Online state space model parameter estimation in synchronous machines

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Abstract

The purpose of this paper is to present a new approach based on the Least Squares Error method for estimating the unknown parameters of the nonlinear 3rd order synchronous generator model. The proposed method uses the mathematical relationships between the machine parameters and on-line input/output measurements to estimate the parameters of the nonlinear state space model. The field voltage is considered as the input and the rotor angle and the active power are considered as the generator outputs. In fact, the third order nonlinear state space model is converted to only two linear regression equations. Then, easy-implemented regression equations are used to estimate the unknown parameters of the nonlinear model.

The suggested approach is evaluated for a sample synchronous machine model. Estimated parameters are tested for different inputs at different operating conditions. The effect of noise is also considered in this study. Simulation results show that the proposed approach provides good accuracy for parameter estimation.

Keywords synchronous machine, state space model, identification, nonlinear model, regression equation

1. Introduction

As the power system becomes more complicated and more interconnected, modeling and identification of its elements become more essential. Synchronous generators play an important role in the stability of power systems, so the accurate modeling of synchronous generators is essential for a valid analysis of dynamic and stability performance in power systems [1].

There are three ways for synchronous generators modeling: white box [2-3], grey box [4-6] and black box [7-8]. The first category assumes a known structure for the synchronous machine such as the traditional methods which are specified in IEEE and IEC standards [9]. These approaches are often conducted under off-line condition. The parameters obtained by these methods may not truly characterize the synchronous machine under various loading conditions [5].

Due to the disadvantages of off-line methods, on-line parameter estimation approaches are of more interest in the recent years. The second and the third categories both use online measurements to estimate synchronous generator parameters, but they act in different ways.

In the second type of identification methods, a known mathematical model for synchronous generator is assumed and on-line measurements are used to estimate machine's physical parameters. In the third one, input data set is mapped to the output data set without considering any known structure for the model.

In synchronous generators modeling, we have two kinds of nonlinearities. The first kind of nonlinearities (structured), are those which are modeled in the well-known nonlinear structures of synchronous models. One example is the sine and cosine functions of the rotor angle. The second kind (unstructured), are those which are not usually considered in the structure of the models such as magnetic saturation in the iron parts of the rotor and stator. In fact the unstructured nonlinearity can be considered by adjusting the physical parameters of the nonlinear model using online measurements [10].

In this paper, an analytical identification procedure for the 3rd order model of synchronous generators is suggested. The proposed method estimates physical parameters of synchronous machine. In [1, 6, 10-11] different algorithms for estimation of physical parameters of synchronous machine are presented. In [6] an approach for synchronous generator parameter estimation is suggested which needs to apply a short circuit on the generator terminals. This short circuit test will disturb the normal operation of the machine. In [10] a method is suggested for the synchronous machine parameter estimation, which results in some nonlinear set of equations. In each test, the nonlinear set should be solved numerically. Furthermore, the algorithm does not discuss the conditions in which the system of equations can be solved. In some situations, the algorithm may result in a singular set of equations and no solution can be found. In [11], an analytic approach is suggested which needs to perturb the field voltage with the PRBS signal. This signal is not harmful for the normal operation of the machine. The suggested method uses not only the mathematical relation between machine parameters, but also it needs to identify the system eigenvalues using the Prony approach. Here, we try to find an algorithm which uses the mathematical relations of the parameters and the extra step of Prony is omitted. In other words, we try to extend the method of [11] and find a way to omit its disadvantage.

In this study, we introduce an analytical, straight forward and easy-implemented method which uses online measurements to estimate the unknown parameters. Here we only need to solve two regression equations. Then, using some simple relations all the unknown parameters of 3rd order nonlinear state space model are estimated.

The paper is organized as follows: the synchronous generator model and the identification method are described in section 2 and 3, respectively. The simulation results are provided in section 4. Section 5 concludes the paper.

2. The synchronous generator model

In this paper, the third order nonlinear synchronous generator connected to an infinite bus is considered as the system under study (Fig. 1). In this model, the stator's dynamics and the effects of dampers are neglected, so we have only one electrical equation (the field dynamic). This model can be used for studying low-frequency oscillations and stability analysis of power systems [1,10].

