

# Effects of Non-Ideal Pre-Distorter High Power Amplifiers in WCDMA Using Multi-User Detectors

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**Abstract:** Wide band code division multiple access (WCDMA) signals, transmitted by the base station high power amplifiers (HPAs), show high peak to average power ratios (PAPR), which results in nonlinear distortions. In this paper, using computer simulations effect of using a predistorted HPA on the symbol error rate (SER) of multi-user detectors in terms of output back-off (OBO) in the transmit power is analyzed. As well, using polynomials for modeling predistorters to remove nonlinear distortions of traveling wave tube amplifiers (TWTAs) and of solid state power amplifiers (SSPAs), effect of different degrees of polynomials on the SER is investigated. Simulation results show that a polynomial of degree 4 is a sufficient degree polynomial, which fits to the AM/AM characteristic of the predistorter for TWTAs. As well, for solid state power amplifiers (SSPAs) with different  $p$  values, different approximations are considered and sufficient degree polynomials are found.

**Keywords:** WCDMA, HPA; MUD, Back off, PAPR, TWTA, SSPA.

## 1 Introduction

Wide band code division multiple access (WCDMA) technique is employed in the third generation mobile networks to provide multimedia services with required qualities. A major drawback of WCDMA networks is a high peak to average power ratio (PAPR) problem, which results in nonlinear distortion in the transmitter high power amplifier (HPA). The nonlinear distortion of the HPA degrades the quality of the receiver. The simplest method to overcome the nonlinear distortion is to back-off the quiescent point of the HPA from the saturation to the linear region at the cost of reducing the power efficiency of the HPA. The amount of back-off required for providing a target symbol or bit error rate, in WCDMA networks, depends on the receiver structure and its parameters.

Conti and Dardari analyze effects of high power amplifiers (HPAs) on the downlink CDMA for matched filter detectors in additive white Gaussian noise (AWGN) channels [1]. Rugini and Banelli do a similar work to that of [1], but for decorrelating detector and MMSE detector, in [2] and [3], respectively, where both of them are multi-user detectors (MUDs), as MUDs can significantly improve the performance of

CDMA receivers [4]. A predistorter can alleviate nonlinear distortions of the HPAs. For this reason, in [5], Ryu and Park analyze the symbol error rate (SER) of a predistorted OFDM system over AWGN channels. In [6], the SER performance of MUDs is studied in terms of OBO in the HPA over AWGN plus flat and/or non-flat fading channels. In [7,8] Weerasinghe and Hasimoto propose convolutional spreading of CDMA scheme with cyclic prefix for PAPR controlling of transmitted signal over downlink transmission. Moreover, in a similar work, PAPR reduction method using constraint code in multicode CDMA system has been considered in [9].

This paper analyzes the down link SER performance of either *linear decorrelating detector* (LDD) or *minimum mean square error* (MMSE) detector in a CDMA system with a *predistorted HPA* over AWGN plus flat fading channels using computer simulations. To adjust the predistorter, an accurate model is required for the nonlinear distortion generated by the HPA. In this direction, we consider *traveling wave tube amplifier* (TWTA) and *solid state power amplifier* (SSPA). We model the predistorter of these two amplifiers by polynomials of different degrees, and through computer simulations, we find the best model.

The remainder of this paper is organized as follows. Section II presents the system model, which is analyzed in Section III. In Section IV, nonlinear distortions of HPAs are characterized and effective signal to noise ratio is defined. Simulation results are presented in

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Section V to find the best model for the predistorters, and to study effects of using different predistorted HPAs on the SER performance. The last section presents conclusions.

## 2 System Model

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We consider the downlink of a WCDMA network. The base station transmits signals for all of  $K$  active users under its coverage. The composite signal to be transmitted is given by

$$x(t) = \sum_{k=1}^K \sum_{j=-\infty}^{+\infty} A_k d_k[j] s_k(t - jT), \quad (1)$$

where  $A_k$ ,  $d_k[j]$ ,  $s_k(t)$  are, respectively, the amplitude,  $j^{\text{th}}$  data symbol, and spreading waveform of user  $k$ ; and  $T$  is the symbol time. The spreading waveform  $s_k(t)$  is expressed by

$$s_k(t) = N^{-1/2} \sum_{m=0}^{N-1} c_k[m] p(t - mT_c), \quad (2)$$

where  $N$  is the processing gain,  $T_c = T/N$  is the chip duration,  $p(t)$  is the chip pulse shaping waveform with unit energy, and  $c_k[m]$  is the value of the  $m^{\text{th}}$  chip with  $|c_k[m]| = 1$ . Data symbols of different users have independent identically distributions (i.i.d.).

