Load-Frequency Control in a Deregulated Environment Based on Bisection Search

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Abstract: Recently several robust control designs have been proposed to the Load-Frequency Control (LFC) problem. However, the importance and difficulties in the selection of weighting functions of these approaches and the pole-zero cancellation phenomenon associated with it produces closed loop poles. Also the order of robust controllers is as high as the plant. This gives rise to complex structure of such controllers and reduces their applicability in industry. In addition conventional LFC systems that use classical or trial-and-error approaches to tune the PI controller parameters are more difficult and time-consuming to design. In this paper, a bisection search method is proposed to design well-tuned PI controller in a restructured power system based on the bilateral policy scheme. The new optimized solution has been applied to a 3-area restructured power system with possible contracted scenarios and the results evaluation shows the proposed method achieves good performance compared with recently powerful robust controllers.

Keywords: Bisection Search, Deregulated Environment, Load-Frequency Control.

1 Introduction

One of the important power system control problems for which a lot of studies have been made is load–frequency control (LFC) [1-3].

The main goal of LFC is to maintain zero steady state errors for frequency deviation and good tracking load demands in a multi-area power system, it is also treated as an ancillary service essential for maintaining the electrical system reliability at an adequate level [4].

However, the electric power industry is in transition from large, vertically integrated utilities providing power at regulated rates to an industry that will incorporate competitive companies selling unbundled power at lower rates. Therefore in a deregulated environment, LFC acquires a fundamental role to power system control which there has been various decentralized robust and optimal control methods to provide better conditions for the electricity trading during the last two decades [5-9]. However, most of the above robust and optimal methods need some information of the system states, which are very difficult to know completely. On the other hand, the order of the robust controllers is as high as that of the

plant. This gives rise to complex structure, complex state-feedback or high-order dynamic controllers and reduces their applicability [10].

Then despite the potential of robust control techniques with different structures, they are not practical for industry practices and power system utilities prefer the online tuned PI controller's because of the ease of tuning and the lack of assurance of the stability and easy implementation.

In this paper a new optimization method based on bisection search [11], is used for tuning of PI controller parameters. The bisection search is a very simple and rapidly converging method in mathematics. It is a root-finding approach which repeatedly bisects an interval and then selects a subinterval in which a root must lie for further processing.

The above technique, which is ideally practical for industry, has been applied to a three-control area example as a case study and has been compared with the robust ILMI based controller proposed by [9]. The results show the optimized controller guarantee the robust performance for a wide range of operating conditions as well as full-dynamic H∞ controllers.

In this paper following a brief discussion on a deregulated LFC model, an explanation on bisection based optimization method and how a load–frequency controller can work within this formulation is provided. Simulation studies are performed to illustrate the capability of the proposed control approach. The resulting controllers are shown to minimize the effect of disturbances and achieve acceptable frequency

regulation in the presence of various load change scenarios.

2 Background

In this section, following an introduction to the traditional and a restructured power system LFC models, the proposed control strategy has been characterized.

2.1 Conventional and Generalized LFC Model

Frequency changes in large-scale power systems are a direct result of the imbalance between the electrical load and the power supplied by system connected generators [12]. A change in real power demand at one point of a network is reflected throughout the system by a change in frequency. Therefore, system frequency provides a useful index to indicate system generation and load imbalance [13]. Any short term energy imbalance will result in an instantaneous change in system frequency as the disturbance is initially offset by the kinetic energy of the rotating plant. Significant loss in the generation without an adequate system response can produce extreme frequency excursions outside the working range of the plant. The control of frequency and power generation is commonly referred to LFC which is a major function of Automatic Generation Control (AGC) systems [14].

In this classical AGC system, the balance between connected areas is achieved by detecting the frequency and tie line power deviations to generate the Area Control Error (ACE) signal which is turn utilized in the PI control strategy.

However, towards the end of the twentieth century many countries sought to reduce direct government involvement in, and to increase the economic efficiency of, their electricity industries through a change in industry managements, often described as electricity industry deregulation [4].

Deregulation is the act or process of removing or reducing state regulations. It is therefore opposite of regulation, which refers to the process of the government regulating certain activities. In another word, in contrast to the traditional power system structure that the Vertically Integrated Utility (VIU) no longer exists and the generation, transmission and distribution is owned by a single entity which supplies power to the customers at regulated rates, in an open energy market, Gencos may or may not participate in the LFC task and the common objectives, i.e. restoring the frequency and the net interchanges to their desired values for each control area are remained [7].

Deregulated systems will consist of generation companies (Gencos), distribution companies (Discos), transmission companies (Transcos) and Independent System Operator (ISO) which there can be various combinations of contracts between each Disco and available Gencos [4]. On the other hand, a Disco may contract individually with Gencos for power in different

areas (It has freedom to contract with any available Genco in its own or another control area).

