Solving Environmental/Economic Power Dispatch Problem by a Trust Region Based Augmented Lagrangian Method

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Abstract: This paper proposes a Trust-Region Based Augmented Method (TRALM) to solve a combined Environmental and Economic Power Dispatch (EEPD) problem. The EEPD problem is a multi-objective problem with competing and non-commensurable objectives. The TRALM produces a set of non-dominated Pareto optimal solutions for the problem. Fuzzy set theory is employed to extract a compromise non-dominated solution. The proposed algorithm is applied to the standard IEEE 30 bus six-generator test system. Comparison of TRALM results with the various algorithms, reported in the literature shows that the solutions of the proposed algorithm are very accurate for the EEPD problem.

Keywords: Environmental and economic power dispatch, fuzzy set theory, trust-region augmented Lagrangian method.

1 Introduction

The main objective of Economic Power Dispatch (EPD) is to minimize the operating cost, while satisfying the load demand, and all unit and system equality and inequality constraints. In addition, the increasing public awareness of the environmental protection guidelines and the passage of the Clean Air Act Amendment of 1990 have impelled the utilities to modify their design or operational strategies in order to reduce pollution and atmospheric emissions of thermal power plants [1, 2].

Several strategies have been proposed to reduce the atmospheric emissions [3, 4], some of which are:

1. Planning to reduce the power use of power plants with higher pollution rates and use the power stations with lower emission rates.
2. Installation filters on power plants to purify the pollutant gases.
3. Switching to low emission fuels from high emission ones (e.g., using natural gas instead of mazut).
4. Replacement aged and low efficient fuel-burners and generator units by high efficient ones.

The second to fourth options require installation of new equipments, and need considerable capital investments, and normally are considered as long-term planning. Hence, the first option, that is planning the power dispatch in such a manner that optimizes the fuel cost objective, as well as emission cost objective, individually, and especially simultaneously, is our concern for study.

After deregulation of electricity markets, serious competition has arisen among generating companies [5-7]. In this situation, generating companies try to reduce the cost of energy, to enable them compete in the competitive electricity markets. One of the effective methods of reducing the cost of electric energy is environmental-economic power dispatch (EEPD). In recent years, the EEPD category has considerably been investigated in different ways. In [8-10], the emission was considered as a constraint with a permissible limit, and the problem was reduced to a single objective optimization problem. The problem in this method is that a compromise optimal solution cannot be found between emission and fuel costs.

In [11], a linear programming based optimization procedure was proposed in which the objectives were considered one at a time. A compromise optimal solution is impossible in this method either.

In [12], a fuzzy multi-objective optimization approach for the EEPD problem was proposed. The solutions produced by this technique were suboptimal and the algorithm did not provide a systematic framework to direct the search towards the Pareto optimal set.

Over the past decade, the EEPD problem has received much interest due to the development of a number of multi-objective search strategies. Strength

Iranian Journal of Electrical & Electronic Engineering, 2012.
Paper first received 11 July 2011 and in revised form 18 Jan. 2012.
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Pareto Evolutionary Algorithm (SPEA) [2], Niched Pareto Genetic Algorithm (NPGA) [13], Non-dominated Sorting Genetic Algorithm (NSGA) [14], Multi-objective Stochastic Search Technique (MOSST) [15], Fuzzy Clustering-based Particle Swarm Optimization (FCPSO) [16], Multi-objective Particle Swarm Optimization (MOPSO) [17], Epsilon Constraint (EC) approach [18], etc., constitute the pioneering multi-objective approaches that have been applied to solve the multi-objective EEPD problem.

In the above approaches, the EEPD problem were converted to a single objective problem by using a linear combination of the objectives as a weighted sum with a long range planning like switching to low emission fuels. The positive characteristic of these methods is that a set of Pareto optimal solutions can be obtained by changing the weights. In general, while there are more than one objective function in a problem, especially when these objective functions are non-commensurable or even conflicting, instead of having one optimal solution, a set of optimal solutions are of interest. The reason for the optimality of many solutions is that no one can be considered to be better than any other with respect to all the objective functions. These optimal solutions are known as Pareto optimal solutions.

There are some problems associated with taking a linear combination of different objectives as a weighted sum:

1. The combined objective function may lose significance due to the incorporation of multiple non-commensurable factors into a single function.
2. The lack of sufficient information regarding the operation conditions make it difficult for the decision maker to decide on the preferences of objective in giving the weighting factors.

The first problem can be addressed by a proper selection of the scaling factor λ and multiplying the emission objective by this factor.

To deal with the second problem, fuzzy set theory has been used to efficiently derive a candidate Pareto optimal solution for the decision maker [19]. This approach will be explained later.

2 Problem Statement

The EEPD problem is to minimize two non-commensurable and competing objective functions, fuel cost and emission, while satisfying several equality and inequality constraints [2]. The problem is generally formulated in the following subsections.

2.1 Problem Variables

The variables of the problem are the quantities of real power of committed power plants, that is, P_{Gi}, $i = 1, 2, ..., N$ and N is the number of committed power plants in the interconnected network.

2.2 Problem Objectives

There are two objectives which are minimization of fuel cost and minimization of emission amount.

