Phase-Only Synthesis of Antenna Arrays Using Nonuniform Phased Sampling Method

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Abstract: Nonuniform Phased Sampling method is proposed to phase-only synthesize the power pattern of both linear and planar antenna arrays. This method modifies the conventional sampling method which is used for amplitude-phase synthesis. This method is based on assigning suitable phases to the sampling points of radiation pattern in order to reach desired amplitude of currents. Some examples are given to verify the effectiveness of the proposed method for both pencil-beam and shaped beam patterns.

Keywords: Antenna Array Synthesis, Nonuniform Phased Sampling Method, Phase-Only Synthesis.

1 Introduction

SYNTHESIS of antenna arrays is usually done by changing both amplitude and phase of their antenna currents [1-4]. This conventional approach for array synthesis is called amplitude-phase synthesis. However, in some arrays such as reflectarray antennas having either pencil beams or contoured beam, only the phase of the unit cells is under our control [5-8]. In these antennas, the amplitude of unit cells are specified by the key parameter F/D [5-8]. Moreover, changing the phase of antennas of an array is more practical and easier than changing their amplitudes. Hence, we have to synthesize many linear and planar arrays by changing only the phase of their antennas. This approach for array synthesis is called phase-only synthesis [9-19].

Almost all proposed methods for phase-only synthesis are based on optimization methods in two main categories: local search algorithms; like alternating projections method [9, 13], and the evolutionary algorithms; like genetic algorithm (GA) [12], particle swarm optimization [10, 11], and semidefinite relaxation technique [14-15]. In [16], a scaling factor is introduced in an iterative optimization approach. In [17], a least-square error is minimized by applying gradient-descent optimization. Besides, autocorrelation matching method (AMM) was introduced to phase-only synthesis of linear arrays in [18, 19].

In this paper, a different method is proposed for phase-only synthesis of both linear and planar antenna arrays. This method is called Nonuniform Phased Sampling (NPS) method. The proposed NPS method is a modification of the well-known sampling (or Woodward-Lawson) method. In the conventional sampling method, the phases of the radiation pattern at the sampling points are uniform. However, in the proposed nonuniform sampling method the phases of the radiation pattern at the sampling points are nonuniform. The nonuniform phase distribution of the sampling points is assigned so that the amplitude distribution of the antenna currents approaches the presumed one. Then, the phases of the antenna currents are determined from both the amplitude and assigned phases of the sampled points of the radiation pattern.

The NPS method introduced in this paper for phase-only synthesis of linear arrays is completely different from the AMM method presented in [18-19]. However, it is shown that the results of these two methods are comparable and almost similar. In the AMM method, the amplitude-phase synthesis must be done at first. However, in the introduced NPS method this requirement is not needed.

The paper is organized as follows. In Section 2, the uniform phased sampling method is reviewed. In Section 3, nonuniform phased sampling method is introduced. Some examples for both linear and planar
arrays are presented in Section 4 to verify the performance of the proposed NPS method as a new phase-only synthesis method.

2 Uniform Phased Sampling Method

A linear antenna array comprises \( N \) identical antennas of uniform inter-distances \( d \) on the \( z \) axis. The excitation current of the \( n \)th antenna is \( I_n = |I_n| \exp(j\varphi_n) \). The radiation pattern of linear arrays is written as follows.

\[
F(\psi) = \sum_{n=0}^{N-1} I_n \exp(jn\psi)
\]

where \( \psi = k d \cos(\theta) \) is a real variable in which \( k = 2\pi/\lambda \) and \( \lambda \) is the wavelength in the free space.

A planar antenna array may be comprised of \( M \times N \) antennas on the \( xy \) plane, or \( d_x \) and \( d_y \) in \( x \) and \( y \) directions, respectively. The excitation current of the \( mn \)th antenna is \( I_{mn} = A_{mn} \exp(j\varphi_{mn}) \) for \( 0 \leq m \leq M-1 \) and \( 0 \leq n \leq N-1 \). The radiation pattern of planar arrays is written as follows.

\[
F(\psi_x,\psi_y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I_{mn} \exp(j(m\psi_x + n\psi_y))
\]

where \( \psi_x \) and \( \psi_y \) are real variables defined as \( \psi_x = 2\pi d_x/\lambda \sin \theta \cos \phi \) and \( \psi_y = 2\pi d_y/\lambda \sin \theta \sin \phi \).

