Direct Torque Control of 5-Phase 10/8 Switched Reluctance Motors

M. R. Feyzi* and Y. Ebrahimi*

Abstract: A switched Reluctance motor (SRM) has several desirable features, including simple construction, high reliability and low cost. However, it suffers from large torque ripple, highly non-uniform torque output and magnetization characteristics and large noise. Several studies have succeeded in torque ripple reduction for SRM using Direct Torque Control (DTC) technique. DTC method has many advantages over conventional voltage control and current chopping mode control such as simple algorithm, less torque ripple and instantaneous response to the torque command. In this paper, DTC method is proposed for a 5-phase 10/8 SRM. The performance of the motor is demonstrated through the computer simulation in Matlab/Simulink. Then, the obtained results are verified by comparison with the corresponding results of a 3-phase 6/4 motor performance.

Keywords: Direct Torque Control, Electrical Drives, Switched Reluctance Motor, Torque Ripple.

1 Introduction
A switched Reluctance motor (SRM) has numerous advantages over other types of AC machines due to its simple and solid construction. The rotor has a simple laminated structure with neither permanent magnets nor rotor windings, and rotates by using the reluctance torque produced from magnetic saliency between stator poles and rotor poles. Thus, SRM can operate at high speed and is suitable for applications in high-temperature and hazardous environments. In addition, SRM has similar enough high efficiency as compared with permanent magnet motors because it also not need secondary windings. SRM is now accepted for some applications. However, SRM has been limited to be used because it produces larger torque ripples and larger noise due to non-uniform torque output characteristics and doubly salient structure. Additionally, the highly nonlinear magnetic characteristics of the motor make the control of the motor intricate. This is further complicated by the interaction due to mutual coupling of motor phases and parameters variation of the inductance characteristics [1], [2].

To overcome these problems various studies have been performed. The previous works have fallen in to two main groups: those which used the linear model of motor without taking into account the saturation [3-6] and those which used the nonlinear model which takes into account the motor saturation [7-9]. Taylor [3] used an analytical solution in order to produce a voltage profile that provides a smooth motor torque [4]; they assumed a linear torque characteristic for the motor. In their works, Matsui and Ma [5], [6] expressed the inductance of the motor as a simple sinusoidal function varying with position. Highly simplified models of a motor with the highly nonlinear torque and magnetization characteristics cause inaccuracy in most practical drives in the references above.

Ilić-Spong et al. [7] proposed a feedback linearising control method compensating the magnetic nonlinearities of the motor. This method needs exact knowledge of the motor parameters, so it is difficult to be implemented as a practical drive. In order to obtain a smooth torque [8], [9] generated such a current profile that enables the torque sharing between two phases. However, it requires a complex current waveform that imposes a time consuming computation making the control system too complicated for real-time implementation.

In general, most implicit studies utilize the voltage...
or current command profile in order to torque, speed or position control or torque ripple reduction. Some studies have succeeded in torque ripple reduction of SR motors using DTC method [10]. This method used the concept of short flux pattern that links two separate poles of the stator. This needed a new type of winding configuration that is a dominant disadvantage, because changing of the motor winding configuration is both expensive and inconvenient.

An excellent study of DTC method applied to 3-phase 6/4 SRM with closest concept to the conventional DTC of ac machines was fulfilled in [11]. Authors introduced comparable theory with conventional DTC of ac machines by using motor flux and torque defining equations that will be mentioned summarily in the following sections. In this method no winding configuration changing is required, thus the scheme can be conveniently implemented on any normal type of SRM drive.

Since torque ripple minimization is one of the main objectives of DTC method, it seems that applying this method to a SRM with more pole numbers can intensify the torque ripple reduction. Motors with a low phase number have some disadvantages that notably increase torque ripple [12]. On the other hand, pole number increment imposes some disadvantages on the other performance criteria that should be noticed. If in two SRM with different pole numbers, the iron to air ratio and excitation ampere-turn assumed to be constant, in order to achieve high inductance and flux density, the motor with the fewer pole number is preferable.

In present study, DTC method is applied to a 5-phase 10/8 SRM, where a significant modification should be exerted on mentioned method in [11]. This occurs due to discrepancy of poles and phases number of two motors.