1 Minimization of fuel cost

The generators cost curves are represented by quadratic functions [1,2]. The total *$/h* fuel cost $F(P_G)$ can be expressed as

$$F(P_G) = \sum_{i=1}^{N} a_i + b_i\, P_{Gi} + c_i\, P_{Gi}^2 \tag{1}$$

where N is the number of generators, a_i, b_i, and c_i are the cost coefficients of the ith generator, and P_G is the real power output of the i_{th} generator. P_G is the vector of real power outputs of generators which is defined as

$$P_G = [P_{G_1}, P_{G_2}, ..., P_{G_N}] \tag{2}$$

2 Minimization of emission amount

The total emission $E(P_G)$ in (*ton/h*) atmospheric pollutants such as sulphur oxides (SO_X) and nitrogen oxides (NOx) caused by the operation of fossil–fueled thermal generation can be expressed as [2]:

$$\begin{aligned} E(P_G) = \sum_{i=1}^{N} 10^{-2} & (\alpha_i + \beta_i\, P_{Gi} + \gamma_i\, P_{Gi}^2) \\ & + \zeta_i \exp(\lambda_i P_{Gi}) \end{aligned} \tag{3}$$

where $\alpha_i, \beta_i, \gamma_i, \zeta_i$ and λ_i are coefficients of the ith generator emission characteristics.

2.3 Problem Constraints

2.3.1 Power Balance Constraint

The total power generation must cover the total power demand P_D and the real power loss in transmission lines P_{loss}. Hence,

$$\sum_{i=1}^{N} P_{Gi} - P_D - P_{loss} = 0\ . \tag{4}$$

The real power loss P_{loss} in Eq. (4) is represented by calculation of the AC load flow problem, which has equality constraints on real and reactive power at each bus as follows [2]:

$$P_{Gi} = P_{Di} + V_i \sum_{j=1}^{NB} V_j \left[G_{ij} \cos(\delta_i - \delta_j) - B_{ij} \sin(\delta_i - \delta j)\right] \tag{5}$$

$$\begin{aligned} Q_{Gi_i} = Q_{Di} + V_i \sum_{j=1}^{NB} V_j [& G_{ij} \sin(\delta_i - \delta_j) \\ & + B_{ij} \cos(\delta_i - \delta_j)] \end{aligned} \tag{6}$$

where NB is the number of buses; P_{Gi} and Q_{Gi} are the real and reactive power generated at the ith bus respectively, P_{Di} and Q_{Di} are the i_{th} bus load real and reactive power, respectively, G_{ij} and B_{ij} are the transfer conductance and susceptance between bus i and bus j, respectively, V_i and V_j are the voltage magnitudes at bus i and bus j, respectively, δ_i and δ_j are the voltage angles at bus i and bus j, respectively.

There are several methods of solving the resulting nonlinear system of Eqs. (5) and (6) which the most popular is known as the Newton-Raphson Method. The load flow solution gives all bus voltage magnitudes and angles that can be used to calculate the transmission losses as follows:

$$P_{loss} = \sum_{k=1}^{NL} g_k \left[V_i^2 + V_j^2 - 2V_iV_j \cos(\delta_i - \delta_j)\right] \quad (7)$$

where, NL is the number of transmission lines and g_k is the conductance of the k_{th} line that connects bus i to bus j.

2.3.2 Generation Capacity Constraint

For stable operation, the real power output of each generator is limited by lower and upper limits as follows [2]:

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \quad , \quad i = 1,2,..., N. \quad (8)$$

where, $P_{Gi}^{\min}$ and $P_{Gi}^{\max}$ are the lower limit and upper limit power outputs of i_{th} generator, respectively, and N is the number of generators.

2.3.3 Security Constraint

For secure operation, the transmission line loading S_l is restricted by its upper limit as follows:

$$S_{lk} \le S_{lk}^{\max} \quad , \quad k = 1,..., NL \quad (9)$$

where S_{lk} and $S_{lk}^{\max}$ are respectively the transmission loading and upper limit transmission loading of i_{th} transmission line.

It should be noted that the k_{th} transmission line flow connecting bus i to bus j can be calculated as

$$S_{lk} = (V_i \angle \delta_i) I_{ij}^* \quad (10)$$

where, I_{ij} is the current flow from bus i to bus j and can be calculated as

$$I_{ij} = (V_i \angle \delta_i) \begin{bmatrix} (V_i \angle \delta_i - V_j \angle \delta_j)(y_{ij}) \\ + (V_i \angle \delta_i)(j \dfrac{y}{2}) \end{bmatrix} \quad (11)$$

where y_{ij} is the line admittance, while y is the shunt susceptance of the line [2].

2.4 Problem Formulation

By summing up the aforementioned objectives and constraints, the problem can mathematically be formulated as a nonlinear constrained multi objective optimization problem as follows:

$$Minimize \quad [F(P_G), E(P_G)] \quad (12)$$

Subject to

$$g(P_G) = 0 \quad (13)$$

$$h(P_G) \le 0 \quad (14)$$

where, g is the equality constraint representing the power balance, and h is the inequality constraint representing the power system security and the generator capacity constraints.

