A Family of Variable Step-Size Normalized Subband Adaptive Filter Algorithms Using Statistics of System Impulse Response

Mohammad Shams Esfand Abadi, and Mohammad Saeed Shafiee

***Abstract*—This paper presents a new variable step-size normalized subband adaptive filter (VSS-NSAF) algorithm. The proposed algorithm uses the prior knowledge of the system impulse response statistics and the optimal step-size vector is obtained by minimizing the mean-square deviation(MSD). In comparison with NSAF, the VSS-NSAF algorithm has faster convergence speed and lower MSD. To reduce the computational complexity of VSS-NSAF, the VSS selective partial update NSAF (VSS-SPU-NSAF) is proposed where the filter coefficients are partially updated in each subband at every iteration. We demonstrated the good performance of the proposed algorithms in convergence speed and steady-state MSD for a system identification set-up.**

***Index Terms*—Adaptive filter, variable step-size (VSS), selective partial update (SPU), normalized subband adaptive filter (NSAF).**

I. Introduction

Adaptive filtering has been, and still is, an area of active research that plays an active role in an ever increasing number of applications, such as noise cancellation, channel estimation, channel equalization and acoustic echo cancellation [1], [2], [3], [4], [5]. The least mean square (LMS) and its normalized version (NLMS) are the workhorses of adaptive filtering. In the presence of colored input signals, the LMS and NLMS algorithms have extremely slow convergence rates. Adaptive filtering in subbands has been proposed to improve the convergence behavior of the LMS algorithm [6]. The normalized subband adaptive filter (NSAF) was proposed in [7]. In [8], the selective partial update NSAF (SPU-NSAF) was proposed to reduce the computational complexity. In this algorithm, the filter coefficients are partially updated in each subband at every iteration. This feature leads to the reduction in computational complexity.

In above mentioned algorithms, the selected fixed step-size can change the convergence rate and the steady-state mean square error (MSE). It is well known that the steady-state MSE decreases when the step-size decreases, while the convergence speed increases when the step-size increases. By optimally selecting the step-size during the adaptation, we can obtain both fast convergence rate and low steady-state MSE. In [9], a new variable-step-size control was proposed for the normalized least-mean-square (NLMS) algorithm. A step-size vector with different values for each filter coefficients was used in [9]. In this approach, based on the prior knowledge of the system impulse response statistics, the optimal step-size vector is obtained by minimizing MSD. In this paper, we extend the approach in [9] to NSAF, and SPU-NSAF algorithms and VSS version of these algorithms are proposed. We demonstrate the good performance of the presented algorithms through several simulation results in a system identification scenario. We have

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organized our paper as follows. In Section II, we briefly review NSAF, and SPU-NSAF algorithms. In Section III, the proposed VSS adaptive algorithms is established. Finally, before concluding the paper, we demonstrate the usefulness of these algorithms by presenting several experimental results.

Throughout the paper, the following notations are adopted:

- $|\ .\ |$ Norm of a scalar.
- $\|.\|^2$ Squared Euclidean norm of a vector.
- $\|\mathbf{t}\|^2_{\mathbf{\Sigma}}$ $\mathbf{\Sigma}$-Weighted Euclidean norm of a column vector $\mathbf{t}$ defined as $\mathbf{t}^T\mathbf{\Sigma}\mathbf{t}$.
- $E\{.\}$ Expectation operator.
- $(.)^T$ Transpose of a vector or a matrix.
- $\text{diag}(.)$ Has the same meaning as the MATLAB operator with the same name: If its argument is a vector, a diagonal matrix with the diagonal elements given by the vector argument results. If the argument is a matrix, its diagonal is extracted into a resulting vector.

II. Background on NSAF, and SPU-NSAF Algorithms

Adaptive filtering in subbands has been proposed to improve the convergence behavior of the LMS algorithm [6], [10]. In subband adaptive filtering, the input signal and desired response are band-partitioned into almost mutually exclusive subband signals. This feature of the SAF permits the manipulation of each subband signal, in such a way that their properties can be exploited [2], allowing each subband to converge almost separately for various modes [6], and thus improving the overall convergence behavior. In this section we briefly review NSAF and SPU-NSAF algorithms.

