

# A New Model Considering Uncertainties for Power Market Simulation in Medium-Term Generation Planning

T. Barforoushi\*, M. P. Moghaddam\*, M. H. Javidi\*\* and M. K. Sheik-El-Eslami\*

**Abstract:** Medium-term modeling of electricity market has essential role in generation expansion planning. On the other hand, uncertainties strongly affect modeling and consequently, strategic analysis of generation firms in the medium term. Therefore, models considering these uncertainties are highly required. Among uncertain variables considered in the medium term generation planning, demand and hydro inflows are of the greatest importance. This paper proposes a new approach for simulating the operation of power market in medium-term, taking into account demand and hydro inflows uncertainties. The demand uncertainty is considered using Monte-Carlo simulations. Standard Deviation over Expected Profit (SDEP) of generation firms based on simulation results is introduced as a new index for analyzing the influence of the demand uncertainty on the behavior of market players. The correlation between capacity share of market players and their SDEP is also demonstrated. The uncertainty of inflow as a stochastic variable is dealt using scenario tree representation. Rational uncertainties as strategic behavior of generation firms, intending to maximize their expected profit, is considered and Nash-Equilibrium is determined using the Cournot model game. Market power mitigation effects through financial bilateral contracts as well as demand elasticity are also investigated. Case studies confirm that this representation of electricity market provides robust decisions and precise information about electricity market for market players which can be used in the generation expansion planning framework.

**Keywords:** Electricity Markets, Game Theory, Medium-term Generation Planning, Monte-Carlo Simulations, Nash-Cournot Equilibrium, Uncertainties.

## 1 Introduction

Power industry has experienced drastic changes in the structure of its markets and regulations during the last three decades. This restructuring process which is essentially a transition from traditional vertical integrated system to competitive framework, has intended to promote competition, mainly in generation sector. In the new situation, former approaches in expansion planning and generation scheduling may not be valid any more. Therefore, new approaches considering different time scales are required to be developed [1]. These time scales range from long-term to short-term decisions. One of the most important decisions to be made by generating companies is

operation planning in medium-term horizon, which typically encompasses a few months up to two years [2]. In this time scale, decisions concerning fuel purchase, hydro resource allocation, bilateral and financial contracting is made. Therefore, to address the medium-term strategic analysis of the generating firms an appropriate modeling which represents the medium-term operation of power system is required. On the other hand, modeling itself is strongly affected by uncertainties which in turn affect strategic analysis as well as expansion planning of generation firms. Therefore, generation firms need suitable decision-support mathematical models which consider not only the operation constraints, but also a variety of uncertainties should be taken into account.

There are several sources for uncertainties affecting medium-term generation planning. Among uncertainties; hydro inflows, fuel prices, system demand, generating units' failures and competence behavior are considered to be more dominant [5]. Some of these including hydro inflows, demand and strategic

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\* Taghi Barforoushi, Mohsen P. Moghaddam and Mohammad. K. Sheik-El-Eslami are with the Department of Electrical Engineering, Tarbiat Modares University, Tehran, Iran.

E-mail: [barforoushi@modares.ac.ir](mailto:barforoushi@modares.ac.ir), [parsa@modares.ac.ir](mailto:parsa@modares.ac.ir), [aleslam@ut.ac.ir](mailto:aleslam@ut.ac.ir)

\*\* Mohammad. H. Javidi is with the Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran.

E-mail: [h-javidi@ferdowsi.um.ac.ir](mailto:h-javidi@ferdowsi.um.ac.ir)

behavior of players have more prominent impacts on operation of electricity market.

J. Villar et al. [1] applied game theory for simulating of hydrothermal power market in a short-term period, i.e. 24 hours. Cournot equilibrium, as the most classical model for considering strategic behavior of the market players has been applied in their study. However, they have only considered strategic behavior of players as an uncertainty. R. Kelman et al. [3] introduced a model based on game theory and Stochastic Dynamic Programming (SDP) with the objective of assessing market power in hydrothermal systems and mitigation effects, e.g. bilateral contracts, in medium-term generation planning. In their approach, inflows have been subjected to uncertainty and were represented by stochastic model as scenarios. Also, hydro units have been considered as strategic units. Furthermore, demand uncertainty has not been considered in their model. J. Barquin et al. [4] proposed a new optimization approach based on equivalent minimization problem for computing of market equilibrium. Later they extended their algorithm to consider inflow uncertainty in a stochastic model [5]. In that research, inflow uncertainty is considered by using of scenario-tree. They represented strategic behavior of players using a conjectured-price-response approach, in which the variation of clearing price with respect to each generating firm's production was assumed to be constant and known. The authors of Ref. [5] have recently published a paper in which they have extended their work to include the demand uncertainty by using of scenario tree [6]. However, because of static nature and theoretical difficulties involved in empirical estimation of conjectural variations, they have no longer been applied for modeling of the electricity market [7]. An iterative algorithm representing the bidding process, which considers market structure and interaction among market participants, is presented in [8]. However, no uncertainty is considered in their work.