The high power amplifier (HPA) of the base station transmits a composite of  $K$  signals. The characteristics of the HPA are given by its AM/AM and AM/PM distortion functions denoted by  $\mathbf{G}(\bullet)$  and  $\boldsymbol{\varphi}(\bullet)$ , respectively [10]. Using these two functions, the base-band output voltage signal  $\omega(t)$  in terms of input  $x(t)$  of the HPA is given by

$$\omega(t) = G(|x(t)|) e^{j\phi[|x(t)|] + j\arg[x(t)]}, \quad (3)$$

where,  $\omega(t)$  may be expressed as a linear amplification of the input signal  $x(t)$  plus a nonlinear distortion noise, i.e. [1]

$$\omega(t) = \alpha_0 x(t) + n_d(t). \quad (4)$$

The peak to average power ratio (PAPR) of input signal  $x(t)$  is defined as

$$PAPR = \frac{\max\{x(t)^2\}}{E\{x(t)^2\}}. \quad (5)$$

For the HPA, the input back-off (IBO) is defined as [2]

$$IBO = \frac{P_{x,sat}}{E\{|x(t)|^2\}}, \quad (6)$$

where  $P_{x,sat}$  is the saturated input power of the HPA. Similarly, the output back-off (OBO) is defined as [2]

$$OBO = \frac{P_{w,sat}}{E\{|w(t)|^2\}}. \quad (7)$$

A predistorter may be used for the compensation of the HPA nonlinearity. The amplitude transfer function and the phase transfer function of the predistorter are denoted by  $\mathbf{T}[\bullet]$  and  $\boldsymbol{\theta}[\bullet]$ , respectively, which are given by

$$\begin{cases} \mathbf{T}[\bullet] = \mathbf{G}^{-1}[\bullet] \\ \boldsymbol{\theta}[\bullet] = -\boldsymbol{\varphi}[\bullet] \end{cases}, \quad (8)$$

where  $\mathbf{G}^{-1}[\bullet]$  is the inverse of  $\mathbf{G}[\bullet]$  in non-saturated region of the amplifier. We consider each of the TWTA and/or SSPA as the HPA. In the case of TWTA, the AM/AM and AM/PM conversion functions are, respectively, given by [11]

$$\text{AM/AM: } \mathbf{G}(|u|) = 2u_0 |u| / (1 + |u|^2), \quad (9)$$

$$\text{AM/PM: } \boldsymbol{\varphi}(|u|) = (\pi / 3) |u|^2 / (1 + |u|^2), \quad (10)$$

where  $|x|$  denotes the absolute value of  $x$ ,  $u_0 = \sqrt{E(|x|^2) \times \text{OBO}}$  and  $|u|$  is defined as  $|u| = |x| / u_0$ . The characteristics (AM/AM and AM/PM) of the predistorter for linearizing the TWTA are given by

$$\text{AM/AM: } \mathbf{T}[|x|] = \begin{cases} (u_0 - \sqrt{u_0^2 - u^2}) / u & 0 < |u| < u_0 \\ u_0 & |u| \geq u_0 \end{cases} \quad (11)$$

$$\text{AM/PM: } \boldsymbol{\theta}[|u|] = (-\pi / 3) |u|^2 / (1 + |u|^2). \quad (12)$$

As seen in (11),  $\mathbf{T}[ ]$  has a discontinuity at  $|u| = 0$ .

Thus,  $|u|$  is limited to the interval of zero and  $u_0$ .

The AM/AM and AM/PM characteristic function of SSPA, respectively, are given by [5]

$$\text{AM/AM: } \mathbf{G}(|u|) = \frac{|u|}{(1 + (\frac{|u|}{u_0})^{2p})^{1/2p}}, \quad (13)$$

$$\text{AM/PM: } \boldsymbol{\varphi}(|u|) = 0. \quad (14)$$

The value of parameter  $p$  determines the intensity of nonlinear distortion. Smaller values for  $p$  result in increasing the intensity of nonlinear distortion. The characteristic functions of the predistorter for the above SSPA are given by

$$\text{AM/AM:T}[\mu] = \begin{cases} \frac{u}{2p\sqrt{1-(\frac{u}{u_0})^{2p}}} & 0 < |u| < u_0 \\ u_0 & |u| \geq u_0 \end{cases}, \quad (15)$$

$$\text{AM/PM:\theta}[\mu] = 0. \quad (16)$$

Equation (15) has a discontinuity at  $u = u_0$ . Thus,  $|u|$  is considered to be smaller than  $|u| \leq u_0^* < u_0$ . We consider  $u_0^* = 0.99u_0$ .

