should be noted that the reliability of load points is not completely controllable and consequently, from technical point of view the free riding phenomenon can occur in reliability improvement methods using load point reliability indices. For this reason, [17] explains that RIS can be implemented at any physically separable node in which some consumers are aggregated. In [19], penalizing free riders, if possible to detect, is proposed to decrease the effect of free riding in reliability investments.
2.3.2 Free Riding

In cases discussed in Section 2.2, the reliability level of the upstream loads on a feeder is always higher or equal to that of the downstream considering the load points of the feeder are located at the sections that their failure rates are equal. For instance, in Fig. 2(d):

\[
\text{if } \lambda_1 = \lambda_2 \left\{ \begin{array}{l}
D_1 = \lambda (D_1 + D_2) \\
D_2 = \lambda (D_3 + D_4)
\end{array} \right.
\]

(5)

Based on Eq. (5), in any radial feeder, outage time of upstream load points \((D_1)\) is always less than downstream ones \((D_2)\). This can lead to free riding for upstream load points, which take advantage of the high reliability requested by the downstream users, while requesting a lower reliability level. The question is that if the upstream loads of feeder request a lower reliability in comparison to downstream parts, for example in RIS, how the utility can make such distinction for the reliability of a feeder. It is also a questionable task for other reliability improvement methods. Then, conventional and regulated methods cannot handle the problem completely. But in this paper, proportional to flexibility of system, the proposed method can be employed successfully and the framework give an opportunity to handle this problem on system by solution other than technical one. In the proposed model based on game theory, economical motivation would be added to the reliability improvement program and free riding can be studied from different viewpoint that can influence this behavior.

3 The Analysis of Distribution System Reliability

For any feeder, load points are divided to the following groups based on the position of switches and tie switches (generally, there are more cases):

1. Loads that can be fed through main power supply in feeder (head or the middle of feeder).
2. Loads that can be restored by operation of sectionalizing switches and tie switches (middle or end of feeder).
3. Loads that cannot be restored (middle or end of feeder).

Fig. 3 shows a feeder with three sections in which the loads can be restored through power supply and tie lines (cases 1 and 2). Such feeder is the most flexible state to restore loads and improve the system’s reliability.

For this system, reliability equations can be written and developed as follows:

\[
\begin{align*}
D_1 &= \lambda_1 D_1 + (\lambda_2 + \lambda_3) D_2 \\
D_2 &= \lambda_2 (D_3 + D_4) + \lambda_2 D_3 \\
D_3 &= (\lambda_1 + \lambda_2) D_5 + \lambda_3 D_5
\end{align*}
\]

(6)

and in matrix form and considering \(\lambda = \lambda_1 + \lambda_2 + \lambda_3\):

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix} = \begin{bmatrix}
\lambda_1 & -\lambda_1 & 0 \\
\lambda_2 & -\lambda_2 & \lambda_4 \\
\lambda_3 & -\lambda_3 & \lambda_4 + \lambda_5
\end{bmatrix} \begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
\]

(7)

where \(\lambda\) is failure rate of the entire line and \(\lambda_i\) is failure rate of section \(i\). According to Eqs. (2) and (3), for feeders with sections similar to these three sections regarding the load restoration (feed from the head of the feeder, restorability in the middle of the feeder, or an end load with restoration possibility), we obtained the expected interruption time of each section by the following closed form equation.

\[
D_i = \lambda D_i + (\lambda - \lambda_i) D_i + \left( \sum_{j=1}^{3} \lambda_j \right) D_i
\]

(8)

According to Eq. (8), it is concluded that repair time affect outage time of any section proportional to its length; switching time affect outage time by length of all other sections in which the load is not located and tie switching time effect is proportional to length of upstream sections of each load point.

If all sections have same line type with failure rate of \(\lambda_i\) per km per year, the matrix Eq. (7) can be rewritten to obtain the effect of the switches’ positions on the reliability of each load point.

