

Adaptive Subcarrier Assignment and Power Distribution in Multiuser OFDM Systems with Proportional Data Rate Requirement

A. Falahati* and M.-R. Ardestani*

Abstract: A low complexity dynamic subcarrier and power allocation methodology for downlink communication in an OFDM-based multiuser environment is developed. The problem of maximizing overall capacity with constraints on total power consumption, bit error rate and data rate proportionality among users requiring different QOS specifications is formulated. Assuming perfect knowledge of the instantaneous channel gains for all users, a new simple algorithm is developed to solve the mentioned problem. We compare the sum capacity, proportionality, and computational complexity of the proposed algorithm with the one presented by Wong et al. Numerical results demonstrate that the proposed algorithm offers a performance comparable with Wong's algorithm, yet complexity remains low and proportionality constraint will be tightly satisfied. As well, the proposed algorithm can provide a flexible trade-off between complexity, capacity and proportionality constraint.

Keywords: Adaptive Multiuser OFDM, Multiuser Diversity, Proportionality, Resource Allocation.

1 Introduction

One of the major challenges in broadband data transmission over wireless channels is known as inter-symbol interference (ISI). Orthogonal frequency division multiplexing (OFDM) is a flexible and bandwidth-efficient modulation technique which has the capability to combat ISI, because OFDM systems divide a wide-band channel profile into many narrowband orthogonal subchannels, each carrying a Quadrature Amplitude Modulation (QAM) symbol.

Since the channel fading on each subcarrier is different, adaptive power allocation and modulation scheme can be employed for each subcarrier independently. With the growing demand of wireless data traffic, adaptive resource allocation has been identified as a necessary condition for efficient utilization of the limited power and bandwidth resources in the future wireless networks.

Multiuser OFDM (MU-OFDM), also referred as orthogonal frequency division with multiple access (OFDMA), is based on OFDM and provides an orthogonal multiple access method. OFDMA allows multiple users to share an OFDM symbol, each owning

a mutually disjoint set of subcarriers. Since the different users experience mutually uncorrelated fading, it is possible to obtain multiuser diversity gains by exploiting a channel aware resource allocation scheme which improves the total offered capacity.

Two types of multiuser resource allocation problems are referred as margin adaptive (MA) [3] and rate adaptive (RA) [4,5,6,7]. The margin adaptive objective is to minimize the total transmit power with subject to constraints on the users' data rate and bit error rate (BER). The rate adaptive aims to maximize the overall system capacity with a total power constraint.

Wong et al [1,2,7] extended the rate maximization problem to deal with the proportionality constraint by introducing a predetermined set of priority parameters. The proportionality constraint is an important condition which allows for quality of service (QOS) level differentiation and flexible billing mechanism. The algorithm proposed in [7] is a near optimal solution, but requires solving a set of nonlinear equations for power distribution and hence, is complex. In [1], the previous solution has been simplified by assuming that each user should be assigned a number of subcarriers proportional to its predefined priority index. This assumption results in linear equations for power distribution among users, and hence a large simplification is obtained. On the other hand, however, the recent algorithm does not guarantee a tight fulfillment of proportionality.

Iranian Journal of Electrical & Electronic Engineering, 2008.
Paper first received 23rd October 2006 and in revised form 20th January 2008.

* The Authors are with the Department of Electrical Engineering, Iran University of Science and Technology (IUST), Narmak, Tehran, Iran.
E-mail: afalahati@iust.ac.ir, Ardestani@ee.iust.ac.ir.

In this paper, we have found a new simple allocation method to solve the proportionality-constrained rate maximization problem formulated in [1,7]. This algorithm involves two stages; in the first stage, similar to [1], subcarrier assignment is performed to maximize the total shared capacity by considering the users' priorities. The second stage, iteratively attempts to satisfy the proportionality constraint. This stage includes solving only one nonlinear equation which is performed by a simple bisection search strategy. As well, an adjustable threshold is introduced which provides a flexible trade-off between complexity, capacity and proportionality tightness. A notable feature of the proposed method is reducing the number of unknown powers from N (number of existent subcarriers) to K (number of active users). Finally, we have compared our algorithm with the one proposed in [1] through computer simulations and complexity analysis.

