A New Subdomain Method for Performances Computation in Interior Permanent-Magnet (IPM) Machines

A. Jabbari* (C.A.) and F. Dubas**

Abstract: In this research work, an improved two-dimensional semi-analytical subdomain-based method for performance computation in IPM machine considering infinite-/finite-magnetic material permeability in pseudo-Cartesian coordinates by using hyperbolic functions has been presented. In the developed technique, all subdomains are divided into periodic or non-periodic regions with homogeneous or non-homogeneous boundary conditions (BCs), respectively. Taking into account the appropriate interfaces conditions in the presented coordinates system, the machine performances including magnetic flux density, cogging/electromagnetic torque, back-EMF, and self-/mutual induction have been calculated for three distinct values of soft-magnetic material relative permeability (viz., 200, 800 and ∞). The semi-analytical results are compared and confirmed by the FEA results.

Keywords: IPM Machine, Analytical Model, Soft-Magnetic Material Relative Permeability, Pseudo-Cartesian Coordinates, Subdomain-Based Technique.

1 Introduction

INTERIOR permanent magnet (IPM) machines are a good candidate to replace surface-mounted PM machines in high-speed applications. Accurate analytical computation of the electromagnetic characteristics of the IPM machines is important, especially in design and optimization stage. Due to property variation of iron parts in different directions, intrinsic UMF [1-5] and therefore, a local/global saturation of these parts occurs [6-17]. In general, numerical methods are attractive due to their high precision for estimating the electromagnetic performances in IPM machines [18]. However, the main drawback of these methods is their long computing time [19]. The magnetic vector potential calculation in electrical machines can be found in [4-12] and [15-39].

Subdomain based techniques are well known as exact analytical methods, with a very well accord compared to FEA. However, some definite physical and geometrical presumptions are considered in these techniques. The most important presumption in subdomain approaches is that soft-magnetic materials permeability is considered infinite [27-30]. Some works have covered the distinct values of soft-magnetic material relative permeability and the saturation effect in subdomain based approaches. In non-periodic regions due to the property variation of iron parts in different directions, the boundary conditions are considered non-homogeneous. But then, the harmonic modeling approach is used in a few kinds of research to consider the soft-magnetic material permeability [6-11].

To the best of the authors’ knowledge, some analytical models are available to include the distinct iron parts relative permeability and the saturation effect in semi-analytical approaches using the subdomain technique in [6-17]. A subdomain based analytical model was presented in Cartesian coordinates [13] and polar coordinates [14] by resolution of Laplace’s and Poisson’s equations in an air-/iron-cored coil. The iron-pieces relative permeability is considered during the electromagnetic computation of IPM machines [12], wherein; r-edge integrated circuits are realized using Taylor series expansion. Roubache et al. [15] introduced a subdomain method for the prediction of the electromagnetic performance in electrical machines.
with radial magnetization orientation taking into account the distinct iron parts relative permeability.

Therefore, the main contribution of the proposed work is to improve the semi-analytical model presented in [14] for prediction of electromagnetic performances in IPM machines considering the finite soft-magnetic parts relative permeability at no-load and on-load conditions. In the presented method, Maxwell’s equations have been solved in non-periodic regions in pseudo-Cartesian coordinates system taking into account the non-homogeneous boundary conditions. In order to calculate the integration constants, the interface conditions are applied in different directions. All the results of the proposed method are then compared in amplitudes and waveforms to those found by the 2-D FEA.

2 Problem Definition and Assumption

A schematic representation of the studied machine with the following regions is shown in Fig. 1.

- Subdomain I: the rotor shaft;
- Subdomain II: the rotor yoke;
- Subdomain III: the rotor teeth,
- Subdomain IV: the rotor slot;
- Subdomain I: the PMs;
- Subdomain k: the rotor teeth;
- Subdomain V: the rotor teeth;
- Subdomain VI: the air-gap;
- Subdomain VI: the stator teeth;
- Subdomain VII: the stator slot (on the left);
- Subdomain VIII: the stator slot (on the right);
- and Subdomain X: the stator yoke.

