

# Nonsingular Green's Functions for Multi-Layer Homogeneous Microstrip Lines

M. Khalaj-Amirhosseini\*

**Abstract:** In this article, three types of green's functions are presented for a narrow strip line (not a thin wire) inside or on a homogeneous dielectric, supposing quasi-TEM dominant mode. These functions have no singularity in contrast to so far presented ones, so that they can be used easily to determine the capacitance matrix of multi-layer and single-layer homogeneous coupled microstrip lines. To obtain the green's functions, the Laplace's equation is solved analytically in Fourier integral or Fourier series expressions, taking into account the boundary conditions including the narrow strip. The validity and accuracy of three presented green's functions are verified by some examples.

**Keywords:** Green's Function, Singularity, Coupled Microstrip Transmission Lines.

## 1 Introduction

The multiconductor coupled microstrip transmission lines are used in RF, microwave and high-speed digital circuits extensively. To analyze these transmission lines, one has to find the capacitance matrix of the structure [1]. The capacitance matrix of this structure is determined using conformal mapping transformations [2, 3], variational methods [4, 5], spectral domain techniques [6, 7], finite difference method [8], solving Laplace's equation [9] and the combination of green's function and method of moments [1, 10-13].

The green's functions presented in the literatures are for an infinitesimally thin wire and have singularity on the wire. In this article, some new green's functions are presented for a narrow strip line (not a thin wire) inside or on a homogeneous dielectric. These green's functions have no singularity and can be used to determine the capacitance matrix of multi-layer and single-layer homogeneous coupled microstrip lines. To obtain these green's functions, the Laplace's equation is solved analytically in Fourier integral or Fourier series expressions, considering the boundary conditions including the narrow strip.

In section 2, open multi-layer microstrip structure is introduced and then a closed form green's function is obtained for open single-layer structure, in section 3. In section 4, shielded multi-layer or single-layer microstrip

structures are introduced and then two green's functions are obtained for both of them. Finally, the validity of three presented green's functions is verified by some examples, in section 5.

## 2 Open Multi-Layer Microstrip Structure

Fig. 1 shows the cross-section of a typical open and inhomogeneous  $N$ -layer microstrip line. The relative electric permittivity and top surface of layers are  $\epsilon_r^{(n)}$  and  $y_n$ , respectively, where  $n = 1, 2, \dots, N$ . There is a narrow strip of width  $\Delta w$  whose center is  $(x', y_n)$ . It is assumed that the principal propagation mode is quasi-TEM. Now, solving the two dimensional Laplace's equation gives the voltage distribution in the  $n$ -th region as follows.

$$\begin{aligned} V_n(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{V}_n(k_x, y) \exp(jk_x x) dk_x \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} (A_n(k_x) \exp(-k_x y) + B_n(k_x) \exp(k_x y)) \exp(jk_x x) dk_x \end{aligned} \quad (1)$$

in which  $\tilde{V}_n(k_x, y)$  is the Fourier transform of the voltage  $V_n(x, y)$ , given by:

$$\begin{aligned} \tilde{V}_n(k_x, y) &= \int_{-\infty}^{\infty} V_n(x, y) \exp(-jk_x x) dk_x \\ &= A_n(k_x) \exp(-k_x y) + B_n(k_x) \exp(k_x y) \end{aligned} \quad (2)$$

Also, the surface charge on the top side of region  $n$  is obtained like this.

Iranian Journal of Electrical & Electronic Engineering, 2013.

Paper first received 3 Mar. 2013 and in revised form 15 May 2013.

\* The Author is with the Department of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran.

E-mail: khalaja@iust.ac.ir.

$$\begin{aligned} \rho_s^{(n)}(x) &= \varepsilon_0 \left( \varepsilon_r^{(n)} \frac{\partial V_n(x, y)}{\partial y} \Big|_{y=y_n} - \varepsilon_r^{(n+1)} \frac{\partial V_{n+1}(x, y)}{\partial y} \Big|_{y=y_n} \right) \\ &= \frac{\varepsilon_0}{2\pi} \int_{-\infty}^{\infty} k_x \begin{pmatrix} A_{n+1}(k_x) \varepsilon_r^{(n+1)} \exp(-k_x y_n) \\ -B_{n+1}(k_x) \varepsilon_r^{(n+1)} \exp(k_x y_n) \\ -A_n(k_x) \varepsilon_r^{(n)} \exp(-k_x y_n) \\ +B_n(k_x) \varepsilon_r^{(n)} \exp(k_x y_n) \end{pmatrix} \exp(jk_x x) dk_x \end{aligned} \quad (3)$$