2 System Description

Consider an OFDMA downlink transmission in a single cell with K active users as shown in Fig. 1. The time-varying channels between base station (BS) and different users are assumed to be frequency-selective wireless channels with independent Rayleigh fading. The BS multiplexes users' data on a shared OFDM symbol. The total available bandwidth B is assumed to be divided into N subchannels with flat fading. The transmit power budget of the BS is limited and denoted by P_{total} . For a time division duplex (TDD) scenario, forward channel information is measured by mobile users and is fed back to the BS. The resource allocation algorithm then assigns a mutually disjoint subset of subcarriers to each user and selects different numbers of bits from different users' data stream to form an OFDM symbol. In this article, perfect channel state information (CSI) is assumed to be available at the BS. It is also assumed that the subcarrier and modulation allocation information is sent to each user by a separate channel.

The objective is to reach the maximum data capacity over the shared forward link with constraints on total transmit power, bit error rate and proportionality among users' resultant rates.

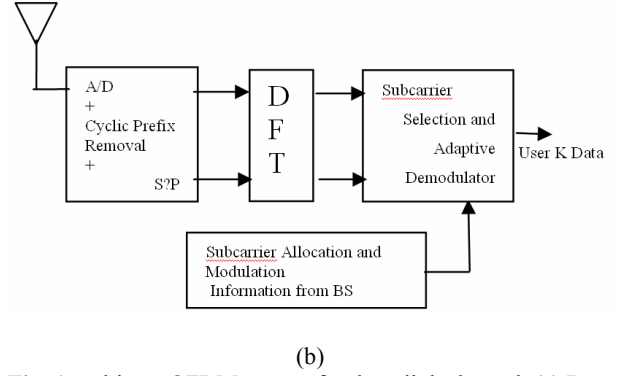
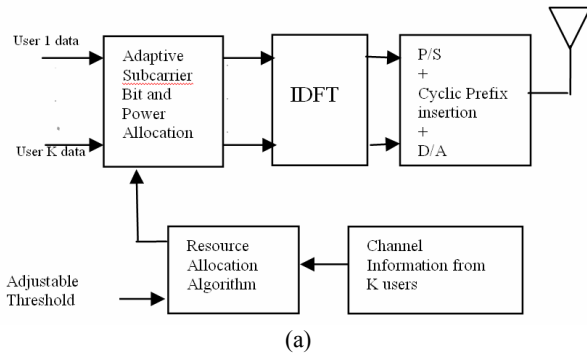


Fig. 1 multiuser OFDM system for downlink channel, (a) Base Station transmitter, (b) mobile user k receiver.

Mathematically, the optimization problem can be formulated as [7]:

$$\max_{\rho_{k,n}} \sum_{k=1}^K \sum_{n=1}^N \frac{\rho_{k,n}}{N} \log_2 \left(1 + \frac{\rho_{k,n} h_{k,n}^2}{\Gamma N_0 \frac{B}{N}} \right) \quad (1)$$

subject to:

$$\sum_{k=1}^K \sum_{n=1}^N \rho_{k,n} \leq P_{total}$$

$$\rho_{k,n} \geq 0, \text{ for all } k, n$$

$$\rho_{k,n} \in \{0,1\}, \text{ for all } k, n$$

$$\sum_{k=1}^K \rho_{k,n} = 1; \text{ for all } n$$

$$R_1 : R_2 : \dots : R_K = \gamma_1 : \gamma_2 : \dots : \gamma_K$$

N_0 is the power spectral density of additive white Gaussian noise. $p_{k,n}$ denotes the power allocated to the n th subcarrier of the k th user. $h_{k,n}$ stands for the channel gain of the n th subcarrier of the k th user. $\Gamma = -\ln(5BER)/1.6$ is named SNR gap. $\rho_{k,n}$ is a binary variable, indicating whether the subcarrier n is assigned to user k or not. The fourth constraint in (1) ensures that each subcarrier to be used only by one user. The capacity of the k th user is expressed as

$$R_k = \sum_{n=1}^N \frac{\rho_{k,n}}{N} \log_2 (1 + p_{k,n} H_{k,n}) \quad (2)$$

The channel-to-noise ratio (CNR) of the n th subcarrier of the k th user is defined as $H_{k,n} = h_{k,n}^2 / (\Gamma N_0 \frac{B}{N})$. Finally, $\{\gamma_k\}_{k=1}^K$ is a set of predetermined values which are used to ensure proportional data rate among various users.