In this paper we propose a new approach for simulating the operation of power market in the medium-term, one year period- taking into account hydro inflows as well as demand uncertainties. The main contribution of this work is as follows:

As the electricity demand is the basic variable for operation and expansion planning of power systems, its uncertainty has important roles on the planning results. On the other hand, consideration of the demand uncertainty only in limited number of realization cases may not results in accurate and robust decisions. Therefore, using an approach which considers more realizations of the demands to get accurate results is essential. Monte-Carlo simulations can include more cases for stochastic variables (e.g. demand). Therefore, in this paper Monte-Carlo simulations have been applied to handle the demand as a stochastic variable with normal probability distribution function. Hydro inflow is treated as a stochastic variable which is

represented through scenario-tree. For each realization of the demand, market equilibrium is computed for all scenarios of the inflow considering their probability in a Cournot game model. Therefore, incorporation of both the demand and inflow uncertainties and their handling by Monte-Carlo simulations are the main contributions of the paper. In addition, a new index, so called, Standard Deviation over Expected Profit (SDEP) of generation firms, which can easily be calculated through the simulation results, is introduced for analyzing the influence of the demand uncertainty on the behavior of market players.

In parallel to above uncertainties, rational uncertainty or strategic behavior of generation firms, intending to maximize their expected profit, is considered and Nash-Equilibrium is determined using the Cournot model game. It is obvious that, as the more uncertainties are included in the model for medium-term generation planning, the more accurate information about electricity price and profit will be achieved. The model presented in this paper provides suitable framework to forecast electricity price for market players by taking into account two important categories of stochastic uncertainties in an equilibrium analysis method [11]. Moreover, the proposed model can be used in generation expansion framework.

## 2 Description of the Proposed Model

### 2.1 Basic Assumptions

The basic assumptions of this study are as follows:

- 1) The electricity market is dominated by a few numbers of price-maker producers, i.e. competition paradigm is oligopoly. In this respect, the Nash-Cournot game is used to model the uniform-pricing market.
- 2) The demand is assumed to be a linear function of electricity price.
- 3) Mixtures of hydro and thermal generating units are considered. In addition, location pricing issues resulting from transmission network constraints are not included.
- 4) As most popular studies, the time horizon of the study is assumed to be two years, [2] and [6].

### 2.2 Modelling of Uncertainties in Electricity Market with Hydrothermal System

To incorporate uncertainties in the medium term generation planning, the general framework presented in Fig. 1 is used which is similar to the framework applied in [12]. However, it should be mentioned that the core of that framework is inflows and rational uncertainties. In other words, the authors have focused on the use of the reservoirs in a context of a power exchange market for inters annual regulation and the demand uncertainty has not been considered. As it can be seen in Fig. 1, uncertainties are classified in two major categories, rational and stochastic.

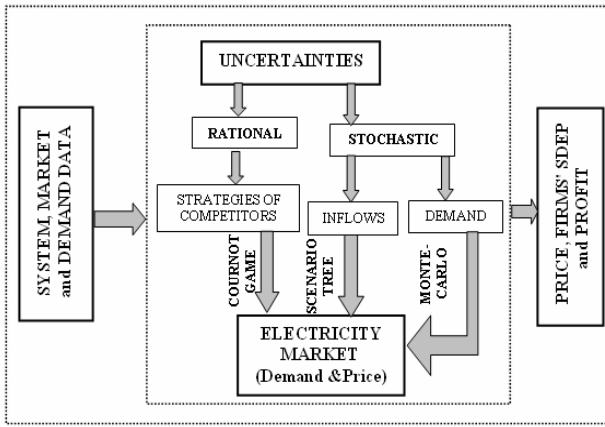


Fig. 1 General Framework of the research.

Rational uncertainties are mainly related to strategic behavior of competitors. Game theory, as an appropriate tool, has been extensively used to analyze the problems of conflict among interacting decision makers [17]. It is considered as a generalization of decision theory to include multiple players or decision makers [18]. The two most popular categories for strategic interaction modeling are Supply Function Equilibrium (SFE) and Cournot strategies. In SFE model, players compete in both quantity and price. Therefore, it is possible for them to link their own bidding prices and quantities in the model [13]. As a result, it can be considered as a more realistic representation of the actual bidding process. However, it doesn't have the flexibility of Cournot model [9]. On the other hand, Cournot model is a more general one for analyzing all types of oligopoly markets and is considered to be a very useful tool to model the strategic behavior of the market players. This model is an upper bound of market equilibrium [1]. In the Cournot model, the competition occurs only in quantities and the product is considered to be homogeneous and not storable. Furthermore, no entry occurs during the game, and firms' operation decisions-making occurs simultaneously [10]. Thus, modeling this type of market equilibrium requires the simultaneous consideration of each firm's profit maximization problem. With starting the game, each player chooses the quantity to produce which maximizes its own profit assuming that the production of the rest of players is constant and known. Electricity price is calculated using the demand curve during any iteration, since a dynamic game is of interest. Therefore, the optimization problems of different generation firms are linked together through the market price resulting from the interaction among them. When no firm can benefit by changing its productions unilaterally, the solution of the game, i.e. Nash-Equilibrium is obtained.

The advantages of the Cournot model are as follows [9]:

- 1) Popularity
- 2) Computational tractability and simplicity
- 3) Flexibility in modeling bilateral contracts and technical limits

- 4) Compatibility with traditional operational planning in power system

Therefore, we have used Cournot model in our simulations. Although, high sensitivity of the Cournot model to the elasticity of the demand curve, especially in low elasticity factor, may result in converging of the solution to an unrealistic equilibrium point, it can be avoided by choosing rational elasticity for the demand. This effect will be reflected in our numerical study.