$$\begin{bmatrix} 1 & -1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ & & \vdots & & \vdots & \vdots & & & \vdots & & & \\ 0 & 0 & \dots & \exp(-k_x y_n) & \exp(k_x y_n) & -\exp(-k_x y_n) & -\exp(k_x y_n) & \dots & 0 & 0 & 0 & \\ 0 & 0 & \dots & -\varepsilon_r^{(n)} \exp(-k_x y_n) & \varepsilon_r^{(n)} \exp(k_x y_n) & \varepsilon_r^{(n+1)} \exp(-k_x y_n) & -\varepsilon_r^{(n+1)} \exp(k_x y_n) & \dots & 0 & 0 & 0 & \\ & & \vdots & & \vdots & \vdots & & & \vdots & & & \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & \exp(-k_x b) & -\exp(k_x b) & 0 & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ \vdots \\ A_n \\ B_n \\ A_{n+1} \\ B_{n+1} \\ \vdots \\ A_{N+1} \\ B_{N+1} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = F_n(k_x, x') \quad (4)$$

in which

$$\begin{aligned} F_n(k_x, x') &= \frac{1}{k_x \varepsilon_0} \int_{-\infty}^{\infty} \rho_s(x, y_n) \exp(-jk_x x) dx \\ &\cong \frac{\rho_l}{k_x \varepsilon_0 \Delta w} \int_{x'-\Delta w/2}^{x'+\Delta w/2} \exp(-jk_x x) dx \\ &= \frac{\rho_l}{k_x \varepsilon_0} \text{sinc}\left(\frac{k_x \Delta w}{2\pi}\right) \exp(-jk_x x') \end{aligned} \quad (5)$$

where  $\rho_l$  is the per-unit-length charge of the strip. After finding the unknown coefficients  $A_n$  and  $B_n$  through Eq. (4), the voltage distributions are obtained using numerical calculating of the integrals in Eq. (1). Finally, the green' function will be in fact  $G(x, y; x', y_n) = V(x, y) / \rho_l$ .

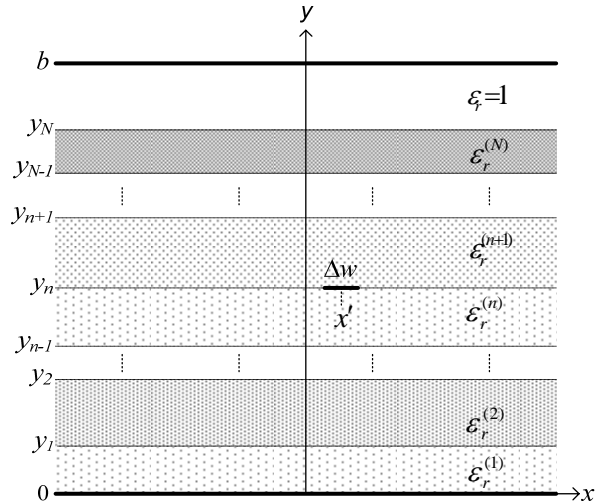
If the electric permittivity of all layers in Fig. 1 are being the same, i.e. a homogeneous dielectric, the Fig. 1 will be reduced to Fig. 2. In fact, there will be only three regions to find potential coefficients. In view of boundary conditions, the Fourier transform of the voltages will be resulted as follows.

$$\tilde{V}(k_x, y) = \begin{cases} A_1 \sinh(k_x y); & 0 \leq y \leq y_n \\ A_2 \sinh(k_x y) + B_2 \cosh(k_x y); & y_n \leq y \leq h \end{cases} \quad (6)$$

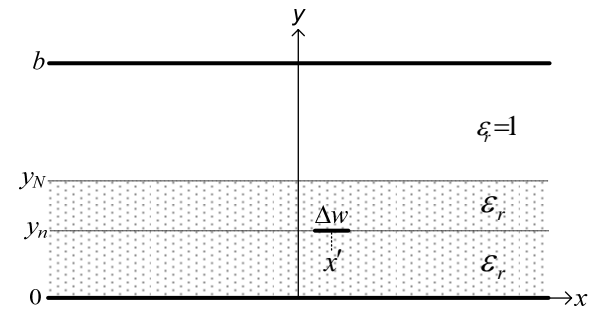
where three desired unknown coefficients are given by

$$\begin{aligned} A_1 &= \frac{\rho_l}{\varepsilon_0 k_x \varepsilon_r} \frac{\text{sinc}\left(\frac{k_x \Delta w}{2\pi}\right) \exp(-jk_x x')}{\coth(k_x h) + \coth(k_x (b-h))} \\ &\times \left[ \frac{1}{\varepsilon_r} \coth(k_x (b-h)) \cosh(k_x y_n) (1 - \tanh(k_x y_n) \coth(k_x h)) \right. \\ &\left. - \sinh(k_x y_n) (1 - \coth(k_x y_n) \coth(k_x h)) \right] \end{aligned} \quad (7)$$