An efficient amplitude-phases method to synthesize linear and planar arrays is the sampling (or Woodward-Lawson) method [1-4]. In the sampling method, the excitation currents of linear and planar arrays are obtained from the samples of their radiation patterns \( F(\psi) \) and \( F(\psi_x,\psi_y) \), respectively, as follows [1-4].

\[
I_n = \sum_{p=-N/2}^{N/2} F(\psi_x) \exp(-jn\psi_x)
\]

\[
I_{mn} = \sum_{p=-M/2}^{M/2} \sum_{q=-N/2}^{N/2} F(\psi_x,\psi_y) \exp(-j(n\psi_x + m\psi_y))
\]

In (3) and (4), \( \psi_x \) and \( \psi_y \) are the sampling points for linear and planar arrays, respectively, where \( \psi_x = 2\pi p/N \) and \( \psi_y = 2\pi q/N \). Also, \( m' = m - 0.5(M-1) \), \( n' = n - 0.5(N-1) \) for \( m = 0, 1, \ldots, M-1 \) and \( n = 0, 1, \ldots, N-1 \).

In conventional sampling method, the value of radiation patterns at sampling points, \( F(\psi_x) \) or \( F(\psi_x,\psi_y) \), are considered real. In fact, all sampling values \( F(\psi_x) \) or \( F(\psi_x,\psi_y) \) are considered to have the same phase like zero. Hence, one may call the conventional sampling method as Uniform Phased Sampling (UPS) method. This well-known method yields us excitation currents that may have undesirable amplitudes.

3 Nonuniform Phased Sampling Method

Here, we modify the conventional or uniform phased sampling method in order to control the amplitudes of the excitation currents. For this purpose, we propose a nonuniform phased sampling (NPS) method. In this proposed method, we assign nonuniform phases to the radiation patterns of linear and planar arrays at the sampling points, as follows.

\[
F(\psi_x) = |F(\psi_x)| \exp(j\varphi_x)
\]

(5)

\[
F(\psi_x,\psi_y) = |F(\psi_x,\psi_y)| \exp(j\varphi_x)
\]

(6)

In (5) and (6), \( \varphi_x \) and \( \varphi_{mn} \) are the phases assigned to the sampling points \( \psi_x \) and \( \psi_x, \psi_y \), respectively. These phases should be chosen so that the amplitudes of the excitation currents \( |I_n| \) and \( |I_{mn}| \) approach as much as possible to the desired amplitudes \( A_n \) and \( A_{mn} \), respectively. This aim can be achieved by some suitable methods among them optimization. One can define the following error functions for the optimization methods for the linear and planar array, respectively.

\[
\text{error}_{\|I_n\|} = \frac{1}{N} \sum_{n=0}^{N-1} |I_n - A_n|^2
\]

(7)

\[
\text{error}_{\|I_{mn}\|} = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |I_{mn} - A_{mn}|^2
\]

(8)

Minimizing these error functions makes amplitudes of the excitation currents \( |I_n| \) and \( |I_{mn}| \) equal to optimized amplitudes \( A_n \) and \( A_{mn} \) which are close to the desired amplitudes \( A_n \) and \( A_{mn} \), respectively. We call these amplitudes as approximately desired amplitudes.

Successive using (5) and (6), Eqs. (3), (4), (7), and (8) yields the optimum values of \( \varphi_x \) and \( \varphi_{mn} \). Finally, the excitation currents of the antennas are obtained by (3) and (4).

It is worth mentioning that to make the error functions defined in (7) and (8) more effective, it had better multiply the amplitudes of the excitation currents \( |I_n| \) and \( |I_{mn}| \) by an appropriate constant so that the following energy equivalence to be hold.

\[
\sum_{n=0}^{N-1} |I_n|^2 = \sum_{n=0}^{N-1} A_n^2
\]

(9)

\[
\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |I_{mn}|^2 = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{mn}^2
\]

(10)

Holding the energy equivalence must be done in each step of the optimization before evaluating error functions.

To summarize, the following steps must be done for NPS phase-only synthesis of linear arrays:

1. Assign phases \( \varphi_x \) to the sampling points of desired radiation pattern \( |F(\psi_x)| \) as indicated in (5).
2. Find the excitation currents \( I_n \) considering the amplitude and (assigned) phases of the radiation...
3. Find the difference between the amplitude of the currents $|I_n|$ and the desired amplitudes $A_n$, through (7). Before this, the energy equivalence must be done as indicated in (9).

