

# Reduced order model for doubly output induction generator in wind park using integral manifold theory

M. Kalantar and M. Sedighzadeh

**Abstract:** A dynamic reduced order model using integral manifold theory has been derived, which can be used to simulate the DOIG wind turbine using a double-winding representation of the generator rotor. The model is suitable for use in transient stability programs that can be used to investigate large power systems. The behavior of a wind farm and the network under various system disturbances was studied using this dynamic model. Simulation results of the proposed method represents that integral manifold method results fit the detailed model results with a higher precision than other methods.

**Keywords:** doubly output induction generator, wind park, reduced order model, integral manifold theory.

## 1 Introduction

The production of a significant amount of power from the wind has required both the development of larger more efficient and reliable wind turbines and the use of more than one machine at each site constituting the popular wind farms. Currently, the most used configuration of wind parks consists of a set of wind turbines each one driving a double output induction generator (DOIG) or synchronous machine [1], which, in turn, are assembled in group(s) and directly coupled to the existing a.c. system.

In order to give a net contribution to solve the problems that are still pending, namely in what concerns the impact of the integration of wind parks in the utility distribution system, some computational tools have been developed with the aim of assisting both wind park and distribution system planners and designers. These computational tools are based on models that are able to accurately simulate the behavior of wind parks under transient situations.

The paper is concerned with the development and application of wind parks reduced order models that are able to simulate relevant dynamics of the park with respect to the utility system, henceforth denoted by wind park reduced order models.

Operation of DOIG is extensively treated in literature, and various models for different applications have been obtained [2]–[8] and reduced order models of DOIG wind turbines for dynamic studies have been published [9]–[14].

Firstly, a wind park detailed model consists of a set of wind turbines each one driving a double output induction generator (DOIG) will be presented. Then, integral manifolds technique will be applied to the model in order to achieve a reduced order model which describe with a reasonable degree of accuracy the transient behavior of the wind park in what concerns the interaction with the grid.

## 2 Wind park full order model

Detailed models previously developed for each component of the system have been conveniently

adapted and linked together in order to form an integrated wind park detailed model. The different elements modeled, namely, wind turbine, DOIG, reactive power compensation system, transformers, interconnection feeder and possible local loads connected to the feeder, are shown in figure 1.

In this paragraph, the equations describing the subsystems of a variable speed wind turbine with DOIG and converter (rectifier+inverter) as depicted in figure 2 will be developed. The equations for the rotor, the generator and the converter will be given here. The equations have been developed using the following assumptions:

- All rotating mass is represented by one element. The so-called ‘lumped-mass’ representation. Elastic shafts and resulting torsional forces are neglected.
- A quasi static approach is used for the description of aerodynamic part of the wind turbine.
- Magnetic saturation in the DOIG is neglected.
- Dynamic phenomena in the converter are neglected.

These assumptions reduce the complexity of the modelling task and the amount of system data is needed. As reliable data are often hard to obtain, this is considered an important advantage. Furthermore, under these assumptions the computation speed can be increased, which is also considered an advantage, particularly when large systems are to be simulated.

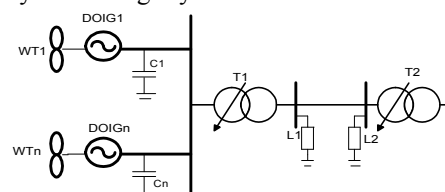


Fig. 1 System studied

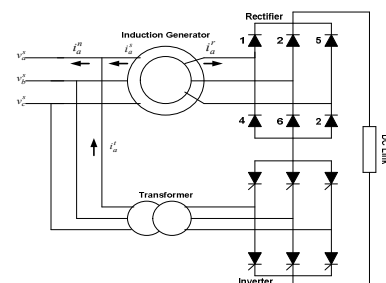


Fig. 2 Schematic representation of DOIG

Iranian Journal of Electrical & Electronic Engineering, 2005.

Paper first received 8th November 2003 and in revised form 10th July 2004.

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### A. wind turbine models

The rotor converts the energy contained by the wind into mechanical energy. The following well known equation between wind speed and power extracted from the wind holds [12].

$$P_w = \frac{\rho}{2} C_p(\lambda, \theta) A_R V_w^3 \quad (1)$$

$$T_m = p_w / \omega_r$$

With  $P_w$  the power extracted from the air flow [W],  $\rho$  the air density [ $\text{kg}/\text{m}^3$ ],  $C_p$  the performance coefficient or power coefficient,  $\lambda$  the tip speed ratio  $V_t/V_w$ , the ratio between blade tip speed  $V_t$  and wind speed upstream the rotor  $V_w$  [m/s],  $\theta$  the pitch angle of rotor blades [deg],  $A_R$  the area covered by the rotor [ $\text{m}^2$ ], and  $\omega_r$  the rotor speed [rad/sec].

The performance coefficient  $C_p$  that is a function of the tip speed ratio  $\lambda$  and pitch angle  $\theta$  will be investigated further. The calculation of the performance coefficient requires the use of blade element theory. As this requires knowledge of aerodynamics and the computations are rather complicated, numerical approximations have been developed.