It is known that the voltage must be continuous on the interfaces between two adjacent regions. Besides, the surface charge on the interfaces between two adjacent regions is zero excepting on the strip which is assumed to be uniform. Considering these boundary conditions, the following  $2N+2$  equation system for  $2N+2$  unknown coefficients are obtained.



**Fig. 1** The cross-section of a typical open inhomogeneous multi-layer microstrip lines.



**Fig. 2** The cross-section of a typical open homogeneous multi-layer microstrip lines.

$$A_2 = \frac{-\rho_l}{\varepsilon_0 k_x} \frac{\sinh(k_x y_n) \operatorname{sinc}\left(\frac{k_x \Delta w}{2\pi}\right) \exp(-jk_x x')}{\varepsilon_r \coth(k_x h) + \coth(k_x (b-h))} \quad (8)$$

$$\times \left[ 1 + \frac{1}{\varepsilon_r} \coth(k_x h) \coth(k_x (b-h)) \right]$$

$$B_2 = \frac{\rho_l}{\varepsilon_0 \varepsilon_r k_x} \sinh(k_x y_n) \operatorname{sinc}\left(\frac{k_x \Delta w}{2\pi}\right) \exp(-jk_x x') \quad (9)$$

### 3 Open Single-Layer Microstrip Structure

If all the strips are situated on an open single-layer homogeneous dielectric of height  $h$ , as shown in Fig. 3, there will be only two different regions. In this case, the Fourier transform of the voltage distribution on dielectric layer will be obtained by considering  $y_n = y = h$  in Eqs. (6)-(9) as follows.

$$\tilde{V}(k_x, h) = \tilde{V}(k_x, y) \frac{\sinh(k_x h)}{\sinh(k_x y)}$$

$$= \frac{\rho_l h}{\varepsilon_0} \frac{\operatorname{sinc}\left(\frac{k_x \Delta w}{2\pi}\right) \exp(-jk_x x')}{(k_x h)(\varepsilon_r \coth(k_x h) + \coth(k_x (b-h)))} \quad (10)$$

$$V(x, h) = \frac{1}{\pi} \int_0^\infty \tilde{V}(k_x, h) \cos(k_x x) dk_x \cong \frac{\rho_l h}{\pi \varepsilon_0 (\varepsilon_r + 1)(\beta_2 - \beta_1) \Delta w} \sum_{i=1}^2 \left\{ (-1)^i \frac{\beta_i - \alpha}{\beta_i} \right.$$

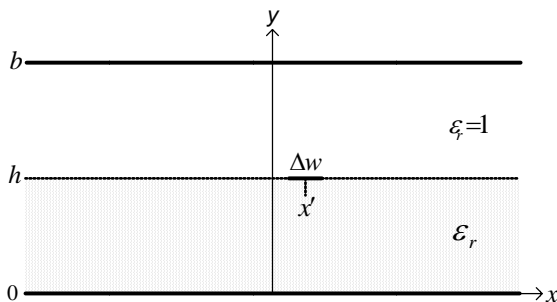
$$\times \left[ Ci\left(\beta_i \frac{|x - x' - \Delta w/2|}{h}\right) \sin\left(\beta_i \frac{x - x' - \Delta w/2}{h}\right) - Ci\left(\beta_i \frac{|x - x' + \Delta w/2|}{h}\right) \sin\left(\beta_i \frac{x - x' + \Delta w/2}{h}\right) \right.$$

$$+ \operatorname{sgn}(x - x' - \Delta w/2) \left( \frac{\pi}{2} - Si\left(\beta_i \frac{|x - x' - \Delta w/2|}{h}\right) \right) \cos\left(\beta_i \frac{x - x' - \Delta w/2}{h}\right)$$

$$- \operatorname{sgn}(x - x' + \Delta w/2) \left( \frac{\pi}{2} - Si\left(\beta_i \frac{|x - x' + \Delta w/2|}{h}\right) \right) \cos\left(\beta_i \frac{x - x' + \Delta w/2}{h}\right)$$

$$\left. \left. + \frac{\pi}{2} (\operatorname{sgn}(x - x' + \Delta w/2) - \operatorname{sgn}(x - x' - \Delta w/2)) \right] \right\} \quad (13)$$

where  $\beta_1$  and  $\beta_2$  are the minus roots of denominator of fraction existing in Eq. (11), given by