2 Main Concepts of DTC Method in 5-Phase SRM

The paper proposed in [11] is a comprehensive reference to perceive DTC scheme in 3-phase 6/4 SRM. Detailed illustrations have been proposed about the electromagnetic torque extraction by using the energy concept instead of co-energy, the stator flux calculation, the voltage space vectors definition and their relation with motor torque and flux control. The power converter configurations, its operation in different states of the switching elements and the voltage space vectors formation then have been presented for 3-phase 6/4 SRM. In the present paper mentioned concepts are expanded for a 5-phase 10/8 SRM summarily.

2.1 Flux Equation and Stator Flux Vector

It is essential to study the machine defining equations in order to extracting of DTC method. The stator flux linkage vector can be expressed as

\[ \phi_s = \int (v_s - R_s i_s) + \phi_{sw} \]  

(1)

where \( \phi_s \) = stator flux vector component, \( \dot{v}_s \) = stator voltage vector component, \( i_s \) = stator current vector component, \( R_s \) = stator resistance and \( \phi_{sw} \) = initial value of the stator flux vector. Except at the low voltage levels, the voltage drop through the stator resistance is dispensable, so from Eq. (1):

\[ \frac{d\phi_s}{dt} \approx \dot{v}_s \]  

(2)

If the time interval is small enough, then (2) can be written as

\[ \Delta \phi_s = \dot{v}_s \Delta t \]  

(3)

This shows that, in order to control of the variation of the stator flux, the voltage space vectors can be utilized. The applied voltage vector produces a stator flux variation which is in the same direction as the voltage vector. Magnitude of the variation is proportional to the voltage vector magnitude and \( \Delta t \). As a result, one of the two basic principles of DTC in SRM is that "The stator flux linkage vector of the motor is kept at constant amplitude by using Eq. (3)".

Selection of the stator flux optimal level for various speeds of the motor has been presented in [13]. In the SRM the voltages are highly non-sinusoidal, so the individual flux linkage of each phase should be firstly found by using Eq. (1). The magnitude of phase flux linkage is variable with time, but its direction is along the related phase stator pole axis. Thus there are five stationary pulsating phase flux vectors in 5-phase 10/8 SRM. To derive the unique stator flux vector, the phases flux vectors are transformed onto a stationary orthogonal two axis \( \alpha - \beta \) reference frame as shown in Fig. 1.

It is supposed that the phase a axis and \( \alpha \) axis are in the same direction, so

\[ \varphi_a = \varphi_1 + \varphi_2 \cos 72^\circ - \varphi_3 \cos 36^\circ - \varphi_4 \cos 36^\circ + \varphi_5 \cos 72^\circ \]

\[ \varphi_b = \varphi_2 \sin 72^\circ + \varphi_3 \sin 36^\circ - \varphi_4 \sin 36^\circ - \varphi_5 \sin 72^\circ \]  

(4)

![Fig. 1. Transformation of phases fluxes on two axis orthogonal stationary reference frame](image)
The magnitude $\phi_s$ and angle $\delta$ of obtained flux vector are

$$\phi_s = \sqrt{\phi_s^2 + \phi_y^2}, \quad \delta = \arctan\left(\frac{\phi_y}{\phi_s}\right)$$

(5)

### 2.2 Motor Torque Equation

Extraction of motor output torque equation has been illustrated in [11] that has defined the motor output torque as a function of system energy instead of co-energy as

$$T = i \frac{\partial \phi(0, i)}{\partial \theta} - \frac{\partial v_{in}}{\partial \theta}$$

(6)

The SRM is usually operated at heavy magnetic saturation in order to achieve a large ratio of torque to mass. It was shown in [14] that contribution of the second term in Eq. (6) is dispensable due to saturation, so

$$T = i \frac{\partial \phi(0, i)}{\partial \theta}$$

(7)

On the other hand relation of the stator current and flux in the $s$ (Laplace) domain can be derived as

$$i = e^{-s\omega} \frac{d\phi}{dt}$$

(8)

where $\omega = \frac{d\theta}{dt}$ and $1 = \frac{d\phi}{dt}$. It can be seen that the stator current has a first order delay relative to $\frac{d\phi}{d\theta}$, thus it can be supposed that the stator current is nearly constant during $\frac{d\phi}{d\theta}$ variations. This means that the motor torque control can be based only on $\frac{d\phi}{d\theta}$ variation. Consequently the second basic principle of DTC in SRM is that "The motor torque is controlled by positive and negative variation of $\frac{d\phi}{d\theta}$ or in other words, by accelerating and decelerating of the stator flux vector".

