Reducing the Sidelobe Level of Reflectarray Antennas Using Phase Perturbation Method

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Abstract: Phase Perturbation Method (PPM) is introduced as a new phase-only synthesis method to design reflectarray antennas so as their sidelobe level is reduced. In this method, only the reflected phase of conventional unit cells are perturbed from their required values. To this end, two approaches namely the conventional Optimization method and newly introduced Phase to Amplitude Approximation (PAA) method are proposed. Finally, a reflectarray antenna is designed and fabricated to have a low sidelobe level and its performance is investigated.

Keywords: Reflectarray Antennas, Phase Perturbation Method, Sidelobe Level Reduction, Phase to Amplitude Approximation, Optimization.

1 Introduction

Reflectarray Antennas (RAAs) are widely investigated and used in recent years. In RAAs, the phases of the reflection coefficient of their unit cells are adjusted so that the radiated wave becomes maximum at a specified direction [1-6]. Reflectarrays are sometimes a replacement for phased arrays and small reflector antennas. Two important challenges of RAAs are their narrow bandwidth and that their Side Lobe Level (SLL) is almost out of control. Reduction of Side Lobe Level (SLL) of the array antennas is of much importance in all applications such as radars and wireless communication. Many references have presented some methods to broaden the bandwidth of RAAs [1, 5, 6]. Also, recently some ones have made efforts to control the SLL of RAAs [7-11].

Two well-known ways to reduce the SLL of an active array of fixed number of radiating elements (transmitters) are using nonuniform excitation of the radiating elements [12-14] and optimally arranging the radiating elements with nonuniform spaces [15-18]. Utilizing these two ways simultaneously, can be another way as well [19].

There are two ways to reduce the SLL of RAAs; using aperiodic unit cells [7] and controlling reflected amplitude of the unit cells [8-11]. In [8], the amplitude is controlled by the power loss in some resistors embedded in the unit cell. In [9], a slot is removed from the ground plane to waste some part of the incident wave. In [10], the cells rotate the polarization of the incident wave so as some of the incident wave is ignored as the cross polarized wave. In [11], the amplitude is controlled by amplifiers placed behind the unit cells. In view of these works, one can infer that the existing methods for controlling amplitude of reflection are either dissipative or complicated.

In this letter, a new method is proposed to reduce the SLL of RAAs without need to dissipative or complicated unit cells, but only using lossless unit cells. In this method, the reflected phases of conventional unit cells are perturbed from their normal values.

2 Phase Perturbation Method

Fig. 1 shows the typical configuration of a RRA in
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which a feed antenna located at the point \((0, y_0, F)\), illuminates a \(D \times D\) aperture consisting of \(N \times N\) square unit cells of dimension \(d_0\). The \(mn\)-th unit cell has a situation whose center is specified by \(x_{mn}\) and \(y_{mn}\). Also, the four following parameters are involved in the problem.

\[
R_0 = \sqrt{y_0^2 + F^2} \tag{1}
\]

\[
R_{mn} = \sqrt{x_{mn}^2 + (y_{mn} - y_0)^2 + F^2} \tag{2}
\]

\[
\theta_f(x, y) = R_f^2 + R_{mn}^2 - (x_{mn} + y_{mn})^2 \over 2R_fR_{mn} \tag{3}
\]

\[
\theta_s(x, y) = \cos^{-1} \left( {F \over R_{mn}} \right) \tag{4}
\]

It is well known that the far field radiation of a RRA in arbitrary direction \((\varphi, \theta)\) on a sphere, can be expressed as follows.

\[
E(\theta, \varphi) = E_0 \cos^\varphi(\theta) \times \sum_{m=1}^{N} \sum_{n=1}^{N} A_{mn} \exp\left\{j(\varphi_{mn} - k_0R_{mn})\right\} \times \exp\left\{j k_0 \sin(\theta)(x_{mn} \cos(\varphi) + y_{mn} \sin(\varphi))\right\} \tag{5}
\]

where \(A_{mn}\) is the amplitude of the \(mn\)-th unit cell as follows.

\[
A_{mn} = \left( { \Gamma_{mn} \over R_{mn} } \right) \cos^\varphi(\theta_f) \cos^\theta(\theta_s) \tag{6}
\]

in which \(\Gamma_{mn} = \exp(\varphi_{mn})\) is the reflection coefficient of the \(mn\)-th cell. Also, \(\varphi_f\) and \(\varphi_s\) are related to transmitting and receiving radiation beams of the feed and the cells, respectively.