A. NSAF Algorithm

Fig. 1 shows the structure of NSAF [7]. In this figure, $\mathbf{f}_0, \mathbf{f}_1, \cdots, \mathbf{f}_{N-1}$, are analysis filter unit impulse responses of an N channel orthogonal perfect reconstruction critically sampled filter bank system. $x_i(n)$ and $d_i(n)$ are nondecimated subband signals. It is important to note that n refers to the index of the original sequences and k denotes the index of the decimated sequences. Similar to the NLMS algorithm, NSAF can be established by the solution of the following optimization problem

$$\min \|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2 \tag{1}$$

subject to the set of N constraints imposed on the decimated filter output

$$d_{i,D}(k) = \mathbf{x}_i^T(k)\mathbf{w}(k+1) \quad for\ \ i = 0, \cdots, N-1 \tag{2}$$

where

$$\mathbf{x}_i(k) = [x_i(kN), x_i(kN-1), \cdots, x_i(kN-M+1)]^T \tag{3}$$

By solving this optimization problem based on the method of Lagrange multipliers, the filter update equation for NSAF can be stated as [7]

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \sum_{i=0}^{N-1} \frac{\mathbf{x}_i(k) e_{i,D}(k)}{||\mathbf{x}_i(k)||^2} \tag{4}$$

where $e_{i,D}(k) = d_{i,D}(k) - \mathbf{w}^T(k)\mathbf{x}_i(k)$ is the decimated subband error signal, and μ is the step size which is chosen in the range $0 < \mu < 2$ [7]. We also assumed a linear data model for the desired signal as

$$d_{i,D}(k) = \mathbf{x}_i^T(k)\mathbf{w}^o + v_{i,D}(k) \tag{5}$$

where $\mathbf{w}^o$ is the true unknown filter vector, and $v_{i,D}(k)$ is partitioned and decimated additive noise with zero mean and variance, $\sigma^2_{v_{i,D}}$. We also assume that $v(n)$ is identically and independently distributed (i.i.d.) and statistically independent of the input data $\mathbf{x}(n)$.

B. SPU-NSAF Algorithm

To reduce the computational complexity of NSAF, SPU-NSAF algorithm was proposed in [8]. Partition $\mathbf{x}_i(k)$ for $0 \leq i \leq N-1$ and $\mathbf{w}(k)$ into B blocks each of length L which are defined as

$$\mathbf{x}_i(k) = [\mathbf{x}^T_{i,1}(k), \mathbf{x}^T_{i,2}(k), \ldots, \mathbf{x}^T_{i,B}(k)]^T \tag{6}$$

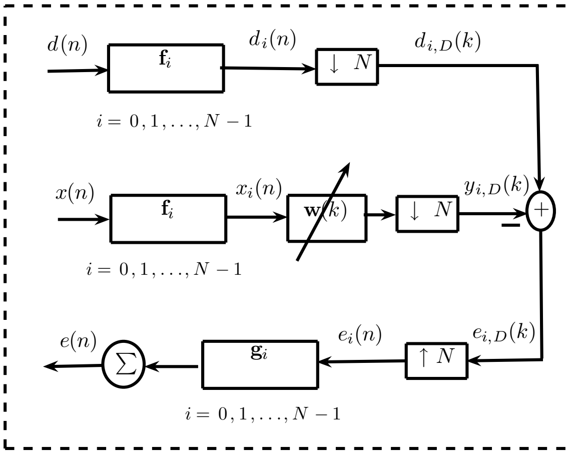

Fig. 1. Structure of NSAF algorithm.

$$\mathbf{w}(k) = [\mathbf{w}_1^T(k), \mathbf{w}_2^T(k), \ldots, \mathbf{w}_B^T(k)]^T. \tag{7}$$

Suppose we want to update S blocks out of B blocks in each subband at every adaptation. Let $F = \{j_1, j_2, \ldots, j_S\}$ denote the indexes of the S blocks out of B blocks. In this case, the optimization problem is defined as

$$\min_{\mathbf{w}_F(k+1)} \|\mathbf{w}_F(k+1) - \mathbf{w}_F(k)\|^2, \tag{8}$$

subject to (2). By using the Lagrange multipliers approach, the filter vector update equation is given by

$$\mathbf{w}_F(k+1) = \mathbf{w}_F(k) + \mu \sum_{i=0}^{N-1} \frac{\mathbf{x}_{i,F}(k) e_{i,D}(k)}{\|\mathbf{x}_{i,F}(k)\|^2} \tag{9}$$

where $\mathbf{x}_{i,F}(k) = [\mathbf{x}_{i,j_1}^T(k), \mathbf{x}_{i,j_2}^T(k), \ldots, \mathbf{x}_{i,j_S}^T(k)]^T$. To reduce the computational complexity associated with the selection of the blocks to update, two alternative simplified criteria were proposed: 1) In the first approach, we compute the following values

$$\sum_{i=0}^{N-1} \|\mathbf{x}_{i,b}(k)\|^2 \text{ for } 1 \le b \le B. \tag{10}$$

The indexes of the set F correspond to the indexes of the S largest values of (10). 2) In the second approach, we identify a set of indexes, correspond to the S smallest values of (11) [8].

$$j = \arg \min_{1 \le b \le B} \{ \sum_{i=0}^{N-1} \frac{|e_{i,D}(k)|^2}{\|\mathbf{x}_{i,b}(k)\|^2} \}. \tag{11}$$

III. Derivation of VSS-NSAF and VSS-SPU-NSAF Algorithms

In this section, we establish the family of VSS-NSAF algorithms based on [9]. This method minimizes MSD and obtains the step-size vector based on the system impulse response statistics.