The power balance constraint is as follows:

$$d_b + \sum_k (B_{bk}\delta_k) - P_{Gb} = 0 \,, \forall b \in B \quad (15)$$

where b and k are number of the buses (nodes) in the electric network, d_b is the demand at bus b, B_{bk} is network susceptance matrix, δ_k is phase angle at bus k, P_{Gb} is the generating unit active power at bus b, and B is the set of all buses.

The forward and backward power system security constraints are:

$$l_{cl} - \sum_k (H_{lk}\delta_k) \ge 0 \,, \forall l \in NL \quad (16)$$

$$l_{cl} + \sum_k (H_{lk}\delta_k) \ge 0 \,, \forall l \in NL \quad (17)$$

where, l is number of the line between bus b and bus k, l_{cl} is the capacity limit of line l, H_{lk} is the network transfer matrix, and NL is the set of all lines.

The generator capacity constraint is as follows:

$$-P_{Gb} + P_{Gb}^{\max} \ge 0 \,, \forall b \in B \quad (18)$$

where, $P_{Gb}^{\max}$ is the maximum generation capacity at bus b.

In order to solve OPF problem it is necessary to select one of the buses as swing (slack) bus with the following relation:

$$-sw_k\delta_k = 0 \,, \forall k \in B \quad (19)$$

where, sw_k is the swing bus vector.

There are two nonnegative decision variables, with the following relations:

$$d_b \ge 0 \,, \forall b \in B \quad (20)$$

$$P_{Gb} \ge 0 \,, \forall b \in B \quad (21)$$

3 Principles of Multi Objective Optimization

Generally, nonlinear constrained multi objective optimization problems can be shown as follows [20]:

$$Minimize \quad f_i(x) \quad i = 1,\ldots,N_{obj} \tag{22}$$

Subject to

$$g_j(x) = 0 \quad j = 1,\ldots,ME \tag{23}$$

$$h_k(x) = 0 \quad k = 1,\ldots,MI \tag{24}$$

where f_i is the *ith* objective function, x is a decision variable vector which representing a solution, N_{obj} is the number of objectives, ME is the number of equality constraints, and MI is the number of inequality constraints.

As stated already, the objective functions often do not have a common scale, and normally compete with each other. For such competing objectives, instead of looking for one optimal solution, a set of optimal solutions is of interest. The reason for the interest in these several optimal solutions is the fact that no solution can be considered to be better than any other one with respect to all objective functions. These optimal solutions are known as Pareto optimal solutions.

In this situation, any two solutions x^1 and x^2 for a multi objective optimization problem can have one of the two following possibilities:

The first solution x^1 dominates or covers the other solution. In this case x^1 is called non dominated (or dominating) solution or vice versa.

In a minimization problem, a solution x^1 covers or dominates x^2 if and only if the following two conditions are satisfied:

$$1.\ \forall i \in \{1,2,\ldots,N_{obj}\} : f_i(x^1) \le f_i(x^2) \tag{25}$$

$$2.\ \exists j \in \{1,2,\ldots,N_{obj}\} : f_j(x^1) < f_j(x^2) \tag{26}$$

The solutions that are non-dominated within the entire search space constitute the Pareto optimal set [2] and [20].

4 Trust Region Based Augmented Lagrangian Method (TRALM)

The Augmented Lagrangian Method (ALM) [21] solves a generic optimization problem

$$\min_X f(X) \tag{27}$$

Subject to

$$H(X) = 0 \tag{28}$$

$$G(X) \le 0 \tag{29}$$

$$X \ge 0 \tag{30}$$

By converting it into a sequence of unconstrained optimization problems with penalty terms as follows:

$$\min_X L^k(X) = f(X) + (\lambda^k)^T H(X) + \frac{1}{2} H(X)^T [W^k] H(X) + \sum_{j=1}^{ni} \frac{1}{2U_j^k} \left\{ (\max[\mu_j^k + U_j^k G_j(X), 0])^2 - (\mu_j^k)^2 \right\} \tag{31}$$

In Eq. (31), is the number of inequality constraints, λ^k and μ^k are trial Lagrange multipliers, and W^k and U^k are penalty parameters. In the so-called "multiplier method", λ^k, μ^k, W^k, and U^k are updated after each round of unconstrained optimization

$$\lambda^{k+1} = \lambda^k + [W^k] H(X^k) \tag{32}$$

$$\mu_j^{k+1} = \max\left\{ \mu_j^k + U_j^k G_j(X^k), 0 \right\} \tag{33}$$

$$W_j^{k+1} = \begin{cases} \beta_W W_j^k & if \left| H_j(X^k) \right| \succ \gamma_W \left| H_j(X^{k-1}) \right| \\ W_j^k & if \left| H_j(X^k) \right| \le \gamma_W \left| H_j(X^{k-1}) \right| \end{cases} \tag{34}$$

$$U_j^{k+1} = \begin{cases} \beta_U U_j^k & if\ G_j(X^k) \succ \gamma_U G_j(X^{k-1}) \\ U_j^k & if\ G_j(X^k) \le \gamma_U G_j(X^{k-1}) \end{cases} \tag{35}$$

where, X^k is the solution of Eq. (31). Convergence is achieved provided that $\gamma_W \succ 0, \gamma_U \prec 1, \beta_W \succ 1, \quad \beta_U \succ 1$, and the following relations are satisfied:

$$\left\| \nabla_X L_X(X^k) \right\| \le \varepsilon^k \tag{36}$$

$$\left\| \lambda^{k+1} - \lambda^k \right\| / (1 + \left\| \lambda^k \right\|_\infty) \le \varepsilon_\lambda \tag{37}$$

$$\left\| \mu^{k+1} - \mu^k \right\| / (1 + \left\| \mu^k \right\|_\infty) \le \varepsilon_\mu \tag{38}$$