### B. DOIG model with double-winding rotor

The DOIG model with double-winding rotor is generally represented by the seventh order differential equations of flux linkages and speed. The flux linkages are represented in the d-q axis reference frame which revolves at synchronous speed, (see nomenclature for notations not defined in the appendix). In these equations,  $v$  and  $i$  are in per unit, but time remains in seconds. The subscripts 1,2 and 3 refer to the stator winding, first winding rotor, and second winding rotor, respectively. In a wind farm that have a number  $n$  of DOIGs, for a DOIG with double-winding rotor is given the following set of complex differential and algebraic equations. If suppose  $\tau_3 = 1/r_3$ , Thus,

$$\begin{aligned} \dot{\varphi}_{1dh}^* &= -(r_{11}/l) \varphi_{1d} + \omega \varphi_{1q} + (r_{13}l_m/l) \varphi_{2d} + (r_{12}l_m/l) \varphi_{3d} + v_{1d} \\ \dot{\varphi}_{1q}^* &= -\omega \varphi_{1d} - (r_{11}/l) \varphi_{1q} + (r_{13}l_m/l) \varphi_{2q} + (r_{12}l_m/l) \varphi_{3q} + v_{1q} \\ \dot{\varphi}_{2d}^* &= (r_{23}l_m/l) \varphi_{1d} - (r_{22}/l) \varphi_{2d} + s \omega \varphi_{2q} + (r_{21}l_m/l) \varphi_{3d} + v_{2d} \\ \dot{\varphi}_{2q}^* &= (r_{23}l_m/l) \varphi_{1q} - s \omega \varphi_{2d} - (r_{22}/l) \varphi_{2q} + (r_{21}l_m/l) \varphi_{3q} + v_{2q} \\ \tau_3 \dot{\varphi}_{3d}^* &= (l_{21}l_m) \varphi_{1d} + (l_{11}l_m) \varphi_{2d} - (l_{33}) \varphi_{3d} + (s \omega l / r_3) \varphi_{3q} \\ \tau_3 \dot{\varphi}_{3q}^* &= (l_{21}l_m) \varphi_{1q} + (l_{11}l_m) \varphi_{2q} - (s \omega l / r_3) \varphi_{3d} - (l_{33}) \varphi_{3q} \\ T_e &= (l_{31}l_m/l) (\varphi_{1d} \varphi_{2q} - \varphi_{1q} \varphi_{2d}) + (l_{21}l_m/l) (\varphi_{1d} \varphi_{3q} - \varphi_{1q} \varphi_{3d}) \quad (3) \end{aligned}$$

The model is developed based on the assumptions that it is possible to decompose the total flux linking each winding into leakage and magnetisation components, and that it is possible to characterise the machine through a unique magnetisation characteristic.

### C. Swing equation

For each unit  $i$ , the rotor slip  $S$  may be evaluated using the swing equation :

$$s^* = -(\omega/2H)(T_m - T_e) \quad (3)$$

### 3 Reduction-order preliminary analysis

In a first step, some of the methodologies currently available to reduce the order of power systems models have been applied to the specific case of wind parks. With the aim of gaining some insight regarding the behavior of each technique, a steady-state preliminary

analysis has been carried out, using linearized models in order to take advantage of its inherent simplicity [15].

Modal truncation is one first reduction schemes that has been applied to electric power systems [16]. This technique is based on the pole location of the system. The state variables are transformed in modal variables and the fast decay poles and/or those associated with high frequencies are neglected, thus enabling a reduction in the order of the system.

Balanced reduction techniques take a slightly different approach, because they are based in the input/output behavior of the system [17]. Actually, the original state-space system is transformed into a new representation that has the property that each state-space variable is both controllable and observable. In order to achieve a reduced order model, states that are strongly influenced by the inputs and strongly connected to the outputs are retained, whereas states that are weakly controllable and observable are truncated.

Another method used in power systems order reduction is the so called optimal Hankel-norm approximation [18]. This criterion tries achieve a compromise between a small worst case error and a small energy error.

Another technique used in power system as singular perturbations decomposes the system according to its fast and slow dynamics and then lowers the model order by first neglecting the fast dynamics phenomena [19]. The effect of fast dynamics are then reintroduced as boundary layer correction calculated in separated time scales, which leads to correct static gains.

This paper employs the manifold concept as a tool for reduced order modeling and decomposition of wind park [20,21,22,23]. Theory of integral manifolds has accuracy over singular perturbation, because nonlinear methods can be classified into two groups: geometric and asymptotic. Among the concepts common to both groups of methods is the concept of integral or invariant manifolds, a nonlinear generalization of the notion of invariant subspace in linear systems and reduced order modeling is treated with a combination of geometric concepts and asymptotic techniques of singular perturbations.