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix} = \begin{bmatrix}
\lambda_1 & -\lambda_1 & 0 \\
\lambda_2 & -\lambda_2 & \lambda_4 \\
\lambda_3 & -\lambda_3 & \lambda_4 + \lambda_5
\end{bmatrix} \begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
\]

(9)

where \(l_i\) is the length of section \(i\) and is determined by switch location on the feeder. Now let \(D_i = 0\) (considering repair group closes tie switch immediately after isolating the fault or it can be remotely closed). In this case, Eqs. (7) and (9) are changed as follows:

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix} = \begin{bmatrix}
\lambda_1 & -\lambda_1 & 0 \\
\lambda_2 & -\lambda_2 & \lambda_4 \\
\lambda_3 & -\lambda_3 & \lambda_4 + \lambda_5
\end{bmatrix} \begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
\]

(10)

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix} = \begin{bmatrix}
l_1 & l_1 & l_1 \\
l_2 & l_2 & l_2 \\
l_3 & l_3 & l_3
\end{bmatrix} \begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
\]

(11)
In Eq. (10), $l$ is the total length of the feeder. Therefore, the length of any section can be obtained based on the outage interruption times of customers.

\[
\begin{bmatrix}
    l_1 \\
    l_2 \\
    l_3
\end{bmatrix}
= \begin{bmatrix}
    1 \\
    \lambda D_1 (D_1 + D_2) - 2D_1^2 \\
    \lambda D_3 (D_3 + D_4) - 2D_3^2
\end{bmatrix}
\times
\begin{bmatrix}
    D_1 \\
    -D_2 \\
    -D_3
\end{bmatrix}
\begin{bmatrix}
    D_1 \\
    D_2 \\
    D_3 \\
    D_4
\end{bmatrix}
+ l_1 + l_2 + l_3 = l_1,
\] (12)

Equation (12) shows that for any reliability levels selected by customers, the utility can find the appropriate position of switches and thus the effect of customers’ decisions on utility’s decision can be understood in the game model. It is what the proposed structure made based on it.

Moreover, by adding the outage time of the entire feeder and considering $l_1 + l_2 + l_3 = l$, expected outage time of system is equal to Eq. (13).

\[
\sum_{i} D_i = D_1 + D_2 + D_3 = \lambda(D_1 + 2D_2) \text{ if } D_i = 0
\] (13)

Furthermore, in general form according to Eq. (8) in any n-bus distribution system with similar reliability characteristic to system presented in Fig. 4, we obtain the SAIDI of system as follows.

\[
\sum_{i} D_i = \sum_{i} (\lambda D_i + (\lambda - \lambda_i)D_i + \sum_{j \neq i} (\lambda_j)D_i)
= \lambda(D_1 + (n - 1)D_2) + \sum_{j = 1}^{n} (\sum_{i = j}^{n} \lambda_i)D_j
\Rightarrow
SAIDI = \frac{1}{n} \sum_{i} D_i = \frac{1}{n} (\lambda(D_1 + (n - 1)D_2))
+ \sum_{j = 1}^{n} (n - j)\lambda_j D_j
\]

\[
SAIDI = \frac{1}{n} (\lambda(D_1 + (n - 1)D_2)) \text{ if } D_i = 0
\] (14)

These equations show that total outage time of the feeder is independent from the locations where switches and tie switches are installed. In other words, if reliability improvement planning is based on the SAIDI of system (or feeder), without notable change in this index, customers can receive different levels of reliability. This clearly shows the reliability planning of conventional methods based on system’s average indices especially SAIDI has weaknesses in terms of customers’ satisfactions. Then, there is a motivation for changing view and determination of new space.

The second point is that these formula show how reliability of each bus changes by operation and performance of utility after a fault (repair and switching times). In other words, sensitivity to utility’s operation in terms of reliability for each bus can be recognized and according to that, different reliability levels can be allocated to different buses (if there is no limitations about operation of system).

4 The Analysis of Distribution System Reliability Considering Customer Preferences by Game Theory

4.1 Case Study 1: Fixed Positions of Switches

In Fig. 2, there are two load points on a feeder and the game approach is investigated on this system with two customers. Table 1 shows aforementioned two customers’ competition in conventional form of two-player game exhibition. It describes a set of strategies of each player and the payoff to each player for any strategy profile (the list of strategies chosen by the players). The concept of Nash equilibrium is the cornerstone in predicting the outcome of a game. In a Nash equilibrium each player's strategy maximizes his utility given the strategies played by the other players [21].

In Table 1, customer 1 is the column player and customer 2 is the row player. It is assumed that load points 1 and 2 on feeder 1, serve as two players in game model, are similar and the imposed costs to customers are equal in any specified outage duration. Therefore, instead of minimizing the cost, it is sufficient to minimize the outage time of customers.