In Eqs. (36-38), ε's are the tolerance parameters and ε^k decreases to a near-zero value ε^∞ as the sub-optimization k increases. Combined with a suitable unconstrained optimization algorithm, the augmented Lagrangian method can solve large-scale nonlinear constrained optimization problems very reliably and generate accurate Lagrangian multipliers.

In the TRALM algorithm, we use a trust region method to solve Eq. (31). Generally trust-region methods are used to solve unconstrained optimization problems [22]. Hence the constrained problems are converted to unconstrained ones by Lagrangian multipliers and are solved by trust-region algorithms. Branch *et al.* proposed a two-dimensional trust-region method for solving large-scale optimization problems [23]. The pseudo code for the trust-region method adopted in TRALM is shown in Fig. 1.

5 Implementation of the Proposed Algorithm

For implementing the proposed algorithm (TRALM), the parameters have been selected as follows:

Let $0 \prec \tau \prec \eta \prec 1$, $0 \prec \gamma_1 \prec \gamma_2$, $\Delta_0 \succ 0$, and X_0 be given,

$k \leftarrow 0$

while $\left\|\nabla_X L(X_k)\right\| \succ \varepsilon$ do

$$\psi_k(S) \equiv \nabla_X L(X_k)^T S + \frac{1}{2} S^T \nabla_X^2 L(X_k) S$$

$$S_k = \arg \min_{\|S\| \le \Delta_k} \psi_k(S)$$

$$\rho_k = \frac{L(X_k + S_k) - L(X_k)}{\psi_k(S_k)}$$

if $\rho_k \succ \tau$, $X_{k+1} \leftarrow X_k + S_k$

else $X_{k+1} \leftarrow X_k$ end if

if $\rho_k \le \tau$, $\Delta_{k+1} \leftarrow \gamma_1 \left\|S_k\right\|$

else if $\rho_k \succ \eta$ and $\left\|S_k\right\| = \Delta_k$, $\Delta_{k+1} \leftarrow \gamma_2 \Delta_k$

else $\Delta_{k+1} \leftarrow \Delta_k$ end if

$k \leftarrow k+1$

end do

Fig. 1 Pseudo code for the trust-region method adopted in TRALM

$\varepsilon_\lambda = 5e-3$, $\varepsilon_\mu = 1e-1$, $\varepsilon^0 = 2e0$, $\varepsilon^\infty = 1e-2$, $\tau = 0.25$

$\eta = 0.75$, $\gamma_1 = 0.1$, $\gamma_2 = 2.0$, $\beta_{w,u} = 3$, $\gamma_{w,u} = 0.33$

Then the TRALM has been implemented in MATLAB on a Pentium 133 MHz PC and was tested on the standard IEEE 30 bus six-generator test system. The single–line diagram and the generator fuel cost and emission coefficients are shown in Fig. 2 and Tables 1 and 2, respectively [2]. The detailed data could be obtained from [2].

To compute the different Pareto optimal solutions, objective functions are linearly combined to constitute a single objective function as follows:

$$Minimize \;\; wF(P_G) + (1-w)\,\lambda\, E(P_G) \qquad (39)$$

where, the scaling factor λ was selected to be 3000 in our study and w is a weighting factor [2].

As can be seen from Eq. (39), when, $w = 0$, the single objective function calculates only the emission amount, and when $w = 1$, it calculates only the fuel cost. When w changes from 0 to 1, for each w, there is a Pareto optimal solution. In order to generate evenly-distributed Pareto optimal solution set, w is increased evenly by a fixed amount Δw in each step from 0 to 1. The number of w counts the number of Pareto optimal solutions. As a matter of fact there is not a definite rule to choose the number of w, but we generated the number of Pareto optimal sets, from 11 to 101 and computed the best compromise solutions. Comparing these best compromise solutions showed that, there is not much difference between the best compromise solutions when the number of w changes from 21 to 101. So 21, was selected for w due to the advantage of less computation time.