### 4 Integral manifolds background [23].

A smooth  $s$ -dimensional surface  $S$  in the  $n$ -dimensional space  $R^n$  is defined by  $m=n-s$  independent algebraic or transcendental scalar equations. In their simplest form, these equations express certain  $m$  coordinates  $z$  as  $m$  explicit functions of the remaining  $s$  coordinates  $x$ , that is they define  $S$  by its graph:

$$S: z=h(x), z \in R^m; x \in R^s; m+s=n \quad (4)$$

It is assumed that, for all  $x$  in a domain of practical interest,  $\partial h/\partial x$  exists and has full rank  $m$ . For approximated constructions of  $h(x)$  pursued in this paper it will also be assumed that higher order derivatives of  $h(x)$  exist and are continuous. In a more general situation the surface  $S$  may vary with time  $t$ , then

$$S_t: z=h(x,t), z \in R^m; x \in R^s; m+s=n \quad (5)$$

It will be assumed that  $\partial h/\partial t$  exists and is continuous over an interval of interest  $t \in (t_0, t_1)$ , preferably infinite:

$$t_1 \rightarrow \infty$$

Let us now use the same coordinates  $z$  and  $x$  to describe a dynamic system  $D_i$  in  $R^n$ :

$$z^* = g(x, z, t), \dots, z \in R^m \quad (6)$$

$$x^* = f(x, z, t), \dots, z \in R^s, m + s = n \quad (7)$$

Where appropriate differentiability assumptions are made about  $g$  and  $f$ . The surface  $S_t$  and the system  $D_t$  have thus been introduced as two entities unrelated to each other. In this paper we explore a particularly useful relationship of  $S_t$  and  $D_t$ , when  $S_t$  is an integral manifold of  $D_t$ . The term invariant manifold will be used when such an integral manifold is time-invariant, that is when  $\partial h / \partial t = 0$  and  $S_t = S$  as in (4).

**Manifold Definition:** Surface  $S_t$  is an integral manifold of  $D_t$  if every solution  $z(t)$ ,  $x(t)$ , of (6) – (7) which is in  $S_t$  at  $t=t_0$ .

$$Z(t_0) = h(x(t_0), t_0) \quad (8)$$

Remains in  $S_t$  for all  $t \in (t_0, t_1)$ , that is

$$z(t) = h(x(t), t), t \in (t_0, t_1) \quad (9)$$

This definition furnishes a condition which can be used to verify whether  $h(x, t)$  in (5) defines an integral manifold of (6)-(7).

**Manifold condition:** If  $h(x, t)$  satisfies the partial differential equation:

$$\frac{\partial h}{\partial t} + \frac{\partial h}{\partial x} f(x, h(x, t), t) = g(x, h(x, t), t) \quad (10)$$

the surface  $S_t$  given by (5) is an integral manifold of the dynamic system (6)-(7).

This condition is simply obtained by differentiating (9) with respect to  $t$ :

$$ez^* = \frac{\partial h}{\partial t} + \frac{\partial h}{\partial x} x^* \quad (11)$$

and then substituting  $x^*$  and  $z^*$  from (6) to (7). Once the existence of an integral manifold  $S_t$  of  $D_t$  has been established and its defining function  $h(x, t)$  has been found, then the restriction of  $D_t$  to the manifold  $S_t$  is given by the  $s$  th-order system

$$x^* = f(x, h(x, t), t), x \in R^s \quad (12)$$

which is obtained by the substitution of  $z=h(x, t)$  into (7).

In addition to being a tool for reduced order modeling, the concept of an integral manifold is also a decomposition tool. A reduced order model (12) is a correct description of the dynamic  $D_t$  only when the initial state is in  $S_t$ , as in (8). When the initial state of  $D_t$  is not in  $S_t$ , the knowledge of the manifold function  $h(x, t)$  continues to be useful by allowing us to replace the  $z$ -coordinates by the “off-manifold” coordinates  $\eta$ .

$$\eta = z - h(x, t) \quad \eta \in R^m \quad (13)$$

In terms of the new coordinates  $\eta$  and  $x$  the original system (6)–(7) becomes:

$$\eta^* = g(x, \eta + h(x, t), t) - \frac{\partial h}{\partial x} f(x, \eta + h(x, t), t) - \frac{\partial h}{\partial t} \quad (14)$$

$$x^* = f(x, h(x, t), t) \quad (15)$$

An advantage of this full order description of  $D_t$  over (6)–(7) is that now the manifold condition is simply  $\eta=0$ .

The decomposition is achieved in the sense that on the surface  $S_t$  the subsystem (14) is at an equilibrium:  $\eta(t_0)=0$  implies  $\eta(t)=0$  for all  $t \in (t_0, t_1)$  and all  $x$ . The off-manifold and in-manifold description (14)–(15) is particularly helpful when the in-manifold behavior of  $D_t$  is of primary interest and the off-manifold variable is evaluated separately as a correction term. The analysis presented in the subsequent sections illustrates both conceptual and computational advantages of this nonlinear decomposition approach.

## 5 Application to the wind park model

The integral manifolds theory outlined in the previous section was applied to the case of the wind park detailed model.

The first step consists in the separation of the time variables in slow variables and fast variables, in order to be able to solve them in the appropriate time scales. The variables stator flux linkage,  $\phi_1$ , first rotor circuit flux linkage,  $\phi_2$ , and speed rotor were considered as slow variables, the remaining ones (variables second rotor circuit flux linkage,  $\phi_3$ ) were assumed as fast variable, thus:

$$x = [\phi_{1d} \dots \phi_{1q} \dots \phi_{2d} \dots \phi_{2q} \dots s] \quad (16)$$