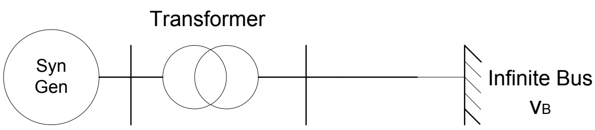

Figure 1: structure of the study system

The third order nonlinear model derived in [13-14] is used in this paper. All the parameters are assumed to be in per unit values. The model is described by the following nonlinear equations:

$$
\begin{aligned}
&\dot{[illegible]} \; [illegible] \\
&\dot{[illegible]} \; \frac{1}{J} [illegible] - T_e - D\omega) \\
&\dot{[illegible]} \; \frac{1}{T''_{do}} [illegible]_{FD} - e'_q - (x_d - x'_d) i_d)
\end{aligned}
\qquad (1)
$$

Where:

$$
\begin{aligned}
i_d &= \frac{e'_q - V_B \cos\delta}{x'_d} \\
i_q &= \frac{V_B \sin\delta}{x_q} \\
T_e \cong P_e &= \frac{V_B}{x'_d} e'_q \sin\delta + \frac{V_B^2}{2}\left(\frac{1}{x_q} - \frac{1}{x'_d}\right)\sin(2\delta)
\end{aligned}
\qquad (2)
$$

In this model, the influence of magnetic saturation is neglected, so x_d, x_q and x'_d are assumed to be constant. x_d, x_q and x'_d are the augmented reactance, i.e. the line and transformer reactances are added with them [11]. The definition of the variables and constants is given in the Appendix.

In our study, it is required to find the state space model of the system, so the above model is converted to the state space one. The system states, inputs and outputs are defined as follows:

$$
X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \delta \\ \omega \\ e'_q \end{bmatrix}, \quad U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} E_{FD} \\ T_m \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \delta \\ P_e \end{bmatrix} \qquad (3)
$$

The state space nonlinear model of the system is given in the following [11]:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ & -\dfrac{D}{J} & 0\\ 0 & 0 & -\dfrac{1}{T'_{d0}}(\dfrac{x_d}{x'_d})\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & \dfrac{1}{J}\\ \dfrac{1}{T'_{d0}} & 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}+\begin{bmatrix}0\\ -\dfrac{1}{J}(\dfrac{V_B}{x'_d}x_3\sin x_1+\dfrac{V_B^2}{2}(\dfrac{1}{x_q}-\dfrac{1}{x'_d})\sin 2x_1))\\ \dfrac{1}{T'_{d0}}(\dfrac{x_d-x'_d}{x'_d})V_B\cos x_1\end{bmatrix} \tag{4}$$

From [11], the linearized model of (4), in the vicinity of an operating point 'o', will conclude the known Heffron-Philips model, which is given below:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ \dfrac{K_1}{J} & -\dfrac{D}{J} & -\dfrac{K_2}{J}\\ -\dfrac{K_4}{T'_{do}} & 0 & -\dfrac{1}{K_3T'_{do}}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & \dfrac{1}{J}\\ \dfrac{1}{T'_{do}} & 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$
$$Y=\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}\Delta\delta\\ \Delta P_e\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ K_1 & 0 & K_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \tag{5}$$

Where:

$$K_1=\frac{x_{30}V_B}{x'_d}\cos x_{10}+V_B^2(\frac{1}{x_q}-\frac{1}{x'_d})\cos 2x_{10} \tag{6}$$

$$K_2=\frac{V_B\sin x_{10}}{x'_d} \tag{7}$$

$$K_3=\frac{x'_d}{x_d} \tag{8}$$

$$K_4=\frac{V_B\sin x_{10}}{x'_d}(x_d-x'_d) \tag{9}$$

To avoid complexity in equations, we define unknown parameters in (5) as $p_1, p_2, ..., p_7$. The linear dynamical model can be written as follows:

$$\dot{X}=\underbrace{\begin{bmatrix}0 & 1 & 0\\ p_1 & p_2 & p_3\\ p_4 & 0 & p_5\end{bmatrix}}_{A}X+\underbrace{\begin{bmatrix}0 & 0\\ 0 & p_7\\ p_6 & 0\end{bmatrix}}_{B}U$$
$$Y=\begin{bmatrix}1 & 0 & 0\\ -\dfrac{p_1}{p_7} & 0 & -\dfrac{p_3}{p_7}\end{bmatrix}X \tag{10}$$

The corresponding nonlinear model will be:

$$\begin{bmatrix} \dot{x} \\ \dot{x} \\ \dot{x} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ p_2 & 0 \\ 0 & p_5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & p_7 \\ p_6 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J}(T_m - \frac{V_B}{x'_d} x_3 \sin x_1 - \frac{V_B^2}{2}(\frac{1}{x_q} - \frac{1}{x'_d}) \sin 2x_1)) \\ -(p_5 + p_6)V \ \cos x_1 \end{bmatrix}$$

$$y = \begin{bmatrix} x_1 \\ \frac{V}{x'_d} x_3 \sin x_1 + \frac{V^2}{2}\left(\frac{1}{x_q} - \frac{1}{x'_d}\right) \sin 2x_1 \end{bmatrix}$$

By comparing the above equations, it is obvious that the linear and the nonlinear models have some common parameters which are independent from the operating point condition. These common parameters are used to identify the nonlinear model.