The optimal solution is obtained as follows [7]. In a system with K users and N subcarriers, there are K^N possible subcarrier assignments, since it is assumed that no subcarrier can be used by more than one user. For a certain subcarrier assignment, an optimal power distribution based on water-filling can be used to maximize the overall capacity, while maintaining proportional fairness. The maximum capacity over all K^N subcarrier assignment schemes is the global maximum and the corresponding subcarrier assignment

and power distribution is the optimal resource allocation scheme. However, the complexity of the optimal solution is terribly high and the system scalability and cost-effective implementation will be impossible.

3 New Proposed Solution

In the proposed algorithm, the following assumptions have been considered to simplify the solving of the previously mentioned problem in (1):

- I. Similar to the algorithm presented in [1], the number of subcarriers to be assigned to each user k , is selected as

$$S_k = \left\lceil N \cdot \gamma_k / \sum_{k=1}^K \gamma_k \right\rceil \quad (3)$$

This may lead to a few unallocated subcarriers which will be assigned to the users with the least number of assigned subcarriers.

- II. All the subcarriers assigned to each user k , will be allocated the same transmit power P_k . Therefore, the number of unknown powers $\{P_k\}_{k=1}^K$ is highly reduced from N (as in [1]) to K .

According to above assumptions, each user k is allocated the total power of $S_k P_k$ and hence, the total transmit power constraint may be expressed as

$$\sum_{k=1}^K S_k P_k = P_{\text{total}} \quad (4)$$

The proposed algorithm consists of two parts, Initial Resource Allocation (IRA) and Iterative Fairness Retrieve (IFR). In the first stage, subcarrier assignment is performed assuming the uniform power distribution among users. The IRA provides a large capacity and satisfies all the constraints in (1), except proportionality. Once a subcarrier allocation has been decided, IFR provides a feasible solution with the required proportionality through a low-complexity iterative power manipulation procedure.

A. Initial Resource Allocation (IRA)

This stage essentially assigns appropriate subcarriers to users in order to maximize capacity. This is done by equal power distribution among users. Similar to the algorithm presented in [1], the number of subcarriers to be assigned to each user is determined as in (3). The best subcarrier of each user is then found to maximize the overall capacity. If there is no conflict between the best subcarriers of individual users, each user can have its desired subcarrier. If there is any conflict, however, the users require to be prioritized. A suitable measure to prioritize the users can be proportionality deviation. The IRA subroutine is as follows:

$$1) \quad U = \{1, \dots, K\}, \quad A = \{1, \dots, N\}, \quad R_k = 0, \quad \rho_{k,n} = 0, \\ \forall k \in U, \quad \forall n \in A, \quad p = P_{\text{total}}/N$$

$$2) \quad S_k = \left\lceil N \cdot \gamma_k / \sum_{k=1}^K \gamma_k \right\rceil, \quad k = 1, \dots, K, \quad \text{allocate}$$

the remainder subcarriers to users with the least S_k 's

$$3) \quad \text{for all } k \in U \quad n_k^* = \arg \max_{n \in A} H_{k,n}$$

4) priority: sort (R_k / γ_k) , $k \in U$, in ascending order and call the ordered U as prior_U

$$5) \quad \text{for all } k \in \text{prior_U}$$

if subcarrier n_k^* not yet assigned to another user

$$\rho_{k,n_k^*} = 1, \quad A = A - \{n_k^*\}, \quad S_k = S_k - 1$$

$$R_k = R_k + \frac{1}{N} \log_2(1 + p \cdot H_{k,n})$$

$$6) \quad U = U - \{\text{users whose } S_k = 0\}$$

if $A = \{\}$ or $U = \{\}$, Exit, otherwise Goto step (3)