To understand how the bidding process and bilateral contracts in a restructured power system are implemented, the "Generation Participation Matrix (GPM)" concept based on the idea presented by [4], is used here.

GPM shows the participation factor of a Genco in the considered control areas (Discos). The rows and columns of the GPM matrix are equal to the total number of Gencos and Discos in the overall power system, respectively. It has the following structure [7],

$$
GPM =
\begin{bmatrix}
gpf_{11} & gpf_{12} & \cdots & gpf_{1(m-1)} & gpf_{1m} \\
gpf_{21} & gpf_{22} & \cdots & gpf_{2(m-1)} & gpf_{2m} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
gpf_{(n-1)1} & gpf_{(n-1)2} & \cdots & gpf_{(n-1)(m-1)} & gpf_{(n-1)m} \\
gpf_{n1} & gpf_{n2} & \cdots & gpf_{n(m-1)} & gpf_{nm}
\end{bmatrix}
\tag{1}
$$

In the above matrix, gpf_{ij} refers to 'generation participation factor' and shows the participation factor of Genco i in the load following of area j based on the appropriate contract.

Also sum of all entries in each column of the GPM matrix according to (2) is unity.

$$
\sum_{i=1}^{n} gpf_{ij} = 1
\tag{2}
$$

Using the GPM matrix concept, the Gencos can submit their ramp rates (Megawatts per minute) and bids to the market operator. After a bidding evaluation, those Gencos selected to provide regulation services must perform their functions according to the ramp rates approved by the responsible organization [9].

For LFC analysis and synthesis in a deregulated environment, we use the generalized dynamical model introduced in [4]. In this scheme each control area has its own AGC and is responsible for tracking its own load and honoring tie-line power exchange contracts with its neighbors.

2.2 Three-Control Area Restructured Power System Example

In this paper, to illustrate the effectiveness of proposed control design, a three-control area power system shown in Fig. 1 (same as example used by [4]) is considered as a test system.

In this model, each control area has its own Disco, two Gencos and a PI controller which is responsible for tracking its own load and honoring tie-line power exchange contracts with its neighbors. For the simulation tests, the rate limit value for each Genco is assumed 0.1, and, 1000 MW is considered as a base for the pu calculations.
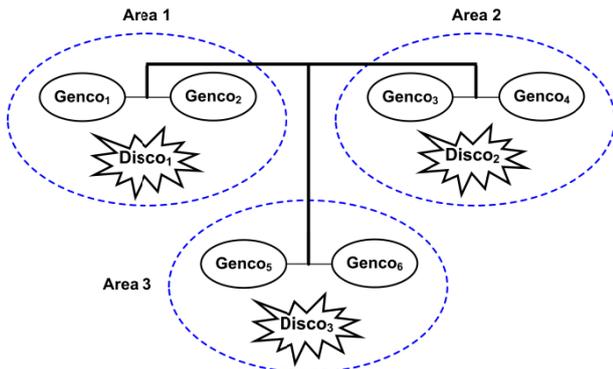
Fig. 1 Three-control area restructured power system.

2.3 Proposed Control Strategy

Most of nonlinear equations are very difficult to solve and some of them are unsolved. Therefore several methods have been proposed to approximate the root of these nonlinear equations. One of the most important methods to approximate the root is the bisection algorithm. The bisection search is a very simple, robust and converging method that is usually used to obtain a rough approximation to a solution. It is an optimization mathematical technique which looks for a root of a function f(x) in [a ,b] (i.e., a value of x such that f(x)=0 and a≤x≤b) by repeatedly bisects an interval (a,b) and selects a subinterval in which a root must lie (see Fig. 2). One way to know that a root lies in this interval is that the sign of f(a) is different from the sign of f(b) [11]. In this method the uniqueness root of the nonlinear equation, is the necessary condition for establishing the bisection search [11, 15].

The necessary definitions, theorems and examples related to the bisection search are as follow,

Definition 1:
Suppose A is a set then,
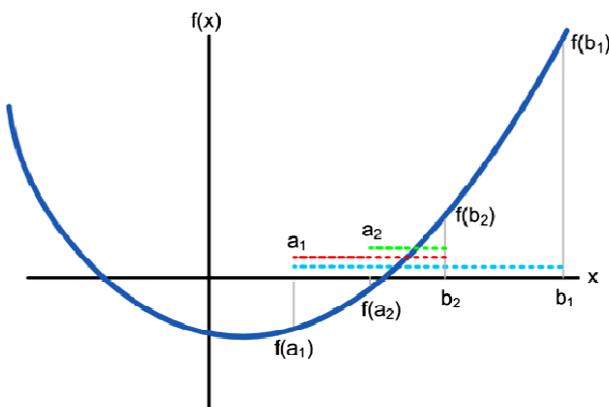$\bar{A} = A \cup A'$ where A' is the limit points of A and $\bar{A}$ is the complement set of A.