The general partial differential equation in an isotopic subdomain can be expressed by

\[ \nabla^2 A = 0 \] in Subdomains I, II, III, k, V, VI, IX, X \hspace{1cm} (1)

\[ \nabla^2 A_z = -\mu_0 \nabla \times M \] in Subdomain I \hspace{1cm} (2)

\[ \nabla^2 A_z = -\mu_0 J \] in Subdomains VII and VIII \hspace{1cm} (3)

where \( \mu_0 \) is the vacuum permeability, \( M \) is the PM magnetization and \( J \) is the current density.

The field vectors \( \mathbf{B} = \{B_r; B_\theta; 0\} \) and \( \mathbf{H} = \{H_r; H_\theta; 0\} \) in all subdomains are paired by the magnetic material equation:

\[ \mathbf{B} = \mu_0 \mathbf{H} \] in Subdomains I, IV, and VI \hspace{1cm} (4)

\[ \mathbf{B} = \mu_0 \mu_m \mathbf{H} + \mu_0 \mu_r \mathbf{M} \] in Subdomain I \hspace{1cm} (5)

\[ \mathbf{B} = \mu_0 \mu_m \mathbf{H} \] in Subdomains II, III, k, V, IX, and X \hspace{1cm} (6)

where \( \mu_m \) and \( \mu_r \) are the relative recoil permeability of PMs and iron pieces, respectively. Using \( \mathbf{B} = \nabla \times \mathbf{A} \) the components of \( \mathbf{B} \) can be concluded by

\[ B_r = \frac{e^i}{R_r} \frac{\partial A_z}{\partial \theta} \] \hspace{1cm} (7)

\[ B_\theta = \frac{e^i}{R_r} \frac{\partial A_z}{\partial t} \] \hspace{1cm} (8)

3 Resolution of Maxwell’s Equations

In this section, firstly, we categorize each subdomain as the periodicity or non-periodicity Subdomain. The resolution of PDE equation in each subdomain has then been performed considering the appropriate BC and IC in order to determine the integration constants.

3.1 Periodic Subdomains

Periodic Subdomains include the stator yoke, the air-gap, the rotor yoke, and the rotor shaft subdomains, with a period interval of 2π. In the periodic Subdomains, the following Laplace’s equation has to be solved:

\[ \frac{\partial^2 A_{\Omega}}{\partial t^2} + \frac{\partial^2 A_{\Omega}}{\partial \theta^2} = 0 \] \hspace{1cm} (9)

where \( \Omega \) is X, VI, II, and I for the Subdomains of the stator yoke, air-gap, rotor yoke and rotor shaft, respectively.
Considering the $\theta \in [0, 2\pi]$ for the machine, (9) becomes

$$
A_{z(t, \theta)} = a_0(t, \theta) + b_0(t, \theta)
$$

$$
+ \sum_{n} a_n(t, \theta) \left( \frac{F(n, t, \theta)}{Sh(n, t, \theta)} + b_n(t, \theta) \left( \frac{F(n, t, \theta)}{Sh(n, t, \theta)} \right) \cos(n\theta) \right)
$$

$$
+ \sum_{n} c_n(t, \theta) \left( \frac{F(n, t, \theta)}{Sh(n, t, \theta)} + d_n(t, \theta) \left( \frac{F(n, t, \theta)}{Sh(n, t, \theta)} \right) \sin(n\theta) \right)
$$

(10)

where $n$ is a positive integer, $F = Sh$, $t \in [t_1, t_2]$ and $b_n(t, \theta) = \delta_n(t, \theta)$ for rotor shaft ($\Omega = I$), $F = Sh$ and $t \in [t_1, t_3]$, for rotor yoke ($\Omega = II$), $F = Sh$, $t \in [t_1, t_5]$, and $b_n(t, \theta) = \delta_n(t, \theta)$ for stator yoke ($\Omega = X$), $F = Ch$ and $t \in [t_1, t_2]$ for air-gap ($\Omega = VI$).

### 3.2 Non-Periodic Subdomains

The stator teeth, the stator slot-opening, the rotor slot, the rotor teeth, the stator slot-right side, the stator slot-left side, and the PM Subdomains are non-periodic Subdomains. Taking into account the Poisson’s equation for the stator slot and the PM Subdomains, Laplace’s equation is considered for the other non-periodic subdomains.

