Abstract: Torque ripple minimization of switched reluctance motor drives is a major subject based on these drives’ extensive use in the industry. In this paper, by using a well-known cascaded torque control structure and taking the machine physical structure characteristics into account, the proposed energy-based (passivity-based) adaptive sliding algorithm derived from the viewpoint of energy dissipation, control stability and algorithm robustness. First, a nonlinear dynamic model is developed and decomposed into separate slow and fast passive subsystems that are interconnected by negative feedbacks. Then, an outer loop speed control is employed by adaptive sliding controller to determine the appropriate torque command. Finally, to reduce torque ripple in switched reluctance motor a high-performance passivity-based current controller is proposed. It can overcome the inherent nonlinear characteristics. The performance of the proposed controller algorithm has been demonstrated in simulation, and experimental using a 4KW, four-phase, 8/6 pole SRM DSP-based drive system.

Keywords: Feedback Dissipative Hamiltonian Realization (FDHR), Passivity-Based Control (PBC), Switched Reluctance Motor (SRM), Torque Ripple Minimization.

1 Introduction
In recent years, there is a growing concern in use of switched reluctance motor. The major reasons for SRM are robustness, high efficiency, low cost, high speed, simple structure, easy to maintain, high controllability, high torque to inertia ratio, simple power converter circuits with reduced number of switches and smaller dimension of the motor in comparison to the other motors [1]. The main problems with SRM include high torque pulsation and noise. Several control methods and schemes have been proposed to overcome these problems. For example, variable structure controller made the SRM drive system insensitive to parameter variations and load disturbance [2]. Artificial neural network and fuzzy controller needs a lot of designer experience [3]. Nonlinear internal model control for SRM drive required very complicated computations and implementation of the system is very difficult [4]. Some studies have succeeded in torque ripple reduction of SRM motors using DTC method [5]. In the methods mentioned above, controller design procedure only done for speed or torque tracking and just speed loop as an outer loop is considered and the current loop is neglected. However, the torque is usually controlled via an inner current loop, with a nonlinear relationship between torque and current. That means the torque ripple, acoustic noise, speed performance and motor efficiency will be fractional affected by improving the current control. Also, for the above-mentioned control strategies of the SRM it is assumed that its parameters are known exactly or the unknown parameters can be identified by the adaptive technique. However, the parameters of the SRM are not exactly known and always vary with current and position. Actually, control is difficult to implement owing to its complex algorithm when considering the structural information of SRM in design. Improving the applicability of the SRM on the basis of taking the structural characteristics into account is a significant step in designing the controller of the SRM. In this paper, a cascaded torque control structure for its well-known advantages is used. First an adaptive sliding mode controller for speed loop control is designed and then compared to a PI controller. Then to achieve high performance torque control a nonlinear feedback current controller using energy-based technique called passivity-based control (PBC) theory which effectively uses the natural energy dissipation properties of the SRM, is proposed. PBC, introduced to define an energy-based nonlinear controller design methodology which achieves stabilization by
passivation [6]. The algorithms is first simulated by MATLAB/SIMULINK and then tested experimentally on a four-phase 8/6 pole 4kW  outlook SRM.

2 Nonlinear Construction and Modeling of SRM
Owing to the doubly salient construction, the SRM presents a highly nonlinear load to the current controller, thus the design of a high-performance current controller for an SRM drive is a challenge [7]. For SRM drives, the mutual coupling between phases is usually neglected for low-speed applications, so the phase currents can be controlled independently. Voltage equation for one phase of an SRM is [8]:

\[ v_k = \tau_k i_k + \frac{d\lambda_k(\theta, i_k)}{dt} \]
\[ = \tau_k i_k + L_k(\theta, i_k) \frac{di_k}{dt} + \frac{\partial \lambda_k(\theta, i_k)}{\partial \theta} \omega \]  

