Design Modification of Rogowski Coil for Current Measurement in Low Frequency

M. Rezaee* and H. Heydari*

Abstract: The principle object of this paper is to offer a modified design of Rogowski coil based on its frequency response. The improvement of the integrator circuit to nullify the phase difference between the waveforms of the measured-current and the corresponding terminal voltage is a further object of this investigation. This paper addresses an accurate, yet more efficient measuring and protecting device for low frequency applications. This requires verification for the simulations by physical descriptions and experimental results. These validate the superior performance of Rogowski coils over conventional current transformers.

Keywords: Current Transformer, Frequency Response, Integrator Circuit, Mutual Inductance, Rogowski Coil, Terminal Resistor.

1 Introduction

The Rogowski coil’s technique possesses many features which offer an advantage over iron-cored current measuring devices and these are well illustrated by making some changes to its parameters for more precise measurements.

A wide range of industrial applications requires AC current transducers able to match both metrological requirements, such as accuracy, linearity, bandwidth, etc., and application-oriented requirements, such as high-level galvanic insulation, ruggedness, lightweight, etc. Unfortunately, none of the commercially available AC current transducers is able to comply with the above requirement. Resistive shunts are characterized by a wide bandwidth and linearity, but they do not assure galvanic insulation. Hall Effect based on current transducers feature wide bandwidth, compact size and galvanic insulation, but become inadequate when wide band, compact size and lightweight requirements are necessary. However, any choice of available AC current transducers represents a compromise between different possibilities, depending on the particular field of application [1, 2].

Emerging modern protective relays constructed by microprocessor equipments increases the applications of Rogowski coil for measuring-current in power systems. Due to non-ferromagnetic material existed in the core of Rogowski coil, this coil is not driven into saturation and it operates as well as a linear device, against current transformer (CT). Contrary to CT, Rogowski coil does not need any converter for adapting to microprocessor's input signal of modern relays.

D. Ward and J. Exton showed that Rogowski coils have better performance in a vast majority applications; sudden short circuit test, lightning test and partial discharge monitoring, as compared to the other current-measuring devices [3].

Ramboz introduced a new construction of Rogowski coil, Machinable Rogowski Coil made of a rigid nonmagnetic coil and a single-layer winding [4].

Kojovic [5] and Karrer and Hofer-Noser [6] proposed a new configuration of Rogowski coil based on printed circuit board (PCB) being useful in transient current measurement. In that study, the amplitude of the coil's output signal is increased without decreasing its bandwidth, by increasing the number of PCB coils.

In the other prior research, for low-frequency applications a CT model, analyzing hysteresis phenomenon and remnant flux on its performance in fault conditions is presented [7]. it is reported that in low frequencies (25 Hz to 400 Hz), the output of the CT in fault state is less accurate and requires more correction factors.

Kojovic compares performance characteristics of conventional CTs and Rogowski coils used for protective relaying purposes [8]. Rogowski coils do not
create hazard for personnel with open secondary like conventional CTs and their output is a voltage proportional to the rate of change of measured current, requiring appropriately designed relays to accept these types of signals, he believed that these coils can replace CTs for both measurement and protection applications in power systems [8].

This paper mainly describes the mathematical analysis of design modification, based on frequency response of Rogowski coil for 50Hz current measurements ranging from tenths to several thousands of amperes in electrical power industry. The mathematical analysis and simulation results are verified by physical descriptions and experimental results.

For comparative purpose, the results prove the superiority of Rogowski Coils over current transformers.

2 Construction and Transformer Function of Rogowski Coil

Figure 1 shows that Rogowski coil being formed with helical winding located on a toroidal rigid coil made of a non-magnetic material. It is shown that the compensating return loop (incremental pitch advancement) of the helical winding sums over its circumferential length to create an undesirable one-turn loop normal to the axis of the coil [4], [9]. The corresponding equivalent circuit consisted of a coil’s equivalent resistance \(R_c\), self inductance \(L_c\), mutual inductance \(M_c\), self capacitance \(C_c\), terminal resistance \(R_t\), and voltage source \(e_{ind}\), which is acting as a coupling circuit, is shown in Fig. 2.

According to Faraday’s law \(e_{ind} \propto \frac{dI}{dt}\), for the compatibility of the current and induced voltage signals, the coil terminals are connected to an integrator circuit. It is reported that the passive integrator circuit is more competent than the active one [10].

Thus, by connecting the passive integrator circuit, the equivalent circuit is replaced by Fig. 3, in which the self capacitance of Rogowski coil is omitted for low frequency applications.

The corresponding passive integrator circuit shown in Fig. 3 consisted of an integrator’s equivalent resistance \(R_i\) and capacitance \(C_i\).