In the TWTA and SSPA models, amplitudes larger than  $u_0$  (i.e.  $|u| \geq u_0$ ) are clipped by  $u_0$ . This characteristic is called soft clipping or soft limiting. This soft clipping causes the SER performance has been degraded. However, if the input signal resides in the regions of interest, i.e.  $0 < |u| \leq u_0$  for the TWTA model and  $|u| \leq u_0^*$  for the SSPA model, the SER is not degraded.

Implementation of the predistorters given by (11) and (12) for the TWTA model, and by (15) and (16) for the SSPA, are quite difficult due to having complicated mathematical operations of powering to 0.5 and/or to  $1/2p$ . To overcome this problem, we propose using different degrees of polynomials to approximate the exact predistorter model. This approximation causes some distortion in the expense of using a simple model. The coefficients of these polynomials are derived using a minimum square error (MSE) criterion. Square error function between  $\mathbf{T}[\bullet]$  and its model,  $\tilde{\mathbf{T}}[\bullet]$  is

$$\mathbf{e}(\bullet) = [\mathbf{T}[\bullet] - \tilde{\mathbf{T}}[\bullet]]^2. \quad (17)$$

The characteristics of the combined predistorter and HPA, in the ideal case and with using polynomial model, are  $\mathbf{L}(\bullet) = \mathbf{G}[\mathbf{T}[\bullet]]$  and  $\tilde{\mathbf{L}}(\bullet) = \mathbf{G}[\tilde{\mathbf{T}}[\bullet]]$ , respectively. In Section V, a sufficient degree polynomial to approximate each of the above two predistorters is found numerically. By sufficient degree polynomial, we mean a minimal degree polynomial, which fits to the *ideal predistorted amplifier* (IPA) so that it results in similar SER as that of the IPA. The optimization has been done on the model of predistorter of TWTA and SSPA. The distribution of the amplitude at the input of the HPAs is considered uniform.

The composite signal of the down link CDMA system, after amplifying by the predistorted amplifier passes through the two different channels: AWGN, AWGN plus flat Rayleigh fading. In a flat Rayleigh fading channel, the voltage gain  $\beta$  and phase  $\theta$  of the channel have Rayleigh and uniform probability density functions (PDF).

### 3 System Analysis

Now, the analysis of the predistorted amplified composite signal received by a mobile station, through a channel having an impulse response denoted by  $g(\tau, t)$  whose output is added by the thermal noise  $n(t)$ , is presented. The output of the predistorted amplifier,  $\omega(t)$  given by (4), after passing through the channel denoted by  $r(t)$ , that is [2]

$$\begin{aligned} r(t) &= \int_{-\infty}^{+\infty} g(\tau, t)\omega(t-\tau)d\tau + n(t) \\ &= r_{SIG}(t) + r_{NL}(t) + n(t), \end{aligned} \quad (18)$$

where  $r_{SIG}(t) = \beta(t)\alpha_0 e^{j\theta(t)}x(t)$  denotes the useful signal,  $r_{NL}(t) = \beta(t)e^{j\theta(t)}n_d(t)$  is the nonlinear distortion noise, and  $n(t)$  is the AWGN.

We assume a receiver with coherent demodulation, perfect time and frequency synchronization, and phase-shift recovery. The signal  $r(t)$  is filtered by a chip-matched filter whose output for the  $n^{\text{th}}$  chip of the  $l^{\text{th}}$  symbol is [2]

$$\begin{aligned} r_n[l] &= \int_{-\infty}^{+\infty} r(t)p^*(t-lT-nT_c)dt \\ &= r_{n,SIG}[l] + r_{n,NL}[l] + r_{n,AWGN}[l]. \end{aligned} \quad (19)$$

The above received sample,  $r_n[l]$  has three additive

parts:  $r_{n,SIG}[l] = \int_{-\infty}^{+\infty} r_{SIG}(t)p^*(t-lT-nT_c)dt$  is the useful component related to  $x(t)$ ;

$r_{n,NL}[l] = \int_{-\infty}^{+\infty} r_{NL}(t)p^*(t-lT-nT_c)dt$  is the in-band nonlinear distortion, which is denoted by the quantity

$n_d(t)$ ; and  $r_{n,AWGN}[l] = \int_{-\infty}^{+\infty} n(t)p^*(t-lT-nT_c)dt$  is the in-band AWGN with power  $\sigma_{AWGN}^2 =$