Stochastic uncertainties are related to economical and environmental variables beyond the control of market players. These uncertainties may be categorized into different classes. In our investigation, we have classified them in two sub-classes:

- I. Those related to hydro units, and
  - II. Those related to the demand.
- I) Inflow uncertainty is considered to be an important one in class I of uncertainties. While it has an important role in operation and expansion planning of hydrothermal systems, it is highly dependent on the weather condition. One of the methods accounting for the uncertainty of inflow is that a set of scenarios are defined and market equilibrium is computed for each scenario separately. Then, average values of respected variables, e.g., electricity price and firms' profit, can be calculated. However, it should be mentioned that accurate results may not be achieved by this method. To obtain more accurate results, one should incorporate the probability of each scenario in different time intervals. Here, we have assumed three different scenarios for hydro inflows including wet, medium and dry for 2<sup>nd</sup> and 3<sup>rd</sup> season with the associated probabilities. Therefore, scenario-tree of the problem can be depicted as Fig. 2 in which W, M and D correspond to Wet, Medium and Dry scenarios, respectively. There are nine states of inflows with respected probabilities due to the independency of the events.

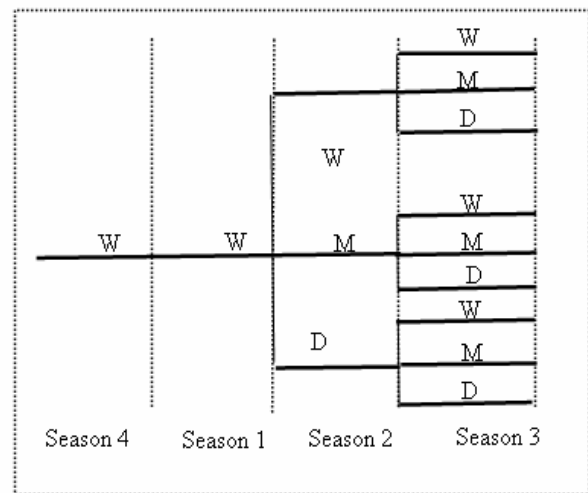


Fig. 2 Representation of the inflow uncertainty by Scenario-tree.

II) Demand uncertainty: According to experiences which have been obtained so far, demand has critical role in operation and expansion planning of power systems. Also uncertainty of the demand has important impact on power system planning results. In our study, uncertainty for each level of the demand is represented by the normal probability distribution function which is specified by mean and standard deviation quantities. The probability density function for the demand can be estimated using historical data.

Monte-Carlo method provides approximate solutions to a variety of mathematical problems by performing statistical sampling experiments on a computer. The method is directly applicable to problems with inherent probabilistic structures. In this method the system must be described by probability density function (pdf's) and necessitates a fast and effective way to generate random numbers normally distributed. The averages and standard deviations of related variables i.e. electricity price and generating firms' profit, are calculated after a definite number of simulations.

### 2.3 Mathematical Formulation of the Proposed Model

As it was mentioned earlier, in this study Cournot game model is applied to simulate the electricity market. In this section, mathematical formulation of the proposed model is presented. Notations and symbols used throughout this paper are stated in Appendix for quick reference.

#### 2.3.1 Firms' Optimization Problem

The objective function of each generation firm is to maximize its profit, i.e. market revenues minus operating cost and CO2 tax for the pollution resulted from generation by the generating units. Consequently, for the whole time horizon, each generation firm is faced with the following optimization problem:

Max:

$$B_e = \sum_{s=1}^{N_s} \sum_{l=1}^{N_l} d_{s,l} \cdot (g_{e,s,l} - q_{e,s,l}) \cdot \pi_{s,l} + \sum_{s=1}^{N_s} \sum_{l=1}^{N_l} d_{s,l} \cdot q_{e,s,l} \cdot \pi'_{s,l} - \sum_{t=1}^{N_{e,t}} v_t \cdot H_t(p_{t,s,l}) - \sum_{t=1}^{N_{e,t}} TR_t \cdot E_t(p_{t,s,l}) \quad (1)$$

S.T.:

$$\sum_{l=1}^{N_l} d_{s,l} \cdot p_{h,s,l} \leq A_{h,s} \text{ for } h = 1, \dots, N_h \quad (2)$$

$$g_{e,s,l} \leq D_{s,l} \quad (3)$$

$$p_{t \min} \leq p_{t,s,l} \leq p_{t \max} \quad (4)$$

$$p_{h \min} \leq p_{h,s,l} \leq p_{h \max} \quad (5)$$

$$g_{e,s,l} = \sum_{t=1}^{N_t} p_{t,s,l} + \sum_{h=1}^{N_{e,h}} p_{h,s,l} \quad (6)$$

The objective function represented by equation (1) represents the profit of the firm in the planning period and is composed of four components. Revenues of the firm in the spot and contractual markets are represented by 1<sup>st</sup> and 2<sup>nd</sup> terms, respectively. The 3<sup>rd</sup> component corresponds to the operational costs of the generation and the 4<sup>th</sup> term represents the CO2-tax related costs. The limitations of available energy for hydro units in each season are represented by equation (2) as constraints. The constraint (3) is incorporated in the problem as demand constraint, because the generation firms are not responsible to meet the whole demand of the market. The constraints (4)-(5) are the boundaries of the decision variables. Equation (6) is an auxiliary constraint which represents the total generation of each firm in each season and load level.