As it is seen from (5) and (6), there is no practical way to control the SLL of RAAs because the magnitude of the reflection coefficients are nearly constant, i.e., \(|\Gamma_{mn}| \equiv 1\). Hence, we propose to perturb the phases of the unit cells, i.e. \(\varphi_{mn}\), so that SLL decreases while the direction of maximum radiation, i.e. \((\varphi = \pi/2, \theta = \theta_m)\), is not changed.

The required phases of reflection coefficient of the unit cells for having maximum radiation in the direction \((\varphi = \pi/2, \theta = \theta_m)\) along with a low SLL, is proposed as follows.

\[
\phi_{mn} = (k_0R_{mn} - k_0y_{mn} \sin(\theta_m) + \phi_0 + \delta\varphi_{mn}) \tag{7}
\]

where \(\phi_0\) is an arbitrary constant phase. The value in parenthesis is the same value for conventional design of RAAs [1-3]. Here, \(\delta\varphi_{mn}\) is the added phase perturbation for the \(mn\)-th unit cell, we have proposed to reduce the SLL of designed RAA.

It is worth noting that the introduced Phase Perturbation Method neither waste the incident power of feed, like in [8-10], nor need complex and costly structure, like in [11].

3  Finding Phase Perturbations

Here, two approaches are introduced to find the phase perturbations of the unit cells:

1. Optimization,
2. Phase to Amplitude Approximation (PAA).

In optimization approach, the error function of the optimization process is defined as follows.

\[
\text{Error} = \left| {E_{\text{max}}^d \over E_{\text{max}}} - |\text{SLL}(d)| \right| \tag{8}
\]

where \(SLL(d)\) is the desired SLL. Also, \(|E_{\text{max}}|\) and \(|E_{\text{max}}^d|\) are the main and second maxima of the radiation pattern, respectively. This error function should be subject to a constraint to limit the broadening of 3-dB beamwidth. Here, we have used the “trust-region-reflective” optimization algorithm which is available in MATLAB environment.

In newly introduced PAA approach, one can consider the following first order approximation for small value of \(\delta\varphi_{mn}\).

\[
\exp(j \delta\varphi_{mn}) = \cos(\delta\varphi_{mn}) + j \sin(\delta\varphi_{mn}) \equiv \cos(\delta\varphi_{mn}) \tag{9}
\]

It is understood from (9) that the phase perturbation of each unit cell, can approximately play the role of a part of the amplitude of that unit cell. In fact the term \(\exp(j \delta\varphi_{mn})\) can be considered as \(\cos(\delta\varphi_{mn})\) which its value is less than one. Hence, one can use this approximation \(|\Gamma_{mn}^{(d)}| = \cos(\delta\varphi_{mn}) \equiv \exp(j \delta\varphi_{mn})\) where \(|\Gamma_{mn}^{(d)}|\) is the desired reflection coefficient magnitude of the \(mn\)-th unit cell. Therefore, one can use the following identity

\[
\delta\varphi_{mn} = -\cos^{-1}( |\Gamma_{mn}^{(d)}| ) \tag{10}
\]

The desired reflection coefficient \(|\Gamma_{mn}^{(d)}|\) is related to the desired amplitude of the \(mn\)-th unit cell, \(A_{mn}^{(d)}\), by the following relation obtained from (6).
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\[ |\Gamma^{(d)}_{\text{mn}}| = k \frac{R_{\text{mn}}}{\cos^2(\theta')} \cos(\theta') A_{\text{mn}}^{(d)} \] (11)

in which $k$ is a normalizing constant so that $\max(|\Gamma^{(d)}_{\text{mn}}|) = 1$. The desired amplitude, $A_{\text{mn}}^{(d)}$, could be obtained by any arbitrary synthesis method like the Chebyshev pattern [12], which has been adopted in this work.