$$z = [\phi_{3d} \dots \phi_{3q}] \quad (17)$$

To produce a fifth order model for DOIG it is necessary to eliminate  $\phi_{3d}^*$  and  $\phi_{3q}^*$  from equations (2). This can be done by finding relations for  $\phi_{3d}$  and  $\phi_{3q}$  as a function of  $\phi_{1d}, \phi_{1q}, \phi_{2d}, \phi_{2q}, s$ . If  $\tau_3$  is zero, such relationships can be obtained by equating the  $\tau_3 \phi_{3d}^*$  and  $\tau_3 \phi_{3q}^*$  with zero. When  $\tau_3$  is non zero but small, We let  $\tau_3 = \varepsilon$  and search for the unknown functions:

$$\phi_{3d} = h(\phi_{1d}, \phi_{1q}, \phi_{2d}, \phi_{2q}, s, \varepsilon) \quad (18)$$

$$\phi_{3q} = p(\phi_{1d}, \phi_{1q}, \phi_{2d}, \phi_{2q}, s, \varepsilon) \quad (19)$$

Using two power series in  $\varepsilon$  about  $\varepsilon = 0$ , namely,

$$h = h_0 + \varepsilon h_1 + \varepsilon^2 h_2 + \dots \quad (20)$$

$$p = p_0 + \varepsilon p_1 + \varepsilon^2 p_2 + \dots \quad (21)$$

To find the terms  $h_0, h_1, \dots$  and  $p_0, p_1, \dots$  of the series, we use the fact the functions  $h$  and  $p$  must satisfy (11). In view of (18) and (19), these give,

$$\varepsilon \frac{\partial h}{\partial \phi_{1d}} \frac{d\phi_{1d}}{dt} + \varepsilon \frac{\partial h}{\partial \phi_{1q}} \frac{d\phi_{1q}}{dt} + \varepsilon \frac{\partial h}{\partial \phi_{2d}} \frac{d\phi_{2d}}{dt} + \varepsilon \frac{\partial h}{\partial \phi_{2q}} \frac{d\phi_{2q}}{dt} + \varepsilon \frac{\partial h}{\partial s} \frac{ds}{dt} = (l_2 l_m) \phi_{1d} + (l_1 l_m) \phi_{2d} - l_{33} h + (s \alpha l / r_3) p \quad (22)$$

$$\varepsilon \frac{\partial p}{\partial \phi_{1d}} \frac{d\phi_{1d}}{dt} + \varepsilon \frac{\partial p}{\partial \phi_{1q}} \frac{d\phi_{1q}}{dt} + \varepsilon \frac{\partial p}{\partial \phi_{2d}} \frac{d\phi_{2d}}{dt} + \varepsilon \frac{\partial p}{\partial \phi_{2q}} \frac{d\phi_{2q}}{dt} + \varepsilon \frac{\partial p}{\partial s} \frac{ds}{dt} = (l_2 l_m) \phi_{1q} + (l_1 l_m) \phi_{2q} - l_{33} p - (s \alpha l / r_3) h \quad (23)$$

Which are partial differential equations that must be satisfied by the series (20) and (21) as identities for all  $\varepsilon$  near zero. With using (2), (18)-(21) and substituting into (22) and (23), we obtain expressions in terms of  $\varepsilon^0, \varepsilon^1, \varepsilon^2, \dots$ .

Equating coefficients of  $\varepsilon$  gives the identities to be satisfied by each  $h_i$  and  $p_i$ . For  $h_0$  and  $p_0$  we equate all

the terms not containing  $\varepsilon$  giving,

$$h_0 = (1/1 + s'^2)[(l_2 l_m / l_{33})(\varphi_{1d} + s' \varphi_{1q}) + (l_1 l_m / l_{33})(\varphi_{2d} + s' \varphi_{2q})] \quad (24)$$

$$p_0 = (1/1 + s'^2)[(l_2 l_m / l_{33})(\varphi_{1q} - s' \varphi_{1d}) + (l_1 l_m / l_{33})(\varphi_{2q} - s' \varphi_{2d})] \quad (25)$$

where

$$s' = (\omega l / r_3 l_{33}) s \quad (26)$$

Which is the same as obtained by setting  $\varepsilon = \tau_3 = 0$  in (2).

Equating coefficients of  $\varepsilon^1$  gives

$$h_1 = -(1/1 + s'^2)[(l_2 l_m / l_{33}^2)(R_{11} \varphi_{1d} + L_{11} \varphi_{1q} + R_{12} \varphi_{2d} + L_{12} \varphi_{2q} + v_{1d}) + (l_1 l_m / l_{33}^2)(R_{21} \varphi_{1d} + L_{21} \varphi_{1q} + R_{22} \varphi_{2d} + L_{22} \varphi_{2q} + v_{2d}) + (s' k_1 + k_2) s'^*] \quad (27)$$

$$p_1 = -(1/1 + s'^2)[(l_2 l_m / l_{33}^2)(L_{11} \varphi_{1d} + R_{11} \varphi_{1q} - L_{12} \varphi_{2d} + R_{12} \varphi_{2q} + v_{1q}) + (l_1 l_m / l_{33}^2)(-L_{21} \varphi_{1d} + R_{21} \varphi_{1q} - L_{22} \varphi_{2d} + R_{22} \varphi_{2q} + v_{2q}) + (s' k_1 + k_2) s'^*] \quad (28)$$