Investigation of the proposed game on this two-bus system has some benefits. Firstly, exhibition of game for more than two players cannot be illustrated on one table and in conventional exhibition of game only two customers are considered. Secondly, as mentioned before, there is free riding opportunity for upstream loads and two-bus system can show this characteristic without adding complexity to the problem. Thirdly, similarity of two customers and the fixed position of switches means that RIS and other reliability improvement methods have no solution for assigning different reliability to customers and free riding would be unavoidable.

The players can arbitrarily select two reliability levels as their actions (High and Low) and according to their choices, utility uses one of strategies mentioned in Section 2.2. In Table 1, the two rows correspond to the two possible actions of player 1 and the two columns correspond to the two possible actions of player 2. Each box shows one possible action profile selected by players and the payoff of player 1 is listed first. If both of customers choose low levels of reliability (L, L), there is no effort for improving reliability, similar to case 1 in Section 2.2. In other strategy profiles selected by players, one of four strategies in Section 2.2 is assigned by the utility that related outcome can be obtained from Table 1.

For the numeric simulation, $\lambda$ is equal to 0.1
Table 1 Two distribution customers game represented in convenient form of game model.

<table>
<thead>
<tr>
<th>(D_1, D_2)</th>
<th>Player 2 (Column Player)</th>
<th>Player 1 (Row Player)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>2 \lambda D_1, 2 \lambda D_2, \lambda(D_1 + D_2), \lambda(D_1 + D_2 + D_1)</td>
<td>L</td>
</tr>
<tr>
<td>H</td>
<td>\lambda(D_1 + D_2), 2 \lambda D_2, \lambda(D_1 + D_2), \lambda(D_1 + D_2 + D_1)</td>
<td>H</td>
</tr>
</tbody>
</table>

Table 2 Reliability parameters of 2 bus distribution system.

<table>
<thead>
<tr>
<th>Reliability improvement activity</th>
<th>Duration</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>sectionalizing switch</td>
<td>30 min</td>
<td>35</td>
</tr>
<tr>
<td>tie switch</td>
<td>30 min</td>
<td>55</td>
</tr>
<tr>
<td>Repair</td>
<td>2 hours</td>
<td>12$</td>
</tr>
<tr>
<td>Breaker</td>
<td>0</td>
<td>20$</td>
</tr>
</tbody>
</table>

Table 3 Nash equilibrium in base case.

<table>
<thead>
<tr>
<th>(D_1, D_2)</th>
<th>L</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0.4, 0.4</td>
<td>0.25, 0.3</td>
</tr>
<tr>
<td>H</td>
<td>0.25, 0.4</td>
<td>0.25, 0.3</td>
</tr>
</tbody>
</table>

Table 4 Nash equilibrium when the utility uses strategy 1.

<table>
<thead>
<tr>
<th>(D_1, D_2)</th>
<th>L</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0.4, 0.4</td>
<td>0.4, 0.3</td>
</tr>
<tr>
<td>H</td>
<td>0.25, 0.4</td>
<td>0.25, 0.3</td>
</tr>
</tbody>
</table>

Table 5 Nash equilibrium when the utility uses strategy 2.

<table>
<thead>
<tr>
<th>(D_1, D_2)</th>
<th>L</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0.4, 0.4</td>
<td>0.4, 0.3</td>
</tr>
<tr>
<td>H</td>
<td>0.25, 0.4</td>
<td>0.2, 0.3</td>
</tr>
</tbody>
</table>

Table 6 Nash equilibrium in base case 2.

<table>
<thead>
<tr>
<th>(C_1, C_2, C\text{utility})</th>
<th>L</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>4, 4, 2.4</td>
<td>3.325, 3.9, 1.775</td>
</tr>
<tr>
<td>H</td>
<td>3.4, 4, 1.8</td>
<td>3.4, 3.9, 1.7</td>
</tr>
</tbody>
</table>

In this case, it is assumed that the reliability value of the customers is high enough that the current equipment costs has no effect on their decision, unless the utility requires a new investment. As Table 3 shows, according to the concepts of game theory, both customers seek to minimize their outage times and the output of game. Nash equilibrium, is (L, H). This outcome indicates free riding possibility of first customer by benefiting from the choice of customer on the second bus.