#### 3.2.1 General solution of Laplace’s Equation

Using the principle of superposition [14], the resolution of Laplace’s equation in non-periodic Subdomains is given by:

$$
V_{L(t, \theta)} = h \pi / \Theta
$$

$$
V_{L(t, \theta)} = k \pi / \theta
$$

$$
A_{z(t, \theta)} = a_0(t, \theta) + b_0(t, \theta)
$$

$$
+ \sum_{k, l} a_{k,l}(t, \theta) \left( \frac{F_{k,l}(t, \theta)}{Sh_{k,l}(t, \theta)} \right) \cos(k\pi, \theta) \right)
$$

$$
+ \sum_{k, l} b_{k,l}(t, \theta) \left( \frac{F_{k,l}(t, \theta)}{Sh_{k,l}(t, \theta)} \right) \sin(k\pi, \theta) \right)
$$

(11)

where $t \in [t_1, t_2]$, $\Theta = \Theta$, $\theta \in [\theta, \theta + 2\pi]$, and $\Omega(\Theta) = \Xi(m)$ and $F = Ch$ for $m$-th stator teeth, $t \in [t_1, t_2]$, $\Theta = \Theta$, $\theta \in [\theta, \theta + 2\pi]$, $\Omega(\Theta) = \Xi(m)$ and $F = Sh$ and $\Omega = \Xi(k)$ for $k$-th rotor teeth, $t \in [t_1, t_2]$, $\Theta = \Theta$, $\theta \in [\theta, \theta + 2\pi]$, $\Omega(\Theta) = \Xi(m)$ and $F = Ch$ and $\Omega = \Xi(k)$ for $j$-th rotor slot.

### 3.2.2 Resolution of Poisson’s Equation

#### A. Stator Slot

In the Subdomain VII(m) and the Subdomain VIII(m), we have to solve (12), that is the Poisson’s equations,

$$
\frac{\partial^2 A_{\Omega(t, \theta)}}{\partial t^2} + \frac{\partial^2 A_{\Omega(t, \theta)}}{\partial \theta^2} = -\mu_0 R_e e^{-\theta/j} J_a
$$

(12)

where $J_a$ is the current density.

Using the principle of superposition [14] and the separation of variables method, (12) becomes

$$
A_{z(t, \theta)}(t, \theta) = a_0(t, \theta) + b_0(t, \theta) \frac{1}{4} \mu_0 R_e e^{-\theta/j} P_0
$$

$$
+ \sum_{k, l} a_{k,l}(t, \theta) \left( \frac{F_{k,l}(t, \theta)}{Sh_{k,l}(t, \theta)} \right) \cos(k\pi, \theta) \right)
$$

$$
+ \sum_{k, l} b_{k,l}(t, \theta) \left( \frac{F_{k,l}(t, \theta)}{Sh_{k,l}(t, \theta)} \right) \sin(k\pi, \theta) \right)
$$

(13)

where $\theta \in [\theta, \theta + 2\pi]$ and $\Omega(\theta) = \Xi(m)$ for $m$-th stator slot-right side and $\theta \in [\theta + 2\pi, \theta + 2\pi]$ and $\Omega(\theta) = \Xi(m)$ for $m$-th stator slot-left side.

#### B. Permanent Magnet Subdomain (Subdomain I)

In the Subdomain I(i), the Poisson’s equation can be rewritten as,

$$
\frac{\partial^2 A_{\Omega(t, \theta)}}{\partial t^2} + \frac{\partial^2 A_{\Omega(t, \theta)}}{\partial \theta^2} = \mu_0 M_r e^{-\theta/j} \frac{\partial M_r}{\partial \theta}
$$

(14)

where $M_r$ is radial components of magnetization.