Fig. 2 Bisection search.

Definition 2:
A , B are separated sets if,

$$\bar{A} \cap B = \emptyset,$$
$$\bar{B} \cap A = \emptyset \tag{3}$$

Definition 3:
A is a connected set if it is not the union of two separated sets.

Also, the following theorems and examples guarantee that the equation $f(x) = 0$ has just a unique root.

Theorem 1: [16]
If f be a continuous function in closed interval $[a, b]$ and $f(a)f(b) < 0$ then $f(x) = 0$ has at least a root in (a, b).
Proof:
Since f is a continuous function, then $f([a, b])$ is connected. Now let $E = f([a, b])$, if there is no any x so that $f(x) = 0$ then $0 \notin f([a, b])$.
Now let $A = E \cap (-\infty, 0), B = E \cap (0, +\infty)$ where $f(a) \in A, f(b) \in B, E = A \cup B$.
According to the above assumptions, since A , B are separated, then E is not connected and this is opposite the assumption. Then proof is complete.

Theorem 2: [16]
If f be a continuous function in closed interval $[a, b]$ and is a differentiable function in (a, b), then $f(x) = 0$ has at most a root in (a, b). The proof of this theorem was given by [16].
Now through the following two examples we can check the above theorems applications,

Example 1:
Let $f(x) = \sin x + x$ in $[-\frac{\pi}{4}, \frac{\pi}{4}]$. Since f is a continuous function and $f(-\pi/4)f(\pi/4) < 0$ according to the following equations,

$$f\left(-\frac{\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) - \frac{\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\pi}{4} < 0$$
$$\tag{4}$$

$$f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) + \frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\pi}{4} > 0$$

Then $f(x) = \sin x + x$ has at least one root in the given interval according to the theorem 1.

Example 2:
Consider the function $f'(x) = \cos x + 1$. Obviously, $f'(x) > 0$ in $[-\frac{\pi}{4}, \frac{\pi}{4}]$, therefore according to the theorem 2, f has at most one root in the given interval.

2.4 Bisection Algorithm Description

After the above definitions, the bisection search algorithm explanation is as follow, suppose f is a continuous function defined on an interval $[a\ b]$. Each

iteration of the bisection algorithm evaluates the function at the midpoint $c = (a + b)/2$. Based on the sign of the evaluation, either a or b is replaced by c to retain different signs on $f(a)$ and $f(b)$. Explicitly, if $f(a)f(c) < 0$ then the subinterval $[a, c]$ is selected and the method sets $b = c$ however if $f(c)f(b) < 0$ the subinterval $[c, b]$ is selected and the method sets $a = c$. If $f(a), f(b)$ and $f(c)$ have the same signs, the bisection method selects the interval which produces the smaller value for f (i.e. *if* $f(a)f(c) < f(c)f(b)$ *then* $b = c$ *otherwise* $a = c$) [11].

The bisection algorithm repeats this iteration until the interval between a and b and, hence, the resolution of the root of $f(x)$ is as small as desired.

If ε is the desired root resolution then the algorithm will terminated at most in $[log_2(|b - a|/\varepsilon)]$ iterations, or when one of the following conditions will be true [11],

1. $|p_{n+1} - p| < \varepsilon$ which p_{n+1}, p are the midpoints of the interval in $(n + 1)^{th}$ step and the midpoint of the initial interval respectively.
2. $|p_{n+1} - p_n| < \varepsilon$, which p_{n+1}, p_n are the midpoints of the interval in $(n + 1)^{th}$ and n^{th} step.
3. $|f(p_n)| < \varepsilon$, which ε is a given very small and positive number in all conditions.

Then this algorithm has the following steps and following theorems,

Step1: input a, b
Step 2: let $= (a + b)/2$: print x.
Step 3: if $ABS(f(x)) < EPS$ then end.
Step 4: if $f(a)f(x) < 0$ then let $b = x$ else let $a = x$.
Step 5: GOTO 2.
Step 6: END.