The phases of the SRM are exited independently, thus total torque of motor is the sum of the five phases torques. As a result the individual torque of each phase $T_j(t, i, \theta)$ should firstly be obtained by using look-up tables. After extracting each phase torque, total torque of the motor is expressed as

$$T_m = \sum_{j=1}^{n} T_j$$

(9)

where $n$ is the number of phases.

### 2.3 Power Converter Configuration

The circuit most commonly used for switched reluctance drives is the asymmetric half bridge circuit Fig. 2. With this configuration there is a separate half bridge for each phase, so the voltages and currents supplied to each phase are completely independent of all other phases. It is necessary to control of current in one phase and force demagnetizing to some other phases of the motor at the same time. This is essential for reduction of the torque ripple. Phase current in asymmetrical torque Ripple. Phase current in asymmetrical half bridge converter is controlled by selecting from three possible states:

i) Both switches in a phase leg are on, and phase is energized from power supply (magnetizing stage).

ii) Both switches in a phase leg are off. Phase current commutates to the diodes and decays rapidly (demagnetizing stage).

iii) Only one of the switches is off. The voltage across winding is near zero and phase current decays slowly (free wheeling).

Phase voltage state $S_j$ can be defined as

$$S_j = \begin{cases} 
1 & \text{both switches in a phase leg are on} \\
-1 & \text{both switches in a phase leg are off} \\
0 & \text{one switch is on and another is off}
\end{cases}$$

(10)

So according to Fig. 2 when phase voltage state is 1, 0 or -1 the positive voltage of dc link, a zero voltage or the negative voltage of dc link is applied to phase respectively. It should be noticed that there are 5 legs in the power converter of 10/8 SRM.

### 2.4 Voltage Space Vectors

When the positive voltage of inverter dc link is applied to the phase winding, a voltage space vector can be defined in the related phase pole axis direction (e.g. vector Phase 1(+) in Fig. 3). By applying negative voltage same vector is definable but in the reverse direction (e.g. vector Phase 1(-)). Now with proper arrangement of the phases voltage states, the voltage space vectors $v_i(i=1,2,...,10)$ suitable for motor torque and flux control can be derived as shown in Fig. 3. As seen, only 10 vectors are selected from 35 possible vectors. Each vector lies in the center of decuple sectors with $2\pi/10$ radians width. Another vector $v_{10}$ is definable when a zero voltage is applied to all five phases, in other words, $v_{10}$ is achieved when each leg of the power converter is in the zero voltage state. This vector is utilized when it is not required to apply simultaneous changes in the torque and the flux.

### 2.5 Switching Table

As mentioned above, basic principles of DTC method in SRM are:

1) The stator flux linkage vector of the motor is kept at constant amplitude by using Eq. (3).

2) The motor torque is controlled by positive and negative variation of $\frac{d\phi}{d\theta}$ or in other words, by accelerating and decelerating of the stator flux vector.

The stator flux and the motor total torque levels are controlled within two separate hysteresis bands, so when the instantaneous torque and flux cross the related band limits, they need to increase or decrease in order to
remain inside the bands. To remove these requirements, the voltage space vectors and two mentioned basic principles of DTC method in SRM are utilized. Table 1 shows the switching table and required voltage space vector selection in each situation, where \( \hat{T} = \) Required variation in torque, \( \hat{\phi} = \) Required variation in flux, \( N = \) Stator flux sector, \( NL = \) Negative Large, \( NS = \) Negative Small, \( ZE = \) Zero, \( PS = \) Positive Small and \( PL = \) Positive Large. As an example when \( \hat{T} = NL \) and \( \hat{\phi} = NS \), it means that motor torque needs to large reduction and stator flux needs to small reduction, so vector \( v_{NL} \) will be able to respond to the system requirement. \( v_0 \) is a vector that has no influence on the stator flux, and it causes the motor torque to reduce very gently. The vectors in circlets are not completely able to comply with the simultaneous requests of torque and flux, but they are the best available vectors. As an example for \( N=1, \hat{T} = NS \) and \( \hat{\phi} = NL \), vector \( v_4 \) can not satisfy the system requirements that are small reduction in torque and large reduction in flux. Motor torque ripple minimization is one of the main aims of DTC method in SRM, so responding to torque requests by selecting of the suitable voltage space vector is prior to others. Namely a voltage space vector should be selected that causes to small reduction (NS) in motor torque. Available vectors are \( v_1, v_4, v_9 \), but only \( v_4 \) can reduce (N) the flux, and \( v_s, v_t \) cause to flux increment (P) that is against the system request. As a result, responding to the torque request is prior to the flux request but the singe of the flux variation (N or P) should be considered.