3. Identification Method

In the identification methods, the signals are sampled and the samples are used to identify the model. On the other hand, the discrete equations are difference not differential type; this causes the discrete equations to be easy for implementation. Because of these two reasons, we discretize the synchronous generator model with sample time ‘h’ and use the discrete model instead of the continuous one.

The discrete model of the system in (11) is given in the following:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & h & 0 \\ hp_1 & 1+hp_2 & hp_3 \\ hp_4 & 0 & 1+hp_5 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & hp_7 \\ hp_6 & 0 \end{bmatrix}\begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{p_1}{p_7} & 0 & -\frac{p_3}{p_7} \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} \tag{12}$$

Since our goal is to find the relation between the machine’s parameters and its outputs, we write the states in terms of the machine’s outputs.

$$x_1(k) = y_1(k) \tag{13}$$

$$x_2(k) = \frac{y_1(k+1) - y_1(k)}{h} \tag{14}$$

$$x_3(k) = -\frac{p_7}{p_3}(y_2(k) + \frac{p_1}{p_7} y_1(k)) \tag{15}$$

Substitute the values of $x_2(k), x_3(k)$ from (14) and (15) in (11). We will have (16) and (17).

$$\frac{y_1(k+2)-y_1(k+1)}{h} = hp_1 y_1(k) + \frac{1+hp_2}{h}(y_1(k+1)-y_1(k))$$

$$+hp_3\left(-\frac{p_7}{p_3}\right)\left(y_2(k)+\frac{p_1}{p_7}y_1(k)\right)+hp_7 u_2(k) \tag{16}$$

$$-\frac{p_7}{p_3}\left(y_2(k+1)+\frac{p_1}{p_7}y_1(k+1)\right) = hp_4 y_1(k) - (1+hp_5)\frac{p_7}{p_3}\left(y_2(k)+\frac{p_1}{p_7}y_1(k)\right)+hp_6 u_1(k) \tag{17}$$

The equation (16) can be written as follows:

$$\frac{y_1(k+2)-y_1(k+1)}{h} = \begin{bmatrix} \frac{y_1(k+1)-y_1(k)}{h} & h(u_2(k)-y_2(k)) \end{bmatrix} \begin{bmatrix} 1+hp_2 \\ p_7 \end{bmatrix} \tag{18}$$

Since (18) is a linear regression equation, we can use the Least Square Error (LSE) method to identify the unknown parameters p_2, p_7. If we apply N different time points to the above formula, the following equation which is a proper format for LSE method will be obtained.

$$\underbrace{\begin{bmatrix} \frac{y_1(3)-y_1(2)}{h} \\ \vdots \\ \frac{y_1(k+2)-y_1(k+1)}{h} \\ \vdots \\ \frac{y_1(N)-y_1(N-1)}{h} \end{bmatrix}}_{Y_1} = \underbrace{\begin{bmatrix} \frac{y_1(2)-y_1(1)}{h} & h(u_2(1)-y_2(1)) \\ \vdots & \vdots \\ \frac{y_1(k+1)-y_1(k)}{h} & h(u_2(k)-y_2(k)) \\ \vdots & \vdots \\ \frac{y_1(N-1)-y_1(N-2)}{h} & h(u_2(N-2)-y_2(N-2)) \end{bmatrix}}_{H_1} \underbrace{\begin{bmatrix} 1+hp_2 \\ p_7 \end{bmatrix}}_{\theta_1} \tag{19}$$