where \( u_k \) is the phase voltage, \( i_k \) is the phase current, \( \tau_k \) is the phase resistance, \( k = a, b, c, d \) is the active phase, \( \theta \) is the rotor position and \( \lambda_k(\theta, i_k) \) is the flux linkage, \( L_k(\theta, i_k) = \frac{\partial \lambda_k(\theta, i_k)}{\partial i_k} \) is the incremental inductance, \( \omega = d\theta/dt \) is the motor speed and \( e = \omega \frac{\partial \lambda_k(\theta, i_k)}{\partial \theta} \) is the back EMF. Fig. 1 (a) and (b) shows the experimentally measured static flux linkage and torque characteristics of a four-phase 8/6 SRM whose parameters are specified in the Table 1 (in the Appendix) and used in this investigation. Using numerical methods, the incremental inductance and back-EMF characteristics of the SRM can be obtained from the measured flux linkage data. Fig. 1(c) and 1(d) shows the curves of calculated incremental inductance and back-EMF coefficient \( \left( \frac{\partial \lambda_k(\theta, i_k)}{\partial \theta} \right) \) against rotor position and phase current. Both incremental inductance and back-EMF coefficient are nonlinear functions of rotor position and phase current.

3 The Proposed Nonlinear Controller Design
In this section, we present a passivity-based adaptive sliding mode control method for designing SRM controller. A complete model of an SRM possesses two-time-scale characteristics and can be decomposed as the feedback interconnection of the two linked subsystems (electrical and mechanical subsystem) [9]. Also, in [10] it is proven that these two subsystems are passive linked subsystems as:

electrical subsystem: \[
\begin{bmatrix}
v_k \\
-\omega \\
\end{bmatrix} \rightarrow \begin{bmatrix}
i_k \\
\end{bmatrix}
\]

mechanical subsystem: \[
\begin{bmatrix}
T_e - T_c \\
\end{bmatrix} \rightarrow -\dot{\theta}
\]

The overall control block diagram, shown in Fig. 2 is separated into the inner loop and the outer loop controller. Based on the assumption that stator current
i_k as well as rotor speed \( \omega \) are available for measurement, controller design procedures can be divided into three steps. First step is to design an adaptive sliding speed controller for speed tracking of the overall system. The next step is passivity-based current control of the electrical subsystem by injecting a nonlinear electrical damping term and a set of reference current vectors \( i_\ast \) are found out to achieve current tracking. The sake of the outer loop is to generate the appropriate torque command fed for the inner loop. Finally, passivity-based inner loop current controller will produce the appropriate switching functions.

### 3.1 Speed Loop Controller Design

A sliding mode control scheme is proposed for the speed control and then compared to PI controller. The conventional adaptive sliding mode control estimates the unknown uncertainty upper boundary which causes the chattering phenomenon. Therefore, a method has been adapted to estimate the unknown uncertainties of lump without using \( \text{sgn}(\cdot) \) that it reduces chattering [11]. For speed controller design, consider the mechanical equation \( \dot{\phi} = (1/J)[T_\ell(0,i_k) - T_\ell - D\phi] \) with uncertainties as:

\[
\frac{d\phi}{dt} = (a + \Delta a)\phi + (b + \Delta b)(T_\ell - T_\ell) \tag{3}
\]

where \( a = -B/j, b = I/j \). Defining the state variable of speed error as \( e = \omega^* - \omega \), switching surface as \( S_\omega = \sigma e + \dot{e}, x = [x_1, x_2]^T = [e, \dot{\phi}]^T \) and choosing Lyapunov function as \( V_\phi = 1/2 S_\omega^2 \), we have:

\[
\dot{V}_\phi = S_\omega \dot{S}_\omega
\]

\[
= S_\omega \left[ -(\sigma + a)\dot{\phi} - bu + b \dot{T}_\ell \right] - b(\ddot{\phi} + \dot{\dot{P}})
\]

Where

\[
P(t) = 1/b \left[ \Delta a \cdot \phi + \Delta b(\dot{T}_\ell - \dot{T}_\ell) \right], \dot{P}(t) = P(t) - \dot{P}(t)
\]

The estimated value of the lumped uncertainty is \( \dot{\hat{P}}(t) \) and \( \ddot{\phi} \) is the estimated error between the actual value \( P(t) \) and the estimated value of the lumped uncertainty. Therefore, the new candidate function is

\[
\dot{V}_\phi = 0.5 \left( S_\omega^2 + (1/\rho) \dot{\dot{P}}^2 \right), \text{ then we have}
\]

\[
\dot{V}_\phi = S_\phi \left[ -(\sigma + a)\dot{\phi} - b(u + \dot{\phi}) + b \dot{T}_\ell + K S_\omega \right]
\]

\[
- \dot{\hat{P}} \left[ (1/\rho) \dot{\dot{P}} + b S_\omega \right] - K S_\omega^2
\]