B. Iterative Fairness Retrieve (IFR)

By swapping allocated power between users, IFR iteratively reduces the maximum proportionality deviation. Define max_user as the user with the largest (R_k / γ_k) ratio at each iteration and min_user as the user with the least one. In addition, define S_{max} (S_{min}) as the number of subcarriers assigned to max_user (min_user). Now we can form the maximum proportionality deviation as

$$\Delta = \frac{R_{\text{max_user}}}{\gamma_{\text{max_user}}} - \frac{R_{\text{min_user}}}{\gamma_{\text{min_user}}} \quad (5)$$

Suppose we take the power x from each subcarrier of the user max_user and give the power y to each subcarrier of the user min_user . For the total power to be constant, it is necessary that

$$S_{\text{min}} \cdot y = S_{\text{max}} \cdot x \quad (6)$$

It is possible to force Δ in (5) becoming zero through appropriate selection of power x according to the following equation

$$f(x) = \left(\frac{1}{\gamma_{\text{max_user}}} \right) \sum_{n \in \Omega_{\text{max_user}}} \log_2(1 + (P_{\text{max_user}} - x) H_{\text{max_user},n}) - \\ \left(\frac{1}{\gamma_{\text{min_user}}} \right) \sum_{n \in \Omega_{\text{min_user}}} \log_2 \left(1 + (P_{\text{min_user}} + \frac{S_{\text{max}}}{S_{\text{min}}} x) H_{\text{min_user},n} \right) = 0 \quad (7)$$

where $\Omega_{\text{max_user}}$ ($\Omega_{\text{min_user}}$) is the subset of allocated subcarriers to max_user (min_user). After solving the equation (7) for x , the users' power can be updated as

$$\begin{aligned}
P_{\max_user} &= P_{\max_user} - x \\
P_{\min_user} &= P_{\min_user} + \frac{S_{\max}}{S_{\min}} x
\end{aligned} \quad (8)$$

This procedure will continue iteratively until Δ becomes less than a desired threshold. By setting a proper threshold, it is possible to make a trade-off between the number of loop iterations, the tightness of proportionality constraint and the resultant overall capacity.

The complexity of IFR stage depends on solving the equation (7). Note that $0 < x < P_{\max_user}$. Additionally, $f(x=0) = \Delta > 0$ and $f(x=P_{\max_user}) < 0$. Therefore, $f(x)$ in (7) has certainly a root in the interval $x \in (0, P_{\max_user})$. The root can be found using a "bisection search" strategy. For this purpose, it is necessary to evaluate function $f(x)$ at the middle point of the interval $(0, P_{\max_user})$. If $f(P_{\max_user}/2) < 0$, then root is in the left half-interval, otherwise in the right half-interval. This procedure can be continued until the desired precision obtained. Each halving the interval requires only one function evaluation and one zero-threshold comparison. Usually 4 or 5 iterations is enough (for $E=5$ iterations, precision is $P_{\max_user}/32$).

4 Complexity Analysis

We have computed the approximate number of required operations for each step of both the algorithm proposed in [1] and our proposed method. The results are shown in Table 1. These operations consist of comparison and arithmetic operations. Finding the maximum (and minimum) of an array including N elements requires $(N-1)$ comparison operations at most. As well, sorting an N -element array needs $N(N-1)/2$ comparison operations using bubble sort method.

Precisely speaking, the number of operations depends on the chosen implementation, but the results in Table 1 can be used to estimate and compare the complexity of the two algorithms.

Table 1. complexity parameters and calculations

operation	Method in [1]	proposed method
Summation(+)	$4N$	$2N(1+E.n)/K$
Subtraction(-)	$2N+3K$	$N+6E.n$
Division(+)	$NK+4N+7K-K^2$	$N+nK$
Multiplication(x)	$2N+10K$	$K+N(1+2E.n)/K$
Exponent(^)	K	—
comparison	$N^2/2+KN-K^2$	$NK/2+N^2/2-N+2n.K$

5 Simulation Results

We have simulated a 6-ray frequency selective channel with maximum delay spread of $5\mu\text{sec}$ and maximum Doppler frequency of 50Hz . The power delay profile decays exponentially with $\exp(-2\tau)$. Each multipath

component experiences Rayleigh fading modeled by Clark's spectrum. The total power budget is 1W and the total bandwidth is 1MHz which is divided into 256 subchannels. Target BER is 10^{-3} and power spectral density of the additive white Gaussian noise is -60dBW/Hz . The users are uniformly distributed in the cell.