From (Norton, 1986) we can estimate the parameters as follows:

$$\theta_1 = (H_1^T H_1)^{-1} H_1^T Y_1 \tag{20}$$

From (19) and (20), the values of D and J can be calculated as follows:

$$J = \frac{1}{\theta_1(2)} \tag{21}$$

$$D = \frac{-\theta_1(1)+1}{h\theta_1(2)} \tag{22}$$

After estimating the mechanical parameters (D and J), consider (17) and divide it by p_6 , then rewrite it in a suitable form for the LSE method, as follows:

$$hu_1(k) = \begin{bmatrix} y_2(k)-y_2(k+1) & y_1(k)-y_1(k+1) & hy_2(k) & hy_1(k) \end{bmatrix} \begin{bmatrix} \frac{p_7}{p_3 p_6} \\ \frac{p_1}{p_3 p_6} \\ \frac{p_5 p_7}{p_3 p_6} \\ \frac{p_1 p_5}{p_3 p_6} - \frac{p_4}{p_6} \end{bmatrix} \tag{23}$$

To reduce the effect of measurement noise, the formula is considered for N different time points.

$$\underbrace{\begin{bmatrix} hu_1(1) \\ \vdots \\ hu_1(k) \\ \vdots \\ hu_1(N-1) \end{bmatrix}}_{Y_2} = \underbrace{\begin{bmatrix} y_2(1)-y_2(2) & y_1(1)-y_1(2) & hy_2(1) & hy_1(1) \\ \vdots & \vdots & \vdots & \vdots \\ y_2(k)-y_2(k+1) & y_1(k)-y_1(k+1) & hy_2(k) & hy_1(k) \\ \vdots & \vdots & \vdots & \vdots \\ y_2(N-1)-y_2(N) & y_1(N-1)-y_1(N) & hy_2(N-1) & hy_1(N-1) \end{bmatrix}}_{H_2} \underbrace{\begin{bmatrix} \frac{p_7}{p_3p_6} \\ \frac{p_1}{p_3p_6} \\ \frac{p_5p_7}{p_3p_6} \\ \frac{p_1p_5}{p_3p_6}-\frac{p_4}{p_6} \end{bmatrix}}_{\theta_2} \tag{24}$$

The above equation is a linear regression equation. We can write:

$$\theta_2 = (H_2^T H_2)^{-1} H_2^T Y_2$$

Where:

$$\theta_2^T = [\frac{p_7}{p_3p_6}, \frac{p_1}{p_3p_6}, \frac{p_5p_7}{p_3p_6}, \frac{p_1p_5}{p_3p_6} - \frac{p_4}{p_6}] \tag{25}$$

Now we use θ_2 to estimate the unknown parameters of synchronous machine. Due to (21) and (25), we have:

$$p_3p_6 = \frac{\theta_1(2)}{\theta_2(1)} \tag{26}$$

$$p_5 = \frac{\theta_2(3)}{\theta_2(1)} \tag{27}$$

From equations (5), (7), (8) and (10), p_5 and p_3p_6 can be written in terms of the machine's parameters. We can write:

$$\frac{p_5}{p_3p_6} = \frac{-\frac{1}{K_3T'_{d0}}}{-\frac{K_2}{J}\frac{1}{T'_{do}}} = \frac{-\frac{1}{T'_{d0}}\frac{x'_d}{x_d}}{-\frac{1}{T'_{do}}\frac{\frac{V_B \sin x_{10}}{x'_d}}{J}} = \frac{J}{\frac{V_B \sin x_{10}}{x'_d}\frac{x'_d}{x_d}} = x_d \frac{J}{V_B \sin y_{10}}$$

From the above equation we can estimate x_d as follows:

$$x_d = \frac{p_5}{p_3p_6}\frac{V_B \sin y_{10}}{J} \tag{28}$$

It should be noted that we can substitute the values of p_5, p_3p_6 and J from (26), (27) and (21), respectively.

Moreover, looking at $\theta_2(2)$, $\theta_2(1)$ and $\theta_2(4)$ in (25), p_1 and $\frac{p_4}{p_6}$ can be calculated as follows:

$$p_1 = \frac{\theta_2(2)\theta_1(2)}{\theta_2(1)} \tag{29}$$

$$\frac{p_4}{p_6} = \frac{\theta_2(3)\theta_2(2)}{\theta_2(1)} - \theta_2(4) \tag{30}$$

Using (30) and (9)-(10), p_4 and p_6 can be written in terms of the machine's parameters. Their ratio is given below:

$$\frac{p_4}{p_6} = -(\frac{x_d - x'_d}{x'_d})V_B \sin x_{10} \tag{31}$$

In the above equation, $V_B \sin x_{10}, x_d$ is known and x_d and $\frac{p_4}{p_6}$ are estimated before, so it can be used to estimate x'_d.