Now, in order to prevent such phenomenon, in case of occurring a fault in line 2, the utility can avoid restoring load point 1 before repairing the line (strategy 1). Therefore, this competition, shown in Table 4, finds a new equilibrium that is (H, H) and that allows the company to prevent free riding.

There can be two other solutions instead of preventing improvement in reliability of bus 1. One solution is to close tie switch 2 and open breaker 1 to change the feed path and create a new configuration for power supply. In this case, load point 1 becomes incapable of free riding; however, the reliability of both load points is reduced in comparison to normal state. This cannot be logical since the goal of this structure is improving reliability again. Another solution is to increase the reliability of bus 1 so high that the upstream user can profit more by selecting a higher reliability level rather than free riding. This can be achieved by replacing a breaker instead of switch 1 in the middle of feeder (strategy 2). It is assumed that reliability value is higher than the cost by which reliability of each customer is improved and thus in this case, according to Table 5, Nash equilibrium changes to (H, H).

Therefore, in general, we can determine an appropriate configuration within existing distribution facilities corresponding to the customers’ preferences to manage their reliability.

4.2 Case Study 2: Fixed Positions of Switches

In this case, reliability costs are also considered in the game model shown in Table 6 and cost allocation and game rules are defined as follows.

1. If customers select high or low reliability levels, respectively a half and a fourth of the switching cost is assigned to them. Moreover, no expense is received for repairs.
2. If reliability is improved, for each bus, half the profit of reliability improvement is paid to the utility.

Following these rules, customers decide about their reliability levels. For both customers, loads are similar, reliability values are equal, and cost of power outage is $10/kW. Also the utility cost (C\text{utility}) related to each strategy profile of two players is calculated and shown in Table 6.

Table 6 shows that the costs of providing reliability for two similar loads in a same feeder are different and in this model, according to this cost, any customer has to pay some expenses. Higher cost of the second load in comparison to the first load in all cases indicates that this load takes more advantage of system’s utilities to...
improve its reliability and pay more expenses accordingly; while, in the regulatory structure, the cost of reliability improvement for all loads are equal.

The equilibrium of this game is (L, H); while the appropriate game equilibrium for the company is (H, H).

As it was mentioned, if the company does not use the reliability improvement option for bus 1 (using switch S1), while bus 1 selects a low level, the game is changed as Table 7 shows.

In this case the company’s cost increases, i.e., it ignores maximum profit (company’s cost using this strategy increases from 1.775 to 2.3) and acts somehow irrationally; however, this action can signal load point 1 and change the equilibrium to a state where more profit can be achieved by the company and free riding is also prevented in the long term.

Moreover, if breaker installation option is used, the game is presented in Table 8.

In this case, reliability value of buses is not high enough to cover the cost of installing a breaker and the Nash equilibrium is (L, H); however, increasing the reliability value of customer 1 allows moving the equilibrium point to where both players select reliability levels based on their needs.

These results are obtained when failure rate, load type, and reliability value of both buses are similar. It has been designed in this way to create a free riding potential for upstream load to be investigated in proposed game. Moreover, allocating costs is performed in a certain way that any customer pays half of operation cost and the company receives half (or a quarter) of customers’ profits. Any changes in game rules and any difference in the customers type can change the system’s behavior and output of the game; however, it is clear that different tools and parameters can be employed to prevent undesired phenomenon like free riding by changing the equilibrium point. Such approaches are strategic solutions and not regulatory strategies to detect and penalize.

5 Game Theory Approach for Reliability Provision

5.1 Model Formulation

In this section, a new formulation for distribution system reliability provision based on game theory is presented.

It is supposed that the utility offers three level of reliability (Low, Medium, and High) to customers. Each level includes an interval of reliability (D_L, D_M and D_H) and the customers pay for reliability provision cost corresponding to their chosen reliability level. Then, each customer has three possible actions in the game model and we would analyze how the customers’ strategy profiles (a set of all possible actions) affect the distribution system reliability and their costs. The reliability options and their values are presented in Table 9.

According to this table, the maximum interruption time for a high level of reliability should be less than D_{hi}. Furthermore, for a medium level of reliability the utility should provide an outage time between D_{im} to D_{hm} hours at load points and the customers with low chosen reliability option may receive more than D_{il} hours of interruption time.

To create the reliability options similar to Table 9, a utility should study its network to analyze how much cost is needed to provide different levels of reliability in the network. The result of this study would determine reliability intervals and their corresponding costs.