With

$$k_1 = 2s'h_0 + (l_2 l_m / l_{33}) \varphi_{1d} + (l_1 l_m / l_{33}) \varphi_{2d}$$

$$k_2 = -2s'p_0 + (l_2 l_m / l_{33}) \varphi_{1q} + (l_1 l_m / l_{33}) \varphi_{2q}$$

$$R_{11} = -(r_1 l_{11} / l) + (r_1 l_m^2 / l_{33}) / (1 + s'^2)$$

$$R_{22} = -(r_2 l_{22} / l) + (r_2 l_m^2 / l_{33}) / (1 + s'^2)$$

$$R_{12} = -(r_1 l_3 l_m / l) + (r_1 l_2 l_m^2 / l_{33}) / (1 + s'^2)$$

$$R_{21} = (r_2 / r_1) R_{12}$$

$$L_{11} = \omega + (r_1 l_m^2 / l_{33}) s' / (1 + s'^2)$$

$$L_{22} = s\omega + (r_2 l_m^2 / l_{33}) s' / (1 + s'^2)$$

$$L_{12} = (r_1 l_2 l_m^2 / l_{33}) s' / (1 + s'^2)$$

$$L_{21} = (r_2 / r_1) L_{12}$$

This process can be continued to obtain higher order terms if desired. Stopping with two terms, the approximate manifold expressions (20) and (21) are,

$$\varphi_{3d} = h = h_0 + \varepsilon h_1 \quad (29)$$

$$\varphi_{3q} = p = p_0 + \varepsilon p_1 \quad (30)$$

If initial conditions for  $\varphi_{1d}, \varphi_{1q}, \varphi_{2d}, \varphi_{2q}, s$  satisfy manifold functions, thus:

$$\varphi_{3d}(0) = h(\varphi_{1d}(0), \varphi_{1q}(0), \varphi_{2d}(0), \varphi_{2q}(0), s(0)) \quad (31)$$

$$\varphi_{3q}(0) = p(\varphi_{1d}(0), \varphi_{1q}(0), \varphi_{2d}(0), \varphi_{2q}(0), s(0)) \quad (32)$$

Thus, (29) and (30) are exact solutions for  $\varphi_{3d}^*, \varphi_{3q}^*$  and with substituting (29) and (30) into  $\varphi_{1d}^*, \varphi_{1q}^*, \varphi_{2d}^*, \varphi_{2q}^*$  and  $s^*$  in (2) a reduced order model fifth order is obtained.

But if initial conditions don't meet manifold conditions, we seek two expression for the deviation introduced the initial condition in the second winding by finding,

$$\eta_d = \varphi_{3d} - h \quad (33)$$

$$\eta_q = \varphi_{3q} - p \quad (34)$$

where  $\eta_d$  and  $\eta_q$  are off-manifolds variables.

The differential equation describing this variables can be found from the original equations as,

$$\varepsilon \frac{d\eta_d}{dt} = \varepsilon \frac{d\varphi_{3d}}{dt} - \varepsilon \frac{dh}{dt} \quad (35)$$

$$\varepsilon \frac{d\eta_q}{dt} = \varepsilon \frac{d\varphi_{3q}}{dt} - \varepsilon \frac{dp}{dt} \quad (36)$$

Where  $\varepsilon = \tau_3$  as before.

Substituting for  $\varepsilon \varphi_{3d}^*$  and  $\varepsilon \varphi_{3q}^*$  from (2) and (33)-(34) and neglecting  $\varepsilon^2$  (consistent with the previous of the manifold series), The equations to be solved are,

$$\varepsilon \frac{d\eta_d}{dt} = l_2 l_m \varphi_{1d} + l_1 l_m \varphi_{2d} - l_{33} (\eta_d + h_0) - l_{33} s' (\eta_d + p_0) \quad (37)$$

$$\varepsilon \frac{d\eta_q}{dt} = l_2 l_m \varphi_{1q} + l_1 l_m \varphi_{2q} - l_{33} (\eta_q + p_0) - l_{33} s' (\eta_d + h_0) \quad (38)$$

where ,

$$\eta_d(0) = \varphi_{3d}(0) - h(0)$$

$$\eta_q(0) = \varphi_{3q}(0) - p(0) \quad (39)$$

These off-manifold dynamics normally are difficult to compute because they require  $\varphi_{1d}, \varphi_{1q}, \varphi_{2d}, \varphi_{2q}, s$ . As a approximation, (37) and (38) could be solved using  $\varphi_{1d}, \varphi_{1q}, \varphi_{2d}, \varphi_{2q}, s$  as constants equal to its initial conditions. This is a reasonably good approximation because the off manifold dynamics should decay (if they are stable) before  $\varphi_{1d}, \varphi_{1q}, \varphi_{2d}, \varphi_{2q}, s$  change significantly.

## 6 Linearized model

Each of the two nonlinear models full order and fifth order can be linearized around an operating point if it is assume that the variables have sufficiently small deviations from the operating point. For example this assumption is made in dynamic stability studies of power systems where it is customary to use a linearized model so that linear system analysis methods can be conveniently applied.

The linearization process could be directly applied to the fifth order machine model. However, the coefficients of the resulting equations, particularly for the fifth-order model, would have rather complicated algebraic expressions. An equivalent approach is through linearizing the complete seventh order model and then numerically deriving the linearized fifth model by following process. Let the linearized seventh-order equation be partitioned as

$$z^* = Dz + Cx \quad (40)$$

$$x^* = Bz + Ax + u \quad (41)$$

Where  $z$  represents the variable whose transients are to be ignored and  $x$  denotes the remaining variables.