$$x'_d = \frac{x_d}{1 - (\frac{p_4}{p_6})(\frac{1}{V_B \sin y_{10}})} \tag{32}$$

Since x_d and x'_d are known, (5)-(10) can be used to calculate T'_{do} as follows:

$$T'_{do} = -\frac{x_d}{p_5 x'_d} \tag{33}$$

The only unknown remained parameter is x_q. From (2), we have:

$$P_e = \frac{V_B}{x'_d} e'_q \sin\delta + \frac{V_B^2}{2}(\frac{1}{x_q} - \frac{1}{x'_d})\sin(2\delta)$$
$$e'_q = V_B \cos\delta + x'_d i_d$$

Substituting e'_q into P_e and rewrite it into the following form:

$$P_e - V_B i_d \sin\delta = \frac{V_B^2}{2}\frac{1}{x_q}\sin(2\delta)$$

Thus we will achieve the following formula:

$$x_q = \frac{V_B^2 \sin(2\delta)}{2(P_e - V_B i_d \sin\delta)} \tag{34}$$

Writing (34) in operating point o, x_q is calculated as follows:

$$x_q = \frac{V_B^2 \sin(2x_{10})}{2(P_0 - V_B i_{d0} \sin x_{10})} \tag{35}$$

In the above equation, P_o is the active power, i_{do} is the direct axis current and x_{10} is the value of rotor angle at the operating point. It is assumed that the value of P_o and x_{10} are measured and i_{do} is calculated from the following relations [13-14]:

$$\varphi = \arctan(\frac{Q}{P}), I = \frac{\sqrt{P^2 + Q^2}}{V_B}$$
$$i_d = I \sin(\delta + \varphi)$$

$$i_{d0} = \frac{\sqrt{P_0^2 + Q_0^2}}{V_B} \sin(x_{10} + \arctan(\frac{Q_0}{P_0}))$$

In the above equation, Q_o is the reactive power at the operating point.

Since the equations in (19) and (24) can be updated online, we can use the whole algorithm for online identification of the machine parameters. It means that we can estimate the machine parameters when it is in service and we do not need to turn it off.

4. Simulation Results

In this section, a simulated model of a synchronous machine is used to test the suggested method. The estimated parameters are compared to the simulated ones and the model is validated under different operating conditions. Finally, the effect of noise on the parameters is examined.

4.1. Model simulation

Our goal is to find a practical approach for synchronous generator model identification, so the mechanical torque is assumed to be constant and only the field voltage, which can be perturbed more easily, is perturbed. To evaluate the proposed method, a Pseudo-Random Binary Sequence (PRBS) with amplitude of 5% of the nominal value is applied to the field voltage [12]. Since the change in the field voltage is very small compared to the nominal value, it will not disturb the normal operation of the system too much.

When the field voltage is perturbed with the PRBS, the active output power, the rotor angle and the field voltage are sampled with sample time h=2 millisecond. The operating point is considered to be P=0.9, Q=0.1, v_t=0.98. The input/output data collected from the system model in this operating point are shown in Fig.2.

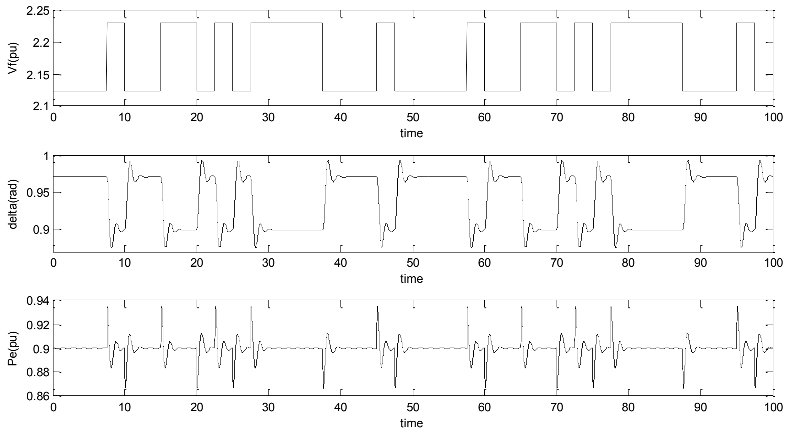

Figure 2: data collected at operating point P=0.9, Q=0.1, v_t=0.98

After simulating the model at operating point P=0.9, Q=0.1 and v_t=0.98, the field voltage, the rotor angle and the active power signals are sampled, then matrixes in the equations (19) and (24) are obtained. Afterward, θ_1 and θ_2 are estimated using LSE method. Finally, using the procedure of the previous section, the machine parameters can be calculated. The results of this procedure are given in Table 1.