The reliability costs in Table 9 are dependent to the total power demand on the feeder (P_i) and each customer pay for reliability based on the desired chosen reliability level and its power demand as Eq. (15) shows:

\[ C_{rel,i} = p_i \frac{C_{R_{ki}}}{P_i} \]

where \( p_i \) is the power demand at load point \( i \), \( k \) is the customers’ possible actions by which they determine their desire reliability level and willingness to pay for reliability and \( C_{R_k} \) is the reliability provision cost for reliability option \( k \) in Table 9.

In the proposed game model, the customers select their desired reliability level and then the utility would minimize its objective function (payoff) to provide reliability for customers. The customer action and what level of reliability they choose affect both utility and the other customers’ costs. This game that the customers decide first and then the other player (utility) take an action is categorized as a Stakelberg game.

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Nash equilibrium when the utility uses strategy 1 in case 2.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C1,C2,Cutility)</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>4, 4, 2.4</td>
</tr>
<tr>
<td>H</td>
<td>3.4, 4, 1.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 8</th>
<th>Nash equilibrium when the utility uses strategy 2 in case 2.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C1,C2,Cutility)</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>4, 4, 2.4</td>
</tr>
<tr>
<td>H</td>
<td>3.4, 4, 1.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 9</th>
<th>Reliability options by utility.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability Level</td>
<td>High</td>
</tr>
<tr>
<td>Reliability Interval (h)</td>
<td>D_{hi} \leq D_{ni}</td>
</tr>
<tr>
<td>Reliability Cost ($/kW)</td>
<td>C_{R_{ki}}/P_i</td>
</tr>
</tbody>
</table>
The customers and the utility payoff functions is presented in Eqs. (16) and (18). The customer payoff function in Eq. (16) includes four parts: the damage cost, EENS price, the reliability cost according to the chosen reliability level and the utility payment to customer if the minimum reliability criteria, corresponding to the customer chosen reliability level, is violated.

\[ C_i = \sum_p e \cdot D_i \cdot EENS_i(D_i) + C_{rel,i} \]

where \( e \) is the energy price per kWh, \( v_p \) is the penalty factor (per kW) for violation of maximum interruption time in chosen reliability level for each customer and step of \( x \) is a function that returns 1 if \( x > 0 \) and 0 if \( x \leq 0 \). In addition, the customer damage cost in Eq. (16), i.e. CDF at load point \( i \) is considered a quadratic function as

\[ CDF(D_i) = a_i \cdot D_i^2 + b_i \cdot D_i + c_i \] [\$/kWh]

If a customer choose a medium level of reliability according to Table 9 and then the final interruption duration of that customer, which is determined by the utility optimization problem, is greater than \( D_{DM} \), then the utility would pay some money according to last term in Eq. (16) to that customer. According of this term, how much violation of the maximum allowable interruption time of a chosen reliability level is greater, more penalty should be paid to the customer.

The utility objective function which is presented in Eq. (18) includes the investment and operation costs for reliability provision, total EENS of the system, the customers’ payment for reliability and the penalty given to customers for violation of the chosen reliability margin of customers.

\[ C_u = C_{rel,u} + \sum_p e \cdot EENS_i(D_i) + \sum_k C_{rel,k} \]

\[ + \sum_v v_p \cdot CDF_i(D_i - D_{MK}), \quad k \in \{L,M,H\} \] (18)

where \( C_{rel,u} \) represents the total investment and operation cost of reliability provision in distribution system.

### 5.2 Numerical Example

A 3-bus radial feeder, as shown in Fig. 3, is studied in this section in terms of reliability improvement. It is supposed that the utility have three options to invest for reliability provision in the system including switch placement, switch automations and tie line installment. These three are the most relevant tools for reliability enhancement of distribution networks. The capital investment cost for a sectionalizing switch is considered to be $3000. The annual operation and maintenance cost of a switch is considered to be 2% of the capital investment cost [22]. Automation cost of each sectionalizing switch and the capital cost for a tie line installment are $5000 and $25000, respectively. The utility would be allowed to recover the investment costs in 5 years, through a 15% discount rate. Furthermore, the energy price is considered to be $0.3/kWh.