Thus, for the fifth-order model,  $z$  represents the second case flux linkage. For a linear time-invariant system

with constant input,  $u$ , the integral manifold is sought in the form

$$z = px + q(u) \quad (42)$$

The substitution in to (40) and (41) yields:

$$p(Ax + B(px + q(u) + u)) = D(px + q(u)) + Cx \quad (43)$$

Collecting the  $x$ -dependent terms we require that the constant matrix  $p$  be a solution of

$$pA - Dp + pBp - C = 0 \quad (44)$$

With such a  $p$ , the  $u$ -dependent terms require that

$$(D - pB)q(u) - pu = 0 \quad (45)$$

which provided  $(D - pB)^{-1}$  exists, is satisfied by

$$q(u) = (D - pB)^{-1} pu \quad (46)$$

The description of the system (40) and (41) restricted to the manifold (42) is given by the reduced order model:

$$\dot{x}^* = (A + pB)x + (B(D - pB)^{-1}p + I)u \quad (47)$$

If initial conditions for  $x$  and  $z$  satisfies in (42), thus, reduced order model is (47). But if initial conditions don't meet manifold conditions, we seek two expressions similar to nonlinear model.

The accuracy of the linearized reduced-order model can be verified by comparing the sets of eigenvalues with that of the linearized full-order model at the operating point since the set of eigenvalues of a linear time-invariant system generally characterizes the system transient behavior. Comparison of the sets of eigenvalues as listed in table 1.

**Table 1** Eigenvalues associated with stator winding, first and second rotor winding.

Variable Associated with Eigen values	Ful order	Fifth order order(integral manifold)	Fifth order order(singular perturbation)
Speed	-6.912	-6.951	-7.18
Stator winding flux linkage	-12.94+j311.74 -12.94-j311.74	-11.34+j312.54 -11.34-j312.54	-10+j313.76 -10-j313.76
First winding flux linkage	-6.62+j5.84 -6.62-j5.84	-6.52+j5.98 -6.52-j5.98	-6.412+j6.01 -6.412-j6.01
Second winding flux linkage	-317.1+j0.902 -317.1-j0.902	.....	.....

## 7 Validation results

In order to evaluate the performance of the integral manifolds reduced order model, that defined by the first four equations of (2), and equations (3), (29) and (30) in describing the wind park transient behavior, some simulations have been carried on and the results compared with singular perturbation reduced order model [19] and full order non-linear detailed model.

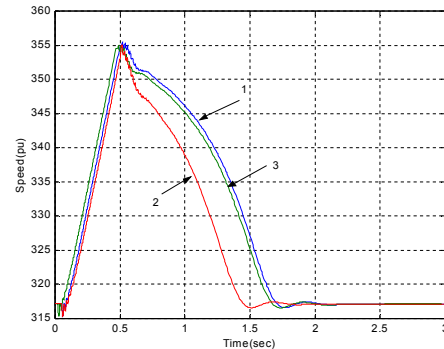
The relevant variables are generally the instantaneous stator current, electromagnetic torque, and the speed for transient behavior.

The first selected case study targets the simulation of a fault in the a.c. system which causes the voltage dip in generator terminal at 50 msec and normal operation occurring 500 msec after. The second chosen case study the simulation of a disturbance in wind power input which generates the mechanical torque decrease 50% at 50 msec and normal performance arising 500 msec after. It has been assumed a three units wind park

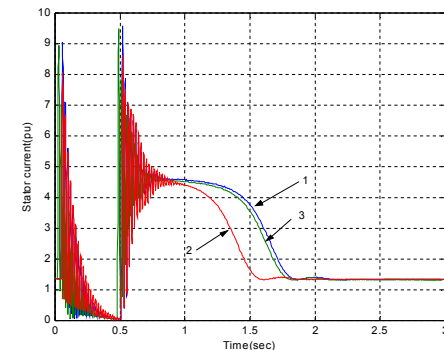
subject to correlated wind input.

Figures (3) and (4) display the rotor speed, stator current, and electromagnetic torque for a given generator, using both the full detailed model and the slow sub-system derived from the application of the integral manifolds method.

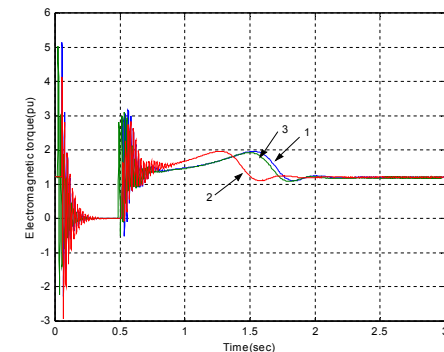
The linearized fifth order model response to the same fault condition with a voltage dip of 40% was also obtained and displayed along with the response of the non linearized fifth order in figure (5). Finally the linearized and non linearized fifth order model simulation were repeated but now with a voltage dip of 15% and responses plotted in figure. (6). The model parameters used for the simulation were listed in table 2 and table 3.