#### 2.3.1 Market Clearing Model

After solving the optimization problem by each firm, electricity price is calculated through equation (7) – a linear price-dependent demand and supply function– for each load level.

$$D_{s,l}(\pi_{s,l}) = -A_{s,l} \cdot \pi_{s,l} + B_{s,l} \quad (7)$$

where  $D_{s,l}(\pi_{s,l})$  is as a function of the associated electricity price in each season and load level. The electricity price  $\pi_{s,l}$  can be calculated by substituting the total power produced by the firms in equation (7). This is due to the fact that balance of demand and supply is essential in the market. Equation (7) also represents the demand elasticity of the market and determines the final market's demand and price. Therefore, the electricity price is affected by the total output of all generation firms in the market. As a result, optimization problems of generation firms are linked together through the market price [19]. As it was mentioned earlier, in this paper, the optimization problems are solved in a Cournot model game and Nash-Equilibrium (i.e. power produced by generation firm and associated electricity price  $\pi_{eq}$  and also firms' expected profit) is calculated. To find the constants ( $A_{s,l}$ ,  $B_{s,l}$ ) of (7), we use reference points ( $D_{base,s,l}$ ,  $\pi_{base,s,l}$ ), i.e. forecasted demand,

reference electricity price for each season, and load level, respectively [1], [6]:

$$A_{s,l} = \varepsilon \cdot \frac{D_{base,s,l}}{\pi_{base,s,l}} \quad (8)$$

$$B_{s,l} = D_{base,s,l} \cdot (1 + \varepsilon) \quad (9)$$

The reference electricity prices  $\pi_{base,s,l}$  are calculated from traditional hydrothermal unit commitment [16].

### 2.3.2 Incorporation of the Inflow and Demand Uncertainty into the Model

As it was mentioned, scenario-tree is used to incorporate the inflow uncertainty into the model. Then, optimization problem (1)-(7) is solved and the Nash-Equilibrium of the game is calculated for each scenario of the inflow. Therefore, the expected profit of each firm and also the expected electricity price can be calculated by equations (10) and (11), respectively.

$$EB_e = \sum_{n=1}^{N_{mf}} p_n \cdot B_e \quad (10)$$

$$E\pi_{eq} = \sum_{n=1}^{N_{mf}} p_n \cdot \pi_{eq} \quad (11)$$

Monte-Carlo simulation is applied to take into considerations the demand uncertainty. In this respect, the demand is assumed to be a stochastic variable with normal probability distribution function. Random sampling of the demand is done and electricity market is simulated for each realization of the demand considering inflow uncertainty. Finally, the average and the standard deviation of the expected profit for generation firms are calculated by equations (12) and (13), respectively.

$$AEB_e = \frac{\sum_{N_{mc}=1}^{N_{mcf}} EB_{e,N_{mc}}}{N_{mcf}} \quad (12)$$

$$SDEB_e = \sqrt{\frac{\sum_{N_{mc}=1}^{N_{mcf}} (EB_{e,N_{mc}} - AEB_e)^2}{N_{mcf}}} \quad (13)$$

Also, expected electricity price and its standard deviation in the planning horizon are calculated by equations (14) and (15), respectively.

$$AE\pi_{eq} = \frac{\sum_{N_{mc}=1}^{N_{mcf}} E\pi_{eq,N_{mc}}}{N_{mcf}} \quad (14)$$

$$SDAE\pi_{eq} = \sqrt{\frac{\sum_{N_{mc}=1}^{N_{mcf}} (E\pi_{eq,N_{mc}} - AE\pi_{eq})^2}{N_{mcf}}} \quad (15)$$

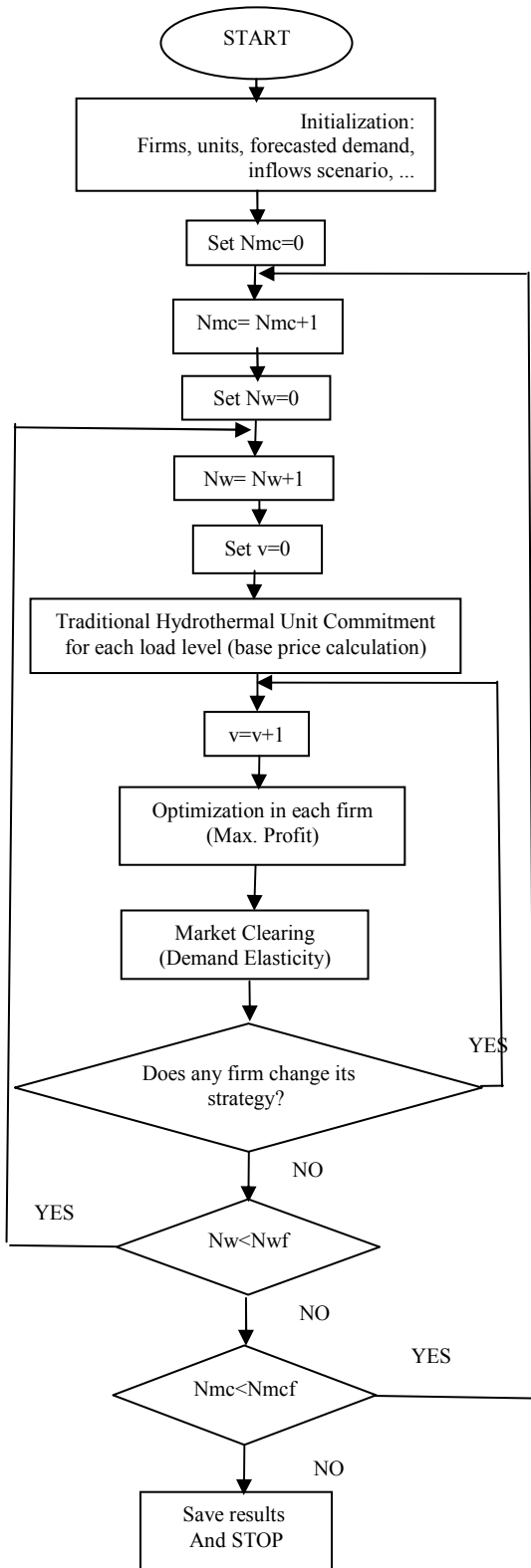
For the sake of clarity, detailed implementation of the proposed model is depicted in Fig. 3. There are three loops in the proposed flowchart. The inner loop corresponds to solving optimization problem in a repeated Cournot game to find Nash-equilibrium point. Average of decision variables i.e. generation levels of units and other market variables such as price and firms' profits are calculated at the end of this loop. The midst loop is associated to inflow uncertainty implementation in the model. It is repeated with the number of states of the scenario-tree. The outset loop is associated to demand uncertainty consideration, i.e. the Monte-Carlo simulation.

