

# A Simple Analytic Method to Model and Detect Non-Uniform Air-Gaps in Synchronous Generators

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**Abstract:** The air-gap of electrical machines may become non-uniform due to low accuracy of the manufacturing machinery, in assembling processes, or by aging. Detection and monitoring of this phenomenon is very important and of interest. There are several methods to model non-uniform air-gaps and to detect them by monitoring systems. One of the most widely used methods is by the analysis of the line currents. In this paper a new, simple and comprehensive method is presented to model and detect non-uniform air-gaps in synchronous generators with skewed rotors. The influence of non-uniform air-gaps on the harmonics of the induced voltage of the stator is investigated by the proposed method. Simulations are performed for three cases: uniform air-gap, static rotor eccentricity, and stator ovality in a two phase generator. The experimental results are also presented. The good correspondence between the simulation and the experimental results clearly validates the theoretical findings put forward in this paper.

**Keywords:** Non-Uniform Air-Gaps, Output Voltage Harmonics, Static Rotor Eccentricity, Stator Ovality, Synchronous Generator.

## 1 Introduction

Rotating electrical machines play an essential role in the world's industrial life. The failure of critical rotating machines even if it is temporary, may lead to huge damages. Hence, monitoring and detection of faults in electrical machines is very crucial. References [1] and [2] have more details and refer to more references on different monitoring techniques of large motors and generators.

Several methods have also been proposed to model and detect non-uniform air-gaps in induction machines [3-8]. This subject has been less considered in synchronous machines. References [9, 10] have modeled this phenomenon in synchronous generators by both the modified winding function approach (MWFA) and the finite element methods and investigated dynamic air-gap eccentricity by analyzing stator-current-induced harmonics. References [11] and [12] have used the neutral line current and inductive sensors to detect non-uniform air-gaps in large synchronous generators respectively. Here, the authors are proposing

to use the induced phase voltage of the stator to detect static rotor eccentricity and stator ovality on salient-pole synchronous machines. This approach is different from the one proposed by Htsui and Stein [13], which utilizes the shaft voltage signals to detect rotor-related problems on salient-pole synchronous machines.

Many papers have been published in the literature about modeling non-uniform air-gaps such as: finite element, MWFA and decomposition of the air-gap between each winding into harmonic components in the equivalent electrical circuit [9, 10], [14-16]. One can deduce from the papers that the methods are complex and time consuming. In this paper a new and simple analytic method is proposed to model a synchronous generator with a skewed rotor. By this method the influences of a non-uniform air-gap on the harmonics of the output phase voltage are investigated. Output voltage analysis for fault detection in comparison with line current analysis has the advantage that its measurement is easier than the line current. Simulation and experimental results verify the applicability of the method to detect faults.

## 2 Case Study

The generator for this case study is a two-phase, salient pole synchronous generator with a 16-slot stator as shown in Fig. 1. In order to generate a sinusoidal output voltage, the stator windings are distributed sinusoidally and the coils are connected to each other in

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a spiral form as shown in Fig. 2(a). Fig. 2(b) illustrates magneto-motive force distribution in the air-gap space. To eliminate the slot harmonics the rotor is skewed as in Fig. 3.

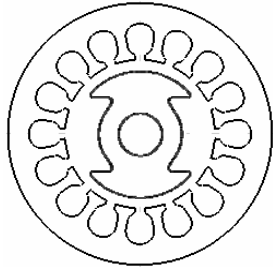


Fig. 1 Cross section of the rotor and the stator

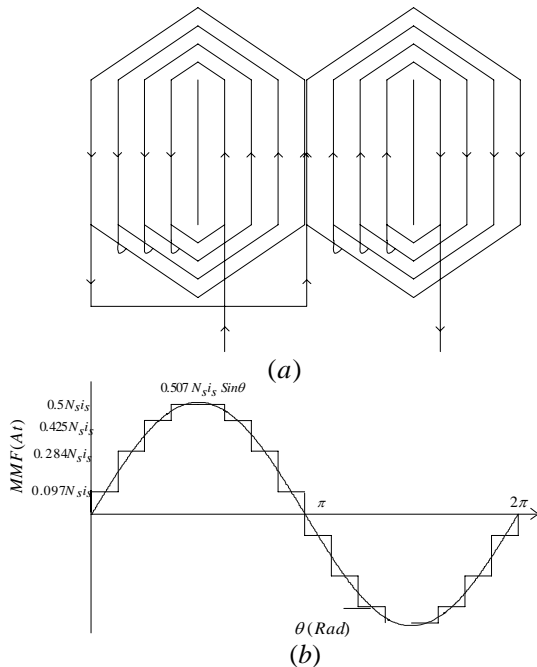


Fig. 2 (a) The winding connection diagram (b) The mmf distribution in air-gap space