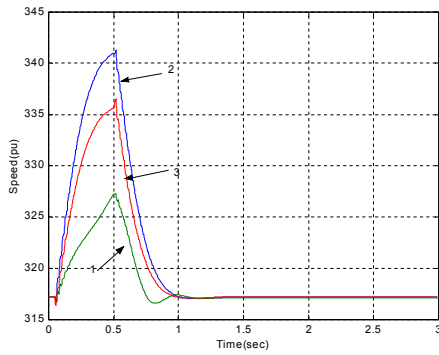
**Fig. 3a.** DOIG no. 2 rotor speed response to temporary three phase fault (1) detailed model, (2) singular perturbation model, (3) integral manifolds model.



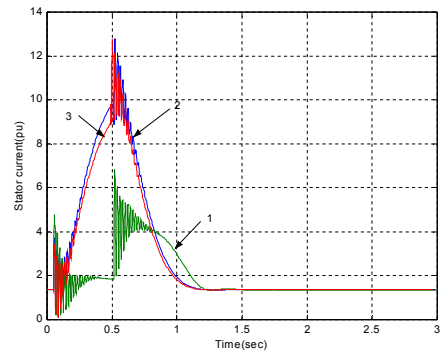
**Fig. 3b.** DOIG no. 2 stator current response to temporary three phase fault (1) detailed model, (2) singular perturbation model, (3) integral manifolds model.



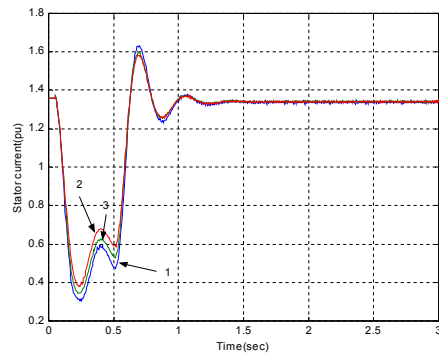
**Fig. 3c.** DOIG no. 2 Electromagnetic Torque response to temporary three phase fault (1) detailed model, (2) singular perturbation model, (3) integral manifolds model



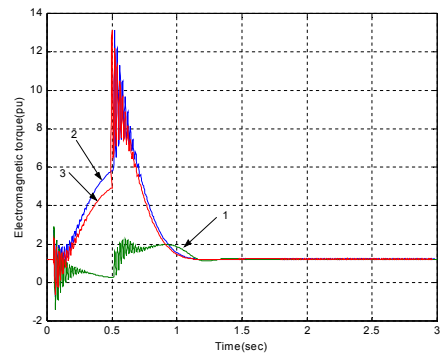
**Fig. 4a.** DOIG no. 2 rotor speed response to temporary 50% mechanical torque decrease (1) detailed model, (2) sigular perturbation model, (3) integral manifolds model.



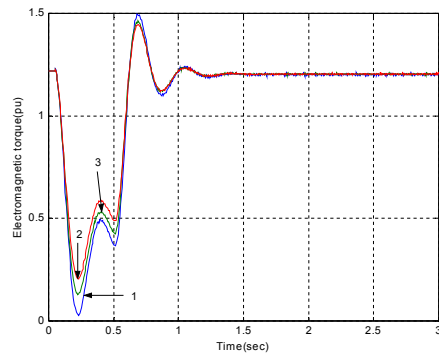
**Fig. 5b** DOIG no. 2 Stator current response to temporary 40% voltage dip , (1)non linear fifth order model, (2)linear sigular perturbation model, (3)linear integral manifolds model



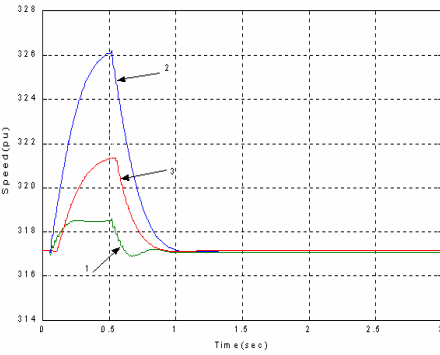
**Fig. 4b.** DOIG no. 2 Stator current response to temporary 50% mechanical torque decrease (1) detailed model, (2) sigular perturbation model, (3) integral manifolds model.



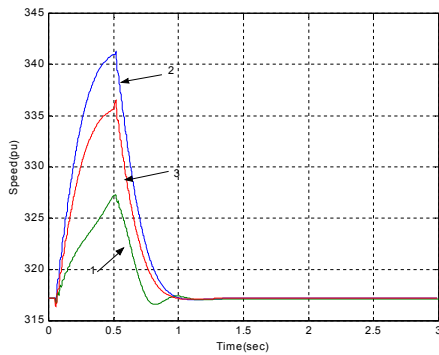
**Fig. 5c** DOIG no. 2 Electromagnetic Torque response to temporary 40% voltage dip , (1)non linear fifth order model, (2)linear sigular perturbation model, (3)linear integral manifolds model.



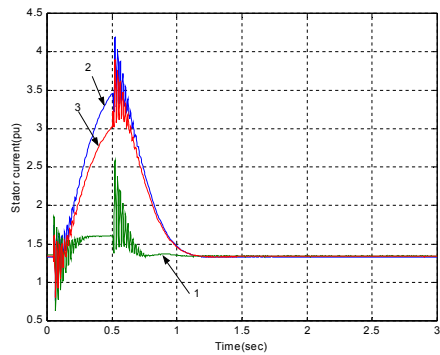
**Fig. 4c** DOIG no. 2 Electromagnetic Torque response to temporary 50% mechanical torque decrease (1) detailed model, (2) sigular perturbation model, (3) integral manifolds model.