A. VSS-NSAF Algorithm

The update equation for VSS-NSAF is introduced as

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mathbf{U}(k) \sum_{i=0}^{N-1} \frac{\mathbf{x}_i(k) e_{i,D}(k)}{||\mathbf{x}_i(k)||^2} \tag{12}$$

where $\mathbf{U}(k) = \text{diag}[\mu_0(k), ..., \mu_{M-1}(k)]$ is the variable step-size matrix. During the adaptation, this matrix changes to obtain faster convergence speed and lower steady-state MSD. Therefore, the objective of this paper is to design the step size matrix $\mathbf{U}(k)$ to improve the performance of the NSAF algorithm. By defining the weight error vector as

$$\tilde{\mathbf{w}}(k) = \mathbf{w}^\circ - \mathbf{w}(k), \tag{13}$$

and using the definition of $e_{i,D}(k)$, the decimated error signal can be rewritten as

$$e_{i,D}(k) = \tilde{\mathbf{w}}^T(k)\mathbf{x}_i(k) + v_{i,D}(k). \tag{14}$$

Now by substituting (13) and (14) into (12), we obtain

$$\tilde{\mathbf{w}}(k+1) = \mathbf{A}(k)\tilde{\mathbf{w}}(k) - \mathbf{U}(k) \sum_{i=0}^{N-1} \frac{\mathbf{x}_i(k) v_{i,D}(k)}{||\mathbf{x}_i(k)||^2} \tag{15}$$

where

$$\mathbf{A}(k) = I_M - \mathbf{U}(k) \sum_{i=0}^{N-1} \frac{\mathbf{x}_i(k)\mathbf{x}_i^T(k)}{||\mathbf{x}_i(k)||^2}, \tag{16}$$

and I_M is the $M \times M$ identity matrix. To quantitatively evaluate the mis-adjustment of the filter coefficients, the MSD is taken as a figure of merit, which is defined as

$$\Lambda(k) = E\left[||\tilde{\mathbf{w}}(k)||^2\right], \tag{17}$$

Note that at each iteration, the MSD depends on $\mu_j(k)$. Combining (15) and (17), we obtain

$$\Lambda(k+1) = E\left[\tilde{\mathbf{w}}^T(k)\mathbf{A}^T(k)\mathbf{A}(k)\tilde{\mathbf{w}}(k)\right] + \gamma, \tag{18}$$

where

$$\gamma = \frac{\mathrm{Tr}\left[\mathbf{U}^2(k)\right]}{M^2}\sum_{i=0}^{N-1}\frac{\sigma^2_{v_{i,D}}}{\sigma^2_{x_i}}, \tag{19}$$

Similar to [11], we assume again that $x_i(kN)$ and $v_{i,D}(k)$ are zero-mean i.i.d. stationary with variance $\sigma^2_{x_i}$ and $\sigma^2_{v_{i,D}}$, respectively; $\tilde{\mathbf{w}}(k)$, $x_i(kN)$, and $v_{i,D}(k)$ are mutually independent and $\mathbf{x}_i^T(k)\mathbf{x}_i(k) \approx M\sigma^2_{x_i}$ for $M \gg 1$. Therefore, we obtain

$$E[\mathbf{A}^T(k)\mathbf{A}(k)] = \left\{1 + \frac{N\mathrm{Tr}[\mathbf{U}^2(k)]}{M^2}\right\} I_M - \frac{2N}{M}\mathbf{U}(k) \tag{20}$$

Combining (18) and (20), we get

$$\Lambda(k+1) = \left\{1 + \frac{N\mathrm{Tr}[\mathbf{U}^2(k)]}{M^2}\right\} E\left[||\tilde{\mathbf{w}}(k)||^2\right] - \frac{2N}{M}E\left[\tilde{\mathbf{w}}^T(k)\mathbf{U}(k)\tilde{\mathbf{w}}(k)\right] + \gamma. \tag{21}$$