3 Frequency Response Analysis of Rogowski Coil

By Laplace transform, the transfer function of the equivalent circuit (Fig. 3) can be written as:

\[
\frac{V(s)}{I(s)} = \frac{M_c R_s s}{A s^2 + B s + C}
\]

where, \(A\), \(B\) and \(C\) are:

\[
A = L_c C_c (R_t + R_s)
B = [L_c + R_s C_c (R_t + R_s) + R_s R_t C_c]
C = (R_t + R_s)
\]

By converting \(s\) to \(j\omega\) in (1), the phase and amplitude function of the equivalent circuit can be obtained:

\[
|H(j\omega)| = \frac{M_c R_s \omega}{\sqrt{(C - A\omega^2)^2 + \omega^2 B^2}}
\]

\[
\angle H(j\omega) = \tan^{-1} \frac{C - A\omega^2}{\omega B}
\]

Figure 4 shows the Bode diagram of the coil’s transfer function with corresponding typical parameters of Rogowski coil and its integrator circuit listed in Table 1.

The parameters shown in Table 1 can be divided as:

a) Dependent parameters: \(R_c\), \(L_c\) and \(M_c\) influenced by structural variations.

b) Independent parameters: \(R_t\), \(C_i\) and \(R_i\). Initially, prior to the design modification, the values of the dependent parameters (Table 1) were taken from Ramboz design values [4] while the independent values are selected arbitrarily.

![Fig. 1 Construction of Rogowski coil.](image)

![Fig. 2 Equivalent circuit of Rogowski coil.](image)

![Fig. 3 Extended Equivalent circuit of Rogowski coil with integrator circuit.](image)
Table 1 Typical Parameters of Rogowski Coil and its Integrator Circuit.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_c$</td>
<td>206.7 mΩ</td>
</tr>
<tr>
<td>$L_c$</td>
<td>34.502 µH</td>
</tr>
<tr>
<td>$M_c$</td>
<td>0.2654 µH</td>
</tr>
<tr>
<td>$R_t$</td>
<td>2 Ω</td>
</tr>
<tr>
<td>$C_i$</td>
<td>1 µF</td>
</tr>
<tr>
<td>$R_i$</td>
<td>100 Ω</td>
</tr>
</tbody>
</table>

Figure 4 shows the variations of amplitude and phase function with frequency for Rogowski coil. It is clearly evident that the phase angle between output voltage and input current is very inconsistent. Basically, in time domain this difference is regarded as a time delay that is, the output voltage does not track the input current exactly. This inconsistency can lead to a couple of undesirable effects in the system connected to the coil:

a) creating a time delay in the protection process during fault conditions.

b) unrecognizing the phase difference between voltage obtained from the potential transformer (PT) and current received from Rogowski coil. This may lead to error in measuring active or reactive power.

To overcome these undesirable effects, the independent parameters of the phase function of the coil in (5) must be changed to suit the modified design. Hence:

$$ (R_c + R_t) - \omega^2 L_c C_i (R_c + R_t) = 0 $$

$$ \Rightarrow (R_c + R_t) = \frac{(R_c + R_t)}{\omega^2 L_c C_i} $$

However, any change in the structure of the coil could lead to undesirable effect in all the dependent parameters in (5). Thus, keeping the values of $R_c$, $R_t$, $L_c$ and $\omega$ in (5) unchanged, it can be rewritten as:

$$ 2.2067 - 3.4052 C_i (R_t + 2) = 0 $$

$$ \Rightarrow C_i (R_t + 2) = 0.648 $$

Based on (6) and keeping $R_c$ constant, the variations of resistance with capacitance is shown in Fig. 5 from which optimal parameters for the integrator circuit can be obtained so as to nullify the time delay of the coil. Fig. 6 shows the same but with three different values of the terminal resistor, $R_t$.

It is evident from (5) that by decreasing $R_t$, $C_i - R_t$ curve is declined. For example, if $C_i = 40$ µF, for $R_t = 1$ Ω, 2 Ω and 5 Ω, the corresponding values of $R_t$ will be 8.779 kΩ, 16.2 kΩ and 38.22 kΩ, respectively. This implies that for an optimal design, the independent parameters should be selected such that nullifying the phase function while the amplitude function is maximum. Hence, the amplitude function is:

$$ |H(j\omega)| = \frac{MR_i}{L_c + R_c C_i (R_c + R_i) + R_c R_i C_i} $$

Fig. 4 Bode diagram of whole system.
Fig. 5 Drawn curve based on (6).
Fig. 6 Variation of the $C_i - R_t$ curve in $R_t = 1$ Ω, 2 Ω and 5 Ω.
Substituting (5) into (7), will yield to:

\[ |H(j\omega)| = \frac{MR_i}{L_i + \frac{(R_i + R_c)^2}{L_i \omega^2} - R_i^2 C_i} \quad (8) \]

By using (8), there are three options for increasing the output of the coil.

a) increasing the mutual inductance, \( M_c \) of the coil as described in [11]. However, the change of the coil's structure is the drawback of this option.

b) increasing the capacitance \( C_i \) of the integrator circuit, thereby increasing the amplitude function to some extent.