### 3 Case Study

The proposed model has been tested using a hydrothermal power system based on the 1996 IEEE RTS [15]. The considered electricity market consists of five price maker generation firms which own 4, 7, 9, 3 and 8 units, respectively. Data regarding the type and ownership of the units is shown in Table 1. Some of the generating unit specifications are detailed in Table 2. The full data required can be found in [15]. Fuel Prices of thermal units have been extracted from [9]. Figure 4 depicts the share of each firm's capacity in the system. The time horizon for the study is assumed to be one year. This scope has been divided into four period representing hydro seasons. Each season is split into three load levels (off-peak, medium and peak) with their corresponding durations. Table 3 shows the data of forecasted demands and their durations in the planning horizon. In order to study influences of different parameters on firms' strategies and electricity price, six scenarios have been defined. These scenarios are summarized in Table 4.

It is assumed that the inflows uncertainties exist only in two seasons, i.e. 2<sup>nd</sup> and 3<sup>rd</sup>. Besides, three different scenarios are considered for hydro inflows in each season. These scenarios are: wet, medium and dry with probabilities 0.1, 0.4, 0.5 and hydro energy in reservoirs of 200, 100, and 70 GWh, respectively.

The results of simulations for different scenarios are summarized in Table 5. This table consists of expected electricity price and expected profits of the firms, as two important variables which are calculated from the Nash-Equilibrium of the game.



**Fig. 3** Detailed Implementation of the proposed model.

Electricity price is the main reference result in any study for an electricity market both for decision and forecast purposes [6].

Profit is also important result for generation firms when planning in the medium-term. In this paper, expected competitive price calculated from traditional

hydrothermal unit commitment and also demand elasticity factor are shown in Table 5. In scenario No. 1 as the base case, the market and competitive electricity price are equal to 30.41 and 28.53 [\$/MWh], respectively, in the absence of hydro energy limitations and uncertainties. The price rise-up with respect to competitive one (i.e. 6.59%) implies the existence of market power. The market power is defined as the ability of market players to unilaterally manipulate electricity price to get further profit which has been also verified through the previous researches [1-3].

The influence of hydro energy limitation on the market variables is studied in scenario No. 2. This scenario is also divided into six sub-scenarios to investigate impacts of demand elasticity on electricity price and exercising market power by the players. Moreover, incorporation of hydro energy limitation is the difference between the first and second scenarios.

The results of simulations in sub-scenario 2-d indicate that limitations of hydro energy, increases the market and competitive electricity prices by 17.99% and 6.91%, respectively. The expected profits of firms with No. 1 to No. 4 are also increased by 9.01%, 8.02%, 5.66%, 6.47%, respectively. On the contrary, the expected profit of firm No. 5 has been decreased by 21.5%, because it has dominant hydro units, i.e. 56.49% of its capacity, which results in less generation level compared with scenario No. 1.

**Table 1** Types and number of units owned by generation firms.

	Generation Firm				
	1	2	3	4	5
Type A	2	1	4	1	-
Type B	-	-	-	-	6
Type C	1	2	2	1	1
Type D	-	2	-	-	-
Type E	1	1	2	-	1
Type F	-	-	1	-	-
Type G	-	-	-	1	-
Type H	-	1	-	-	-
Total	4	7	9	3	8

**Table 2** Capacity and fuel price of generating units.

Type	A	B	C	D	E	F	G	H
Pmax [MW]	12	50	76	100	155	197	350	400
Pmin [MW]	2.4	0.0	15.2	25	54.25	68.95	140	100
Fuel Price [\$/MBTU]	5.27	0.0	1.68	5.27	1.68	5.27	1.68	1.65

**Table 3** Forecasted Load of the system and their durations in each season and load level.

Season	Load level and Duration [MW]&[hrs.]		
	Min (Duration)	Medium (Duration)	Max (Duration)
1	950(876)	1800(985.5)	2300(328.5)
2	1200(876)	1650(985.5)	2370(328.5)
3	1300(766.5)	1900(876)	2500(547.5)
4	1000(876)	1550(985.5)	2250(328.5)

**Table 4** Scenario definition.

Sc. No.	Inflow Constraint	Bilateral contract	Inflow Uncertainty	Co2 Tax	Demand Uncertainty
1	NO	NO	NO	NO	NO
2	YES	NO	NO	NO	NO
3	YES	YES	NO	NO	NO
4	YES	YES	YES	NO	NO
5	YES	YES	YES	YES	NO
6	YES	YES	YES	YES	YES