Using the separation of variables method, resolution of (14) for $j$-th PM Subdomain led to

$$
A_{z(t, \theta)}(t, \theta) = a_0(t, \theta) + b_0(t, \theta) \frac{1}{4} \mu_0 R_e e^{-\theta/j} \left( -1 \right) \right) \left( -1 \right) \right)
$$

$$
+ \sum_{k, l} a_{k,l}(t, \theta) \left( \frac{F_{k,l}(t, \theta)}{Sh_{k,l}(t, \theta)} \right) \cos(k\pi, \theta) \right)
$$

$$
+ \sum_{k, l} b_{k,l}(t, \theta) \left( \frac{F_{k,l}(t, \theta)}{Sh_{k,l}(t, \theta)} \right) \sin(k\pi, \theta) \right)
$$

(15)

where $k$ is a positive integer and the coefficients $a_{k,l}(\theta)$, $b_{k,l}(\theta)$, $a_{k,l}(\theta)$, and $b_{k,l}(\theta)$ are determined based on the continuity and ICs.
4 Results and Evaluation

The two-dimensional semi-analytical model for IPM machine considering the distinct iron parts relative permeability is applied to predict the magnetic vector potential components, the back electromotive force, the cogging torque, the electromagnetic torque and the self-/mutual inductances in three distinct machines. The topology of each studied machine is shown in Fig. 2. These machines are powered by a sinusoidal current. The machines’ parameters and geometrical dimensions are listed in Table 1.

Flux lines distribution in three studied machines is presented in Fig. 3. The effect of the three distinct values of soft-magnetic relative permeability (viz., 200, 800 and \(\infty\)) on the performances of the studied machines, are estimated analytically and compared by 2-D FEA. The \(t\)- and \(\theta\)-component of the magnetic flux density waveform in the pitch circle of the air-gap (i.e., Subdomain VI) are calculated with a harmonic number of \(n = 100\) at no-/on-load conditions in M1, M2 and M3 machines as presented in Figs. 4-6, respectively. The analytical estimation of magnetic flux distribution is done taking into account the same relative permeability in all iron parts. It is obvious that a good accordance is obtained for the \(t\)- and \(\theta\)-components of the magnetic flux density.

![Fig. 2 The investigated IPM machines; a) M1, b) M2, and c) M3.](image)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Definition</th>
<th>Value M1</th>
<th>Value M2</th>
<th>Value M3</th>
</tr>
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<tr>
<td>(B_{rm})</td>
<td>T</td>
<td>Remanence flux density of PMs</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu_{rm})</td>
<td></td>
<td>PMs’ relative permeability</td>
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<td></td>
<td></td>
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<tr>
<td>(N_c)</td>
<td></td>
<td>Number of wires per slot</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(I_0)</td>
<td>A</td>
<td>Phase current amplitude</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Q_s)</td>
<td></td>
<td>Stator slots number</td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>deg.</td>
<td>Stator slot-opening</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(p)</td>
<td></td>
<td>Number of pole pairs</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_1)</td>
<td>mm</td>
<td>Radius of stator slots</td>
<td>82.75</td>
<td></td>
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<tr>
<td>(R_2)</td>
<td>mm</td>
<td>Radius of the stator inner surface</td>
<td>38.9</td>
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</tr>
<tr>
<td>(R_{1})</td>
<td>mm</td>
<td>Radius of the rotor outer surface at the PM surface</td>
<td>27.72</td>
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<td>(R_3)</td>
<td>mm</td>
<td>Radius of the rotor inner surface at the PM bottom</td>
<td>15.92</td>
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<tr>
<td>(g)</td>
<td>mm</td>
<td>Air-gap length</td>
<td>1</td>
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<tr>
<td>(L_u)</td>
<td>mm</td>
<td>Axial length</td>
<td>80</td>
<td></td>
<td></td>
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<tr>
<td>(n)</td>
<td>rpm</td>
<td>Mechanical pulse of synchronism</td>
<td>8000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stator/rotor core material</td>
<td>M19-0.5mm</td>
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</tbody>
</table>
Fig. 3 Flux lines distribution in a) M1, b) M2, and c) M3 machines.

Fig. 4 The $t$- and $\theta$- component of the magnetic flux density waveform in the pitch circle of the air-gap (i.e., Subdomain VI) at a) no-load, and b) on-load condition in M1 machine.