If control input \( u \) is chosen as:

\[
u = \frac{1}{b} \left[ -(\sigma + a)\dot{\phi} - b \dot{\phi} + b \dot{T}_\ell + K S_\phi \right] \tag{6}
\]

One can obtain \( \dot{V}_\phi = -K S_\phi^2 \leq 0 \) and it can be achieved that \( \int_0^\infty W(t)dt = V[S_\phi(t)] - V[S_\phi(0)] \). Since \( S_\phi(t) \) and \( S_\phi(0) \) are bounded, according to Barbalet lemma [12], can be obtained that \( \lim_{t \to \infty} W(t) = 0 \). It means that \( e \to 0 \) as \( t \to \infty \) and as a result, the proposed SRM drive system is stable. Finally in this step a proportional-integral (PI) controller is designed here to compare with the nonlinear controller. The parameters of the PI controller are tuned by pole placement [13]. By assumption of \( \zeta = 1.25 \) and \( \omega_n = 0.7 \text{ rad/sec} \) the parameters of the PI controller obtained as \( K_p = 0.466, K_i = 7.47 \).

### 3.2 Passivity-Based Current Control Design using Feedback Dissipative Hamiltonian Realization

As mentioned in previous section the electrical subsystem is passive. Therefore, in this section the feedback dissipative Hamiltonian realization method for passivity-based current controller design of SRM is proposed. Control design procedures can be divided into two steps. First, a suitable control that transfers dynamic model to a dissipative Hamiltonian system is fined. Finally, this section presents the PBC technique and then the current control law obtained. The feedback dissipative Hamiltonian realization method considers the problem of designing a state-space controller for the stabilization of a desired equilibrium point of a nonlinear system [14]:

\[
\dot{x} = f(x) + g(x)u \tag{7}
\]

where \( x \in \mathbb{R}^n \) are system states, \( u \in \mathbb{R}^m \) are system inputs. System (7) is said to have a feedback Hamiltonian realization if there exists a control law \( u = \varphi(x) \). Such that the closed loop is of the form:

\[
\dot{x} = F_\varphi(x) \frac{\partial}{\partial x} H_\varphi(x) \tag{8}
\]

System (7) is said to have a feedback dissipative Hamiltonian realization if the closed loop is a dissipative Hamiltonian system, its structure matrix satisfies:

\[
F_\varphi(x)^T + F_\varphi(x) \leq 0 \tag{9}
\]

where \( F_\varphi \in \mathbb{R}^{(n \times m)} \) are called the structure matrices and \( H_\varphi(x) \) the Hamiltonian function. The key idea behind the technique is to match the closed-loop dynamics to a port-controlled Hamiltonian system form:

\[
f(x) + g(x)u = F_\varphi(x) \frac{\partial}{\partial x} H_\varphi(x) \tag{10}
\]
To enforce dissipativity, the constraint of (9) on the $F_d$ matrix is required. In this case, $F_d$ decomposed as $F_d(x) = J_d(x) - R_d(x)$. The interconnection structure is captured in the matrix $J_d(x) = -J_d^T(x)$. The dissipation effects are captured by the matrix $R_d = R_d^T \geq 0$. In general, this leads to a set of partial differential equations. But for a real physical system, according to its physical meaning and the control objectives, we may find a natural candidate Hamiltonian function, then (9) becomes a set of algebraic equations.

Lemma 1: Necessary and sufficient condition for the existence of feedback dissipative Hamiltonian realization for fixed $F_d(x)$, which satisfy (10), and for fixed Hamiltonian function $H_d(x)$, is that there exists a feedback such that (10) holds if and only if the projected matching equation:

$$
\sum_{i=1}^{d} \frac{dL_i(\theta_i, x)}{\partial \theta_i} x_i^2 - T - \frac{B}{j} x_i = 0
$$

holds for an arbitrary full-rank left annihilator $g(x)$. By defining state an input vector respectively as $x = [L_i i_a L_i i_b L_i i_c L_i i_d \text{jo}]^T$ and using (1), then dynamic system for current loop controller design can be represented as:

$$
\dot{x} = f(x) + g(x) u
$$

For convenience, first following pre-feedback to the original system is used to simplify the controller design:

$$
u_i = \frac{r_i}{L_i} x_i + \frac{dL_i}{\partial \theta} x_i x_i + v_k$$

So, affine dynamic system (12) converts to:

$$
\dot{x} = f(x) + g(x) v = \begin{pmatrix}
0 \\
\frac{1}{2} \sum_{i=1}^{d} \frac{dL_i(\theta_i, x)}{\partial \theta} x_i^2 - T - \frac{B}{j} x_i \\
+ g(x) v
\end{pmatrix}
$$