Fig. 2(a) shows the overall capacity vs. the number of users. All γ_k 's are assumed to be unity and the average subchannel SNR is 25dB . The proposed method nearly provides the same capacity as the method in [1]. Both of these dynamic schemes achieve significantly higher capacity than static TDMA. When the number of users increases, the system capacity increases because of multiuser diversity gains.

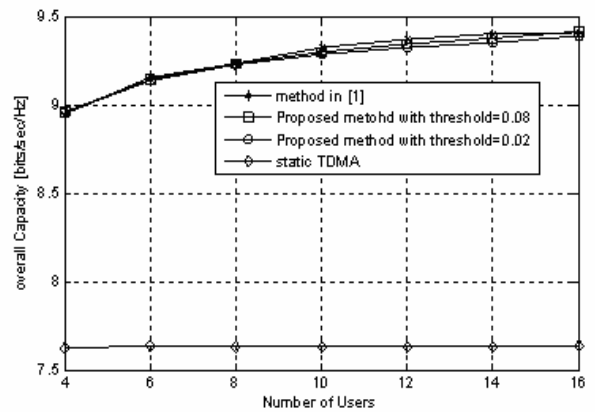


Fig. 2(a). The overall capacity performance of the proposed method.

Fig. 2(c) shows the number of loop iterations required by IFR subroutine to reach Δ less than the preset threshold. From Fig. 2(b),(c), when the threshold is 0.02, the proposed algorithm satisfies rate proportionality constraint much tighter than method in [1] and for this purpose, it requires about 4 iterations. When the threshold increases to 0.08, the number of required iterations reaches one, but at the cost of less rate proportionality tightness. Fig. 2(b) shows the final maximum proportionality deviation defined in (5).

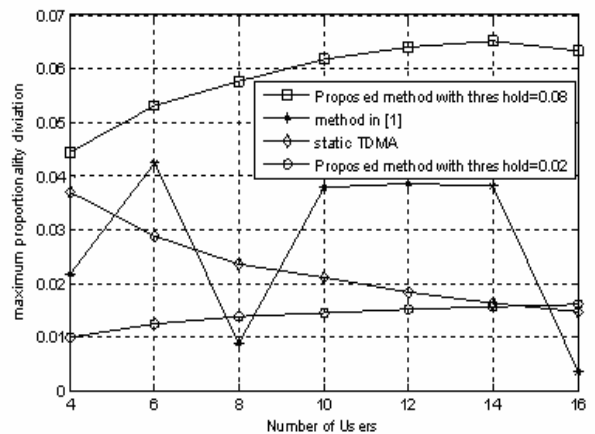


Fig. 2(b). Final maximum proportionality deviation (5).

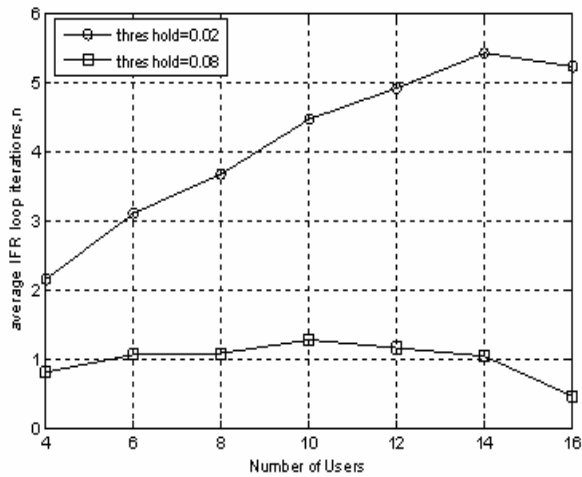


Fig. 2(c) The number of loop iterations required by IFR subroutine to reach Δ less than the preset threshold.