$E\{[r_{n,AWGN}[l]]^2\}$ . Defining,  $\beta[l] = \beta(lT)$ ,

$\phi[l] = \theta(lT) + \arg(\alpha_0)$ ,  $\mathbf{c}_k = N^{-1/2}[c_k[0] \dots c_k[N-1]]$ ,

$\mathbf{r}[l] = [r_0[l] \dots r_{N-1}[l]]^T$ ,  $\mathbf{A} = \text{diag}(\mathbf{A}_1, \dots, \mathbf{A}_K)$ ,  $\mathbf{d}[l] = [d_1[l] \dots d_K[l]]^T$ ,  $\mathbf{C} = [c_1 \dots c_K]$ , it is easy to derive [2]

$$\begin{aligned} \mathbf{r}[l] &= \mathbf{r}_{SIG}[l] + \mathbf{r}_{NL}[l] + \mathbf{r}_{AWGN}[l] \\ &= \beta[l]|\alpha_0|e^{j\phi[l]}\mathbf{C}\mathbf{A}\mathbf{d}[l] + \mathbf{r}_{NL}[l] + \mathbf{r}_{AWGN}[l]. \end{aligned} \quad (20)$$

For any linear detector, the decision variable is obtained by a linear combination of the elements of received vector  $\mathbf{r}[l]$  in (20). The receiver decision is  $\hat{\mathbf{d}}[l]$ , which can be expressed as [2]

$$\hat{\mathbf{d}}[l] = \text{sgn}\{\text{Re}(\mathbf{D} \mathbf{r}[l] \exp(-j\phi[l]))\}, \quad (21)$$

where the matrix  $\mathbf{D}$  represents the operation of multi-user detector (will be defined in the next paragraph). The channel phase,  $\theta$ , and the mean HPA phase-shift,  $\arg(\alpha_0)$ , are assumed to be perfectly known to the receiver.

The operation of the linear decorrelating detector (LDD) is obtained as the Moore-Penrose generalized inverse of the code matrix  $\mathbf{C}$  [4]:

$$\mathbf{D}_{\text{LDD}} = (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T = \mathbf{R}^{-1} \mathbf{C}^T, \quad (22)$$

where  $\mathbf{R} = \mathbf{C}^T \mathbf{C}$  is the  $K \times K$  matrix containing the cross correlation coefficients of the user's spreading sequences. In (22), it is assumed that spreading sequences of different users are linearly independent, to guarantee the existence of  $\mathbf{R}^{-1}$ . It is also considered that the number of active users  $K$  is time-invariant (see [12] for the LDD analysis in a user dynamic environment) and smaller than the processing gain  $N$ .

The operation of scaled version of the MMSE receiver can be expressed by [3],

$$\begin{aligned} \mathbf{D}_{\text{MMSE}} &= (\mathbf{I}_K + |\alpha_0 \mathbf{g}_0|^2 \mathbf{A} \mathbf{C}^H \mathbf{W}^{-1} \mathbf{C} \mathbf{A})^{-1} \mathbf{A} \mathbf{C}^H \mathbf{W}^{-1} \\ &= \mathbf{A} \mathbf{C}^H \left( |\alpha_0 \mathbf{g}_0|^2 \mathbf{C} \mathbf{A}^2 \mathbf{C}^H + \mathbf{W} \right)^{-1}, \end{aligned} \quad (23)$$

and  $\mathbf{W}$  is the covariance matrix of the total colored noise term  $\mathbf{r}_{\text{NL}}[l] + \mathbf{r}_{\text{AWGN}}[l]$ .

#### 4 Characterization of the Nonlinear Distortion

Under the assumptions of interference is Gaussian [1] and the SIR at input and output of downlink channel remains unchanged, the HPA input signal  $x(t)$ , in (1), can be approximated by a cyclostationary Gaussian process. This makes it possible to apply the Busgang theory [13]. Using this theory, the linear and nonlinear components, (i.e.  $\alpha_0 x(t)$  and  $n_d(t)$  in (3)) are mutually uncorrelated, and the autocorrelation of the HPA output signal  $\omega(t)$  can be evaluated [14].