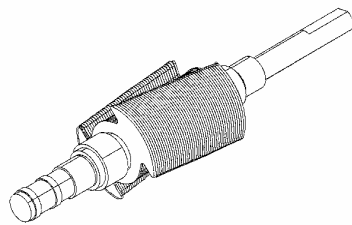


Fig. 3 The skewed rotor

### 3 Modeling

The procedure of modeling method can be summarized as follows:

To obtain the output voltage of a non-uniform air-gap generator, the induced voltage of each stator conductor should first be calculated by Lorentz law and then, properly added together. The skewing of a rotor will change the waveform of the induced voltage from a rectangular shape to a trapezoidal one as will be described later.

The induced voltage in each stator conductor in a rotating period of a rotor can be obtained by Lorentz law as in Eq. (1).

$$e = B \cdot R_0 \cdot \omega \cdot l(t) \quad (1)$$

where  $e$ ,  $B$ ,  $\omega$ ,  $R_0$  and  $l(t)$  are the induced voltage in each conductor, the magnetic flux density, the angular velocity, the inner radius of stator and the length of conductor that is subjected to the magnetic flux of the rotor in a period, respectively.  $l(t)$  Can be defined as below:

If  $\alpha$  and  $\beta$  are respectively the rotor skew angle and the width of the pole face of the rotor as shown in Fig. 4, then the waveform of  $l(t)$  will be as depicted in Fig. 5. Here,  $T$  and  $L$  are the rotating period and the axial length of the rotor respectively. In the study,  $\alpha$  is assumed 22.5 degrees to eliminate slot harmonics and  $\beta$  is assumed 100 degrees to eliminate the third harmonic [17, 18].

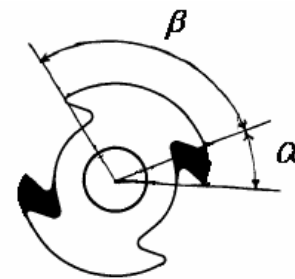


Fig. 4 The illustration for  $\alpha$  and  $\beta$

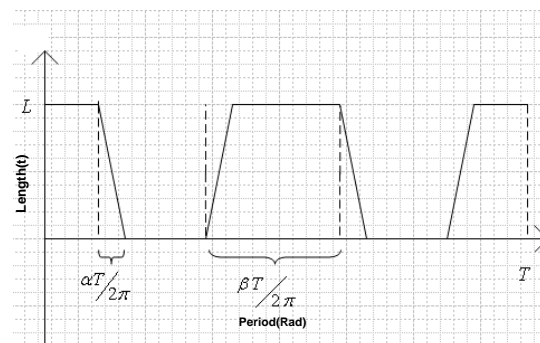


Fig. 5 Length of conductor that is subjected to the magnetic flux in a period

By using Eq. (1), the induced voltage in each coil-side is as in Eq. (2).

$$e = n.B.R_0.\omega.l(t) \quad (2)$$

where  $n$  is the number of turns in the coil.

To obtain the magnetic flux in the air-gap, Eqs. (3) and (4) can be used.

$$mmf = n_r.i_r = \phi.\mathfrak{R} = B.A.\mathfrak{R} \quad (3)$$

$$B = \frac{n_r.i_r}{A.\mathfrak{R}} = \frac{\mu_0}{g} n_r.i_r \quad (4)$$

where  $n_r$ ,  $i_r$ ,  $\phi$ ,  $\mathfrak{R}$ ,  $A$ ,  $\mu_0$ , and  $g$  are the number of turns in rotor coil, the rotor current, the magnetic flux of rotor, the magnetic reluctance, the cross section area of rotor, the permeability of air and the length of air-gap of the machine, respectively. In the non-uniform air-gap case,  $g$  is a function of  $\theta$  and should be stated by  $g(\theta)$ , where  $\theta$  is the angle of the related coil-side slot.