The optimal step size is obtained by minimizing the MSD at each iteration. Taking the first-order partial derivative of $\Lambda(k+1)$ with respect to $\mu_j(k)$ $(j = 0, ..., M-1)$,

$$\frac{\partial\Lambda(k+1)}{\partial\mu_j(k)} = E\left[||\tilde{\mathbf{w}}(k)||^2\right]\frac{2N\mu_j(k)}{M^2} - \frac{2N}{M}E\left[\tilde{w}_j^2(k)\right] + \frac{2}{M^2}\mu_j(k)\sum_{i=0}^{N-1}\frac{\sigma^2_{v_{i,D}}}{\sigma^2_{x_i}} \tag{22}$$

setting it to zero, we obtain

$$\mu_j(k) = \frac{NME\left[\tilde{w}_j^2(k)\right]}{NE\left[||\tilde{\mathbf{w}}(k)||^2\right] + \sum_{i=0}^{N-1}\frac{\sigma^2_{v_{i,D}}}{\sigma^2_{x_i}}} \tag{23}$$

To update $E[\tilde{w}_j^2(k)]$, we use the following equation obtained by taking the mean square of the k th entry in (15)

$$E\left[\tilde{w}_j^2(k+1)\right] = \left[1 - \frac{2N}{M}\mu_j(k)\right]E\left[\tilde{w}_j^2(k)\right] + \frac{N\mu_j^2(k)}{M^2}E\left[||\tilde{\mathbf{w}}(k)||^2\right] + \frac{\mu_j^2(k)}{M^2}\sum_{i=0}^{N-1}\frac{\sigma^2_{v_{i,D}}}{\sigma^2_{x_i}}. \tag{24}$$

From (5), and using the definition of $e_{i,D}(k)$, we obtain

$$E\left[e^2_{i,D}(k)\right] = \sigma^2_{x_i}E\left[||\tilde{\mathbf{w}}(k)||^2\right] + \sigma^2_{v_{i,D}} \tag{25}$$

Substitution of (25) into (24) leads to

$$E\left[\tilde{w}_j^2(k+1)\right] = \left[1 - \frac{2N}{M}\mu_j(k)\right]E\left[\tilde{w}_j^2(k)\right] + \frac{\mu_j^2(k)}{M^2}\sum_{i=0}^{N-1}\frac{E\left[e^2_{i,D}(k)\right]}{\sigma^2_{x_i}} \tag{26}$$

It is straightforward to estimate $E[e^2_{i,D}(k)]$ by a moving average of $e^2_{i,D}(k)$:

$$\hat{\sigma}^2_{e_{i,D}}(k) = \lambda\hat{\sigma}^2_{e_{i,D}}(k-1) + (1-\lambda)e^2_{i,D}(k) \tag{27}$$

similar for $x_i(kN)$:

$$\hat{\sigma}^2_{x_i}(k) = \lambda\hat{\sigma}^2_{x_i}(k-1) + (1-\lambda)x_i^2(kN) \tag{28}$$

where $0 < \lambda < 1$ is the forgetting factor.The initial $E[\tilde{w}_j^2(0)]$ is obtained by the second-order statistics of the channel response, i.e. $E[w_j^{\circ 2}]$ from (13). Finally, the entire adaptive algorithm is described by (12), (23), (26), (27), and (28). Table I summarizes VSS-NSAF algorithm.

B. VSS-SPU-NSAF Algorithm

The filter coefficients update for VSS-SPU-NSAF is introduced as

$$\mathbf{w}_F(k+1) = \mathbf{w}_F(k) + \mathbf{U}_F(k)\sum_{i=0}^{N-1}\frac{\mathbf{x}_{i,F}(k)e_{i,D}(k)}{||\mathbf{x}_{i,F}(k)||^2} \tag{29}$$

where $\mathbf{U}_F(k) = \mathrm{diag}[\mu_{j_1}(k), \ldots, \mu_{j_S}(k)]$. Using the approximation for $e_{i,D}(k)$ as $e_{i,D}(k) \approx \mathbf{x}^T_{i,F}(k)\tilde{\mathbf{w}}_F(k) + v_{i,D}(k)$ and substituting it into (29), we obtain

$$\mathbf{w}_F(k+1) = \mathbf{w}_F(k) + \mathbf{U}_F(k)\sum_{i=0}^{N-1}\frac{\mathbf{x}_{i,F}(k)(\mathbf{x}^T_{i,F}(k)\tilde{\mathbf{w}}_F(k) + v_{i,D}(k))}{||\mathbf{x}_{i,F}(k)||^2} \tag{30}$$