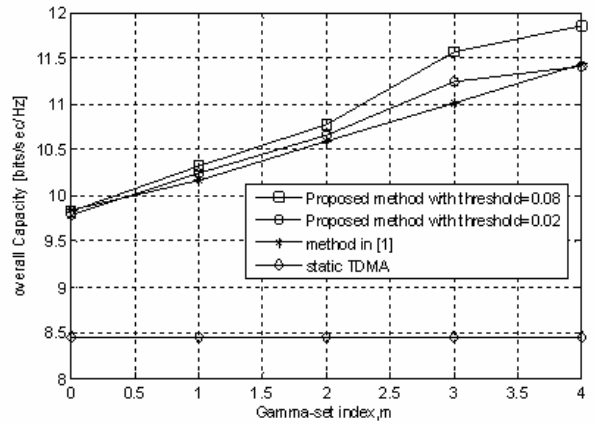


Fig. 3(a) The total capacity in a 16-user OFDM system vs. Γ -set index m [7].

Fig. 3(a) shows the total capacity in a 16-user OFDM system vs. Γ -set index m [7]. It is assumed that $\gamma_1 = \dots = \gamma_4 = 2^m$ and $\gamma_5 = \dots = \gamma_{16} = 1$.

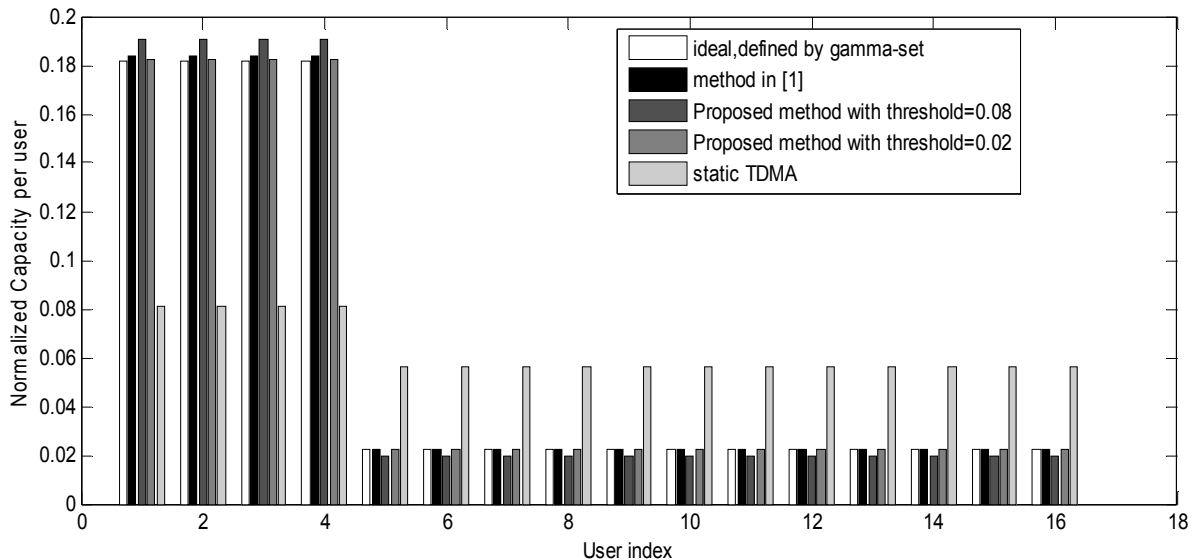


Fig. 3(b) The normalized capacity for different values of threshold.

Average channel SNR of the first four users are assumed to be 10dB higher than the other 12 users. As more priority is allocated to the first four users, i.e. as the Γ -set index increases, higher capacity is achieved for both dynamic schemes. This is because the first four users have higher average channel gain and hence can utilize the available resources more efficiently. TDMA capacity cannot change by varying Γ -set index, because no proportionality control mechanism exists.

From Fig. 3(a), the capacity gain of the proposed method compared with the method in [1] becomes more significant as the threshold increases. Fig. 3(b) shows the normalized sum capacity ($R_k / \sum R_k$) distribution among users for $m=3$, i.e. $\gamma_1 = \dots = \gamma_4 = 8$ and $\gamma_5 = \dots = \gamma_{16} = 1$. Ideal capacity distribution is defined as $\gamma_k / \sum \gamma_k$. The proposed algorithm with threshold of 0.02 distributes capacity very well among users even better

than method in [1]. However, when threshold in IFR subroutine is 0.08, the capacity distribution is a little different from the ideal case.