It is supposed that the power demand on the system (\( P_i \)) is equal to 1200 kW. This power is distributed at the load points in order to the customers and utility behaviors are analyzed in different scenarios of simulation. But in the base case, it is assumed that the power demands on the feeder are 200, 300 and 700 kW respectively. In this study, the expected outage time of load points is considered as reliability index.

It is supposed that customers on the feeder are industrial and reliability cost of an industrial customer is constructed by curve fitting based on CDF presented in [18]. The parameters of CDFs and other related parameters for simulation are presented in Table 10.

To run the game, first an analysis is done on the case study to determine the investment costs for each level of reliability. Such an analysis is necessary to the utility’s reliability options is made according to Table 9. For this study, all possible options for reliability provision, i.e. switch placement, switch automations and tie line instalment, are investigated and the minimum cost needed for provision of each level of reliability (SAIDI) is reported in Fig. 4.
Furthermore, in this figure, the equipment needed to satisfy each level of reliability is specified. S and T indicate sectionalizing switch and tie line installations respectively. Sa and Ta indicate that an automated switch or a tie line with automated switch are placed in the system. For example, in lowest level of reliability, the figure shows that by placement of one switch the SAIDI of the system can be improved by 2.58 h in an optimal manner. Based on this figure, reliability margins and the reliability charges (according to Table 9) are obtained and presented in Table 11. \(C_{Rk}\) in the table is obtained supposing that the utility behaviors in distribution system reliability provision, the game model and conventional reliability improvement methods are tested in a 3-bus distribution system.

The models are tested in two different power demand arrangement along the feeder: In first case (CS1), the power demands are 700, 300 and 200 kW from upstream to downstream customers, respectively and in the second case (CS2), the power demands is distributed by 200, 300 and 700 kW in opposite order of CS1. The total power on the feeder (\(P_f\)) in both case is equal to 1200 kW. These arrangements are designed to both heavy and light demands to be located in upstream and downstream parts of the feeder.

In CS1, before any improvements in the system reliability, the imposed costs to customers are equal to $19048, $8163 and $5442, respectively. In addition, the utility’s reliability cost is $1440 which is because of the supplied energy. In such a case, if the utility wants to invest for reliability improvement and do that by minimization of its own cost (\(C_u\)), according to the results in Table 13, the utility’s reliability cost increases to $1733.5 and SAIDI of system decreases from 4 to 2.95 hours. What is obvious in the simulation results is that in both CS1 and CS2 the utility cost minimization (\(C_u\)) tends to give better reliability to

of distribution system i.e. the utility reliability provision cost minimization (\(C_u\)) and the social cost optimization (\(SC\)) are inspected. Also, based on the location of load points, switches positions are constrained.

To test the game model and assess the customers and utility behaviors in distribution system reliability provision, the game model and conventional reliability improvement methods are tested in a 3-bus distribution system.

Two conventional optimization for reliability planning

### Table 11 Reliability options by utility.

<table>
<thead>
<tr>
<th>Reliability Level</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability Interval [h]</td>
<td>(D_1 \leq 1.5)</td>
<td>(1.5 &lt; D_2 \leq 2.5)</td>
<td>(2.5 &lt; D_3 \leq 3.5)</td>
</tr>
<tr>
<td>Reliability Cost [$/kW]</td>
<td>$13844/P_f$</td>
<td>$9368/P_f$</td>
<td>$955/P_f$</td>
</tr>
</tbody>
</table>

### Table 12 Reliability level selected by each customer in case studies.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_{i,\text{action}})</td>
<td>2.5</td>
<td>3.5</td>
</tr>
<tr>
<td>(D_{2,\text{action}})</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>(D_{1,\text{action}})</td>
<td>2.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