Theorem 3: *[15]*
The bisection method is convergent in the interval $[a, b]$ if $f(a)f(b) < 0$ and f is continuous.
Proof:
If p_i is the midpoint of the interval $[a, b]$ in the ith step and p is the problem solution, then absolute error in the nth iterations are calculated as follow,

$$|p_1 - p| < \frac{b - a}{2}$$

$$|p_2 - p| < \frac{\frac{b-a}{2}}{2} = \frac{b-a}{2^2} \tag{5}$$

$$\vdots$$

$$0 \le |p_n - p| < \frac{b - a}{2^n}$$

As we know:

$$\lim_{n \to \infty} \frac{1}{2^n} = 0 \tag{6}$$

Consequently we have:

$$\lim_{n \to \infty} \frac{1}{2^n} = 0 \Rightarrow \lim_{n \to \infty} \frac{b - a}{2^n} = 0 \Rightarrow$$

$$\lim_{n \to \infty} |p_n - p| = 0 \Rightarrow \lim_{n \to \infty} p_n = p \tag{7}$$

Therefore, the produced sequence by the bisection algorithm is finally convergent to the root of f. Following examples show the applicability of the above theorems and bisection algorithm in finding function's roots.

Example 3:
Suppose we are going to solve the following simple equation by the bisection algorithm,

$$x^2 + x = 1 \tag{8}$$

To solve this equation firstly we manipulate it that right side be zero. Then we have,

$$\boldsymbol{x^2 + x - 1 = 0} \tag{9}$$

Equivalently the goal is finding the root of function: $f(x) = x^2 + x - 1$. Now we guess two different numbers a, b so that $f(a)f(b) < 0$. Let $a = 0$ and $b = 1$ then $f(a) = -1$ and $f(b) = 1$ therefore $f(a)f(b) = (-1) \times 1 < 0$.

Whereas f is polynomial then it is continuous in every interval of real numbers particularly in [0,1]. Therefore f has conditions of theorem 1 then f has at least one root in [0,1].

Derivative of function f is equal to:

$$f'(x) = 2x + 1 \tag{10}$$

Obviously, $f'(x)$ is positive in (0,1), therefore $f'(x) > 0$ and f has conditions of theorem 2 too. Namely $f(x) = 0$ has at most a root in (0,1).

According to the theorem 1 and 2, $f(x) = 0$ has just one root in (0,1). Now we can use the bisection algorithm to find the root of $f(x) = x^2 + x - 1$ in interval [0,1].

The Table 1 shows summary of the bisection method to solve this example at five iterations. According the Table 1, root of $f(x) = x^2 + x - 1$ in [0 1] approximately is equal to .5973.

Example 4:
Suppose we are going to approximate the root of following equation by the bisection algorithm until $|f(x_n)| < 0.01$.

$$x^2 - (1 - x)^5 = 0 \tag{11}$$

Table 1 Bisection method iteration for Example 3

Iterations	a	b	x_n	sign of $f(a)f(x_n)$
1	0	1	0.5	+
2	0.5	1	0.75	−
3	0.5	0.75	0.625	−
4	0.5	0.625	0.5625	+
5	0.5625	0.625	0.5937	+

In fact we want to find root of function $f(x) = x^2 - (1-x)^5$.

Now we guess two different numbers such as a, b so that $f(a)f(b) < 0$. Let $a = 0$ and $b = 1$ then $f(a) = -1$ and $f(b) = 1$ therefore $f(a)f(b) = (-1) \times 1 < 0$. Because f is a polynomial, then it is continuous in every interval of real numbers particularly in [0,1]. Therefore f has conditions of theorem 1 then f has at least one root in [0,1].

Derivative of function f is equal to,

$$f'(x) = 2x + 5(1-x)^4 \tag{12}$$

Obviously, $f'(x)$ is positive in $(0,1)$, therefore f has conditions of theorem 2 too. Namely $f(x) = 0$ has at most a root in $(0,1)$.

According to the theorem 1 and theorem 2, $f(x) = 0$ has just one root in $(0,1)$. Now we can use the bisection algorithm to find the root of $f(x) = x^2 - (1-x)^5$ in interval [0,1].

The Table 2 shows summary of the bisection method to solve this example at five iterations.
As it is clear from the Table 2, the root of $f(x) = x^2 - (1-x)^5$ in [0,1] is approximately equal to .3437.

According to the above examples, although the bisection is a slow algorithm to approximate the root of equations, however it is so simple and unlike the most of other searching methods, it is a very convergent algorithm.

Table 2 Bisection method iteration for Example 4

It.	a	b	x_n	sign of $f(a)f(x_n)$	$|f(x_n)|$
1	0	1	0.5	$-$	0.2167
2	0	0.5	0.25	$+$	0.1748
3	0.25	0.5	0.375	$-$	0.0452
4	0.25	0.375	0.3125	$+$	0.0559
5	0.3125	0.375	0.3437	$+$	0.0035

3 Problem Formulation

In this paper, a bisection method obtains the approximate best values of PI controller parameters. In each control area P and I parameters have been tuned according to the absolute value of Area Control Error ($|ACE|$) signal as their evaluation function (f). The aim of the optimization method is to tune P and I parameters according to gain the smallest value of the evaluation function.