Since the nonlinear distortion noise is generated at the transmitter, the apparent signal-to-noise ratio  $\text{SNR}_{\text{app}}$  measured at the decorrelator input is given by [2]

$$\text{SNR}_{\text{app}} = \frac{\sigma_{\text{SIG}}^2 + \sigma_{\text{NL}}^2}{\sigma_{\text{AWGN}}^2}, \quad (24)$$

where  $\sigma_{\text{SIG}}^2 = E\{|r_{\text{SIG}}[l]|^2\}$ ,  $\sigma_{\text{NL}}^2 = E\{|r_{\text{NL}}[l]|^2\}$  and  $\sigma_{\text{AWGN}}^2 = E\{|r_{\text{AWGN}}[l]|^2\}$ . In (24), an increase in NLD

through  $\sigma_{\text{NL}}^2$  results in increasing  $\text{SNR}_{\text{app}}$  but, it degrades the SER. Therefore, we should define the effective SNR, i.e.  $\text{SNR}_{\text{eff}}$ , as [2]

$$\text{SNR}_{\text{eff}} = \frac{\sigma_{\text{SIG}}^2}{\sigma_{\text{AWGN}}^2 + \sigma_{\text{NL}}^2}. \quad (25)$$

The total degradation (denoted by TD) to obtain a target SER can be defined as [2]

$$[\text{TD}]_{\text{dB}} = \left( [\text{SNR}_{\text{app}}]_{\text{dB}} - [\text{SNR}_{\text{eff}}]_{\text{dB}} \right) + [\text{OBO}]_{\text{dB}}, \quad (26)$$

where the difference of the terms inside the parentheses indicates the power penalty with respect to the linear scenario, and the OBO indicates the power penalty with respect to the saturating output power of the amplifier. There is a trade-off between these two power penalties, since an increase in OBO results in a smaller distortion.

#### 4 Citations and References

In this section, we first evaluate the SER performance of multi-user detectors over AWGN channels when the predistorter for each of the TWTA and SSPA is modeled by polynomial functions, and we compare these predistorted power amplifiers with the *ideal predistorted amplifier* (IPA). We use MATLAB for simulation of the system. The IPA model having input  $x$  characterized by  $G(x) = |x|$  for  $|x| < A_{\text{sat}}$  and  $G(x) = A_{\text{sat}}$  when  $|x| > A_{\text{sat}}$ , and  $\phi(x) = 0$ , where  $A_{\text{sat}}$  denotes the peak amplitude of the IPA. The TWTA and SSPA models, respectively, are given by a pair of equations (9), (10); and (15), (16). We assume a rectangular pulse shaping waveform  $p(t)$  and QPSK for modulation of data. The base station transmits data with equal amplitude for each user. Maximal length sequences with length of 63 have been considered for

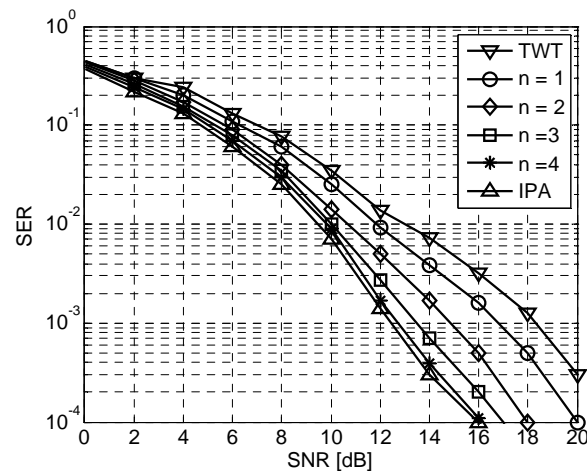
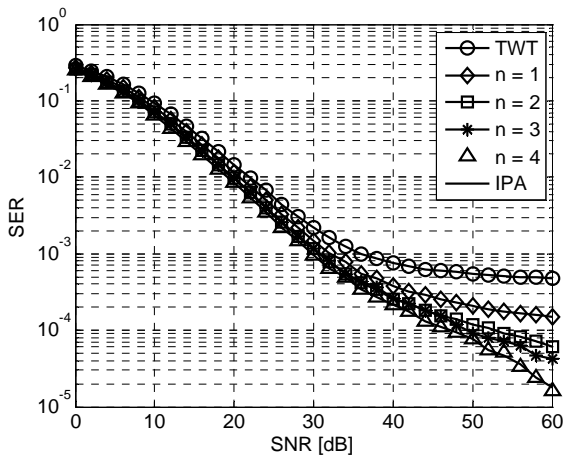


Fig. 1 SER performance of the LDD in AWGN channels for the TWTA with different values of  $n$ . and OBO = 3.82 dB.



**Fig. 2** SER performance of the LDD in flat fading channels for the TWTA with different values of  $n$ , and  $OBO = 3.82$  dB.