Considering Eqs. (2) and (4) and Fig. 5 for  $l(t)$ , one can calculate the induced voltage in each coil-side and then, add them up properly to obtain the output voltage. One should note that in adding up the coil-side voltages, not only the sign but also the  $2\pi/k$  phase difference should be considered. Here,  $k$  is the number of stator slots.

It should be mentioned that this method of modeling is not limited to skewed salient-pole rotors. With the proposed method it is easy to model other rotor types such as salient-pole, wound, skewed, and non-skewed. For example if the rotor studied in this paper was not skewed, Fig. 5 would be changed to a step shape (not inclined shape). If rotor is of wound type, the produced magnetic flux density used in Eq. (2) is not constant. In this case, the flux density would have a multi-step wave form according to the rotor winding distribution.

In this research, a MATLAB/SIMULINK program is implemented to calculate the induced output voltages. Modeling is done for three cases: uniform air-gap, stator ovality, and static rotor eccentricity. Moreover, the effects of each case on the harmonics of the output voltage are investigated.

### 3.1 Uniform Air-Gap

For the uniform air-gap,  $g(\theta)$  is constant and the induced output voltage can easily be obtained.

### 3.2 Stator Ovality

As it is depicted in Fig. 6, the stator ovality causes the air-gap between the rotor and the stator to become non-uniform.

By using the ellipse equation in polar coordinates, and written in Eq. (5), one can calculate the length of the air-gap corresponding to each stator slot at an angle of  $\theta$ , as in the following:

$$\frac{(R \cos(\theta))^2}{a^2} + \frac{(R \sin(\theta))^2}{b^2} = 1 \quad (5)$$

$$g(\theta) = \sqrt{\frac{1}{\frac{\cos^2 \theta}{(R_0 + dR)^2} + \frac{\sin^2 \theta}{R_0^2}}} - r_0 \quad (6)$$

where  $a$ ,  $b$ ,  $R_0$ ,  $R$ ,  $dR$  and  $r_0$  are the small and the large radii of ellipse, the inner radius of stator, the distance of inner points of the stator to the center, the increased magnitude in stator radius, and the outer radius of rotor, respectively as shown in Fig. 6. It is also assumed that  $a = R_0$  and  $b = R_0 + dR$ . Now by knowing  $g(\theta)$ , one can obtain the induced voltage in each coil-side.

### 3.3 Static Rotor Eccentricity

Static rotor eccentricity causes a nonuniformity in the air-gap as shown in Fig. 7.

The equation for the small circle in polar coordinates is as bellow:

$$(r \cos \theta - dx)^2 + (r \sin \theta - dy)^2 = r_0^2 \quad (7)$$

$$r = dx \cos \theta + dy \sin \theta + \sqrt{(dx \cos \theta + dy \sin \theta)^2 - dx^2 - dy^2 + r_0^2} \quad (8)$$

where  $r$  is the distance of outer points of rotor to the stator center and  $dx$  and  $dy$  are the rotor deviations from the center with respect to  $x$  and  $y$  axes, respectively. Now  $g(\theta)$  can be written as in Eq. (9) and therefore, the induced voltage corresponding to each coil-side can be obtained as before.

$$g(\theta) = R_0 - r = R_0 - (dx \cos \theta + dy \sin \theta + \sqrt{(dx \cos \theta + dy \sin \theta)^2 - dx^2 - dy^2 + r_0^2}) \quad (9)$$

## 4 Simulation Results

Here, the simulation results for the three mentioned cases are presented. In the stator ovality case, calculations are performed for the condition that stator is deformed and extended in the  $x$  axis by 50% of the air-gap length. In the case of rotor eccentricity, rotor is moved in the  $x$  and  $y$  axes up to 50% of the air-gap length. For each case the output voltage waveform along with its Fourier analysis is presented. The Fourier diagrams show the percentage of each harmonics in terms of the fundamental component, and for clarity each diagram is cut at 4%. Figs. 8, 9 and 10 depict the results for uniform air-gap, the stator ovality and the static rotor eccentricity, respectively.

From Figs. 9 and 10 one can deduce that non-uniformity of air-gap causes a significant increase in the third harmonic of the output voltage in both cases of stator ovality and static rotor eccentricity. It is also noticed that compared to the stator ovality case, the fifth harmonic increases slightly in the static rotor eccentricity case. The comparison of the percentages of the odd harmonics for the three cases is summarized in Table 1.