Assume P_i and I_i are the controller parameters of control area i respectively which $0 \leq P_i \leq 1$ and $0 \leq I_i \leq 1$; The bisection evaluation function of area i is sum of all ACE instances over simulation time t based on the specified value of P and I parameters (x_{Pi}, x_{Ii}) as follow,

$$f_{xi}(x_{Pi}, x_{Ii}) = \sum_{t=1}^{n} |ACE_{i,t}| \tag{13}$$

where $ACE_{i,t} = \Delta f_{i,t} + \Delta Ptie_{i,t}$ in which $\Delta f_{i,t}$ is the frequency deviation and $\Delta Ptie_{i,t}$ is the power tie line between area i and other areas.

The bisection search for P and I parameters is performed as following algorithm,

Step 1: Define [0 1] as lower and upper bound criterions for solution values of P and I parameters of area i respectively, then $a_{Pi} = 0$, $b_{Pi} = 1$ and $a_{Ii} = 0$, $b_{Ii} = 1$.

Step 2: In each iteration two different midpoints are calculated for control area i, $c_{Pi} = (a_{Pi} + b_{Pi})/2$ and $c_{Ii} = (a_{Ii} + b_{Ii})/2$ then the 3-control area example simulation is run according to the new solutions of P and I, $[c_{Pi}, c_{Ii}]$ for each area.

Step 3: After the simulation is done, next points are selected according to the bisection evaluation function (3), if $f_{ai}(a_{Pi}, a_{Ii})f_{ci}(c_{Pi}, c_{Ii}) < 0$ the subinterval $[a, c]$ is selected and the method sets $b = c$ however if $f_{bi}(b_{Pi}, b_{Ii})f_{ci}(c_{Pi}, c_{Ii}) < 0$ the subinterval $[c, b]$ is selected and the method sets $a = c$ then go to the Step 2 to run the next iteration.

The procedure is terminated when $f_{ci}(c_{Pi}, c_{Ii}) < 0.001$. In this case (c_{Pi}, c_{Ii}) is an optimal value for P and I parameters of area i.

4 Experiments

In order to demonstrate the effectiveness of the proposed strategy, it is examined in the presence of a sequence of step load changes for the various possible scenarios of bilateral contracts and load disturbances. In these simulations, the proposed optimization technique were applied to the controller of the 3-control area power system described in Background Section and the performance of it is compared with the performance of the ILMI robust controller introduced in [4].

4.1 Case Study 1: Poolco-Based Transactions

The first test case study is based on the possible contracts under practical operating conditions and large load demands (a step increase in demand) by Discos of area 1, 2, and 3 as $\Delta P_{L1} = 100\ MW, \Delta P_{L2} = 70\ MW, \Delta P_{L3} = 60\ MW$.

A case of Poolco based contracts between Discos and available Gencos is simulated based on the following GPM. In this scenario Gencos participate only in load following control of their areas.

$$GPM = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \\ 0 & 0 & 0.5 \end{bmatrix} \tag{14}$$

Frequency deviation (Δf), area control error (ACE) and actual tie-line power flow (ΔP_{tie}) for the closed loop system are shown in Fig. 3. In this figure, solid line is used for the current solution and dashed line is used for ILMI based method.

As shown in Fig. 3, using the proposed method, the area control error and frequency deviation of all areas are quickly driven back to zero and have small overshoots. Since there are no contracts between areas, the scheduled steady state power flow over the tie-lines is zero as well as ILMI robust controller.

4.2 Case Study 2: Combination of Poolco and Bilateral-Based Transactions

In this case the transaction is based on free contracts. Then consider larger demands by Disco 2 and Disco 3, i.e. $\Delta P_{L1} = 100\ MW, \Delta P_{L2} = 100\ MW, \Delta P_{L3} = 100\ MW$.

And assume Discos have the freedom to have a contract with any Gencos in their areas and other areas according to the following GPM,

$$GPM = \begin{bmatrix} 0.25 & 0.25 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0.25 & 0.75 \\ 0.25 & 0.25 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{bmatrix} \qquad (15)$$

All Gencos participate in the LFC task. The closed-loop responses are shown in Fig. 4.

The simulation results show the same values in the steady state. It is worth noting that the small differences seen in simulation results between ILMI and the current solution generated signals. Also using the proposed method, the frequency deviation of all areas quickly driven back to zero and has a good dynamic response too.