4. Go to step 2 and repeat steps 2, 3, and 4 until the error in (7) reaches a value as small as possible.

5. The phases of the excitation currents found in step 3, are the same phases $\phi_n$ we are looking for them. The same procedure can be done for phase-only synthesis of planar arrays.

### 4 Examples and Discussion

To verify the proposed NPS method, several examples are presented in three different groups, i.e., pencil, flat-topped, and cosecant beams. The desired amplitudes $A_n$ is presumed as symmetric linear variation with respect to $n$. The characteristic of the desired amplitude excitations is Amplitude Dynamic Range (ADR) which is defined as the maximum amplitude to minimum one. The initial values of phases of $\phi_p$ and $\phi_p^q$ in the optimization process are produced by a random generator of $[-\pi, \pi]$. The initial values of phases $\phi_p$ should be regenerated several times to find the best solution which has the least error functions (7) or (8) among all solutions. The values of $N$ and $M$ are considered constant as 21 and therefore the sample points would be at multiples of $2\pi/21 = 0.299$.

#### 4.1 Pencil-Beam Patterns

Two linear arrays with $N = 21$ antennas are designed to have pencil-beam patterns of Chebyshev and Taylor of $\bar{n} = 4$ types with sidelobe level of SLL = $-20$ dB. The amplitude dynamic range is considered ADR = 2.5.

Figs. 1 and 2, show the resultant patterns obtained by amplitude-phase (UPS) and phase-only (NPS) methods. It is seen that satisfactory patterns have been obtained because the resulted SLLs are not larger than $-20$ dB. Besides, the Taylor pattern has been synthesized better than the Chebyshev one. This may be due to smoother behavior of Taylor pattern and its excitation currents in amplitude-phase synthesis.

Fig. 3 illustrates the amplitude of currents for amplitude-phase and NPS methods, both desired amplitudes $A_n$ and approximately desired amplitudes $A_n^\prime$. The value of error defined in (7) amounts to 0.0755 and 0.0433 for Chebyshev and Taylor patterns, respectively. Fig. 4 illustrates the nonuniform phases assigned to the sampling points. Finally, Fig. 5 shows the required phases of the phase-only synthesized currents obtained by NPS method. All the required phases for amplitude-phase synthesis are zero.

Now, a linear arrays with $N = 31$ antennas are designed to have a pencil-beam pattern of Chebyshev type with sidelobe level of SLL = $-20$ dB and having a deep null about $\psi = 2.5$. The excitations of such array factor could be found as explained in [3] by setting the pattern $F(\psi_p)$, through (3).

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**Fig. 1** Amplitude-phase and phase-only synthesized Chebyshev pencil-beam patterns of SLL = $-20$ dB and ADR = 2.5.

**Fig. 2** Amplitude-phase and phase-only synthesized Taylor pencil-beam patterns of SLL = $-20$ dB, $n$-bar = 4 and ADR = 2.5.

**Fig. 3** Amplitude of the currents for both amplitude-phase and phase-only (NPS) pencil-beam patterns of Chebyshev and Taylor types.

**Fig. 4** Nonuniform phases at sampling points for the pencil-beam patterns of Chebyshev and Taylor types.
sixth sidelobe to be −40 dB.
The amplitude dynamic range is considered ADR = 2.5 and the value of error amounts to 0.0831. Fig. 6 compares the resultant pattern obtained by phase-only (NPS) method with the desired one. It is seen that a deep null has been created around $\psi = 2.5$ while SLL is somewhat larger than −20 dB. In this example, both amplitude-phase and phase-only patterns have complex currents. Figs. 7 and 8 show the amplitude and phase of these currents, respectively.

4.2 Flat-Top Beam Pattern
A linear array with $N = 21$ antennas is designed to have a flat-top beam between $\psi = -0.898$ to $\psi = +0.898$ supposing $F(±0.898) = 0.5F(0)$. The amplitude dynamic range is considered ADR = 2.5 and 5.

Figs. 9 and 10, show the resultant patterns obtained by amplitude-phase (UPS) and phase-only (NPS) methods for ADR = 2.5 and 5, respectively. In NPS method, the amplitudes of currents $|I_n|$ are considered equal to both desired amplitudes $A_n$ or approximately desired amplitudes $\tilde{A}_n$. It is seen that satisfactory patterns have been obtained, specifically when $|I_n|$ are imposed to be equal to the desired amplitudes $A_n$. Besides, the case of ADR = 5 gives a more satisfactory pattern than the case of ADR = 2.5. It is worthy to mention that the synthesized patterns by both UPS and NPS are equal to the desired pattern at $N$ sampling points $\psi_p$.