Rewrite (30) as a weight error vector

$$\tilde{\mathbf{w}}_F(k+1) = \mathbf{Q}(k)\tilde{\mathbf{w}}_F(k) - \mathbf{U}_F(k)\sum_{i=0}^{N-1}\frac{\mathbf{x}_{i,F}(k)v_{i,D}(k)}{||\mathbf{x}_{i,F}(k)||^2}, \tag{31}$$

where

$$\mathbf{Q}(k) = \mathbf{I}_{SL} - \mathbf{U}_F(k)\sum_{i=0}^{N-1}\frac{\mathbf{x}_{i,F}(k)\mathbf{x}^T_{i,F}(k)}{||\mathbf{x}_{i,F}(k)||^2}, \tag{32}$$

and $\mathbf{I}_{SL}$ is the $SL \times SL$ identity matrix. To obtain MSD we can write

$$\Lambda(k) = E\left[||\tilde{\mathbf{w}}(k)||^2\right] = E\left[||\tilde{\mathbf{w}}_F(k)||^2\right] + E\left[||\tilde{\mathbf{w}}_{\acute{F}}(k)||^2\right] \tag{33}$$

where $\left[||\tilde{\mathbf{w}}_{\acute{F}}(k)||^2\right]$ are weights that are not selected to update. Therefore

$$\left[||\tilde{\mathbf{w}}_{\acute{F}}(k+1)||^2\right] = \left[||\tilde{\mathbf{w}}_{\acute{F}}(k)||^2\right] \tag{34}$$

Combining (31) and (33) leads to

$$\Lambda(k+1) = E\left[\tilde{\mathbf{w}}_F^T(k)\mathbf{Q}^T(k)\mathbf{Q}(k)\tilde{\mathbf{w}}_F(k)\right] + \frac{\mathrm{Tr}\left[\mathbf{U}_F^2(k)\right]}{(SL)^2}\sum_{i=0}^{N-1}\frac{\sigma^2_{v_{i,D}}}{\sigma^2_{x_i}} + E\left[||\tilde{\mathbf{w}}_{\acute{F}}(k)||^2\right] \tag{35}$$

Similar to [11], we assume again that $x_i(kN)$ and $v_{i,D}(k)$ are zero-mean i.i.d. stationary with variance $\sigma^2_{x_i}$ and $\sigma^2_{v_{i,D}}$, respectively; $\tilde{\mathbf{w}}(k)$, $x_i(kN)$, and $v_{i,D}(k)$ are mutually independent and $\mathbf{x}^T_{i,F}(k)\mathbf{x}_{i,F}(k) \approx SL\sigma^2_{x_i}$ for $SL \gg 1$. Therefore, we obtain

$$E[\mathbf{Q}^T(k)\mathbf{Q}(k)] = \left\{1 + \frac{N\mathrm{Tr}[\mathbf{U}_F^2(k)]}{(SL)^2}\right\}\mathbf{I}_{SL} - \frac{2N}{SL}\mathbf{U}_F(k). \tag{36}$$

By combining (35) and (36), we get

$$\Lambda(k+1) = \left\{1 + \frac{N\mathrm{Tr}[\mathbf{U}_F^2(k)]}{(SL)^2}\right\}E\left[||\tilde{\mathbf{w}}_F(k)||^2\right] - \frac{2N}{(SL)}E\left[\tilde{\mathbf{w}}_F^T(k)\mathbf{U}_F(k)\tilde{\mathbf{w}}_F(k)\right] + \frac{\mathrm{Tr}\left[\mathbf{U}_F^2(k)\right]}{(SL)^2}\sum_{i=0}^{N-1}\frac{\sigma^2_{v_{i,D}}}{\sigma^2_{x_i}} + E\left[||\tilde{\mathbf{w}}_{\acute{F}}(k)||^2\right] \tag{37}$$

Taking the first-order partial derivative of $\Lambda(k+1)$ with respect to $\mu_j(k)(j = 0, \ldots, SL-1)$, and setting it to zero for $j \in F$, we obtain

$$\frac{\partial\Lambda(k+1)}{\partial\mu_j(k)} = E\left[||\tilde{\mathbf{w}}_F(k)||^2\right] + \frac{2N\mu_j(k)}{(SL)^2} - \frac{2N}{SL}E\left[\tilde{w}_j^2(k)\right] + \frac{2}{(SL)^2}\mu_j(k)\sum_{i=0}^{N-1}\frac{\sigma_{v^2_{i,D}}}{\sigma^2_{x_i}} = 0 \tag{38}$$

Therefore

$$\mu_j(k) = \frac{N(SL)E\left[\tilde{w}_j^2(k)\right]}{NE\left[||\tilde{\mathbf{w}}_F(k)||^2\right] + \sum_{i=0}^{N-1}\frac{\sigma^2_{v_{i,D}}}{\sigma^2_{x_i}}} \tag{39}$$