Obviously, achieving the regulating objective is equivalent to asymptotically stabilizing the equilibrium $x^*$. In order to stabilize the desired equilibrium point, one can use a Hamiltonian function as:

$$
H_d(x) = \frac{1}{2} (x - x^*)^T \lambda (x - x^*)
$$

The feedback law to make desired equilibrium point $x^*$ asymptotically stable is used as:

$$
v = \phi(x) = \left(\phi_1(x_i), \ldots, \phi_d(x_i)\right)^T
$$

Such that $f(x) + g(x) \phi = F_d(x) \partial H_d(x)$. Where $F_d(x)$ is a $n \times n$ dissipative matrix. According to Lemma 1, such a feedback exists if and only if (16) holds. The $F_d(x)$ matrix is chosen in order to facilitate the solution of the resulting algebraic equations. Furthermore, in order to simplify the control structure, we want to assign one output to each control action and also to obtain a controller as simpler and robust (minimum parameter dependence) as possible. In the stability analysis, the damping coefficient (which is present in all real machines) plays a fundamental role to ensure convergence to the equilibrium point. The presented method allows decoupling the outputs,
improving the robustness and facilitating the gain tuning [15]. Therefore, \( F_d(x) \) matrix is chosen in order to render the resulting Hamiltonian system dissipative using (9) and facilitate the solution of the resulting algebraic equations as:

\[
F_d(x) = \begin{pmatrix}
-\Gamma_1 & 0 & 0 & 0 & -\beta_1 \\
0 & -\Gamma_2 & 0 & 0 & -\beta_2 \\
0 & 0 & -\Gamma_3 & 0 & -\beta_3 \\
-\beta_1 & \beta_2 & \beta_3 & \beta_4 & \alpha \\
\end{pmatrix}
\]  

(17)

where \( \Gamma_i > 0 \) are arbitrary positive constants. From (12) and assuming \( B_0 \cong \beta \), a particular solution is:

\[
\beta_i = \frac{x_i}{2L_i^2 \lambda_i} \frac{dL_k}{d\theta}, \alpha = 0
\]  

(18)

We can solve for the feedback \( v = \varphi(x) \) from (10) as:

\[
\varphi_i(x_i) = -\Gamma_i \frac{\partial H_d}{\partial x_i} = \frac{1}{2L_i^2 \lambda_i} \frac{dL_k}{d\theta} x_i
\]  

(19)

The resulting closed-loop system is:

\[
\dot{x} = (J_d(x) - R_d(x)) \frac{\partial H_d}{\partial x}
\]  

(20)

in which

\[
J_d(x) = \frac{1}{2} \left( F_d(x) - F_d^T(x) \right)
\]  

and

\[
R_d = -\frac{1}{2} \left( F_d + F_d^T \right)
\]

In order to prove that \( x^* \) is asymptotically stable, now calculate the derivative of \( H_d(X) \) along the trajectories of the closed loop as follows:

\[
\dot{H}_d = -\left[ \frac{\partial H_d}{\partial x} \right]^T R_d \frac{\partial H_d}{\partial x}
\]

\[
= -\lambda_i \Gamma_i (x_i - x_i^*)^2 - \cdots - \lambda_d \Gamma_d (x_d - x_d^*)^2 \leq 0
\]

(21)

Hence, \( H_d(X) \) qualifies as a Lyapunov function. Closed loop system (8) with \( x^* \) (locally) stable equilibrium is asymptotically stable if, in addition, the largest invariant set under the closed–loop dynamics (8) contained in \( \{ x \in \mathbb{R}^n \mid \left[ \frac{\partial H_d}{\partial x} \right]^T R_d \frac{\partial H_d}{\partial x} = 0 \} \) equals \( x^* \). Asymptotic stability follows immediately invoking LaSalle’s invariance principle and the above condition. Finally, combining the two controls (13) and (19), it is easy to obtain:

\[
u_k = -\Gamma_i \lambda_i (i_k - i_k^*) + r_i i_k - \frac{dL_k}{d\theta} i_k \omega - \frac{1}{2L_i^2 \lambda_i} \frac{dL_k}{d\theta} i_k j \lambda_i (\omega - \omega^*)
\]