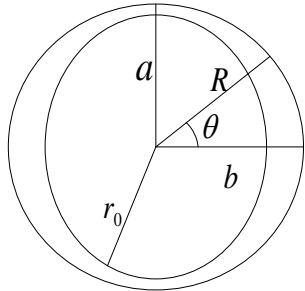


Fig. 6 A non-uniform air-gap caused by stator ovality

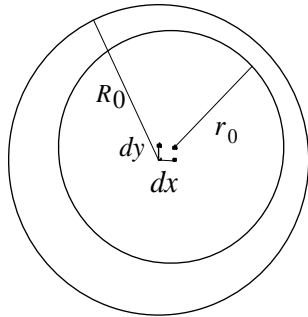


Fig. 7 A non-uniform air-gap caused by static rotor eccentricity

Table 1 The percentages of the odd harmonics in the simulated output voltages

	3rd	5th	7th	9th
Uniform Air-gap	0.07	0.04	0.01	0.02
Rotor Eccentricity	3.59	0.94	0.05	0.06
Stator Ovality	2.16	0.18	0.01	0.02

The results are not limited to the case of skewed salient-pole rotors. The skewing of rotor and non-uniformity of air-gap affects harmonics differently. For the proposed case study, skewing has an impact on slot harmonic numbers 15 and 17, whereas non-uniformity of the air-gap influences the 3rd and 5th harmonics. On the other hand, implementing a sinusoidal distributed wound rotor produces a more sinusoidal output voltage and does not add any additional harmonics. Thus, the two different types of rotors produce similar results.

## 5 Experimental Results

To verify the expected effects of non-uniform air-gap on synchronous generators, a skewed rotor was designed and constructed inside the stator of a two-phase generator as shown in Fig. 11.

The rotor was then excited with 20 mA of current and rotated at a speed of 1200 rpm. Fig. 12 shows the open circuit output voltage of the generator on a scope. Tests were performed for three cases: uniform air-gap, static rotor eccentricity and stator ovality. The output

voltages and Fourier diagrams of those three cases are respectively shown in Figs. 13, 14 and 15.

The diagrams and the experimental data summarized in Table 2 verified simulation results and showed that

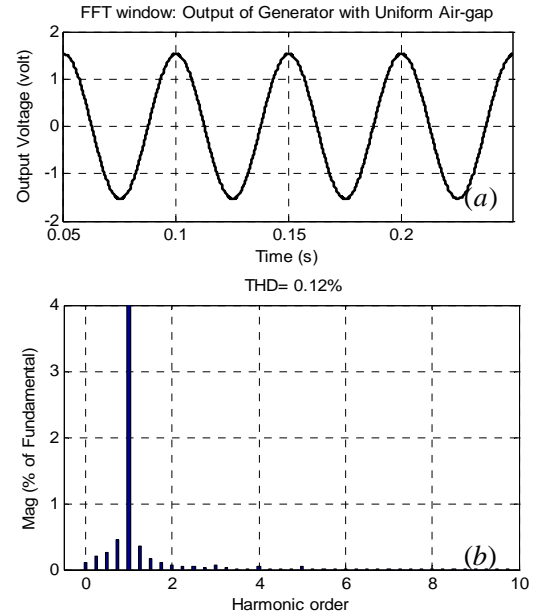


Fig. 8 (a) The waveform and (b) The Fourier diagram of the output voltage for the uniform air-gap case