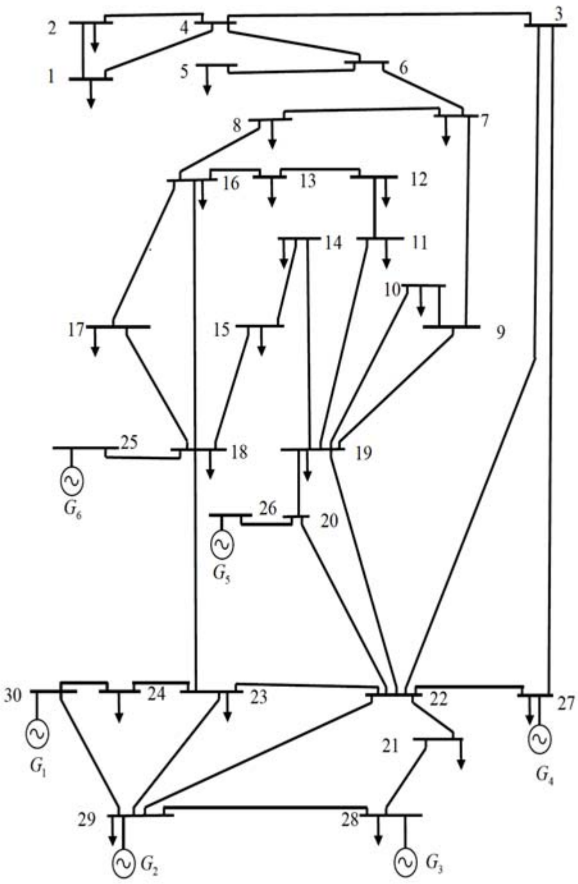

Fig. 2 Single-line diagram of IEEE 30 bus test system

Table 1 Generator fuel cost coefficients

Gen No	$F = a + bP_G + cP_G^2$ \$/h			$P_{G\max}$ Per 100 MW	$P_{G\min}$ Per 100 MW
	a	*b*	*c*		
1	10	200	100	1.5	0.05
2	10	150	120	1.5	0.05
3	20	180	40	1.5	0.05
4	10	100	60	1.5	0.05
5	20	180	40	1.5	0.05
6	10	150	100	1.5	0.05

Table 2 Generator emission coefficients

Gen No	$E = 10^{-2}(\alpha + \beta P_G + \gamma P_G^2) + \xi \exp(\lambda P_G)(ton/h)$				
	α	β	γ	ξ	λ
1	4.091	-5.554	6.490	2.0 E-4	2.857
2	2.543	-6.047	5.638	5.0 E-4	3.333
3	4.258	-5.094	4.586	1.0 E-6	8.000
4	5.326	-3.550	3.380	2.0 E-3	2.000
5	4.258	-5.094	4.586	1.0 E-6	8.000
6	6.131	-5.551	5.151	1.0 E-5	6.667

By varying w from 0 to 1 with the abovementioned procedure, the compromise solution set has been computed using TRALM, and the results are shown in Figs. 3, 4 and 5. In Fig. 3, the system is considered as lossless and the security is released (Case1). In Fig. 4, the transmission power loss has been taken into account and the security is released (Case2). At last in Fig. 5 all three constraints have been taken into account (Case3).