**Fig. 3** The cross-section of a typical open homogeneous single-layer microstrip lines.

After some mathematical manipulations, a reasonable approximation can be obtained for the Eq. (10) as is seen in the following relation.

$$\tilde{V}(k_x, h) \cong \frac{\rho_l h}{\varepsilon_0} \times \frac{|k_x h| + \alpha}{(\varepsilon_r + 1) |k_x h|^2 + (\varepsilon_r + h/(b-h)) |k_x h| + \alpha(\varepsilon_r + h/(b-h))}$$

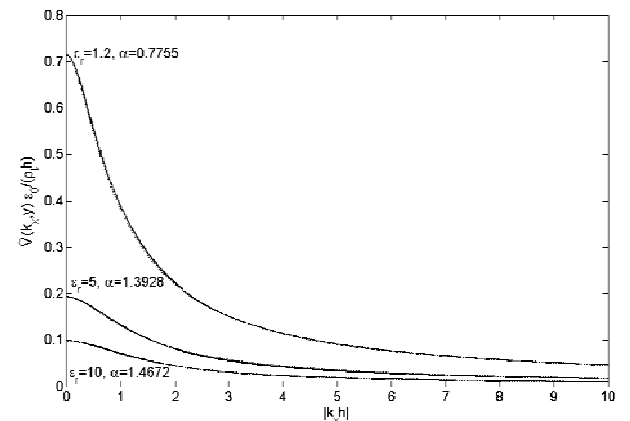
$$\times \operatorname{sinc}\left(\frac{k_x \Delta w}{2\pi}\right) \exp(-jk_x x') \quad (11)$$

In the fractional relation (11),  $\alpha$  is a constant around 1.0 approximated as follows

$$\alpha \cong 1.4 \tanh(\varepsilon_r / 3) + (1.75 - 1.4 \tanh(\varepsilon_r / 3)) h / (b - h) \quad (12)$$

Fig. 4 shows the excellent agreement between two Eqs. (10) and (11) for  $h/(b-h)=1/5$  versus  $k_x h$ .

Substituting the approximate Eq. (11) in Eq. (1) and doing some mathematical manipulations, leads us to voltage distribution on the surface of dielectric as follows.



**Fig. 4** Comparing two relations 10 (---) and 11 (...).

$$\beta_{1,2} = \frac{1}{2(\varepsilon_r + 1)} \left[ (\varepsilon_r + h/(b-h)) \mp \sqrt{(\varepsilon_r + h/(b-h))^2 - 4\alpha(\varepsilon_r + 1)(\varepsilon_r + h/(b-h))} \right] \quad (14)$$

Also, the function  $\text{sgn}(\cdot)$  is the signum function and the functions  $Ci(\cdot)$  and  $Si(\cdot)$  are the sine and cosine integrals, respectively, as follows:

$$Si(x) = \int_0^x \frac{\sin(u)}{u} du \quad (15)$$

$$Ci(x) = \gamma + \ln(x) + \int_0^x \frac{\cos(u) - 1}{u} du \quad (16)$$

where  $\gamma$  is Euler's constant equal to 0.5772... . One can see that the green's function (13) has no singularity at the center of narrow strip.

If the width  $\Delta w$  approaches zero, the voltage function will be reduced to the following relation, by equating the sinc function in Eq. (11) to 1.

$$V(x, h) \cong \frac{\rho_l}{\pi \varepsilon_0 (\varepsilon_r + 1) (\beta_2 - \beta_1)} \sum_{i=1}^2 \left\{ (-1)^{i+1} (\beta_i - \alpha) \times \left[ Ci\left(\beta_i \frac{|x-x'|}{h}\right) \cos\left(\beta_i \frac{x-x'}{h}\right) - \left(\frac{\pi}{2} - Si\left(\beta_i \frac{|x-x'|}{h}\right)\right) \sin\left(\beta_i \frac{|x-x'|}{h}\right) \right] \right\} \quad (17)$$

#### 4 Shielded Microstrip Structures

In numerous applications, the microstrip lines are fully shielded by two lateral walls at  $x = 0$  and  $x = a$ , as shown in Figs. 5-7. The green's function of shielded structures can be readily obtained by considering images of the strip with respect to the left and right walls as seen in Table 1.