Table 1. Switching table

<table>
<thead>
<tr>
<th>( \hat{T} )</th>
<th>( \hat{\phi} )</th>
<th>( N )</th>
<th>( ZE )</th>
<th>( PS )</th>
<th>( PL )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( NL )</td>
<td>( v_{NL} )</td>
<td>( v_{NL2} )</td>
<td>( v_{NL3} )</td>
<td>( v_{NL4} )</td>
<td>( v_{NL5} )</td>
</tr>
<tr>
<td>( NS )</td>
<td>( v_{NS} )</td>
<td>( v_{NS2} )</td>
<td>( v_{NS3} )</td>
<td>( v_{NS4} )</td>
<td>( v_{NS5} )</td>
</tr>
<tr>
<td>( ZE )</td>
<td>( v_{ZE} )</td>
<td>( v_{ZE2} )</td>
<td>( v_{ZE3} )</td>
<td>( v_{ZE4} )</td>
<td>( v_{ZE5} )</td>
</tr>
<tr>
<td>( PS )</td>
<td>( v_{PS} )</td>
<td>( v_{PS2} )</td>
<td>( v_{PS3} )</td>
<td>( v_{PS4} )</td>
<td>( v_{PS5} )</td>
</tr>
<tr>
<td>( PL )</td>
<td>( v_{PL} )</td>
<td>( v_{PL2} )</td>
<td>( v_{PL3} )</td>
<td>( v_{PL4} )</td>
<td>( v_{PL5} )</td>
</tr>
</tbody>
</table>

3 The Phases Transposition Phenomenon

The rotor instantaneous position calculation for each phase \( \theta_j (j = a, b, c, d, e) \) and conversion it to the rotor normalized position \( \theta_{norm,j} \) suitable to be applied to the torque and flux look-up tables is not a new topic, but in order to illustrate the phases transposition phenomenon in a 5-phase 10/8 SRM it is necessary to discuss about it. The rotor normalized position for each phase \( \theta_{norm,j} \) is always a number inside the range of

\[
0 \leq \theta_{norm,j} \leq \pi/N,
\]

where, \( N_r \) is the rotor poles number. Thus the nonlinear electromagnetic torque characteristic \( T_e(t, i, \theta_j) \) can be used easily. To obtain the phase instantaneous torque, separate look-up tables should be utilized for each phase, so it is needed three similar look-up table for 6/4 motor and five similar ones for 10/8 motor. The zero point of the rotor position encoder can be defined as the rotor zero axis. Assume that the rotor position is equal to zero when the rotor zero axis and the phase a axis are in the same direction. Now the rotor position with respect to other phases can be obtained by using Fig. 4, where

\[
k = \begin{cases} 
0 & \text{for phase } a \\
1 & \text{for phase } b \\
2 & \text{for phase } c \\
3 & \text{for phase } d \\
4 & \text{for phase } e 
\end{cases}
\]
The rotor positions are shown in Fig. 5 for all phases. These are not adequate to apply to the look-up tables, and have to be reformed properly. During rotor rotation within poles replacement with each other (e.g. 0 up to $2\pi/\text{Nr}$), the positive torque is produced during first half (0 up to $\pi/\text{Nr}$), and the negative torque is produced during second half ($\pi/\text{Nr}$ up to $2\pi/\text{Nr}$). Therefore the rotor circumference can be divided to 16 separate sections as shown in Fig. 6 where symbols + and - refer to positive and negative torque generation respectively while the rotor zero axis passes through the relative section. Thus the rotor position axis $\theta$ of the 5-phase SRM output torque characteristic is scaled within zero and $360^\circ/8*2=22.5^\circ$, where 8 is the rotor poles number. With description above, while the rotor zero axis lies in the odd numbered sections of Fig. 6-(a), the rotor normalized position is:

$$\theta_{\text{norm},j} = \theta_j - (M_j - 1)*22.5^\circ$$  \hspace{1cm} (13)\)