Fig. 4 shows the performance vs. the worst SNR (WSNR) which is related to the nearest user to the cell border. In the graphs of Fig. 4, the path loss exponent is considered as 4 and the maximum path loss difference between users to be 10dB. 16 users are uniformly distributed in the cell and all γ_k 's are assumed to be unity. From Fig. 4(a), it is seen that the capacity of the proposed algorithm and the method in [1] are closely matched and both are higher than non-adaptive TDMA. From Fig. 4(b),(c) the trade-off between rate proportionality constraint and the number of required loop iterations for IFR can be exploited as already mentioned.

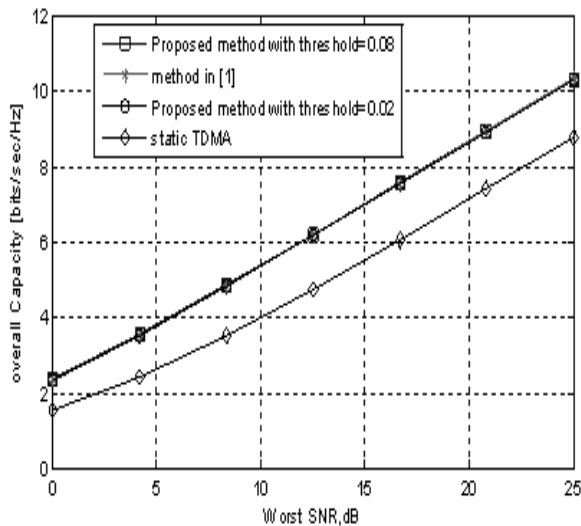


Fig. 4(a) Overall capacity vs. WSNR.

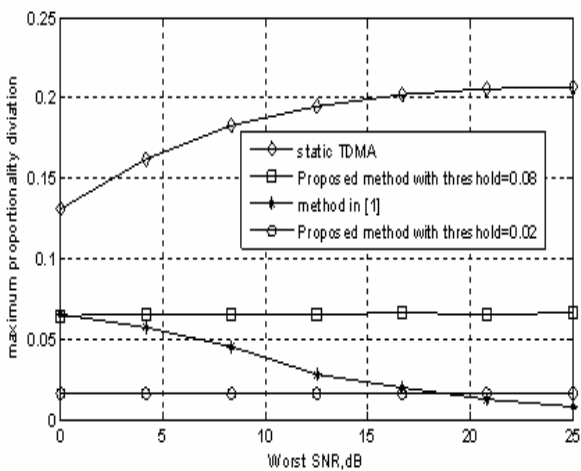


Fig. 4(b) Proposed method with different thresholds.

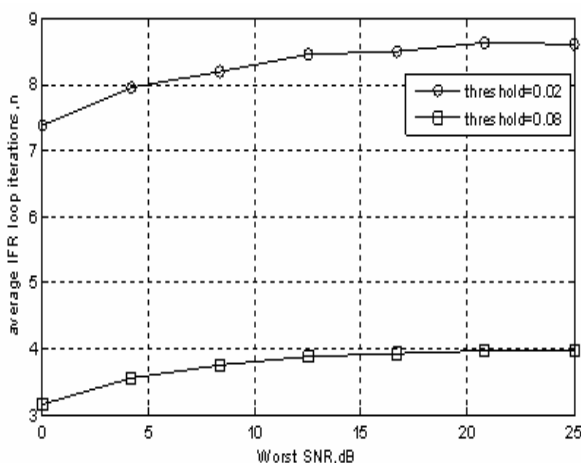


Fig. 4(c) Proposed method IFR loop iterations with the desired threshold.

6 Conclusion

In this paper, we propose a multiuser resource allocation scheme for OFDMA systems which is based on rate

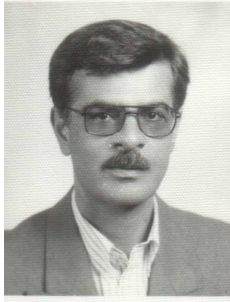
maximization idea and achieve the required proportional rate constraints. Adjustable proportionality among users' data rates provides a flexible multimedia environment which can present different QOS specifications. The proposed algorithm contains two stages which subcarrier assignment is performed in the first stage. The second stage provides a feasible solution with the required proportionality through a low-complexity iterative power manipulation procedure.