For shielded homogeneous multi-layer microstrips, i.e. Fig. 6, also there can be found the voltage distribution by Fourier series expansion and well-known boundary conditions (four walls and voltage and surface charge on  $y = y_n$ ). In view of boundary conditions and performing some mathematical efforts like as Eqs. (4) and (5), the voltage distribution will result in as follows.

$$V(x, y) = \begin{cases} \sum_{m=1}^{\infty} a_m \sinh\left(m\pi \frac{y}{a}\right) \sin\left(m\pi \frac{x}{a}\right); & 0 \leq y \leq y_n \\ \sum_{m=1}^{\infty} \left[ b_m \sinh\left(m\pi \frac{y}{a}\right) + c_m \cosh\left(m\pi \frac{y}{a}\right) \right] \sin\left(m\pi \frac{x}{a}\right); & y_n \leq y \leq h \end{cases} \quad (18)$$

where three desired unknown coefficients are given by:

$$a_m = \frac{2\rho_l}{\varepsilon_0} \frac{\text{sinc}\left(m\frac{\Delta w}{2a}\right) \sinh\left(m\pi \frac{y_n}{a}\right) \sin\left(m\pi \frac{x'}{a}\right)}{m\pi \left( \varepsilon_r \coth\left(m\pi \frac{h}{a}\right) + \coth\left(m\pi \frac{(b-h)}{a}\right) \right)} \times \left[ \frac{1}{\varepsilon_r} \coth\left(m\pi \frac{(b-h)}{a}\right) \coth\left(m\pi \frac{y_n}{a}\right) \times \left( 1 - \tanh\left(m\pi \frac{y_n}{a}\right) \coth\left(m\pi \frac{h}{a}\right) \right) - \left( 1 - \coth\left(m\pi \frac{y_n}{a}\right) \coth\left(m\pi \frac{h}{a}\right) \right) \right] \quad (19)$$

$$b_m = \frac{-2\rho_l}{\varepsilon_0} \frac{\text{sinc}\left(m\frac{\Delta w}{2a}\right) \sinh\left(m\pi \frac{y_n}{a}\right) \sin\left(m\pi \frac{x'}{a}\right)}{m\pi \left( \varepsilon_r \coth\left(m\pi \frac{h}{a}\right) + \coth\left(m\pi \frac{(b-h)}{a}\right) \right)} \times \left[ 1 + \frac{1}{\varepsilon_r} \coth\left(m\pi \frac{h}{a}\right) \coth\left(m\pi \frac{(b-h)}{a}\right) \right] \quad (20)$$

$$c_m = \frac{2\rho_l}{\varepsilon_0 \varepsilon_r m\pi} \text{sinc}\left(m\frac{\Delta w}{2a}\right) \sinh\left(m\pi \frac{y_n}{a}\right) \sin\left(m\pi \frac{x'}{a}\right) \quad (21)$$

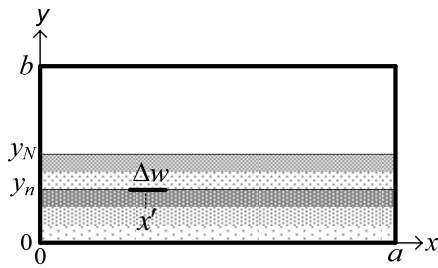
The voltage distribution of shielded homogeneous single-layer microstrips, i.e. Fig. 7, can be found from Eqs. (18) and (19) assuming  $y = y_n = h$  as the following relation.

$$V(x, h) = \frac{2\rho_l}{\varepsilon_0} \times \sum_{n=1}^{\infty} \frac{\text{sinc}\left(m\frac{\Delta w}{2a}\right) \sin\left(m\pi \frac{x'}{a}\right)}{m\pi \left( \varepsilon_r \coth\left(m\pi \frac{h}{a}\right) + \coth\left(m\pi \frac{(b-h)}{a}\right) \right)} \sin\left(m\pi \frac{x}{a}\right) \quad (22)$$

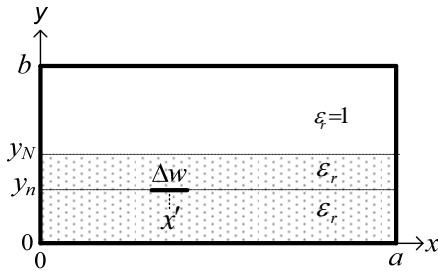
One can see that the green's functions (18) and (22) have no singularity at the center of narrow strip. Of course, the summations in Eqs. (18) and (22) can be truncated to  $M$  terms so as the last term is being very smaller than the first term. This gives us the following condition for Eq. (22).