4.3 Case Study 3

The purpose of this scenario is to test the performance of proposed controllers against large and random load disturbances.

Consider the GPM of scenario 2 again. Assume a bounded random load changes (Fig. 5) as an uncontracted local demand, is applied to each control area as follow:

$$-50\ MW \ \leq \ \Delta P_{di} \ \leq \ +50\ MW \qquad (16)$$

The corresponded frequency deviations and tie-line power changes are shown in Fig. 6. This figure demonstrates that the designed controllers track the load fluctuations, effectively as well as ILMI based controllers.

The above simulation results show that the proposed simple and easy optimization method achieve good robust performance as well as powerful ILMI robust controller technique with complex structure for the possible contracted scenarios in the presence of system nonlinearities. Furthermore the higher flexibility, model independency and simple structure of the proposed solution for a wide range of load disturbances and possible bilateral contract scenarios are investigated.

For more investigation and to demonstrate the robustness of the proposed control strategy, the average value of ACE_i over three minutes is used as a performance index for comparison of the proposed control scheme and ILMI design.

As shown in Table 3 the current solution presents relatively better performance than the complex robust ILMI based design.
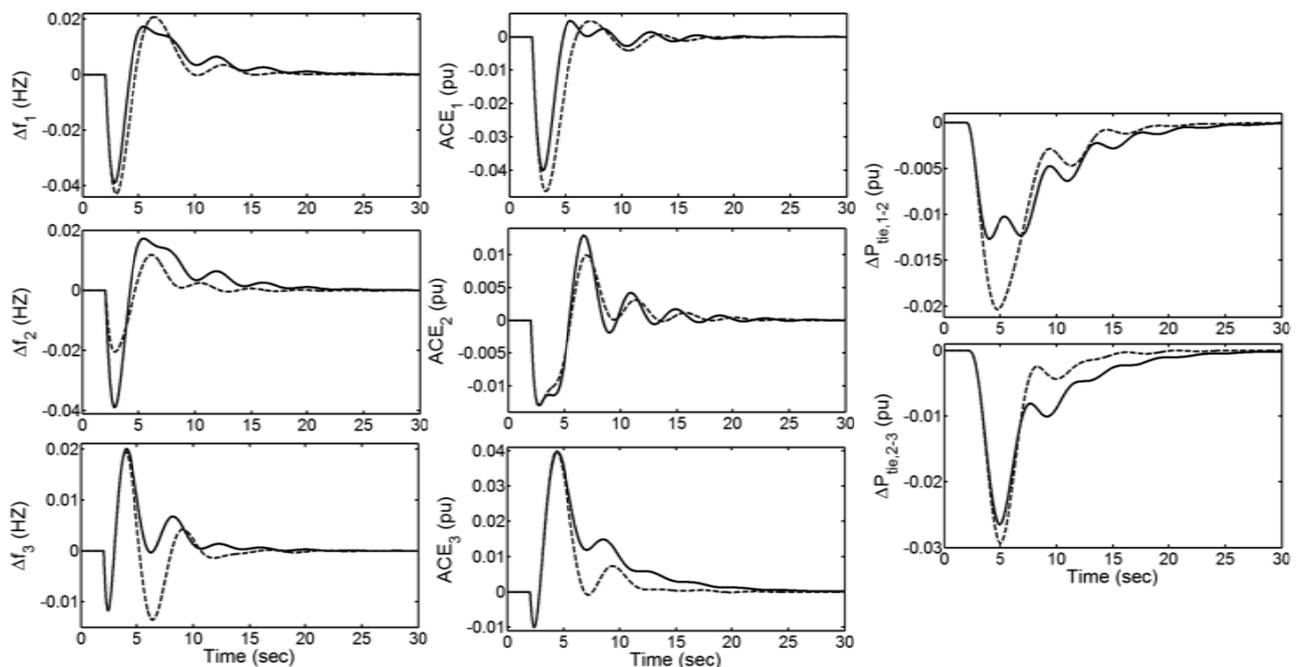