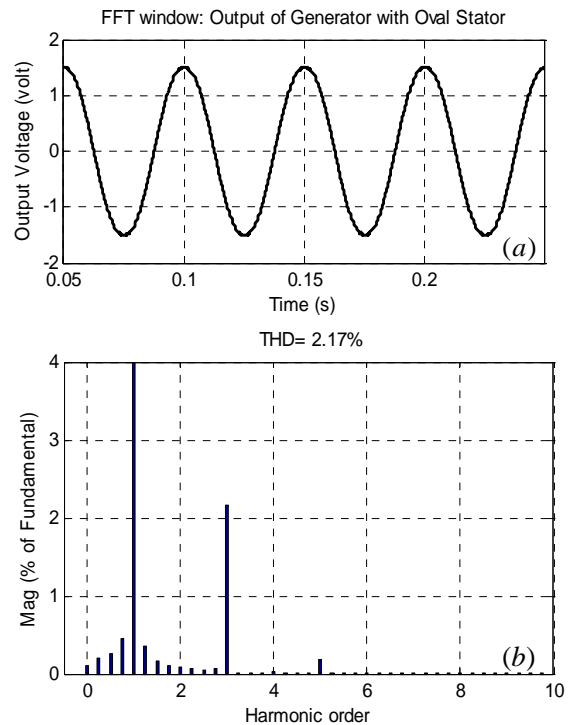
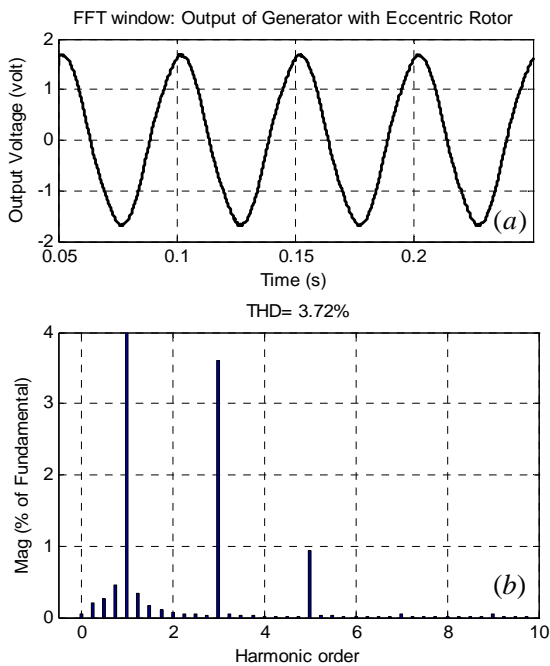


Fig. 9 (a) The waveform and (b) The Fourier diagram of the output voltage for the stator ovality case

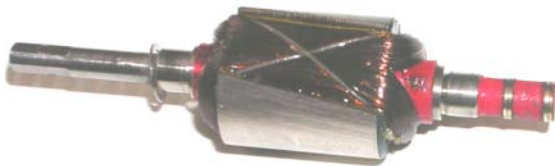
non-uniform air-gap caused a marked increase in the third harmonic of the output voltage. Small discrepancies between the experimental and simulation results might be due to mechanical and/or electrical measurements, non-uniformity of magnetic material and saturation that were not considered in the simulation.

**Table 2** The percentages of the odd harmonics in the experimental output voltages

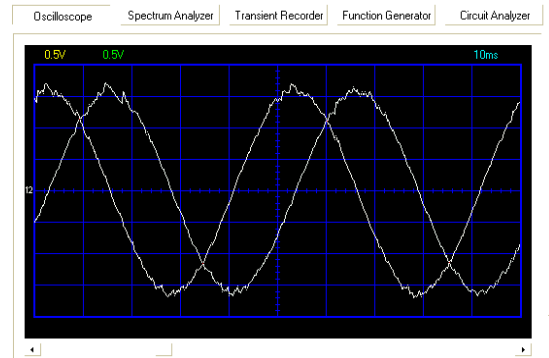
	3rd	5th	7th	9th
<b>Uniform Air-gap</b>	0.14	0.36	0.1	0.23
<b>Rotor Eccentricity</b>	3.43	1.33	0.34	0.91
<b>Stator Ovality</b>	2.28	0.36	0.15	0.15



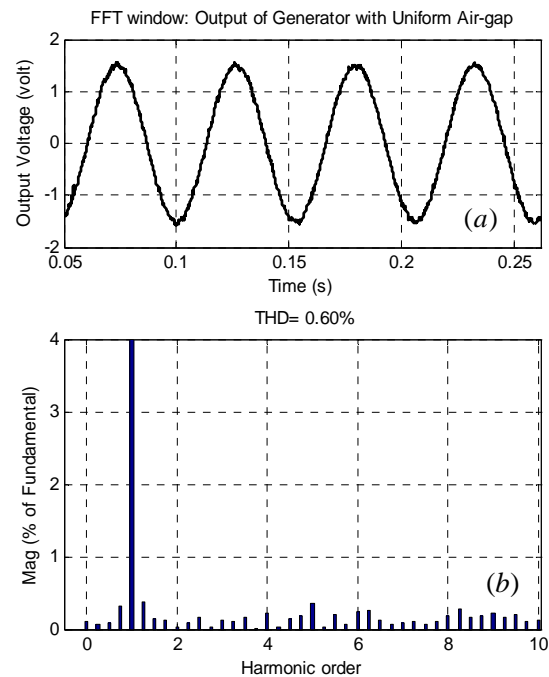
**Fig. 10** (a) The waveform and (b) The Fourier diagram of the output voltage for the static rotor eccentricity case