To update $\left[\tilde{w}_j^2(k)\right]$, the following equation is obtained by taking the mean square of the jth entry in (31)

$$E\left[\tilde{w}_j^2(k+1)\right] = \left[1 - \frac{2N}{SL}\mu_j(k)\right]E\left[\tilde{w}_j^2(k)\right] + \frac{N\mu_j^2(k)}{(SL)^2}E\left[||\tilde{\mathbf{w}}_F(k)||^2\right] + \frac{\mu_j^2(k)}{(SL)^2}\sum_{i=0}^{N-1}\frac{\sigma^2_{v_{i,D}}}{\sigma^2_{x_i}} \tag{40}$$

Using the following assumption as

$$E\left[e^2_{i,D}(k)\right] \approx \sigma^2_{x_i}E\left[||\tilde{\mathbf{w}}_F(k)||^2\right] + \sigma^2_{v_{i,D}} \tag{41}$$

and substituting it into (40), the following relation is obtained

$$E\left[\tilde{w}_j^2(k+1)\right] = \left[1 - \frac{2N}{SL}\mu_j(k)\right]E\left[\tilde{w}_j^2(k)\right] + \frac{\mu_j^2(k)}{(SL)^2}\sum_{i=0}^{N-1}\frac{E\left[e^2_{i,D}(k)\right]}{\sigma^2_{x_i}} \tag{42}$$

where $E[e^2_{i,D}(k)]$ and $\sigma^2_{x_i}$ are estimated according to (27) and (28). Table II summarizes the VSS-SPU-NSAF algorithm.

TABLE I

SUMMARY OF VSS-NSAF ALGORITHM

For $k = 0, 1, 2, \ldots$

$$e_{i,D}(k) = d_{i,D}(k) - \mathbf{w}^T(k)\mathbf{x}_i(k)$$

$$\hat{\sigma}^2_{e_{i,D}}(k) = \lambda\hat{\sigma}^2_{e_{i,D}}(k-1) + (1-\lambda)e^2_{i,D}(k)$$

$$\hat{\sigma}^2_{x_i}(k) = \lambda\hat{\sigma}^2_{x_i}(k-1) + (1-\lambda)x_i^2(kN)$$

$$E[\tilde{w}_j^2(k+1)] = [1 - \tfrac{2N}{M}\mu_j(k)]E[\tilde{w}_j^2(k)] + \frac{\mu_j^2(k)}{M^2}\sum_{i=0}^{N-1}\frac{\hat{\sigma}^2_{e_{i,D}}(k)}{\hat{\sigma}^2_{x_i}(k)}$$

$$\mu_j(k) = \frac{NME\left[\tilde{w}_j^2(k)\right]}{NE\left[||\tilde{\mathbf{w}}(k)||^2\right] + \sum_{i=0}^{N-1}\frac{\sigma^2_{v_{i,D}}}{\sigma^2_{x_i}}}$$

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mathbf{U}(k)\sum_{i=0}^{N-1}\frac{\mathbf{x}_i(k)e_{i,D}(k)}{||\mathbf{x}_i(k)||^2}$$

end

TABLE II

SUMMARY OF VSS-SPU-NSAF ALGORITHM

For $k = 0, 1, 2, \ldots$

$$e_{i,D}(k) = d_{i,D}(k) - \mathbf{w}^T(k)\mathbf{x}_i(k)$$

$$\hat{\sigma}^2_{e_{i,D}}(k) = \lambda\hat{\sigma}^2_{e_{i,D}}(k-1) + (1-\lambda)e^2_{i,D}(k)$$

$$\hat{\sigma}^2_{x_i}(k) = \lambda\hat{\sigma}^2_{x_i}(k-1) + (1-\lambda)x_i^2(kN)$$

$$E[\tilde{w}_j^2(k+1)] = [1 - \tfrac{2N}{SL}\mu_j(k)]E[\tilde{w}_j^2(k)] + \frac{\mu_j^2(k)}{(SL)^2}\sum_{i=0}^{N-1}\frac{\hat{\sigma}^2_{e_{i,D}}(k)}{\hat{\sigma}^2_{x_i}(k)}$$

$$\mu_j(k) = \frac{N(SL)E\left[\tilde{w}_j^2(k)\right]}{NE\left[||\tilde{\mathbf{w}}_F(k)||^2\right] + \sum_{i=0}^{N-1}\frac{\sigma^2_{v_{i,D}}}{\sigma^2_{x_i}}}$$

$$\mathbf{w}_F(k+1) = \mathbf{w}_F(k) + \mathbf{U}_F(k)\sum_{i=0}^{N-1}\frac{\mathbf{x}_{i,F}(k)e_{i,D}(k)}{||\mathbf{x}_{i,F}(k)||^2}$$

end

IV. COMPUTATIONAL COMPLEXITY

Table III shows the number of multiplications, divisions, and comparisons of different adaptive algorithms. The compu-