As it was mentioned above, in order to investigate the impact of elasticity of the demand on electricity price, six sub-scenarios were defined. The electricity price and firms' expected profit have been increased extremely in scenario No. 2-a with respect to scenario No. 2-d. For instance, the electricity price in scenario No. 2-a is 4.07 times of its value in the scenario No. 2-d. This raising of market price with respect to competitive price means exercising of market power by players. Decreasing of electricity price resulting from increasing of elasticity factor is illustrated in sub-scenario No. 2-b to scenario No. 2-f. The more elasticity factor, the less market power and the less difference between market and competitive price will be observed. Figure 5 depicts the variation of expected price versus demand elasticity, which shows an approximate exponential relation. Therefore, the results are very sensitive to the elasticity and market demand when Cournot game is applied for modeling strategic interaction among market players. Specifically, electricity price tends to be very high and uncertain in actual power market in which the elasticity is very low.

One of the popular methods for market power mitigation is facilitating bilateral contracts by regulatory measures. This approach is modeled by pre-purchasing 10% of installed system capacity with a 25[\$/MWh] constant price by customers in scenario No. 3. Simulation results show the market price decreases by 23.72% compared with scenario No. 2. The profits of generation firms are also decreased significantly. In addition to mitigate market power in short and medium-term, bilateral contracts can affect the investment behavior of market participants in long-term. Therefore, from the market stability point of view, they are effective and important measures [14].

In scenario No. 4, in which inflow uncertainty is considered, both expected value of the competitive price and the market price increase by 20.90% and 19.0%, respectively as compared with scenario No. 3. In addition, the profits of firm No. 1 to firm No. 5 also increase by 25.34%, 24.59%, 22.42%, 25.85% and 5.80%, respectively. The growth of profit of firm No.5 is less than its rivals. The reason is that it is the only firm having hydro units, being affected by the inflow uncertainty.

Inclusion of CO2-tax in scenario No. 5 decreases the expected profits of generation firms, but not on the

electricity price and firms' strategy. In this simulation the CO2-tax rate was assumed to be 0.01[\$/lbs Co2] and units' emission data were selected from [15].

The effects of demand uncertainty on electricity price and firms' profits are investigated in scenario No. 6, where Monte-Carlo simulations have been used. The forecasted demand is described as a normal probability distribution function with specified average and standard deviation. The mean values of the forecasted demand are those mentioned in Table 3 and with standard deviation is assumed to be 5%. Then, market simulations have been conducted for each realization of the demand which is selected through random sampling from its probability distribution function.

In order to analyze the impact of demand uncertainty on firms' profits, we introduce Standard Deviation over Expected Profit (SDEP) of generation firms as a new index. This has been chosen because of the fact that there is a good correlation between Standard deviation of any parameter with its uncertainty. On the other hand, to normalize this quantity, it has been divided to its expected value. This index is calculated through equation (13).

$$SDEP_e = \frac{\sqrt{\frac{\sum_{N_{mc}=1}^{N_{mef}} (EB_{e,N_{mc}} - AEB_e)^2}{N_{mef}}}}{\frac{\sum_{N_{mc}=1}^{N_{mef}} EB_{e,N_{mc}}}{N_{mef}}} \quad (13)$$

Reduction of the average firms' profit with respect to scenario No. 5, and also their SDEP are presented in Table 6. Among all thermal generation firms (i.e. firm No. 1 to firm No. 4), those which have less capacity share; have experienced more reduction of their expected profit than the others. Therefore, it seems that there is a negative correlation between capacity share and expected profit reduction of the mentioned firms. Regarding to this assumption, the correlation coefficient between two variables has been obtained equal to -0.8773, which shows approximately high negative correlation between these variables. A similar result has been obtained for the SDEP and the capacity share of all thermal generation firms. In other words, these firms have experienced more profit fluctuations than the others. The correlation coefficient between these variables has been calculated equal to -0.8446, which also shows relatively high negative correlation. Thus, this is in consistent with the fact that in a market consisting of all thermal units, the more capacity share of a firm, the less fluctuation of its profit will happen. However, this is not applicable for firm No. 5 with dominant hydro units. This means that the demand

uncertainty has less effect on this firm's profit. The variations of expected electricity price, firms' profit in the Monte-Carlo simulations and expected electricity price in different scenarios are depicted in figures 6 – 8, respectively.

Consequently, the results of the case study confirm that incorporating of the demand uncertainty in medium-term generation planning is very useful and can provide more information about firms' profit and electricity price and increases robustness of decisions. Finally, the obtained results can be used directly by firms in their generation expansion framework as a decision- support.

**Table 5** Results obtained by simulations for each scenario.

Scenario No.	Firms' Profit [M\$]					Expected Price [\$/MWh]	Expected Competitive Price [\$/MWh]	Demand Elasticity Factor ( $\epsilon$ )	
	1	2	3	4	5				
1	38.06	126.58	89.03	75.74	109.12	30.41	28.53	1.00	
2	a	113.09	242.77	185.24	161.89	177.61	30.50	0.10	
	b	70.16	182.40	134.85	116.59	122.44		0.25	
	c	48.72	144.60	101.96	87.96	94.15		0.50	
	d	41.49	136.73	94.07	80.64	86.37		1.00	
	e	40.41	136.24	93.16	79.95	85.64		1.20	
	f	37.43	132.42	88.82	75.91	81.93		2.00	
3	23.32	76.31	57.85	45.26	53.85	27.37	21.05	1.00	
4	29.23	95.08	70.82	56.96	56.98	33.09	25.05		
5	2.88	59.01	18.95	15.09	34.26	33.09			
6	1.93(3.93)*	55.90(11.23)	16.59(7.25)	13.13(7.33)	31.35(5.68)	32.47(2.61)			24.17(2.09)

\*The numbers in parentheses are standard deviation of the respected variable

Besides, an approximate exponential relation between elasticity and equilibrium price was observed.