Fig. 3 Power system response to case study 1: Solid line (proposed strategy), Dashed line (ILMI based approach).
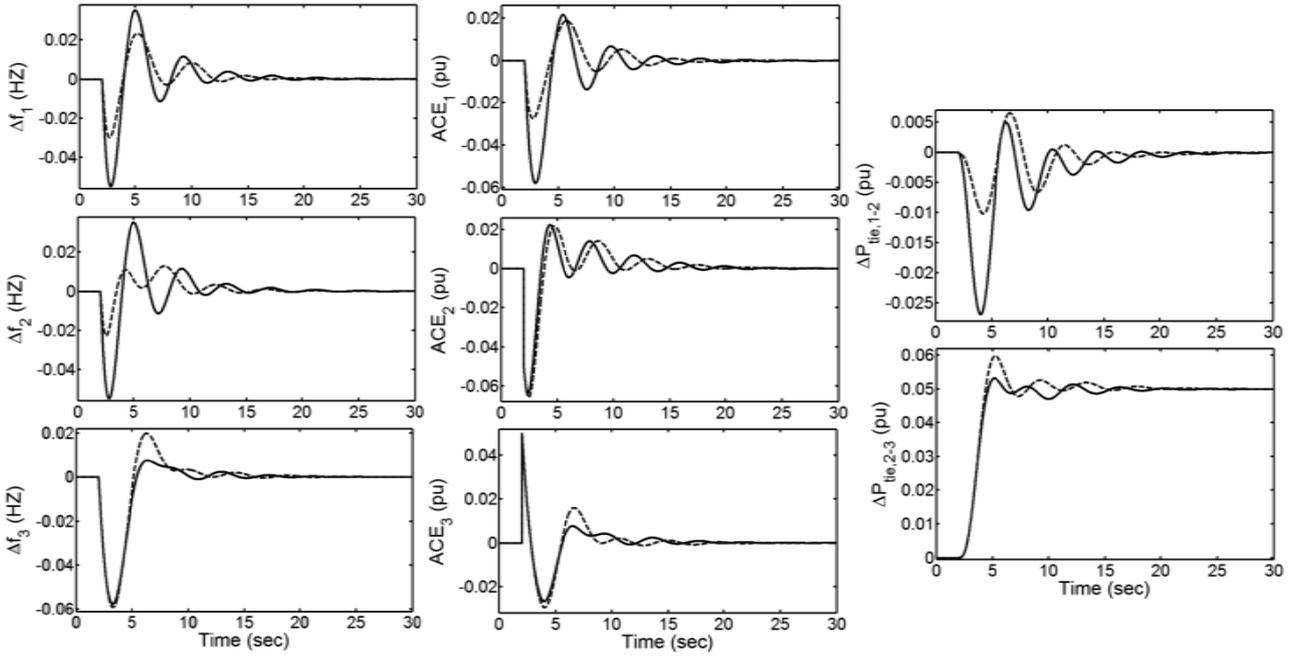
Fig. 4 Power system response to case study 2: Solid line (proposed strategy), Dashed line (ILMI based approach).

Table 3 Performance Evaluation.

Method	Case Study 1			Case Study 2			Case Study 3		
	$|ACE_1|_{avg}$	$|ACE_2|_{avg}$	$|ACE_3|_{avg}$	$|ACE_1|_{avg}$	$|ACE_2|_{avg}$	$|ACE_3|_{avg}$	$|ACE_1|_{avg}$	$|ACE_2|_{avg}$	$|ACE_3|_{avg}$
Proposed Controller	0.0013	0.0024	0.0044	0.0034	0.0032	0.0022	0.0063	0.0110	0.0126
ILMI-based	0.0025	0.0025	0.0023	0.0020	0.0038	0.0031	0.0098	0.0130	0.0118

5 Conclusion

In this paper, an easy implemented optimization technique for the LFC design, using the bisection search has been proposed in a deregulated power system. The proposed method is a very simple, robust and converging technique and was applied to a 3-control area power system with different possible scenarios. In this new scheme, in addition to the regulating area frequency, the AGC system should control the net interchange power with neighboring areas at scheduled values. Therefore, a desirable AGC performance is achieved by effective adjusting of generation to minimize frequency deviation and regulate tie-line power flows. The AGC system realizes generation changes by sending signals to the under control
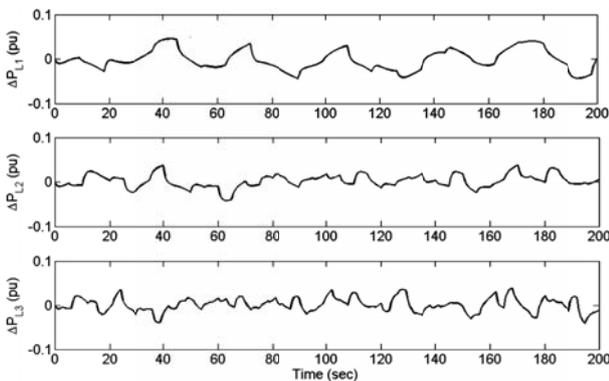