As an example, for phase a equivalent angle of $\theta_j = 110^\circ$ positioned in sector $M_j = 5$ is $20^\circ$, so angle $\theta_j = 110^\circ$ is replaced with angle $\theta_j = 20^\circ$. When the rotor zero axis lies within the even numbered sections of Fig. 6-(a), the rotor normalized position is:

$$\theta_{\text{norm},j} = 22.5^\circ - [\theta_j - (M_j - 1)*22.5^\circ]$$ \hspace{1cm} (14)

As an example, equivalent angle of $\theta_j = 120^\circ$ positioned in sector $M_j = 6$ is $7.5^\circ$, so angle $\theta_j = 120^\circ$ is replaced with angle $\theta_j = 7.5^\circ$ (Fig. 6-(a)). Examples above are for phase a. Fig. 6-(a) changes to Fig. 6-(b) for phase b, so angle $\theta_j = 110^\circ$ in the above example is equal to $\theta_j = 38^\circ$ positioned in sector $M_j = 2$. Consequently according to Eq. (14), $\theta_{\text{norm},j} = 7^\circ$.

Similarly, rotor normalized angles can be obtained for other phases. Applying Eqs. (13) and (14) to Fig. 5 results Fig. 7 which shows the relation of the rotor position and the rotor normalized position of one phase. If Fig. 7 is plotted for all five phases, the rotor normalized position of all phases will be derived as shown in Fig. 8. As shown in Fig. 5 each phase graph follows its previous phase graph. Namely phase b comes after a, c after b, d after c, e after d and a after e, but in Fig. 8 phases sequence is different in comparison with Fig. 5. It means that the controller senses phase c instead of b, e instead of c, b instead of d and d instead of e. As a result, while applying $\theta_{\text{norm},j}$ and $i_j$ to the look-up table in order to obtain each phase instantaneous torque, phases order should be considered as following:

- $\theta_{\text{norm},a}$ is applied to look-up table of phase a,
- $\theta_{\text{norm},b}$ is applied to look-up table of phase d,
- $\theta_{\text{norm},c}$ is applied to look-up table of phase b,
- $\theta_{\text{norm},d}$ is applied to look-up table of phase e and
- $\theta_{\text{norm},e}$ is applied to look-up table of phase c.

![Fig. 5. Rotor position $\theta_j$ for all five phases](image)

![Fig. 6. Rotor position sectors](image)
Fig. 7. Rotor position $\theta_j$ and rotor normalized position $\theta_{\text{norm}, j}$ (Numbers are for phase a)

Fig. 8. Rotor normalized position $\theta_{\text{norm}, j}$ for all five phases

These changes are not required for 3-phase 6/4 SRM, because phase transposition is not occurred in the 6/4 SRM. Phase transposition phenomenon has such a considerable significance that it is not possible to apply DTC to the 10/8 SRM without taking it into account. It should be noticed applying $\theta_{\text{norm}, b}$ and $i_d$ to the look-up table of phase d does not mean that two different phases parameters are used to derive $T_j(t, i, \theta)$. In fact this approach is continuation of deriving $\theta_{\text{norm}, j}$ from the encoder output rotor position $\theta$ by using Fig. 4 and Eqs. (13) and (14).

4 Simulation Results

To confirm the validity of the proposed method, a model of the system was constructed in Matlab/Simulink. Obtained results were compared with the results of DTC method in the 3-phase 6/4 SRM. In both motors, both static torque $T(t, i, \theta)$ and flux $\varphi(t, i, \theta)$ characteristics are supposed to be same. In all below figures upper ones are for the 5-phase motor.

Fig. 9 shows the motors total torque under steady state condition. Motor torque and stator flux reference levels are adjusted to 4 Nm and 0.35 Wb respectively. As seen, in both cases the motor instantaneous torque follows the reference value within the certain band limits. Torque ripple in the 5-phase motor is about 60% of the one in the 3-phase motor that is the most noticeable advantage of the 5-phase motor.