It was shown that bilateral contracts mitigate the market power exertion. Under inflow uncertainty hydrothermal firms can benefit less than firms having only thermal units.

Although the CO<sub>2</sub>-tax regulations didn't affect firms' strategies and market price, but because of its influence on firms' profit in the mid-term, it can affect their expansion strategies. Monte-Carlo simulations showed that all thermal firms owning low capacity share, have been more affected by demand uncertainty as compared with others. Besides, generating firms can learn to know more about their situations in the uncertain environments and therefore can have robust decisions and precise information about the market.

More aspects of demand uncertainties, especially load spikes in developing countries, can be added to the proposed model. Reliability issues such as units' failure and also maintenance outage rate can be included in the proposed model. Finally, the proposed model in this paper which has provided a comprehensive analysis of uncertainties' effects on medium term generation planning can be used in generation expansion framework. Further research is going on to use the

#### 4 Conclusions

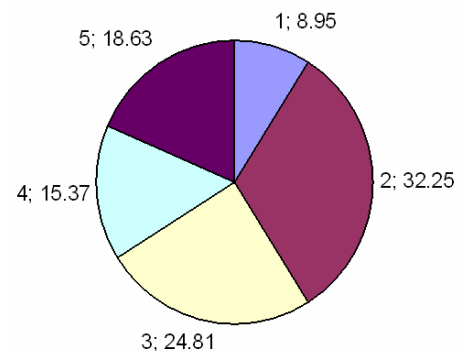
The power market simulating model presented in this paper includes the combination of inflows and demand uncertainties which play important role in medium term generation planning in competitive market environment. A framework comprising Cournot game, scenario-tree and Monte-Carlo simulations were presented to deal with uncertainties. Simulations results confirm that limitation in hydro inflows increases the electricity price, thus, all thermal firms profit and hydrothermal firms loose from such a situation.

The elasticity of the demand has extreme effect on the equilibrium price and market power exertion by players.

proposed model in a new framework for dynamic of investments in generation expansion planning.

**Table 6** comparing the result of Scenario No. 6 with No. 5. (Firms' profit reduction and SDEP)

Generation Firm	Capacity share (%)	Profit Reduction with respect to sc. 5 (%)	SDEP
1	8.95	32.98	2.04
2	32.25	5.27	0.20
3	24.81	12.45	0.44
4	15.37	12.98	0.56
5	18.63	8.49	0.18



**Fig. 4** Capacity share of generation firms.



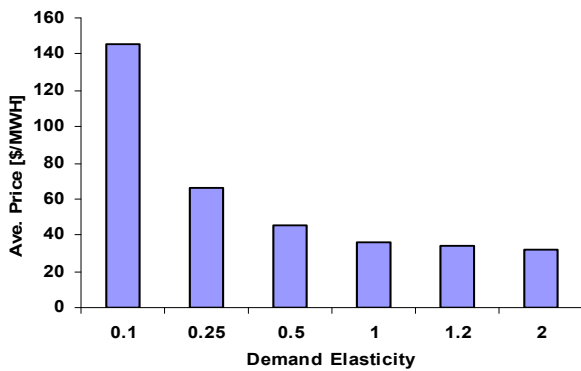


Fig. 5 Variation of electricity price versus demand elasticity.

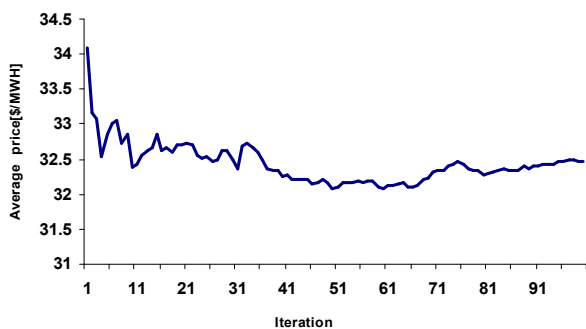


Fig. 6 Expected electricity price resulted in Monte Carlo simulations.

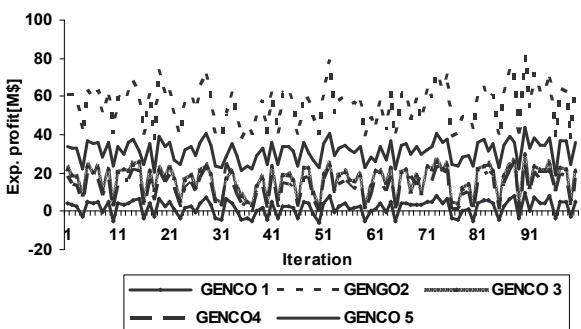


Fig. 7 Expected profit of generation firms in Monte Carlo simulations.

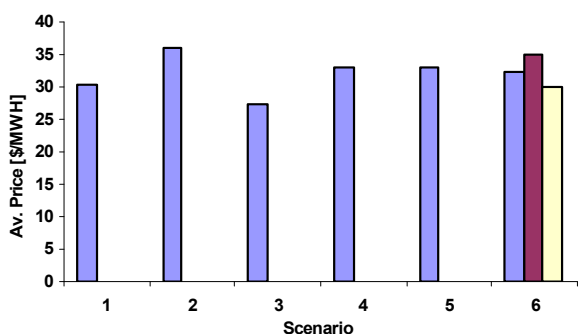


Fig. 8 Expected electricity price for different scenarios.