Table 1: Synchronous Machine Parameters

Parameters	Real values	Estimated values	Error%
J	0.0252	0.0252	0%
D	0.500	0.507	1.4%
x_d	2.072	2.1132	1.988%
x_q	1.559	1.5589	0.0064%
x'_d	0.568	0.592	4.22%
T'_{pdo}	0.131	0.1304	0.06%

According to the third column of the above table, we can conclude that the estimation error is acceptable. In reality, we don't know the real value of the parameters. To validate the results, the outputs of real model and the identified one are compared. In the next part, the procedure of model validation is discussed.

4.2. Model validation

To evaluate the obtained nonlinear state space model, the model is excited by different inputs in the field voltage in different operating conditions and the performance of the model is observed. First, the operating point P=0.8pu, Q=0, v_t=1.05pu is selected. The system is excited by a *PRBS* input and the power and the rotor angle are measured. The results are compared in Fig. 3.

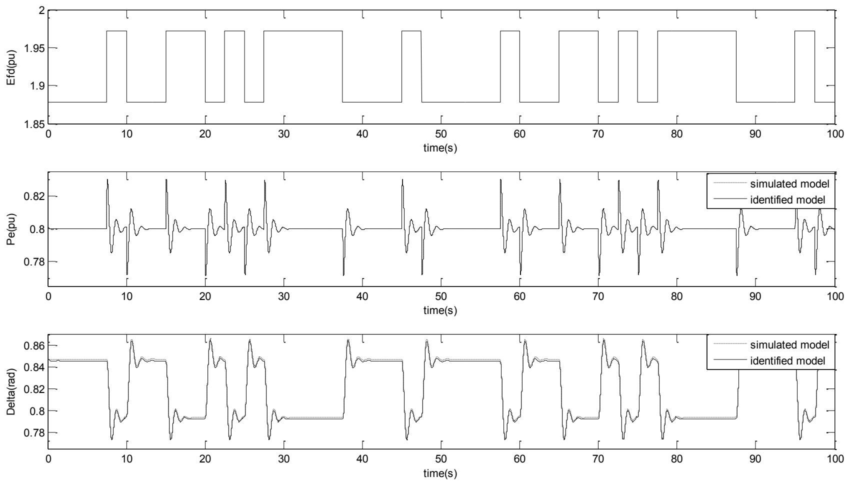

Figure 3: data collected at operating point P=0.8, Q=0, v_t=1.05

To calculate the estimation error, we use the following indexes [15]:

$$I = \frac{1}{N}\sum_{k=1}^{N}(y(k) - \hat{y}(k))^2$$

$$FPE = \frac{N+n}{N(N-n)}\sum_{k=1}^{N}(y(k) - \hat{y}(k))^2$$

$$P_{R_{ee}} = \text{Precentage of } R_{(y-\hat{y})(y-\hat{y})} \text{ in } \textit{the range} \left(-\frac{2}{\sqrt{N}}, \frac{2}{\sqrt{N}}\right)$$

Where, n is the number of estimated parameters, N is the number of samples and R is the autocorrelation function. In the $P_{R_{ee}}$ index, we calculate the autocorrelation function of error. If more than 95% of data remain in the range $\left(-\frac{2}{\sqrt{N}}, \frac{2}{\sqrt{N}}\right)$, we have good accuracy in the estimation procedure. The autocorrelation function for the collected data in Fig. 3 are given in Fig.4.

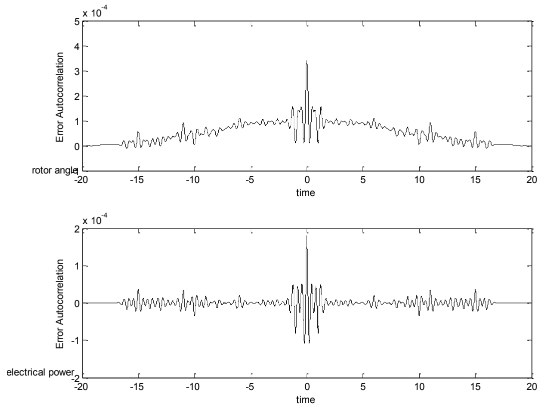

Figure 4: Autocorrelation function of Fig.3

The values of I, FPE and $P_{\mathrm{R}_{ee}}$ FPE for PRBS input would be:

$I_\delta = 1.8521\text{e-}005, \quad I_P = 4.1009\text{e-}006$
$FPE_\delta = 1.8576\text{e-}005, \quad FPE_P = 4.1132\text{e-}006$
$P_{\mathrm{R}_{ee}}\_\delta = 100\%, \quad P_{\mathrm{R}_{ee}}\_P = 100\%$

To show that the identified model has successfully covered the main non-linearities of the system, more studies were carried out. The performance of the identified model and the system at P=1.1pu, Q=0.2pu, v_t=1.02pu, when the system is excited by a step signal in the generator field voltage is compared in Fig. 5. The autocorrelation function for the collected data in Fig. 5 are given in Fig.6.