### Table 13 The results of simulation.

<table>
<thead>
<tr>
<th>Case Study</th>
<th>CS1: (p_1=700, p_2=300, p_3=200)</th>
<th>CS2: (p_1=200, p_2=300, p_3=700)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Conventional Optimization</td>
<td>Game Approach</td>
</tr>
<tr>
<td>Strategy</td>
<td>(D_i)</td>
<td>(D_{2})</td>
</tr>
<tr>
<td>Configuration</td>
<td>Cu</td>
<td>SC</td>
</tr>
<tr>
<td>(D_{1}) [h]</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>(D_{2}) [h]</td>
<td>4</td>
<td>2.25</td>
</tr>
<tr>
<td>(D_{1}) [h]</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>(SAIDI) [h]</td>
<td>2.95</td>
<td>2.37</td>
</tr>
<tr>
<td>(C_i) [$/h]</td>
<td>5607.5</td>
<td>5607.5</td>
</tr>
<tr>
<td>(C_i) [$/h]</td>
<td>8163.3</td>
<td>4827.6</td>
</tr>
<tr>
<td>(C_i) [$/h]</td>
<td>5442.2</td>
<td>5442.2</td>
</tr>
<tr>
<td>(C_i) [$/h]</td>
<td>1735.5</td>
<td>2551</td>
</tr>
<tr>
<td>(EENS) [kW]</td>
<td>2595</td>
<td>2070</td>
</tr>
</tbody>
</table>
upstream load point (UL) and minimum investment costs occurs in this optimization (in both case the utility invest only for one switch over the system). It can be perceived from Eq. (14), where replacement of the switch closed to head of feeder a higher reliability would be accessible. In this approach, the customers’ costs are neglected. Social costs minimization (SC) considers the customers costs but tends to give better reliability to the more expensive part of the feeder if enough switches and other reliability improvements tools exist in the system. This tendency would be restricted if the utility have not invest much in the system. According to Table 13, the most costly configuration is by SC minimization when the downstream load point (UL) is high demanded. However, the point is that none of the two conventional optimizations consider the customers preferences. If a light load need the high level of reliability SC could not respond to such a request even the utility is equipped with much of instrument for reliability improvements. But, in both cases of simulation, the game approach could follow the customers’ reliability requirements. It means that the game model tends to provide reliability according to customers willingness. The main point is that the simulation results of both cases of CS1 and CS2 confirms that conventional reliability improvement i.e. $C_u$ and SC optimizations are not effective to the customers preferences in distribution system reliability provision is considered. It is because of that reliability for upstream customers can be provided in lower cost. As Eq. (8) shows, improvement of reliability for downstream customer needs to tie switching while upstream customer reliability can be provided by only sectionalizing switch operation. In the other hand, according to this equation, even by tie switching for downstream user, its reliability could not reach to reliability of upstream one that is provided by sectionalizing switch operation. The reliability of customers in UL and DL can be same only in the case that tie switching time is negligible. It deteriorate the condition as decrease in tie switching time means a higher investment cost.

Nash equilibrium of the game in CS1 case is (H, L, H) which is possible with two sectionalizing switches placement and a tie line installed in downstream part of the feeder. In this point, the customers’ costs decrease by 28%, 18% and 15%, respectively. Even the utility makes a net benefit which is not possible in conventional reliability cost optimizations. Such an achievement for the utility is so interesting if it is known that if the utility wants to optimize its own cost function by such equipment (two switches and tie line), $C_u$ would be equal to $\$9872$ which make a great difference. In fact, all participants make benefit in the game model and simultaneously, the customer preference is considered in reliability provision program without any estimation made by regulatory or survey by the utility.

Another important point is that in the utility’s cost optimization method, in both CS1 and Cs2, the UL takes advantage of its location and a considerable decrease in UL’s cost is observable. For example in CS1 and CS2, the UL experience 70% and 41% decrease in its costs, respectively while there is no change in the DL’s costs. This property as explained in other sections may create free riding if the customers’ costs is desirable to be considered in reliability planning. Another point is that the social cost optimization, could not represent a fair mechanism for reliability cost allocations to customers. In CS1, the section with the highest load demand (700kW) would impose approximately similar cost in comparison to the section with light power demand (200 kW) or in CS2, DL’s cost with 700 kW of power demand is smaller than the costs of customer 2 with 300 kW power demand. While in the game approach, in all scenarios the customers experience a reasonable decrease in their costs and simultaneously, the reliability is provided according their preferences. It should be valuable to note that from the utility point of view, SC optimization might impose a high cost to the utility only for the location of customer with high demand. In other words, although the power demand of feeder ($P_d$) in both CS1 and CS2 is equal (1200 kW), the utility’s cost increase considerably from $2531 in CS1 to $12704.5 in CS2. Add to this shortness, what is mentioned about the depression in customers costs which is not happen fairly in SC optimization. But, Game approach overcome much of these weaknesses with a fair cost allocations to all participants and considering their preferences for reliability provision.