TABLE III

COMPUTATIONAL COMPLEXITY OF THE FAMILY OF VSS-NSAF ALGORITHMS

Algorithm	Multiplications	Divisions	Additional Multiplications	Comparisons
NSAF [7]	$3M + 3NK + 1$	1	-	-
SPU-NSAF [8]	$2M + SL + 3NK + 1$	1	-	$O(B) + Blog_2(S)$
Proposed VSS-NSAF	$3M + 3NK$	4	$5M + 8$	–
Proposed VSS-SPU-NSAF	$2M + SL + 3NK$	4	$5M + 8$	$O(B) + Blog_2(S)$

tational complexity of NSAF for each input sampling period is exactly $3M+3NK+1$ multiplications and 1 division, where K is the length of the channel filters of the analysis filter bank, M is the number of filter coefficients, and N is the number of subbands. SPU-NSAF needs $2M+SL+3NK+1$ multiplications, 1 division, and $O(B)+Blog_2(S)$ comparisons when using the heapsort algorithm [12]. The proposed VSS-NSAF needs $5M+8$ multiplications and 3 divisions more than conventional NSAF. Using SPU approach in VSS-NSAF leads to the reduction in number of multiplications. The number of multiplications is $7M + SL + 3NK + 8$ in this algorithm. The VSS-SPU-NSAF algorithm needs also 4 divisions and $O(B) + Blog_2(S)$ comparisons.

V. SIMULATION RESULTS

We demonstrate the performance of the proposed algorithms by several computer simulations in a system identification scenario. In the first simulation, we use the real acoustic impulse response with length $M = 256$ as shown in Fig. 2 [13]. The same length is used for the adaptive filter. The colored Gaussian signal is used for the input signal. The input signal is obtained by filtering a white, zero-mean and unit variance Gaussian random sequence through a second-order auto regressive (AR(2)) system with transfer function $T(z) = \frac{1}{1-0.1z^{-1}-0.8z^{-2}}$. The filter bank used in NSAF was the four subband extended lapped transform (ELT) ($N = 4$) [14]. The white zero-mean Gaussian noise was added to the filter output such that the $SNR = 30dB$. In all simulations, we show the normalized MSD, $E\|\mathbf{w}^o-\mathbf{w}(k)\|^2/\|\mathbf{w}^o\|^2$ which is evaluated by ensemble averaging over 20 independent trials. Also, we assume that the noise variance, σ_v^2, is known a priori [15]. For all simulations we consider $\lambda = 0.999$.

Fig. 3 compares the convergence rate of the NSAF algorithm with the proposed VSS-NSAF when the real unknown impulse response should be identified. In NSAF, different step sizes ($1, 0.2$ and 0.05) were chosen. As we can see, the proposed VSS-NSAF has both fast convergence rate and low steady-state MSD features compared with ordinary NSAF. Fig. 4 shows the normalized MSD curves for the proposed VSS-NSAF for $w^\circ = e^{-j\tau}r(j),\ \ j = 0,\cdots,M-1$ where $r(j)$ is a white Gaussian random sequence with zero-mean and variance σ_r^2 of 0.09. In this case, the impulse response length is $M = 200$, and the envelope decay rate τ is set to 0.04. The simulation results show that for low and large values for the step-size, the performance of NSAF is deviated. But the VSS-NSAF has both fast convergence speed and low steady-state MSD due to the strategy of variable step-size.

In Fig. 5, we presented the results for random unknown impulse response. The parameter M is set to 50. The simulation results show that in the case of random unknown system the performance of VSS-NSAF is deviated, but still, the overall performance is better than ordinary NSAF algorithm. Fig. 6 compares the MSD curves of VSS-NSAF, and VSS-SPU-NSAF algorithms when the real unknown impulse response should be identified. The number of blocks (B) was set to 4 and various values for S were selected. By increasing the parameter S, the performance of VSS-SPU-NSAF will be closed to the VSS-NSAF algorithm. Furthermore, the computational complexity of VSS-SPU-NSAF is lower than VSS-NSAF due to partial updates of filter coefficients.

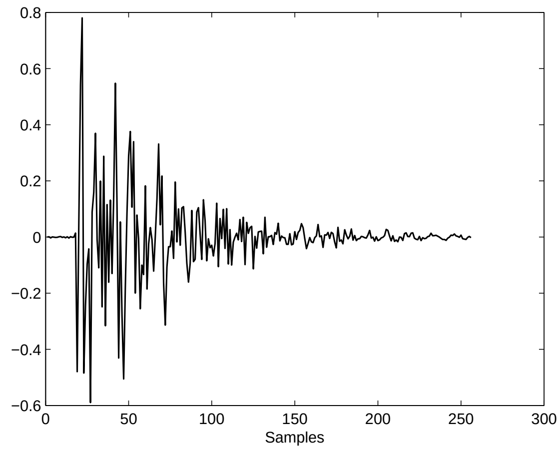

Fig. 2. The impulse response of the car echo path.