(22)

4 Simulation Results

The proposed controllers are simulated using the SIMULINK software. The model takes magnetic saturation into account. The drive system simulations are used for comparison purposes to investigate the performance of the proposed PBC approach at different load conditions. From the imposed pole locations, the gains of current PI controller are computed and the damping parameter values of passivity-based controller have been obtained by using a trial-and-error procedure. The desired rotor speed is set to 200 RPM and the external load torque is suddenly changed at \( t=0.05 \) second from 5 Nm to 10 Nm. Simulation result obtained for proposed controllers shows in Fig. 3(a) and 3(b). As it can be considered higher tracking performance of reference speed achieved in the case of using passivity-based current loop controller with adaptive sliding speed loop control.

5 Experimental Setup and Results

The validity and effectiveness of the proposed control approach are shown by adopting the same objectives as the simulation results. The controller gains are nearly the same as used in the system simulation. A DSP-based drive system using a four-phase 8/6 4KW oulton SRM which has the nonlinear static flux linkage and torque characteristics shown in Fig. 1 is used to test speed controllers experimentally. The work presented here employs a conventional digital-control platform. It is based on the eZdsp F2812 board as a suitable platform for implementing motor controllers. This board is built around the TMS320F2812 digital signal processor (DSP). This platform is compatible with Simulink, and includes four dual pulse-width-modulation (PWM) channels (8 channels total), 4
analog-to-digital converters (ADCs), and a speed-encoder input. The processor is a 32-bit DSP with fixed-point arithmetic; thus, discrete and fixed-point math blocks from Simulink can be used to program it. The complete experimental hardware used for evaluating the 8/6 SRM drive is shown in Fig. 4. A conventional asymmetric converter used for our four-phase SR drive circuit. Simulink is able to compile a block diagram into C code and then call CCS to generate assembly code for the DSP. A project is generated in CCS to be loaded into the DSP. Fig. 5 shows a summary of the setup. The experimental results were obtained for the SRM speed controls shown in Fig. 6. Speed reference tracking improvement was performed through the proposed idea in 200 rpm on 5Nm load condition.

Since a torque sensor was not available, for the electromagnetic torque, only simulation results comparing the response of three controllers are presented. Fig. 7 shows that performance of electromagnetic torque has been increased to a value corresponding to the load and obviously can be observed that torque ripple associated with using passivity-based controller operation is significantly decreased. Fig. 8 shows current and voltage waveforms for case of using passivity-based adaptive sliding mode control algorithm. Experimental waveforms show an acceptable current control mode.
6 Conclusions

A nonlinear controller has been presented for a four-phase SRM drive based on the passivity-based adaptive sliding technique. Complete model of SRM possesses two-time-scale characteristics and decomposed as the feedback interconnection of the two electrical and mechanical passive linked subsystems. Hence, by using cascaded torque control structure, the proposed PBC algorithm is designed. Because of tacking the machine physical structure characteristics into account, it can overcome the inherent nonlinear characteristics of the system. The simulation and experimental results show the proposed controller has improved dynamic performance of rotor speed and torque, also produces lower torque ripple for SRM drives. The controller is not deigning to compensate for the uncertainties caused by the parameter variations and external load disturbance. Future research includes a dynamical extension keeping the Hamiltonian structure to improve the robustness (basically on the electrical parameters) and the performance.

Appendix

Table 1 Motor specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>4 kW</td>
</tr>
<tr>
<td>Rated phase current</td>
<td>9 A</td>
</tr>
<tr>
<td>Rated speed</td>
<td>1500 rpm</td>
</tr>
<tr>
<td>Number of pole</td>
<td>8/6</td>
</tr>
<tr>
<td>Phase resistance</td>
<td>0.75 Ω</td>
</tr>
<tr>
<td>Inertia (J)</td>
<td>0.008 Jms</td>
</tr>
<tr>
<td>Damping Factor (B)</td>
<td>0.00078 Nms</td>
</tr>
</tbody>
</table>

References


Namazi Isfahani et al: Energy-Based Adaptive Sliding Mode Speed Control for …


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