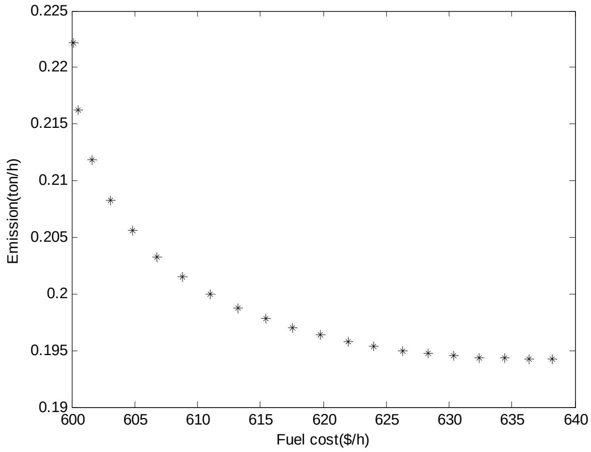

Fig. 3 The solution of TRALM approach for Case 1

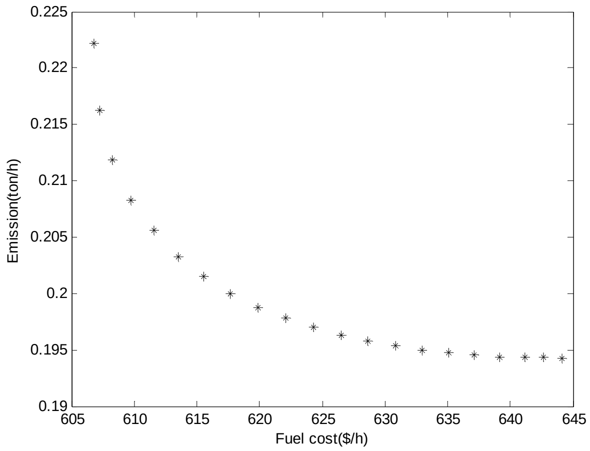

Fig. 4 The solution of TRALM approach for Case 2

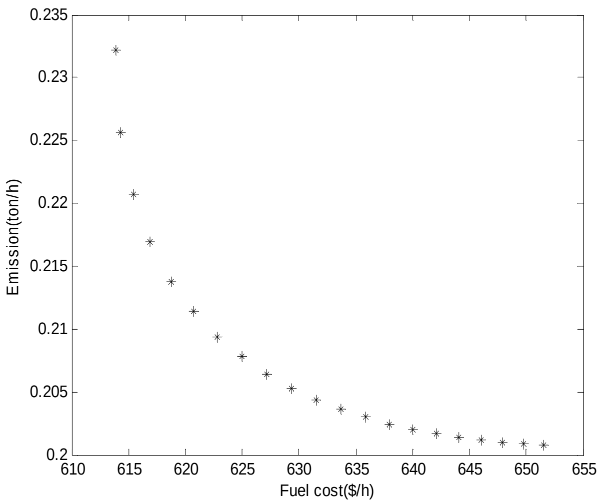

Fig. 5 The solution of TRALM approach for Case 3

6 Best Compromise Solution

Optimization of the formulated objective functions Eqs. (1) and (3) using TRALM yields not a single optimal solution, but a set of Pareto optimal solutions, in which one objective cannot be improved without sacrificing another objective. For practical applications, however, we need to select one solution, satisfying the different goals to some extent. Such a solution is called best compromise solution. One of the challenging factors for the tradeoff decision is the imprecise nature of the decision maker's judgment. For this consideration fuzzy set theory is employed [19]. The i_{th} objective value, F_i corresponding to a solution is represented by a membership function μ_i

$$\mu_i = \begin{cases} 1 & F_i \le F_i^{\min} \\ \dfrac{F_i^{\max} - F_i}{F_i^{\max} - F_i^{\min}} & F_i^{\min} < F_i < F_i^{\max} \\ 0 & F_i \ge F_i^{\max} \end{cases} \tag{40}$$

where, $F_i^{\min}$ is the value of an original objective function i which is supposed to be completely satisfactory, and $F_i^{\max}$, is the value of the objective function which is clearly unsatisfactory to the decision maker. For each non-dominated solution k, the normalized membership function μ^k is calculated as follows:

$$\mu^k = \frac{\sum_{i=1}^{Nobj} \mu_i^k}{\sum_{k=1}^{M} \sum_{i=1}^{Nobj} \mu_i^k} \tag{41}$$

where, M is the number of non-dominated solutions, and N_{obj} is the number of objective functions. The function μ^k in equation Eq. (41) represents a fuzzy cardinal priority ranking of the non-dominated solutions. The solution which attains the maximum membership μ^k in the fuzzy set can be chosen as the best compromise solution or that having the highest cardinal priority ranking.

The values of μ^k for non-dominated solutions of the proposed algorithm have been calculated by a MATLAB program.

7 Results and Discussions

To demonstrate the effectiveness of the proposed algorithm TRALM, our obtained results are compared with the results obtained by the seven other algorithms reported as SPEA [2], LP [11], NPGA[13], NSGA[14], MOSST[15], FCPSO[16], and EC[18] for three Cases as follows:

7.1 Case 1

At this level the system is considered as lossless and only the capacity constraints are considered. The results obtained from the proposed algorithm on the test system, have been shown in Fig. 3.

Table 3 is provided to compare the best fuel costs of various algorithms. As can be seen from this table the fuel cost calculated by the proposed algorithm (TRALM) is less than or at least equal to the other concerned algorithms.

Table 4 has been provided to compare the best emissions of different algorithms. This table shows that the emission amount obtained from the proposed algorithm is less or at least equal to the other algorithms. It is worth to be noted that when the emission of TRALM is equal to other algorithms its fuel cost is less than the ones of the others.

Table 5 has gathered the existing best compromise solutions of the concerned algorithms. Since in compromise solution, decreasing of one objective function occurs in compensation of increasing the other one. So it is not a good measure for comparison, but Table 5 has been provided to show that the compromise solution of the proposed algorithm is quite reasonable.

The run time of TRALM, for Case1 is 11 seconds, which is greater than the run time reported in [18], and less than the one reported in [2].

7.2 Case 2

In this case the transmission losses and power balance constraints are considered, but the security constraints are released. The numerical results obtained from the proposed algorithm on the test system are shown in Fig. 4.

Table 6 is provided to compare the best fuel costs of various algorithms. This table shows that the fuel cost obtained by the proposed algorithm except in EC approach is less than the ones of the other concerned algorithms.

Table 7 has been provided to compare the best emissions of the various algorithms. It shows that the emission amount obtained from the proposed algorithm is less than or at least equal to the other algorithms. It is worth to be noted that when the emission of TRALM is equal to other algorithms its fuel cost is less than the ones of the others.

Table 8 has gathered the existing best compromise solutions of the concerned algorithms. This table represents a reasonable compromise solution for the proposed algorithm.

7.3 Case 3

In this case, all constraints including transmission losses, power balance, and security constraints are considered. The numerical results obtained from the proposed algorithm on the test system are shown in Fig. 5.

Table 9 is provided to compare the best fuel costs of the various algorithms. This table shows that the fuel cost obtained by the proposed algorithm except in EC approach is less than the ones of the other concerned algorithms.

Table 10 has been provided to compare the best emissions of the various algorithms. It shows that the emission amount obtained from the proposed algorithm except in EC approach is less than the ones of other algorithms.

Table 11 has gathered the existing best compromise solutions of the concerned algorithms. This table represents a reasonable compromise solution for the proposed algorithm.