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## Appendix (I)

### Nomenclature

#### A. Indices

e:	Generation firms
t:	Thermal units
h:	Hydro units
s:	Seasons
l:	Load levels

#### B. Parameters

$A_{s,l}$ :	Slope of demand function in sl [MW/\$/MWh]
$B_{s,l}$ :	Total demand at zero price [MW]
$\epsilon$ :	Demand elasticity factor
$A_{h,s}$ :	Hydro energy reserve of hydro unit h at the beginning of season s [MWh]
$D_{s,l}$ :	Total Demand in sl [MW]
$d_{s,l}$ :	Duration of sl [hrs.]
$H_t$ :	Heat rate function of thermal unit t [MBtu/h]
$N_s$ :	Number of seasons
$N_l$ :	Number of load level in each season
$N_{e,t}$ :	Number of thermal units belonging to firm e
$N_{e,h}$ :	Number of hydro units belonging to firm e
$P_{tmin}$ :	Min. generation capacity of thermal unit t [MW]

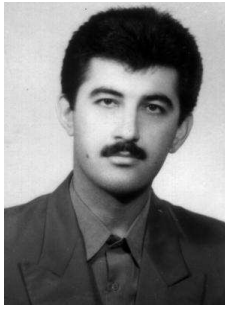
$P_{tmax}$ :	Max. generation capacity of thermal unit t [MW]
$P_{hmin}$ :	Min. generation capacity of hydro unit h [MW]
$P_{hmax}$ :	Max. generation capacity of hydro unit h [MW]
Be:	Profit of firm e in planning horizon [\$]
$q_{e,s,l}$ :	Total power generation contracted by firm e in sl [MW]
$v_t$ :	Fuel price of thermal unit t [\$/MWh]
$TR_t$ :	CO2-tax rate of thermal unit t [\$/lbs CO <sub>2</sub> ]
$\pi'_{s,l}$ :	Contracted electricity price in sl [\$/MWh]
$N_{wf}$ :	Number of inflow scenarios
$p_n$ :	Probability of inflow scenario n
EB <sub>e</sub> :	Expected profit of firm e resulting from to inflow uncertainty [\$]
EB <sub>e,N<sub>mc</sub></sub> :	Expected profit of firm e in $N_{mc}$ th iteration of Monte-Carlo simulation [\$]
SDEB <sub>e</sub> :	Standard deviation of expected profit of firm e due to demand uncertainty [\$]
AEB <sub>e</sub> :	Average of expected profit of firm e due to demand uncertainty [\$]
$N_{mcf}$ :	Number of simulations in Monte-Carlo
$D_{base,s,l}$ :	Forecasted demand in sl
$\Pi_{base,s,l}$ :	Competitive electricity price in sl

### C. Decision Variables

$p_{t,s,l}$ :	Power generation of thermal unit t in sl [MW]
$p_{h,s,l}$ :	Power generation by hydro plant h in sl [MW]

### D. Auxiliary Variables

$g_{e,s,l}$ :	Total power generation of firm e in sl [MW]
$\pi_{s,l}$ :	Electricity price in sl [\$/MWh]
$\pi_{eq}$ :	Equilibrium electricity price in the planning horizon [\$/MWh]
$E\pi_{eq}$ :	Expected equilibrium electricity price in the planning horizon [\$/MWh]
$AE\pi_{eq}$ :	Average of expected equilibrium electricity price in the planning horizon [\$/MWh]
$SDAE\pi_{eq}$ :	Standard deviation of the average of expected electricity price in the planning horizon [\$/MWh]



**Taghi Barforoushi** received the B.Sc. degree in Electrical Engineering from Iran University of Science and Technology in 1994, the M.Sc. degree from Tarbiat Modares University in 1999. He is currently pursuing the Ph.D. degree from Tarbiat Modares University.

He is the manager of R&D unit of the Mazandaran Regional Electric Company. His research interests are power market simulation, generation and transmission expansion planning.



**Mohsen Parsa Moghaddam** received the B.Sc. degree in electrical engineering from Sharif University of Technology, Tehran, Iran, in 1980, the M.Sc. degree from Toyohashi University of Technology, Japan, in 1985, and the Ph.D. degree from Tohoku University, Japan, in 1988. Currently, he is an

Associate Professor in the Department of Electrical Engineering, Tarbiat Modares University, Tehran. His research interests include power system planning and control, optimization, and restructuring.



**Mohammad Hossein Javidi** received his B.Sc. degree from Tehran University, Tehran, Iran in 1980, M.Sc. degree from Nagoya University, Nagoya, Japan in 1985 and Ph.D. degree from McGill University, Montreal, Canada in 1994 all in Electrical Engineering.

He is currently an associate professor in the Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran. He is a board member as well as the secretary of the electricity regulatory board in Iran. His research interests include power system operation and planning, restructuring and market design, and Artificial Intelligence.



**Mohammad Kazem Sheikh-El-Eslami** received the B.Sc. degree in electrical engineering in 1992 from Tehran University, Tehran, Iran, and both the M.Sc. and Ph.D. degrees in 2001 and 2005, respectively, from Tarbiat Modares University, Tehran. His research interests include power market simulation and generation

expansion planning.