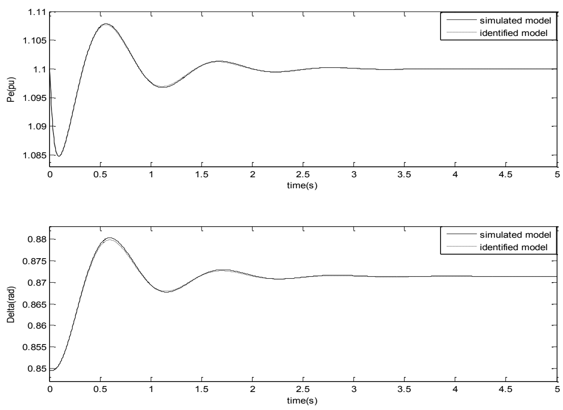

Figure 5: data collected at operating point P=1.1, Q=0.2, v_t=1.02

when the input is step signal

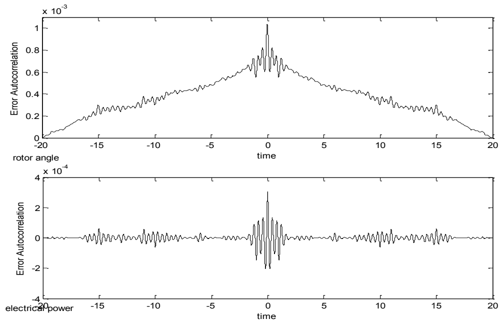

Figure 6: Autocorrelation function of Fig.5

The value of I , FPE and $\mathrm{P}_{\mathrm{R}_{ee}}$ for the step input would be:

$I_{\delta} = 2.4416\text{e-}006, \quad I_{P} = 4.1764\text{e-}006$
$FPE_{\delta} = 2.4637\text{e-}006, \quad FPE_{P} = 4.2142\text{e-}006$
$\mathrm{P}_{\mathrm{R}_{ee}}\_\delta = 100\%, \quad \mathrm{P}_{\mathrm{R}_{ee}}\_P = 100\%$

As mentioned in the beginning of this section, we apply the PRBS to the input of the system. Although, in some papers [10,16], it is shown that PRBS is applied to the input of real power plants, it is not necessary to use this kind of signal. In another example, we apply a triangular signal, which is the integral of PRBS, to the field voltage. The collected data are shown in Fig. 7. The identification procedure is done again. The results are given in table 2.

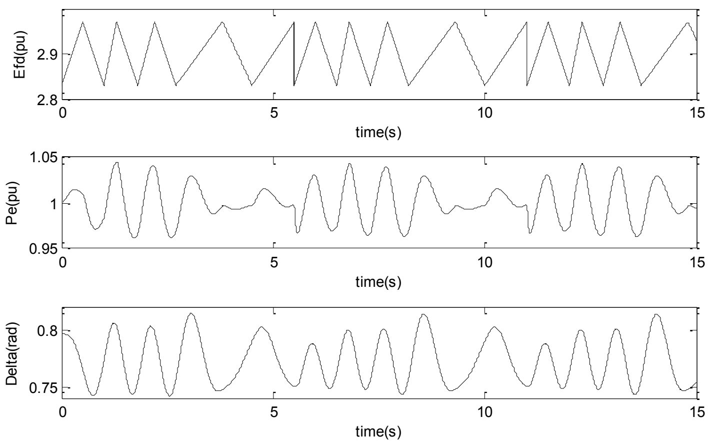

Figure 7: data collected at operating point P=1, Q=0.4, v_t=0.95 when the input signal is triangular

Table 2: Synchronous machine parameters when the input signal is triangular

parameters	Real values	Estimated values	Error%
J	0.0252	0.0252	0%
D	0.05	0.0511	2.2%
x_d	2.072	2.0117	2.91%
x_q	1.559	1.5592	0.0012%
x'_d	0.568	0.5544	2.39%
T'_{pdo}	0.131	0.1344	2.59%

The estimation error in the above table is satisfactory, so we can conclude that the triangular signal can be used in practice.