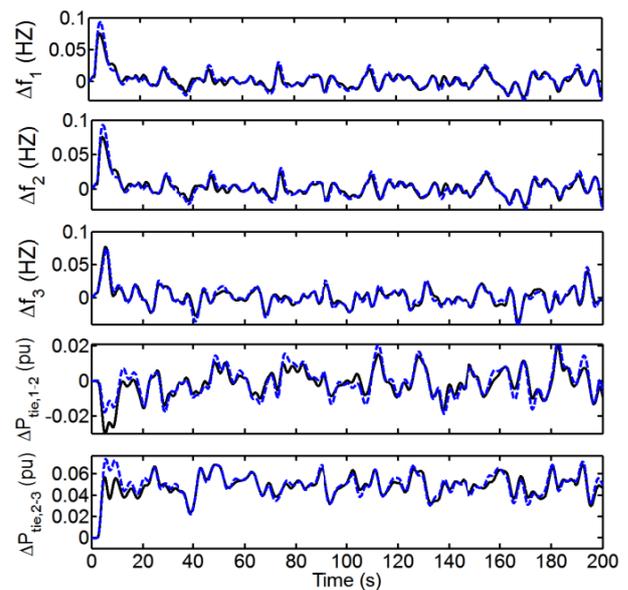**Fig. 5** Random load changes.

Fig. 6 Power system response to case study 3: Solid line (proposed strategy), Dashed line (ILMI based).

generating units. The simulation results in the new model, show that it presents a desirable performance under a wide range of load changes specially compare with robust controllers. Moreover, this newly developed solution has a simple structure, and is fairly easy to implement in comparison to other controllers, which can be useful for the real world complex power systems.

Acknowledgment

References

[1] Ibraheem, Kumar P. and Kothari P., "Recent philosophies of automatic generation control strategies in power systems", *IEEE Trans. on Power Systems*, Vol. 20, No. 1, pp. 346-357, 2005.

[2] Kothari ML., Nanda J., Kothari D. P. and Das D., "Discrete mode AGC of a two area reheat thermal system with new ACE", *IEEE Trans. on Power Systems,* Vol. 4, pp. 730-738, 1989.

[3] Shoults R. R. and Jativa J. A., "Multi area adaptive LFC developed for a comprehensive AGC simulator", *IEEE Trans. on Power Systems*, Vol. 8, pp. 541-547, 1991.

[4] Bevrani H.*, Frequency Response Characteristics and Dynamic Performance*, Robust power system frequency control, 1st edn, Springer Press, 2009, pp. 49-59.

[5] Ishi T., Shirai G. and Fujita G., "Decentralized load frequency based on H-inf control", *Electr. Eng. Jpn.,* Vol. 136, No. 3, pp. 28-38, 2001.

[6] Kazemi M. H., Karrari M. and Menhaj M. B., "Decentralized robust adaptive-output feedback controller for power system load frequency control", *Electr. Eng. J.,* Vol. 84, pp. 75-83, 2002.

[7] Liu F., Song Y. H., Ma J. and Lu Q., "Optimal load frequency control in the restructured power systems", *IEE Proceedings Generation Transmission & Distribution,* Vol. 15, No. 1, pp. 87-95, 2003.

[8] Bevrani H. and Hiyama T., "Robust load-frequency regulation: a real-time laboratory experiment", *Optimal Control Applications and Methods,* Vol. 28, No. 6, pp. 419-433, 2007.

[9] Bevrani H., Mitani Y. and Tsuji K., "Robust decentralised load-frequency control using an iterative linear matrix inequalities algorithm", *IEE Proc. Gener. Transm. Distrib.* Vol. 3, No. 151, pp. 347-354, 2004.

[10] Shayeghi H., Shayanfar H. A. and Jalili A., "Multi-stage fuzzy PID power system automatic generation controller in deregulated environments", *Energy Conversion and Management*, Vol. 47, No. 18-19, pp. 2829-2845, 2006.

[11] Hamming R. W., *Numerical methods for scientists and engineers*. Mc Graw-Hill, New York. 1962.

[12] Daneshfar F. and Bevrani H., "Load-Frequency Control: A GA-based Multi-agent Reinforcement Learning", *IET Generation, Transmission & Distribution*, Vol. 4, No. 1, pp. 13-26, 2010.

[13] Jaleeli N., Ewart D. N. and Fink L. H., "Understanding automatic generation control", *IEEE Trans. Power Syst.* Vol. 7, No. 3, pp. 1106-1112, 1992.

[14] Hiyama T., "Design of decentralised load-frequency regulators for interconnected power systems", *IEE Proc. C Gener. Transm. Distrib.* Vol. 129, pp. 17–23, 1982.

[15] Froberg, CE., *Numerical mathematics, theory and computer applications*. The Benjamin Publishing Company, MA. 1985.

[16] Rudin, W., *Principles of mathematical analysis*. 3rd ed. McGraw-Hill, New York. 1976.

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