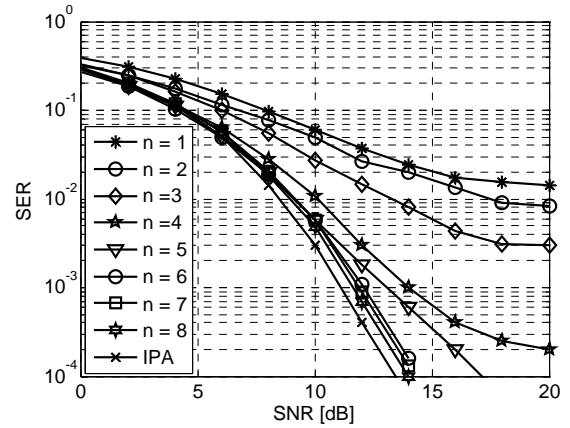
the spreading sequences. The channel model is considered to be AWGN or AWGN plus slow flat Rayleigh fading.

Table 1 shows the polynomials fitted to the predistorter of the TWTA. AM/AM characteristic function. Fig. 1 shows the SER performance of the LDD over an AWGN channel with 25 active users and  $OBO = 3.82$  dB when the TWTA is modeled by different degrees of polynomial functions given in Table 1. As seen in this figure, at SNR of 14 dB, the IPA results in a SER of  $3 \times 10^{-4}$ , while the 4<sup>th</sup> and 3<sup>rd</sup> degrees of the predistorted TWTA results in a SER of  $7 \times 10^{-4}$  and  $1.5 \times 10^{-3}$ , respectively. So, using the 4<sup>th</sup> and 3<sup>rd</sup> degrees polynomials give the sufficient degree polynomials to approximate the predistorted TWTA. Fig. 2 is similar to Fig. 1, but it is for flat Rayleigh fading channels. It can be seen in this figure that the 4<sup>th</sup> degree polynomial gives the sufficient approximation for the predistorted TWTA.

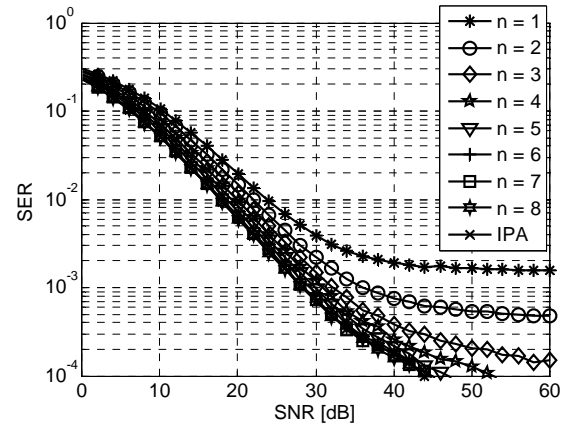
Fig. 3 shows the SER performance of LDDs over AWGN channels when  $p = 1$  for the predistorted SSPA. As it is seen in this figure, the sufficient degree polynomial, which results in the SER close to that of the IPA, is found to be of 8<sup>th</sup> degree, which is computationally inefficient. Fig. 4 is similar to Fig. 3, but it is for flat fading channels. Again, it can be seen in this figure that the sufficient degree polynomial is of 8<sup>th</sup> degree, which is similar to that of AWGN channels.

Table 2 shows the approximated polynomials fitted to the predistorter of the AM/AM characteristic function of the SSPA with  $p = 2$ . In this case, as seen in Figs. 5 and 6 for AWGN and flat Rayleigh fading channels, respectively, a second degree polynomial is a sufficient approximation the modeling of the predistorted SSPA.

Table 3 shows the additional SNR for different values of approximation in order to provide a SER performance close to  $SER = 10^{-3}$  of the IPA. Based on



**Fig. 3** SER performance of the LDD in AWGN channels for the SSPA model ( $p = 1$ ) with different values of  $n$  and  $OBO = 4.82$  dB.



**Fig. 4** SER performance of the LDD in flat fading channels for the SSPA model ( $p = 1$ ) with different values of  $n$  and  $OBO = 4.82$  dB.

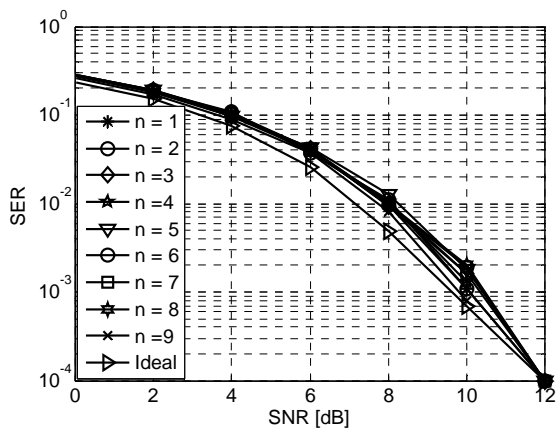
**Table 1** Polynomials fitted to predistorter of the TWTA AM/AM characteristic amplifier.