**Fig. 11** The rotor of the built generator



**Fig. 12** The open circuit output voltage of the generator on a scope

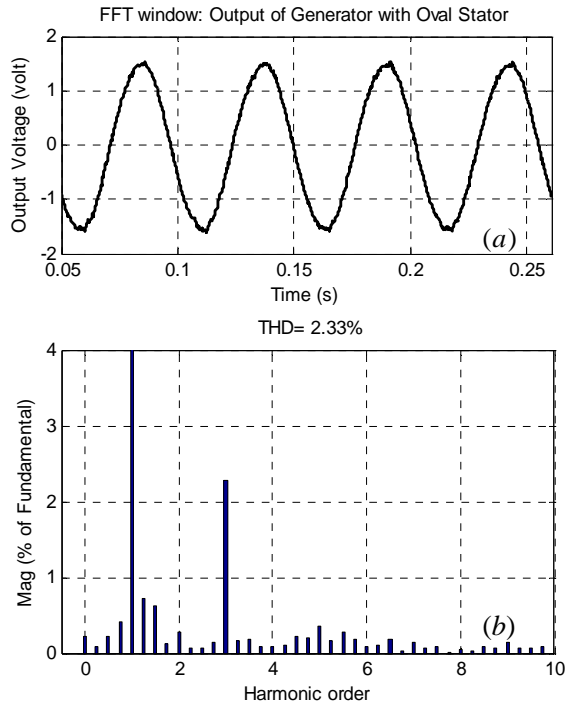


**Fig. 13** (a) The waveform and (b) the Fourier diagram of the output voltage for the uniform air-gap case

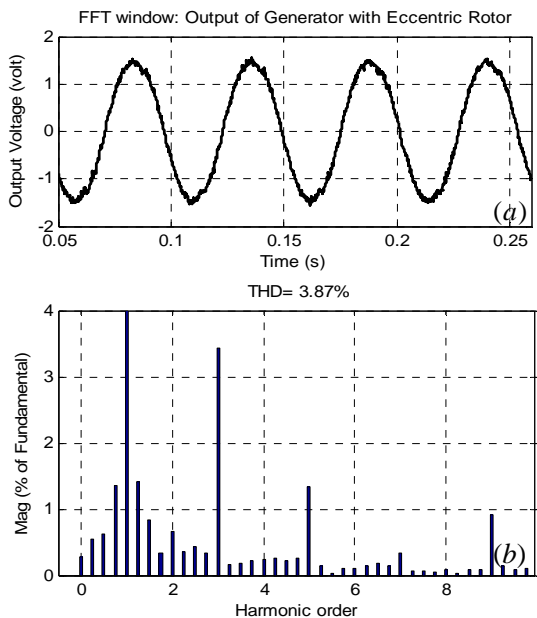
## 6 Conclusion

In this research, the harmonic analysis of the open circuit phase voltage was implemented to monitor and detect the non-uniform air-gap of a synchronous generator. The proposed model was simple, yet accurate enough to simulate and investigate the effects of non-uniformity in the air-gaps of generators with very small computer memory requirements. The results showed that non-uniformity of air-gap caused a significant increase in the third harmonic of output phase voltage. The correspondence between the experimental and simulation results was close, the difference being within the estimated experimental error band.

The authors believe the model can also be easily used to analyze the effects of generator dimensions, number of stator slots, number of turns in stator coils, and the rotor skew angle among others. The studies are still going on and the results will be presented at the next occasion.