The best option is to find an optimum value of terminal resistor (\( R_t \)) thereby increasing the coil's output to maximum extent, as shown in Fig. 7. That is:

\[ \frac{\partial |H(j\omega)|}{\partial R_t} = 0 \Rightarrow R_t = \sqrt{\frac{L_i \omega^2 + R_c^2}{1 - L_i C_i \omega^2}} \quad (9) \]

According to (9), the optimal value of \( R_t \) depends on \( C_i \) as shown in Fig. 7. For making optimal decision on the independent parameters, initially, the highest permissible capacitance should be selected so as to obtain an optimal value of terminal resistor, considering operating frequency. This will lead to calculate the value of \( R_t \) from (5).

It stands to reason that the output voltage of the coil can trace the input current in an exact manner with the maximum amplitude.

An experimental data using Rogowski coil for measuring partial discharge is shown in Fig. 8.

Although it shows the Rogowski coil behavior for high frequency measurement, it reveals information concerning the idea upon which the variation in termination elements (for \( R=0.5 \) ohms and \( R=10 \) ohms) could change the time delay of coil output. As a result, the proposed optimal termination element provides an excellent accuracy for the current measurement particularly, for the phase function.

4 Performance Evaluation of Rogowski Coil and Current Transformer in Fundamental Frequency

It is now an arguable question for the superiority of Rogowski coil over CT for power frequency. In power system, Rogowski coil is used for measurement and protection applications as the current transformer. So, in the performance evaluation, these applications should be considered.

Equipment used for measurement and protection purposes must possess the following features:

- Tracking the actual value in normal and fault conditions.
- Free from electromagnetic interference (EMI) noises.
- Compatible with system requirement.
- Protecting the monitoring devices in fault conditions.

Undoubtedly, the superiority of Rogowski coil in the last two features is conspicuous. However, for the other features, CT's equivalent circuit, based on IEEE and IEC standard [8], shown in Fig. 9 should be analyzed where, \( Z_s \) is nonlinear impedance describing the behavior of CT's core and \( R_{eq} \) and \( X_{eq} \) are the series equivalent resistance and inductance respectively. Also, \( Z_B \) represents the impedance of CT's load (burden).

Based on electromagnetic theory, the magnetic branch of a transformer consists of a resistance and an
inductance with no-load loss and no-load magnetic VA, respectively. Because of large air gap in CT and \( L \propto 1/R \), the inductance, \( L \) is very low. Thus, the resistance of magnetic branch \( (R_{core}) \) can be neglected. The secondary winding is very close to the core. Thus, the CT equivalent circuit becomes as shown in Fig. 10 where, \( X_s \) is a nonlinear inductance. The vector diagram of simplified equivalent circuit of CT is shown in Fig. 11.

Figure 11 shows that there is a difference, in both amplitude and phase values, between the measured current \( I_B \) with the intended measured current \( I_s \). This difference is caused not only by variation of the CT’s burden, but mainly by changing the magnetizing current \( I_e \). Since we have:

\[
|I_s| = |E_s| = \frac{|E_s|}{2\pi f L_m} = \frac{|E_s|}{2\pi f \mu_0 \mu_r N A / l}
\]

where \( f, \mu_0, \mu_r, N, A \) and \( l \) are the operating frequency, air permeability, relative permeability, number of secondary winding, the cross-sectional area of CT’s core and the mean length of the core respectively.

Because of magnetic property of the core material, the relative permeability is considerably changed by the saturation effect. In addition, the output of CT is affected by another property of magnetic material, the remnant flux. Moreover, these phenomena cause many distortions on CT’s output during fault conditions [8]. As such, Rogowski coils are far better than CTs in tracking the actual value in normal and fault conditions.

The Influence of EMI on Rogowski coil is a drawback, if an operational amplifier is connected to its output. In this study, this has been overcome by using the passive integrator circuit.

Since modern protecting or monitoring devices needs voltage signal input thus, Rogowski coils can be directly connected to them. While, CTs output current signal needs a converter with no negative effects on its burden.

Although, saturation effect of CT’s core is beneficial feature for protecting the monitoring device, this leads to error in the measuring process. However, protecting the monitoring device using Rogowski coil can be performed by introducing a zener diode with no error creation.

5 Conclusion

The analysis of the coil’s transfer function in frequency domain shows that the output voltage is either lagging or leading the measured current. The proposed method improves this undesirable phenomenon. In addition, in zero phase difference for obtaining the maximum voltage output, the terminal resistor effects were considered. Consequently, to nullify the phase difference and to maximize the coil’s output, the suitable \( R_e, R_i \) and \( C_i \) were selected for precise current measurement.

It can therefore be concluded that Rogowski coils are yet more efficient measuring and protecting device which can provide comparable performance with conventional CTs, for low frequency applications.

References


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