Fig. 11 compares the amplitude of currents for both...
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Fig. 12 Assigned phases at sampling points for the flat-top beam patterns.

Fig. 13 Required phases of the phase-only synthesized currents for flat-top beam patterns.

UPS and NPS methods. The value of error defined in (7) are equal to 0.0951 and 0.0575 for ADR = 2.5 and 5, respectively. Fig. 12 shows the assigned phases of the sampling points. Eventually, Fig. 13 shows the required phases of the phase-only synthesized currents obtained by NPS method. The required phases for amplitude-phase synthesis are either zero or 180°.

Here, the effect of the number of antennas is studied. For this purpose, desired flat-top beam is synthesized using four arrays having \( N = 15, 21, 29, \) and 35 antennas. Figs. 14 and 15, show the resultant patterns obtained by amplitude-phase (UPS) and phase-only (NPS) methods, respectively. ADR is considered to be equal to 5. To evaluate the similarity of synthesized patterns to the desired one, the following error is defined.

\[
\text{error}_{\text{(UPS)}} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \left( |\hat{F}(\psi)| - |F(\psi)| \right)^2 d\psi}
\]

where \( \hat{F}(\psi) \) and \( F(\psi) \) are synthesized and desired patterns, respectively. Table 1, shows this error for four flat-top synthesized beams. As expected, the error of NPS method is larger than the error of UPS method. However, it is seen that from the last column of this table that as \( N \) increases the efficiency of NPS method decreases slowly with respect to UPS method.

4.3 Cosecant Pattern

A linear array with \( N = 21 \) antennas is designed to have a cosecant beam, i.e. \( 1/\psi \), between \( \psi = 0.299 \) to \( \psi = +2.094 \) supposing \( F(0)=0.6F(0.299) \). The amplitude dynamic range is considered ADR=2.5 and 5.

Figs. 16 and 17, show the resultant patterns obtained by amplitude-phase (UPS) and phase-only (NPS) methods for ADR=2.5 and 5, respectively. It is seen that satisfactory patterns have been obtained, specifically when \( |I_n| \) are imposed to be equal to the desired amplitudes \( A_n \). Again, it is seen that as ADR increases from 2.5 to 5, the phase-only synthesized pattern tends the desired pattern further. Also, the synthesized patterns by both UPS and NPS are equal to the desired pattern at \( N \) sampling points \( \psi_p \).

Fig. 18 compares the amplitude of currents for both amplitude-phase and phase-only methods. The value of error defined in (7) are equal to 0.0698 and 0.0333 for ADR = 2.5 and 5, respectively. Fig. 19 illustrates the nonuniform phases of the sampling points. At last, Fig. 20 shows the required phases of the phase-only synthesized currents obtained by NPS method. This figure also shows the required phases for amplitude-phase synthesis. In fact, the currents of amplitude-phase
Phase-Only Synthesis of Antenna Arrays Using Nonuniform Phased Sampling (NPS) was proposed for phase-only synthesis of power pattern of linear and planar antenna arrays. In this method the conventional sampling method was modified by assigning suitable phases to the sampling points. The effectiveness of the proposed procedure for pencil-beam, flat-top and cosecant patterns was verified. It was seen that as amplitude dynamic range (ADR) increases, the phase-only pattern tends the desired pattern further.

5 Conclusion

Nonuniform Phased Sampling (NPS) was proposed for phase-only synthesis of power pattern of linear and planar antenna arrays. In this method the conventional sampling method was modified by assigning suitable phases to the sampling points. The effectiveness of the proposed procedure for pencil-beam, flat-top and cosecant patterns was verified. It was seen that as amplitude dynamic range (ADR) increases, the phase-only pattern tends the desired pattern further.
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Fig. 21 Amplitude-phase synthesized pencil-beam pattern of Taylor type with nbar = 4 and SLL = −20 dB.

Fig. 22 Amplitude of the currents for amplitude-phase synthesized two-dimensional pencil-beam pattern.

Fig. 23 Desired amplitudes of the currents presumed for phase-only synthesis.

Fig. 24 Approximately desired amplitudes synthesized by NPS method.

Fig. 25 Phase-only synthesized symmetric pencil-beam pattern for ADR = 2.5.

Fig. 26 Required phases of phase-only synthesized currents for two-dimensional pencil-beam pattern.

References


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