**Fig. 14** (a) The waveform and (b) the Fourier diagram of the output voltage for the stator ovality case



**Fig. 15** (a) The waveform and (b) the Fourier diagram of the output voltage for the static rotor eccentricity case

## References

- [1] Tavner P. J., Gaydon B. G. and Ward D. M., "Monitoring generators and large motors," *Proc. Inst. Elect. Eng.*, Vol. 133, pt. B, No. 3, pp. 169–180, May 1986.
- [2] Tavner P. J. and Penman J., *Condition Monitoring of Electrical Machines*, Letchworth, U.K.: Research Press, 1987.
- [3] Long W., Xianghui H., Habetler T. G. and Harley R. G., "Eliminating Load Oscillation Effects for Rotor Eccentricity Detection In Closed-Loop Drive-Connected Induction Motors", *Power Electronics Specialists Conference, PESC 2006*.
- [4] Bossio G., Angelo C. D., Solsona J., Garca G. O. and Valla M. I., "Application of an Additional Excitation in Inverter-Fed Induction Motors for Air-Gap Eccentricity Diagnosis" *Energy Conversion IEEE Transaction*, Vol. 21, No. 4, pp. 839-847, Dec. 2006.
- [5] Antonino-Daviu J., Jover P., Riera M., Arkkio A. and Roger-Folch J., "DWT Analysis of Numerical and Experimental Data for the Diagnosis of Dynamic Eccentricities in Induction Motors", *Mechanical Systems and Signal Processing*, In Press.
- [6] Guldemir H., "Detection of airgap eccentricity using line current spectrum of induction motors", *Electric Power Systems Research*, Vol. 64, No. 2, pp. 109-117, Feb. 2003.
- [7] Nandi S., Ahmed S. and Toliyat H. A. "Detection of rotor slot and other eccentricity related harmonics in a three phase induction motor with different rotor cages", *IEEE Transactions on Energy Conversion*, Vol. 16, No. 3, pp. 253-260, Sep. 2001.
- [8] Hwang D.H., Chang J.H., Kang D. S.; Lee J. H. and Choi K.H., "A Method for Dynamic Simulation and Detection of Air-gap Eccentricity in Induction Motors by Measuring Flux Density", *IEEE Conference on Electromagnetic Field Computation*, INSPEC Accession Number: 8968625, pp. 69-69, 2006.
- [9] Toliyat H. A. and Al-Nuaim N. A., "Simulation and Detection of Dynamic Air-Gap Eccentricity in Salient-Pole Synchronous Machines", *IEEE Transactions on Industry Applications*, Vol. 35, No. 1, pp. 86-93, Jan/Feb 1999.
- [10] Toliyat H. A. and Al-Nuaim N. A. "A Method For Dynamic Simulation And Detection of Dynamic Air-Gap Eccentricity in Synchronous Machines", *Texas A&M University College Station IEEE*, 1997.
- [11] Azuaje C. and Millan A., "Stator Deformation of Large Hydrogenerators and Its Effects on the Machines", *Transmission & Distribution Conference and Exposition: IEEE/PES*, pp. 1-5, Aug. 2006.

- [12] Xuan M. T., Simond J.-J., Wetter R. and Keller S., "A Novel Air-gap Monitoring System for Large Low Speed Hydro-generators", *Power Engineering Society General Meeting, IEEE, INSPEC* Accession Number. 9096394, June 2006.
- [13] Tsui J. S. and Stein J., "Shaft Signals of Salient-pole Synchronous Machines for Eccentricity and Shorted-field-coil Detections", *IEEE Trans. Energy Conversion*, Vol. 9, pp. 572–578, Sep. 1994.
- [14] Debortoli M. J., Salon S. J., Burow D. W. and Slavick C. J., "Effects of Rotor Eccentricity and Parallel Windings on Induction Machine Behavior: A Study Using Finite Element Analysis", *IEEE Transactions on Industry Applications*, Vol. 29, No. 2, pp. 1676-1682, March 1993.
- [15] Bossio G., Deangelo C., Solsona J., Garcia G.O. and Valla M. I., "A Model for Induction Motors With Non-uniform Air-gap," *Latin American Applied Research*, Vol. 35, pp. 77-82, 2005.
- [16] Nabeta S. K. Y. O., Chabu I. E. and Cardoso J. R., "Analytical Modeling of Shaded Pole Motors with Non-uniform Air Gap," *IEEE Electric Machines and Drives Conference IEMDC*, Brazil, Vol. 3, pp. 1656-1662, 2003.
- [17] Sawhney A. K., *A Course in Electrical Machine Design*, Danpat Rai & Co., 1998.
- [18] Wakileh G. J., "Harmonics in rotating machines" *Electric Power Systems Research*, Vol. 66, No. 1, pp. 31-37, July 2003.



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