4 Design Examples

Here we design a RAA at frequency $f_0=10$ GHz, with length of $D = 225$ mm = $7.5\lambda_0$ consisting of $N \times N = 15 \times 15$ unit cells of dimension $d_0 = 15$ mm = $0.5\lambda_0$. The parameters $q_f$ and $q_e$ are assumed to be equal to one. Hence, a small antenna such as a microstrip patch is used as feed at height of $F = 2D = 450$ mm and $y_f = 0$. The maximum radiation angle is supposed to be $\theta_m = 0$, i.e. boresight.

Figs. 2-4 show the calculated radiation patterns of designed RAA supposing desired SLL to be -16dB, -20dB and -25dB, respectively. These figures have been obtained by exploiting MATLAB MathWork. It is seen that SLL of designed RAAs has been reduced from -13.8dB for $\delta_{\text{mn}} = 0$ to -16dB, -20dB and -25dB, by using phase perturbations obtained by optimization approach. The PAA approach is not able to reduce SLL less than about -20dB. Besides, it is seen that reduction of SLL leads to increase the beamwidth of the main lobe as expected.

The phase perturbation of designed RAA with SLL of -20dB, obtained by optimization and PAA approaches are shown in Figs. 5 and 6, respectively. One sees that although they are not the same but are analogous but somewhat different. In fact, PAA method is suitable for little phase perturbation and consequently for little SLL reduction. The advantage of PAA method is its capability to give us the phase perturbation directly via (10) and (11).

5 Simulation and Fabrication

To have phase distribution according to (7), a configuration called phoenix [4] is chosen for the unit cells as shown in Fig. 7. It contains a metal circle of variable radius $r_3$ lying between two constant circular metals, all of them over an air gap of thickness $h_1=8.0$ mm and an FR4 substrate of thickness $h=1.6$ mm and $\epsilon_r = 4.3$. Also, the radii $r_1$ and $r_2$ are fixed to 1.5mm and 7.0mm, respectively.

Fig. 8 shows the phase of the reflection coefficient of the chosen unit cell versus $r_3$ at frequency 10GHz, obtained by simulation with CST. The total coverage of phase is about 360° which suffices for our designed RAAs.

The designed RAA with SLL of -20dB employing the optimization approach is fabricated and measured as shown in Fig. 9. Fig. 10 shows the resulting radiation pattern of this designed RAA. It is seen that the SLL has been reduced from about -13.2dB to -19.6dB, which is near to our design.
Reducing the Sidelobe Level of Reflectarray Antennas Using Phase Perturbation Method (PPM) was introduced as a new phase-only synthesis method to reduce the sidelobe level of reflectarray antennas. To this end, two approaches namely Optimization and Phase to Amplitude Approximation (PAA) were proposed. The PAA approach is capable to reduce SLL slightly but the optimization approach is capable to reduce SLL significantly. However, the advantage of PAA method is its capability to give us the phase perturbation directly. Finally, a reflectarray antenna was designed and fabricated utilizing phoenix type unit cells to have a low sidelobe level about -20 dB and its performance was investigated.

### 6 Conclusion

Phase Perturbation Method (PPM) was introduced as a new phase-only synthesis method to reduce the sidelobe level of reflectarray antennas. To this end, two approaches namely Optimization and Phase to Amplitude Approximation (PAA) were proposed. The PAA approach is capable to reduce SLL slightly but the optimization approach is capable to reduce SLL significantly. However, the advantage of PAA method is its capability to give us the phase perturbation directly. Finally, a reflectarray antenna was designed and fabricated utilizing phoenix type unit cells to have a low sidelobe level about -20 dB and its performance was investigated.

### References


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