Simulation results demonstrate the proposed algorithm outperforms static algorithms, such as TDMA and offers the same capacity as the one proposed in [1]. In addition, it is possible to make a trade-off between complexity, total capacity and rate proportionality tightness by adjusting a flexible threshold. Since the number of unknown powers used by proposed algorithm is highly reduced and power manipulations are done through a short iterative subroutine, it is clear that the complexity is low and the algorithm is realizable in the realistic applications.

References

- [1] Wong I. C., Shen Z., Evans B., and Andrews J., "A low complexity algorithm for proportional resource allocation in OFDMA systems," in *Proc. IEEE Workshop on Signal Processing Systems*, pp. 1–6, Oct. 2004.
- [2] Shen Z., Andrews J., and Evans B., "Optimal power allocation in multiuser OFDM systems," in *Proc. IEEE Global Communications Conference*, San Francisco, CA, pp. 337–341, Dec. 2003.
- [3] Wong C. Y., Cheng R. S., Letaief K. B., and Murch R. D., "Multicarrier OFDM with adaptive subcarrier, bit, and power allocation," *IEEE Journal on Selected Areas in Communications*, Vol. 17, No. 10, pp. 1747–1758, Oct. 1999.
- [4] Jang J. and Lee K. B., "Transmit power adaptation for multiuser OFDM systems," *IEEE Journal on Selected Areas in Communications*, Vol. 21, No. 2, pp. 171–178, Feb. 2003.
- [5] Rhee W. and Cioffi J. M., "Increasing in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE Int. Vehicular Tech. Conf.*, Vol. 2, pp. 1085–1089, 2000.
- [6] Kivanc D. and Liu H., "Subcarrier allocation and power control for OFDMA," *the 34th Asilomar Conference on Signals, Systems, and Computers*, Vol. 1, pp. 147–151, 2000.
- [7] Wong I. C., Shen Z., Evans B., and Andrews J., "Adaptive resource allocation in multiuser OFDM systems with proportional fairness," *IEEE Transactions on Wireless Communications*, Available at:

<http://www.ece.utexas.edu/bevans/papers/2005/multiuserOFDM/AdaptResAllocMUOFDM.pdf>



Abolfazl Falahati received the B.S.c (Hons) in electronics with physics from the University of Warwick (UK) in 1980. He received M.Sc. and Ph.D. degrees in Digital Communication Systems from Loughborough University of Technology (UK) in 1986 and 1990 respectively. During 1990-1994, he was a research fellow at Rutherford

Appleton Laboratory (Oxford-UK) to design equalization and secure techniques over HF radio links employing DSP hardware. Since 1994, he has been at Iran University of Science and Technology where he is currently a Senior Lecturer in Cryptography, Channel Coding, Advanced Mobile Communication Systems and Digital Signal Processing. His main research interests are the analysis and implementations of Coding algorithms and techniques with hardware designs over data transmission systems especially over time-varying and frequency selective fading channels.

Dr. Falahati is an Associate Professor and a Chartered Engineer, Member of the Institute of Electrical Engineers (IET and IEEE Communication Society) and Cryptography Society. He is a DSP devices Consultant at Damovand, ComranFO and Modern Electronics Companies as well as being the executive director of Arian Science Electronics. He is also a Chief Examiner for the Engineering Council.



Majid R. Ardestani received his B.Sc. degree in electrical engineering in 2003 from University of Tehran (UT), and M.Sc. degree in communication systems engineering in 2005 from Iran University of Science and Technology (IUST), Tehran. He is currently pursuing the Ph.D. degree at IUST. He worked as a research assistant at Iran

Telecommunication Research Center (ITRC) in 2002 for eight months.

His research interests include mobile communications, OFDM multicarrier modulation over DVB and Mobile Systems, spread spectrum wireless communications, scalable video compression and network distribution of compressed multimedia content. He is also a co-author of many publications.