Table 3 Comparison of best fuel costs of various algorithms for Case 1

	LP [11]	MOSST [15]	FCPSO [16]	NPGA [13]	NSGA [14]	SPEA [2]	EC [18]	TRALM [Proposed]
P_{G1}	0.1500	0.1097	0.1070	0.1116	0.1038	0.1009	0.1097	0.1097
P_{G2}	0.3000	0.2998	0.2897	0.3153	0.3228	0.3186	0.2998	0.2998
P_{G3}	0.5500	0.5243	0.5250	0.5419	0.5123	0.5400	0.5243	0.5243
P_{G4}	1.0500	1.0162	1.0150	1.0415	1.0387	0.9903	1.0162	1.0162
P_{G5}	0.4600	0.5243	0.5300	0.4726	0.5324	0.5336	0.5243	0.5243
P_{G6}	0.3500	0.3597	0.3673	0.3512	0.3241	0.3507	0.3597	0.3597
Cost ($/h)	606.314	605.8890	600.1315	600.31	600.34	600.22	600.1114	600.1114
Emission (ton/h)	0.2233	0.2222	0.2223	0.2238	0.2241	0.2223	0.2221	0.2222

Table 4 Comparison of best emissions of various algorithms for Case 1

	LP [11]	MOSST [15]	FCPSO [16]	NPGA [13]	NSGA [14]	SPEA [2]	EC [18]	TRALM [Proposed]
P_{G1}	0.4000	0.4095	0.4097	0.40584	0.4072	0.4240	0.4060	0.4054
P_{G2}	0.4500	0.4626	0.4550	0.45915	0.4538	0.4577	0.4590	0.4592
P_{G3}	0.5500	0.5426	0.5363	0.53797	0.4888	0.5301	0.5379	0.5382
P_{G4}	0.4000	0.3884	0.3842	0.38300	0.4302	0.3721	0.3830	0.3832
P_{G5}	0.5500	0.5427	0.5348	0.53791	0.5836	0.5311	0.5380	0.5382
P_{G6}	0.5000	0.5142	0.5140	0.51012	0.4707	0.5190	0.5100	0.5099
Emission (ton/h)	0.19423	0.19418	0.1942	0.1943	0.1946	0.1942	0.1942	0.1942
Cost ($/h)	639.600	644.1118	638.3577	636.04	633.83	640.42	638.2703	638.2387

Table 5 Comparison of best compromise solutions of various algorithms for Case 1

	LP [11]	**MOSST [15]**	**FCPSO [16]**	**NPGA [13]**	**NSGA [14]**	**SPEA [2]**	**EC [18]**	**TRALM [Proposed]**
P_{G1}	Compromise solution was not considered in the paper	Compromise solution was not calculated in details	Compromise solution was not considered in the paper	0.2663	0.2252	0.2623	Compromise solution was not calculated in details	0.2502
P_{G2}				0.3700	0.3622	0.3765		0.3700
P_{G3}				0.5222	0.5222	0.5428		0.5394
P_{G4}				0.7202	0.7660	0.6838		0.7080
P_{G5}				0.5256	0.5397	0.5381		0.5394
P_{G6}				0.4296	0.4187	0.4305		0.4296
Cost($/h)		621.7582		608.90	606.03	610.2977	610.9634	608.8234
Emission (ton/h)		0.1968		0.2015	0.2041	0.2005	0.2000	0.2015

Table 6 Comparison of best fuel costs of various algorithms for Case 2

	L [11]	**MOSST [15]**	**FCPSO [16]**	**NPGA [13]**	**NSGA [14]**	**SPEA [2]**	**EC [18]**	**TRALM [Proposed]**
P_{G1}	Case2 was not considered in the paper	Case2 was not considered in the paper	0.1130	0.1425	0.1447	0.1279	0.1076	0.1129
P_{G2}			0.3145	0.2693	0.3066	0.3163	0.3012	0.3024
P_{G3}			0.5826	0.5908	0.5493	0.5803	0.5970	0.5322
P_{G4}			0.9860	0.9944	0.9894	0.9580	0.9897	1.0215
P_{G5}			0.5264	0.5315	0.5244	0.5258	0.5120	0.5322
P_{G6}			0.3450	0.3392	0.3542	0.3589	0.3511	0.3629
Cost($/h)			607.7862	608.06	607.98	607.86	605.8363	606.8015
Emission (ton/h)			0.2201	0.2207	0.2191	0.2176	0.2208	0.2222

Table 7 Comparison of best emissions of various algorithms for Case 2

	LP [11]	**MOSST [15]**	**FCPSO [16]**	**NPGA [13]**	**NSGA [14]**	**SPEA [2]**	**EC [18]**	**TRALM [Proposed]**
P_{G1}	Case2 was not considered in the paper	Case 2 was not considered in the paper	0.4063	0.4064	0.3929	0.4145	0.4102	0.4061
P_{G2}			0.4586	0.4876	0.3937	0.4450	0.4633	0.4611
P_{G3}			0.5510	0.5251	0.5818	0.5799	0.5447	0.5438
P_{G4}			0.4084	0.4085	0.4316	0.3847	0.3921	0.3966
P_{G5}			0.5432	0.5386	0.5445	0.5384	0.5447	0.5438
P_{G6}			0.4942	0.4992	0.5192	0.5051	0.5152	0.5126
Emission (ton/h)			0.1942	0.1943	0.1947	0.1943	0.1942	0.1942
Cost($/h)			642.8964	644.23	638.98	644.77	646.2203	644.0792