Fig. 10. Motor torque response to step change of torque command (stator flux level is constant)
Motor torque response to the step change of the torque command with a constant stator flux level is shown in Figs. 10 and 11. The 5-phase motor torque is able to follow the command value as fast as the 3-phase one. Similar test is done to check the stator flux step response with a constant level of the motor torque command. Fig. 12 shows that in both cases, stator flux follows the command value precisely, but the 5-phase motor response is faster than the 3-phase one.

Startup test results are shown in Figs. 13, 14 and 15. This test is done in order to examine the low speed performance of the control system. In both motors command torque and flux are set on 4 Nm and 0.4 Wb respectively. As shown in Fig. 13, in the output torque a large unwanted distortion occurs in transient period of t = 0 up to 3 ms that is smaller in 5-phase motor. Stator flux and its trajectory in startup period are shown in Figs. 14 and 15 respectively. After a short transient period, command fluxes are followed precisely in both cases but there is more linear increment in the 5-phase case. More pole numbers in the 5-phase motor causes more flux trajectory rotation in transient period, because the electrical cycle is shorter in the 5-phase motor.

In another test, simultaneous large changes are applied to the torque and flux commands at low speed in order to examine the system performance under difficult condition. The motor torque and flux commands experience a 500% and 100% change respectively at t = 0.03 sec when the motor speed is only 4 rad/sec in 3-phase motor and 10 rad/sec in 5-phase motor. As seen in Figs. 16, 17 and 18 the 5-phase motor is able to follow the torque and flux commands as well as the 3-phase one. Another point to notice is that the stator flux level in the 5-phase motor is higher than the one in the 3-phase motor. In order to have an equal saturation level in two motors, the phase flux magnitude should be equal in both cases. This causes higher stator flux level in 5-phase motor, so selecting the higher stator flux level in 5-phase motor helps to have an equal saturation level. It should be noticed that the higher stator flux level imposes more torque ripple while the lower level reduces the total torque density.

To confirm the validity of this test in all levels of the torque and flux, linear changes in torque and flux commands are applied. Figs. 19 and 20 show that the 5-phase motor instantaneous torque and flux can trace the relative commands like 3-phase one successfully.

5 Conclusion
In this paper the main concepts of DTC method in 5-phase 10/8 SRM were proposed then the phase transposition phenomenon that occurs as a problem in 10/8 SRM and its solution were introduced. Comparable simulation results between two 3-phase and 5-phase motors were presented. From these results, it can be seen that compared with 3-phase motor, the 5-phase one has the advantages such as less torque ripple, faster flux transient response, less phase torque peak value, consequently less phase current peak value. In the following study DTC method by using fuzzy logic controller will be applied to the 5-phase 10/8 SRM, and its performance will be compared with the 5-phase motor performance studied in present paper.

![Fig. 11. Motor torque response to step change of torque command (closed view of Fig. 10)](image1)

![Fig. 12. Stator flux response to step change of flux command (torque level is constant)](image2)

![Fig. 13. Motor torque at startup](image3)
Fig. 14. Stator flux at startup

Fig. 15. Stator flux trajectory at startup

Fig. 16. Torque response to simultaneous step change of torque and flux command at low speed condition

Fig. 17. Flux response to simultaneous step change of torque and flux command at low speed condition

Fig. 18. Stator flux trajectory at simultaneous step change of torque and flux command under low speed condition
References


Mohammad Reza Feyzi received his BSc and MSc in 1975 from university of Tabriz in Iran with honor degree. He worked in the same university during 1975 to 1993. He started his PhD work in the University of Adelaide, Australia in 1993. Soon after his graduation, he rejoined to the University of Tabriz. Currently, he is an associate professor in the same university. His research interests are finite element analysis, design and simulation of electrical machines and transformers.

Yousef Ebrahimi received the B.S. degree in control engineering from Sahand University of Technology, Tabriz, Iran, in 2001, and the M.S. degree in electrical engineering from Tabriz University, Tabriz, Iran, in 2008. Since 2005 he has been with the National Iranian Gas Company (NIGC). His major research interest is design and simulation of electrical motors drive.