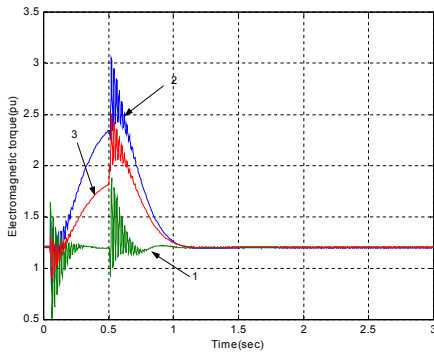
**Fig. 6a** DOIG no. 2 rotor speed response to temporary 15% voltage dip , (1)non linear fifth order model, (2)linear sigular perturbation model, (3)linear integral manifolds model.



**Fig. 5a** DOIG no. 2 rotor speed response to temporary 40% voltage dip , (1)non linear fifth order model, (2)linear sigular perturbation model, (3)linear integral manifolds model.



**Fig. 6b** DOIG no. 2 Stator current response to temporary 15% voltage dip , (1)non linear fifth order model, (2)linear sigular perturbation model, (3)linear integral manifolds model.



**Fig. 6c** DOIG no. 2 Electromagnetic Torque response to temporary 15% voltage dip , (1)non linear fifth order model, (2)linear sigular perturbation model, (3)linear integral manifolds model.

## 8 Conclusions

This paper presents an application of integral manifolds theory to reduce the order of a detailed wind park model. The final aim of this task is to obtain a dynamic equivalent of the wind park that retains the relevant dynamics of the park with respect to the utility grid.

The results achieved allow the conclusion that integral manifolds method is an excellent tool to derive dynamic equivalents of wind parks. The reduced order model reproduces with a high degree of accuracy the transient behavior results provided by the full order model.

From figures(5) and(6), it is evident that the linearized model is only accurate in a sufficiently small region around the operating point.

Moreover, the computing time issue is satisfactory addressed. The slow and fast dynamics are analyzed in separated time scales, which allows the use of different time steps for the integration of the two sub-systems. Computing time for integral manifold method and singular perturbation method are in turn 50% and 62% of computing time detailed model.

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**Appendix**

**A. Nomenclature**

- $l_m, l$  – magnetizing inductance and dynamic inductance
- $r_s$  – resistance
- $v, i, \phi$  – instantaneous values of voltages, currents and flux linkages
- $\omega_r$  – angular velocity of rotor (electrical)
- $H$  – inertia coefficient
- $T_m, T_e$  – mechanical torque and machine torque
- $s$  – slip
- $\omega$  – stator angular frequency
- 1,2,3- subscripts for stator winding, rotor first and second winding

**B. Model parameters(stator circuit)**

**Table 2** Characteristics of DOIG used in calculations

DOIG Characteristic	value
Number of poles	4
Base Frequency	50 Hz
Stator resistance( $r_1$ )	0.01 pu
Stator leakage inductance( $l_1$ )	3.47*e-4 pu
First winding resistance( $r_2$ )	.01 pu
First winding leakage inductance( $l_2$ )	3.58*e-4 pu
Second winding resistance( $r_3$ )	1.36*e-1 pu
Second winding leakage inductance( $l_3$ )	2.63*e-3 pu
Magnetizing inductance( $L_m$ )	3 pu
Total moment of inertia	5.9*e6 kgm2

**Table 3** Characteristics of wind turbine used in calculations

Wind turbine Characteristic	value
Rotor diameter	75 m
Area covered by rotor	4418 m <sup>2</sup>
Rotor speed	9-21 rpm
Nominal power	2 MW
Nominal wind speed	12 m/s
Gear box ratio	1 :100

**C. Auxiliary equations**

The inductance matrix  $l$  in

$$\phi = li \tag{c-1}$$

is written as

$$l = \begin{bmatrix} l_1 + l_m & \dots & l_m & \dots & l_m \\ l_m & \dots & l_2 + l_m & \dots & l_m \\ l_m & \dots & l_m & \dots & l_3 + l_m \end{bmatrix} \tag{c-2}$$

So that

$$i = L^{-1}\phi \tag{c-3}$$

with

$$L^{-1} = (-1/l) \begin{bmatrix} l_{11} & \dots & -l_3 l_m & \dots & -l_2 l_m \\ -l_3 l_m & \dots & l_{22} & \dots & -l_1 l_m \\ -l_2 l_m & \dots & -l_1 l_m & \dots & l_{33} \end{bmatrix} \tag{c-4}$$

where

$$\begin{aligned} l_{11} &= l_2 l_3 + l_2 l_m + l_3 l_m \\ l_{22} &= l_1 l_3 + l_1 l_m + l_3 l_m \\ l_{33} &= l_1 l_2 + l_1 l_m + l_2 l_m \\ l &= l_1 l_2 l_3 + l_1 l_2 l_m + l_1 l_m l_3 + l_m l_2 l_3 \end{aligned} \tag{c-5}$$