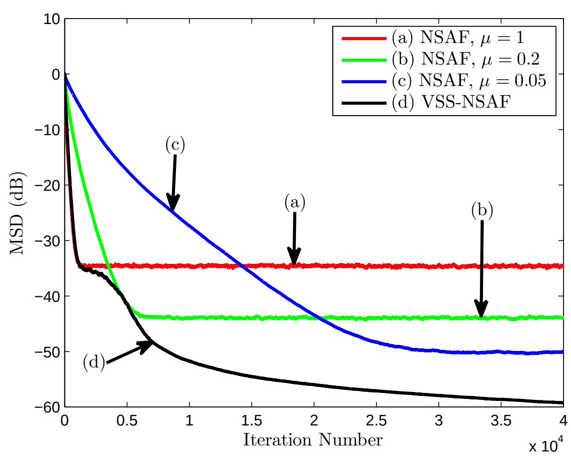

Fig. 3. The MSD curves of VSS-NSAF and conventional NSAF for real unknown impulse response.

VI. Conclusion

In this paper we presented the new variable step-size NSAF algorithm. This algorithm had fast convergence speed and low steady-state MSD compared with ordinary NSAF algorithm. To reduce the computational complexity of VSS-NSAF, the

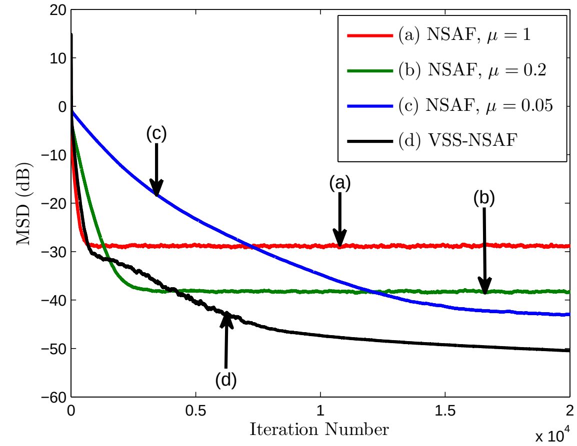

Fig. 4. The MSD curves of VSS-NSAF and conventional NSAF for exponential unknown impulse response.

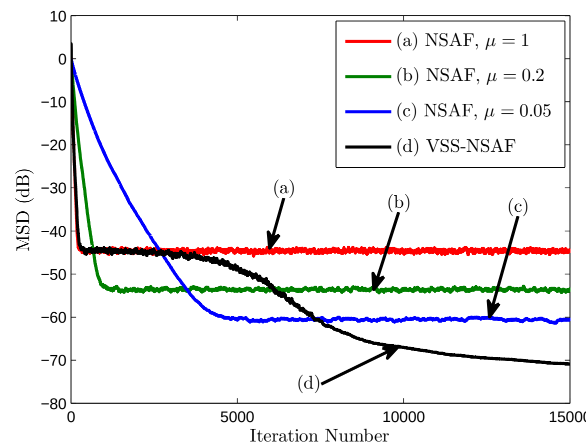

Fig. 5. The MSD curves of VSS-NSAF and conventional NSAF for random unknown impulse response.

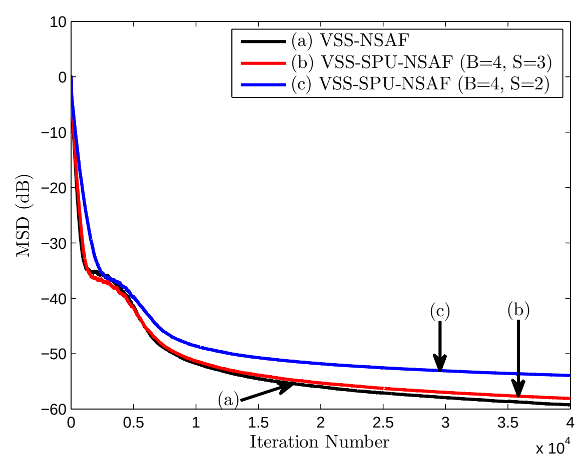

Fig. 6. The MSD curves of VSS-SPU-NSAF with B=4 and S=2, 3, and 4 for real unknown impulse response.

VSS-SPU-NSAF was proposed. We demonstrated the good performance of the presented VSS adaptive algorithms in system identification scenario by several simulation results.

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