Fig. 5 The $t$- and $\theta$- component of the magnetic flux density waveform in the pitch circle of the air-gap (i.e., Subdomain VI) at a) no-load, and b) on-load condition in M2 machine.
The \( x \)- and \( y \)-components of the magnetic flux density waveform in the pitch circle of the air-gap (i.e., Subdomain VI) are calculated with a harmonic number of \( n = 100 \) at no-/on-load conditions in M1, M2 and M3 machines as shown in Figs. 7-9, respectively. It is obvious that a good accordance is obtained for the \( t \)- and \( \theta \)-components of the magnetic flux density.

(a)  
\[ B_x(\theta) \]  
\[ B_y(\theta) \]  
\[ B_z(\theta) \]  
\[ B_r(\theta) \]  
\[ B_t(\theta) \]  

(b)  
\[ R_x(\theta) \]  
\[ R_y(\theta) \]  
\[ R_z(\theta) \]  
\[ R_r(\theta) \]  
\[ R_t(\theta) \]  

Fig. 6 The \( t \)- and \( \theta \)-component of the magnetic flux density waveform in the pitch circle of the air-gap (i.e., Subdomain VI) at a) no-load, and b) on-load condition in M3 machine.

Fig. 7 The \( x \)- and \( y \)-component of the magnetic flux density waveform in the pitch circle of the air-gap at a) no-load, and b) on-load condition in M1 machine.
The $x$- and $y$-components of Maxwell tensor force waveform in the pitch circle of the air-gap at no-/on-load conditions in M1, M2, and M3 machines are computed analytically and compared to those obtained by numerical method for three different soft-magnetic material permeability as presented in Figs. 10-12, respectively. It is obvious that a good accordance is obtained for the $x$- and $y$-components of the Maxwell tensor force.
Fig. 10 The $x$- and $y$-component of Maxwell tensor force waveform in the pitch circle of the air-gap at a) no-load, and b) on-load condition in M1 machine.

Fig. 11 The $x$- and $y$-component of Maxwell tensor force waveform in the pitch circle of the air-gap at a) no-load, and b) on-load condition in M2 machine.

Fig. 12 The $x$- and $y$-component of Maxwell tensor force waveform in the pitch circle of the air-gap at a) no-load, and b) on-load condition in M3 machine.
A waveform comparison of the normal component of Maxwell tensor force in the pitch circle of the air-gap at no-/on-load conditions in M1, M2 and M3 machines are computed analytically and compared to those obtained by numerical method for three different soft-magnetic material permeability as presented in Figs. 13-15, respectively. The obtained results are in close accordance with the results of FEA.

![Fig. 13](image1.png)  
**Fig. 13** The normal component of Maxwell tensor force waveform in the pitch circle of the air-gap at a) no-load, and b) on-load condition in M1 machine.

![Fig. 14](image2.png)  
**Fig. 14** The normal component of Maxwell tensor force waveform in the pitch circle of the air-gap at a) no-load, and b) on-load condition in M2 machine.

![Fig. 15](image3.png)  
**Fig. 15** The normal component of Maxwell tensor force waveform in the pitch circle of the air-gap at a) no-load, and b) on-load condition in M3 machine.

A waveform comparison of Maxwell tensor torque in the pitch circle of the air-gap at no-/on-load conditions in M1, M2 and M3 machines are computed analytically and compared to those obtained by numerical method for three different soft-magnetic material permeability as presented in Figs. 16-18, respectively. The obtained results are in close accordance with the results of FEA.

![Fig. 16](image4.png)  
**Fig. 16** Maxwell tensor torque waveform in the pitch circle of the air-gap at a) no-load, and b) on-load condition in M1 machine.
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7 Conclusions

A subdomain based semi-analytical model was presented in a pseudo-Cartesian coordinates system for the prediction of the no-load/on-load electromagnetic performances in one magnet per pole IPM machines considering the distinct value of relative permeability in periodic/non-periodic subdomains. The general solution of Maxwell’s equations in each periodic/non-periodic region has then been derived by using the principle of superposition. Appropriate BCs at the interface among the different subdomains were defined in two axis (i.e., t- and θ-edges), to compute the integration constants. It is clear that the results of the proposed model are in close accordance with those realized by 2-D FEA.

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Interests: applied mathematics; partial differential equations, separation of variables method; principle of superposition; (semi-)analytical modeling; subdomain technique; magnetic equivalent circuit; electrical machines.

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