Table 8 Comparison of best compromise solutions of various algorithms for Case 2

	LP [11]	**MOSST [15]**	**FCPSO [16]**	**NPGA [13]**	**NSGA [14]**	**SPEA [2]**	**EC [18]**	**TRALM [Proposed]**
P_{G1}	Compromise solution were not considered in the paper	Compromise solution were not considered in the paper for Case 2	Compromise solution were not considered in the paper	0.2976	0.2935	0.2752	Compromise solution were not considered in the paper for Case 2	0.2694
P_{G2}				0.3956	0.3645	0.3752		0.3817
P_{G3}				0.5673	0.5833	0.5796		0.5463
P_{G4}				0.6928	0.6763	0.6770		0.6815
P_{G5}				0.5201	0.5383	0.5283		0.5463
P_{G6}				0.3904	0.4076	0.4282		0.4388
Cost($/h)				617.79	617.80	617.57		617.5106
Emission (ton/h)				0.2004	0.2002	0.2001		0.2001

Table 9 Comparison of best fuel costs of various algorithms for Case 3

	LP [11]	**MOSST [15]**	**FCPSO [16]**	**NPGA [13]**	**NSGA[14]**	**SPEA [2]**	**EC [18]**	**TRALM [Proposed]**
P_{G1}	Case3 was not considered in the paper	Case3 was not considered in the paper	0.1596	0.1127	0.1358	0.1319	0.2183	0.1648
P_{G2}			0.3535	0.3747	0.3151	0.3654	0.3554	0.3456
P_{G3}			0.7974	0.8057	0.8418	0.7791	0.5776	0.6619
P_{G4}			0.9719	0.9031	1.0431	0.9282	0.7590	1.1079
P_{G5}			0.08624	0.1347	0.0631	0.1308	0.5393	0.1691
P_{G6}			0.49609	0.5331	0.4664	0.5292	0.4080	0.4147
Cost($/h)			620.18	620.46	620.87	619.60	611.2198	613.9360
Emission (ton/h)			0.2283	0.2243	0.2368	0.2244	0.2043	0.2322

Table 10 Comparison of best emissions of various algorithms for Case 3

	LP [11]	MOSST [15]	FCPSO [16]	NPGA [13]	NSGA [14]	SPEA [2]	EC [18]	TRALM [Proposed]
P_{G1}	Case3 was not considered in the paper	Case 2 was not considered in the paper	0.47969	0.4753	0.4403	0.4419	0.4122	0.4649
P_{G2}			0.5287	0.5162	0.4940	0.4598	0.4667	0.5164
P_{G3}			0.67116	0.6513	0.7509	0.6944	0.5514	0.6201
P_{G4}			0.5318	0.4363	0.5060	0.4616	0.4059	0.4764
P_{G5}			0.1257	0.1896	0.1375	0.1952	0.5731	0.2091
P_{G6}			0.5299	0.5988	0.55364	0.6131	0.4550	0.5771
Emission (ton/h)			0.2047	0.2017	0.2084	0.2019	0.1944	0.2008
Cost($/h)			651.62	657.57	649.24	651.71	642.70	651.5708

Table 11 Comparison of best compromise solutions of various algorithms for Case 3

	LP [11]	MOSST [15]	FCPSO [16]	NPGA [13]	NSGA [14]	SPEA [2]	EC [18]	TRALM [Proposed]
P_{G1}	Compromise solution was not considered in the paper for Case3	Compromise solution was not considered in the paper for Case 3	Compromise solution was not considered in the paper for Case 3	0.2998	0.2712	0.3052	Compromise solution was not considered in the paper for Case 3	0.3126
P_{G2}				0.4325	0.3670	0.4389		0.4262
P_{G3}				0.7242	0.8099	0.7163		0.6508
P_{G4}				0.6852	0.7550	0.6978		0.7994
P_{G5}				0.1560	0.1357	0.1552		0.1809
P_{G6}				0.5561	0.5239	0.5507		0.4941
Cost ($/h)				630.06	625.71	629.59		622.7865
Emission (ton/h)				0.2079	0.2136	0.2079		0.2094

8 Conclusions

Most papers reported in the literature review, used evolutionary algorithms to solve multi-objective environmental-economic power dispatch problem. The trust region based augmented Lagrangian method (TRALM) is a known and powerful technique for solving constrained nonlinear programming problems. Therefore in this paper the TRALM, was presented and applied to combined environmental / economic power dispatch optimization problem. The problem was formulated as a multi objective optimization problem with competing fuel cost and environmental impact objectives. The two objective functions were linearly combined by weighting factors to constitute a single objective function. By varying the weighting factor the Pareto optimal sets are achieved.

A fuzzy based mechanism was employed to extract the best compromise solution among the Pareto optimal solution set.

To demonstrate the effectiveness of the proposed algorithm, we compared the results obtained by an implementation of our algorithm with the ones obtained by seven different algorithms reported in the literature. The results of the comparisons showed our proposed approach is very competitive in the sense of being accurate.

In the real world EEDP is very important for generating companies to achieve an optimal solution for their installed generating units. This approach can help them find an optimal solution for their generation schedule.

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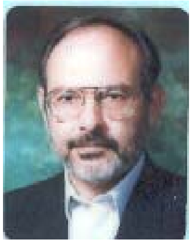

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