In CS2, Nash point is (H, L, M) and the results in game approach is similar to ones in CS1. For example, the model could follow the customers’ reliability preferences and the utility make profit in Nash point in comparison to the case that its own cost function i.e. $C_g$ to be optimized. Consequently, the approach is attractive for both customers and the utility. It should be added that the capability of the utility to follow the different strategy profiles of customers is because of the existence of enough flexibility over the system. Such a flexibility is created by different reliability improvements tools in the system, i.e. switch, tie line and automation. If the system reliability provision cost is restricted, the capability of utility to respond to each desired level of reliability by customers is also limited. As shown in Fig. 5, it is vital that the utility have enough tools, proportional to the customers’ reliability requirements, to be able to respond to the customers’ reliability requests.

In this figure, three levels of reliability requirement are proposed by customers so that all customers make similar requests for High, Medium or Low level of reliability, and then for different possible configurations corresponding to different level of investment cost for reliability improvement, the following equation (AD) is minimized to show how much the utility can be closed
A Game Theory Approach to Distribution System Reliability  … R. Mohammadi and H. Rajabi Mashhadi

\[ \Delta D = \sqrt{\sum_i (D_i - D_{i,req})^2} \]  \hspace{1cm} (19) 

where \( D_{i,req} \) is the requested level of reliability by customer \( i \). As Fig. 5 shows, for higher reliability demand by customers the utility would not satisfy the customers’ requests. For example, for investments under $5000 (two switches with automation option), the utility could follow only the low level requests of customers. By more investment on system, lower distance to the customers’ requirement would be possible. An important point is that such an investment should be performed correspond to the customers reliability values in the system. According to this figure, high investment make no difference for low level requests by customers (red line). In fact, in this case, lower investment give grater interruption times to customers in comparison to the requested level and higher investment provide lower interruption times than what is needed by customers. Consequently, the distance between what is needed and what is provided is similar in both cases. But in later case, the utility’s cost considerably increases. Then, enough flexibility according to customers’ requirements should be provided in the system.

The last point is that based on the formulations in section 4 and the results in Table 13, although SAIDI of the system may have similar value in a same configuration, e.g. Nash point in CS1 and CS2, but the customers’ reliability levels could be different with each other. Consequently, reliability improvement method based on minimization of average reliability value would have weakness to satisfy customers’ preferences. It is clear that based on Eq. (13), if tie switching time is ignored, in such cases, SAIDI will be exactly same; although reliability of load points can be changed.

**6 Conclusions**

In this paper, different methods for distribution system reliability provision are analyzed and their weaknesses and advantages are elaborated. Also, distribution system reliability formulation is developed and accordingly, it was shown that how the conventional reliability planning in distribution systems, which is based on improving social welfare and considering the average reliability indices, may have results that are not satisfactory for customers. Furthermore, the customer’s behavior has been modeled and studied in this paper which rarely had been considered in distribution system reliability researches.

It is shown that according to the relations between the reliability of different points, the proposed game scheme can be applied to the problem of distribution systems’ reliability. In the proposed method, by strategic behaviors which the game model provides, free riding that most approaches are facing to can be handled. The results have shown that the game model could satisfy the customers’ reliability preferences and simultaneously, be beneficial for both customers and utility. It is an important property which encourage the customers and utility to participate in reliability provision of the system.

Finally, the game model is suitably applicable in smart distribution systems which high level of flexibility all over the system meets the high demand of reliability by customers.

**References**


R. Mohammadi was born in Iran. Now he is pursuing the Ph.D. degree in Electrical Engineering at Ferdowsi University of Mashhad, Iran. His areas of interest include distribution system operation and planning, distribution system reliability and smart grid.

H. Rajabi Mashhadi was born in Mashhad, Iran, in 1967. He received the B.Sc. and M.Sc. degrees with honor from the Ferdowsi University of Mashhad, both in Electrical Engineering, and the Ph.D. degree from the Department of Electrical and Computer Engineering of Tehran University, Tehran, Iran, under joint cooperation of Aachen University of Technology, Germany, in 2002. He is as Full Professor of Electrical Engineering at Ferdowsi University of Mashhad. His research interests are power system operation and planning, power system economics, and biological computation.

© 2019 by the authors. Licensee IUST, Tehran, Iran. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution-NonCommercial 4.0 International (CC BY-NC 4.0) license (https://creativecommons.org/licenses/by-nc/4.0/).