To continue the model validation, the field voltage is excited by the pulse input with period T=4s at operating point P=1.2pu, Q=0.15pu, v_t=0.9. Comparison of the performance of the identified model and the system at P=1.2pu, Q=0.15pu, v_t=0.9 when the system is excited by a pulse signal is shown in Fig. 8.

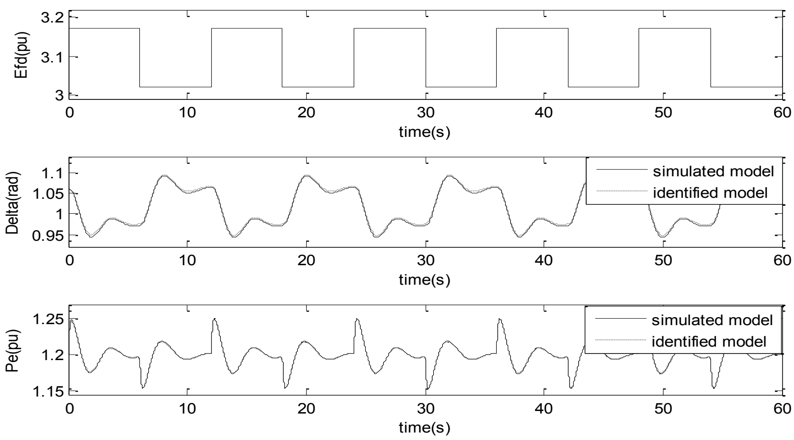

Figure 8: data collected at operating point P=1.2, Q=0.15, v_t=0.9

when the input signal is a pulse signal

The value of I and FPE for pulse input would be:

$I\_\delta = 9.2595\text{e-}007, \quad I\_P = 1.6253\text{e-}007$
$FPE\_\delta = 9.2817\text{e-}007, \quad FPE\_P = 1.6292\text{e-}007$

Since in most real cases the measurements are noisy, in the next section the effect of noise is studied.

4.3. Noise effect

In this part, we examine the effect of white noise on the proposed method. We add the white noise with different SNR (Signal to Noise Ratio) to the measured outputs. The estimation procedure is done for noisy measurements. The results are given in table 3.

Table 3: estimated parameters for noisy measurements with different SNR

SNR / parameter	1000	500	250	150	120	110	100	90
D	0.0511	0.0511	0.051	0.0511	0.0523	0.0629	0.134	0.26
J	0.0252	0.0252	0.0252	0.0252	0.0250	0.0240	0.0174	0.0048
x_q	1.5588	1.5588	1.5588	1.5588	1.5588	1.5588	1.5588	1.5588
x'_d	0.6068	0.6068	0.6068	0.6068	0.6068	0.6070	0.6091	0.631
x_d	2.1164	2.1164	2.116	2.116	2.116	2.1169	2.1207	2.1575
T'_{do}	0.1299	0.1299	0.1299	0.1299	0.1299	0.1297	0.1283	0.115

According to the above table, we conclude that for SNR more than 110 the estimated parameters are close to the real values, but for less than it, the estimation error is increased especially for parameters D and J. In addition, we see that the value of x_q is robust with respect to the noise. Simulation tests admit that the proposed method can be used successfully for identification of synchronous machine model.

5. Conclusion

In this paper, the theoretical relation based approach, which uses the known least square error method, has been presented to identify the nonlinear 3rd order synchronous generator state space model parameters. The most important point about the method is to use the linear least square error method to estimate the nonlinear model. In this paper, the field voltage is considered as the machine's input and the rotor angle and active powers are as the outputs of the machine.

Simulation results show that the proposed method has good accuracy for estimation of state space synchronous generator parameters.

6. Appendix

The main variables of model (1) are:

J, D: rotor inertia and damping factor

T'_{do} : direct-axis transient time constant

x_d : direct axis reactance

x'_d : direct axis transient reactance

x_q : quadrature axis reactance

δ : rotor angle with respect to the machine terminals

ω: rotor speed

T_m : input mechanical torque

T_e : output electric torque

E_{FD}: The equivalent EMF in the excitation coil

e'_q : transient internal voltage of armature

P, Q: terminal active and reactive power per phase

v_t : generator terminal voltage

V_B : Infinite bus voltage

i_d, i_q : direct and quadrature axis stator currents

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