Polynomial degrees	Corresponding Polynomials
1	$0.7553x - 0.0607$
2	$0.5815x^2 + 0.1532x + 0.0365$
3	$1.1411x^3 - 1.1318x^2 + 0.8396x - 0.0209$
4	$2.4172x^4 - 3.6982x^3 + 1.9833x^2 + 0.1458x + 0.0140$

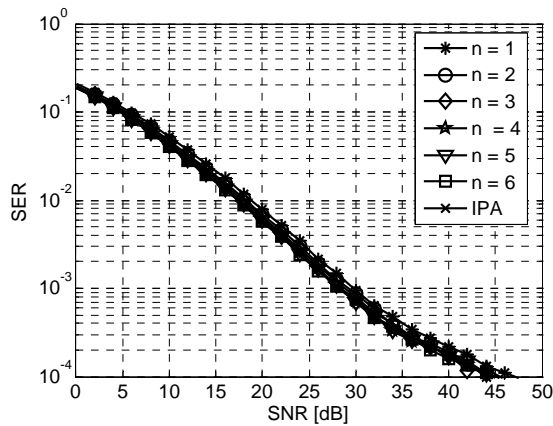
**Table 2** Polynomials fitted to predistorter of the SSPA AM/AM characteristic amplifier with  $p=2$ .

Polynomial degrees	Corresponding Polynomials
1	$1.4730x - 0.1399$
2	$1.6481x^2 - 0.1734x + 0.1339$
3	$4.9637x^3 - 5.7900x^2 + 2.7974x - 0.1128$

this table, we can say that for  $p = 1$ , 8<sup>th</sup> degree polynomial function is the sufficient approximation, since the additional SNR required is only 0.6 dB, and for  $p = 3$  and 6, the first degree polynomial is sufficient



**Fig. 5** SER performance of the LDD in AWGN channels for the SSPA model ( $p = 2$ ) with different values of  $n$  and OBO = 4.82 dB.



**Fig. 6** SER performance of the LDD in flat fading channels for the SSPA model ( $p = 2$ ) with different values of  $n$  and OBO = 4.82 dB.

**Table 3** Additional SNR for the SSPA model with different values of  $p$  and  $n$  at  $SER = 10^{-3}$ .

$p$	$N$	Extra SNR[dB]
1	4	2.9
1	6	1
1	8	0.6
3	1	0.3
3	6	0.1
3	8	0
6	1	0.35
6	6	0.21
6	8	0.1

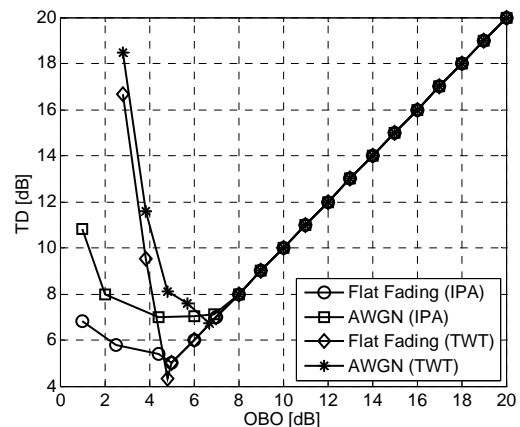
for modeling the predistorter, since in these two cases, the additional SNR required is only 0.3 and 0.35 dB, respectively.

Figures 7 and 8 show the total degradation (TD) when the SER is  $10^{-3}$ . It is clear that for any value of OBO the TD caused by the TWTA model is larger than that of the IPA model. As well, the TD in flat Rayleigh fading scenarios is smaller than those of AWGN channels. This can be intuitively explained by the

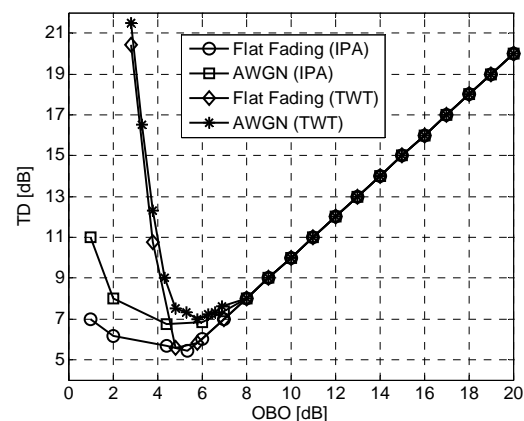
following consideration. In a flat Rayleigh fading scenario, the nonlinear distortion noise increases by increasing the useful signal power. Since most of the errors are committed when the useful signal power is small, the nonlinear distortion noise is partially masked by the thermal noise whose power is independent of the fading gain.

## 6 Conclusions

This paper studied effects of predistorted HPAs on the symbol error rates (SER) of multi-user detectors. Simulation results showed that using a predistorter with the TWTA as the HPA improves the SER of both decorrelating and MMSE detectors. Moreover, it is clear that for any value of OBO the TD in flat Rayleigh fading scenarios is smaller than that of AWGN channels. We employed different degrees of polynomials for implementing a predistorter, and through simulations, we showed that the 4<sup>th</sup> degree polynomial is a sufficient fit to the AM/AM characteristic of the predistorter for the TWTA. For  $p = 1$ , appropriate approximation is a 8<sup>th</sup> degree polynomial function, which is inefficient. For  $p = 2$ , second degree polynomial function is the sufficient approximation. Finally for  $p > 2$ , first degree polynomial function is the sufficient approximation.



**Fig. 7** TD performance for the LDD.



**Fig. 8** TD performance for the MMSE detector.

## References

- [1] Conti A., Dardari D. and Tralli V., "An analytical framework for CDMA systems with a nonlinear amplifier and AWGN," *IEEE Trans. Comm.*, Vol. 50, No. 7, pp. 1110–1120, July 2002.
- [2] Rugini L., Banelli P. and Cacopardi S., "Theoretical analysis and performance of the decorrelating detector for DS-CDMA signals in nonlinear channels," *IEEE Trans. Wireless Comm.*, Vol. 3, No. 2, pp. 367-372, March 2004.
- [3] Rugini L., Banelli P. and Cacopardi S., "SER performance of linear multiuser detectors for DS-CDMA downlink with transmitter nonlinear distortions," *IEEE Trans. Veh. Tech.*, Vol. 53, No. 4, pp. 992-1000, July 2004.
- [4] Verdù S., *Multiuser Detection*, Cambridge Un.Press, 1998.
- [5] Ryu H.-G., Park J. S. and Park J.-S., "Threshold IBO of HPA in the predistorted OFDM communication system," *IEEE Trans. on broadcasting*, Vol. 50, No. 4, pp. 425-428, December 2004.
- [6] Ghavami S. and Abolhassani B., "Performance of Decorrelating and MMSE Detectors with Transmitter Non-Linearity over Non-Flat Fading Channels", *In Proc. of ICTTA*, Vol. 2, pp. 2297-2302, April 2006.
- [7] Weerasinghe N. S. and Hasimoto T., "A convolutional spreading CDMA scheme with cyclic prefix for downlink transmission," *In the proceeding of IEEE International Conference on Communications*, Vol. 4, pp. 2328-2332, May 2005.
- [8] Weerasinghe N. S. and Hasimoto T., "Convolutional spreading CDMA and comparison with the DS-CDMA with RAKE receiver," *IEEE Trans. on Communications*, Vol. 54, No. 11, pp. 1918-1922, Nov. 2006.
- [9] Ryu H. G., Kim S. K, Lee S. E., Wang C. X., "A PAPR Reduction Method using Constraint Code in Multicode CDMA System," *In the proceeding of IEEE Pacific Rim Conference on Communications, Computers and Signal Processing*, pp. 296-299, Sep. 2007.
- [10] Kaye A. R., George D. A. and Eric M. J., "Analysis and compensation of bandpass nonlinearities for communications," *IEEE Trans. Commun.*, Vol. 20, No. 5, pp. 965–972, Oct. 1972.
- [11] Saleh A. A. M., "Frequency-independent and frequency-dependent nonlinear models of TWT amplifiers," *IEEE Trans. Comm.*, Vol. 29, No. 11, pp.1715–1720, Nov. 1981.
- [12] Mitra U. and Poor H. V., "Analysis of an adaptive decorrelating detector for synchronous CDMA channels", *IEEE Trans. on Comm.*, Vol. 44, No. 2, pp. 257-268, Feb. 1996.
- [13] Papoulis A., *Probability, Random Variables and Stochastic Processes*, 4th ed., New York: McGraw-Hill, 2001.
- [14] Rugini L., Banelli P. and Cacopardi S., "An analytical upper bound on MMSE performance using approximated MMSE multiuser detector in flat Rayleigh fading channels," *in Proc. Eur. Wireless*, Vol. 2, Florence, Italy, pp. 952–956, Feb. 2002.



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