**Table 1** The location and sign of images of the shielded strip with respect to the left and right walls.

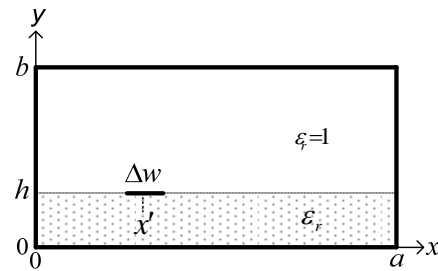
...	$-4a-x'$	$4a+x'$	$-2a-x'$	$-2a+x'$	$-x'$	$x'$	$2a-x'$	$2a+x'$	$4a-x'$	$4a+x'$	...
-+	-	+	-	+	-	+	-	+	-	+	-+



**Fig. 5** The cross-section of a typical shielded inhomogeneous multi-layer microstrip lines.



**Fig. 6** The cross-section of a typical shielded homogeneous multi-layer microstrip lines.



**Fig. 7** The cross-section of a typical shielded homogeneous single-layer microstrip lines.

$$M \gg \sqrt{\frac{\epsilon_r \coth\left(\frac{\pi}{a}h\right) + \coth\left(\frac{\pi}{a}(b-h)\right)}{(\epsilon_r + 1) \sin\left(\pi \frac{\Delta w}{2a}\right) \sin\left(\pi \frac{x'}{a}\right) \sin\left(\pi \frac{x}{a}\right)}} \quad (23)$$

## 5 Examples and Results

In this section we investigate the validity of three presented green's functions (13), (18) and (22) by some examples. To get self and mutual capacitances, the lines are subdivided to  $K$  equi-potential parts of one volt potential and then the Method of Moment is used.

As a first example, consider an open single-layer microstrip transmission line of width and thickness  $w$  and  $h$ , respectively. Table 2 compares the characteristic impedance  $Z_0$  of the line obtained through Eq. (13) considering  $b/h=51$  and  $K=12$  with that of obtained in the references. The characteristic impedance can be obtained the following relation.

$$Z_0 = \frac{1}{c\sqrt{C_1 C}} \quad (24)$$

where  $c$  is the velocity of the light and  $C$  and  $C_1$  are the capacitance of the microstrip line with and without substrate, respectively.

For the second example, consider two identical coupled lines of width  $w$  and gap  $s$  on an open single-layer dielectric of  $\epsilon_r=1$  and  $b=2h$ . Table 3 compares the even and odd mode capacitances of the lines obtained through Eq. (13) considering  $K=20$  with that of obtained in the reference [9].

For the third example, consider two identical coupled lines of width  $w$  lying at points  $(x_1=6.5, y_1=1)$  and  $(x_2=11.5, y_2=0.5)$  in a shielded homogeneous structure of  $\epsilon_r=1$ ,  $a=18$  and  $b=5$ . Table 4 compares the even and odd mode capacitances of this two-layer structure obtained through Eq. (18) considering  $K=10$  and  $M=300$  with that of obtained in the reference [13].

For the fourth example, consider eight identical coupled lines of width  $w = 1/16h$  and gap  $s = 1/16h$  on the middle of a shielded single-layer structure of  $\epsilon_r=12.9$ ,  $a = 175/16h$  and  $b = 7.25h$ . Table 5 compares the 20 different coefficients of the capacitance matrix of the lines obtained through Eq. (22) considering  $K=20$  and  $M=8780$  with that of obtained in the reference [9].

One sees from tables 2-5 that there is an excellent agreement between the results obtained from explicit form green's functions expressed by Eqs. (13, 18) and (22) with those reported in reliable references. Therefore one can conclude the validity of these three presented relations.

**Table 2** An open single-layer microstrip transmission line (Example 1).

$w/h$	$\epsilon_r$	$C$ [pF/m] Eq. (13)	$C_1$ [pF/m] Eq. (13)	$Z_0$ [ $\Omega$ ] Eq. (13)	$Z_0$ [ $\Omega$ ] (Refs.)
0.4	6	71.18	18.40	91.68	91.17 [13]
0.4	9.5	110.30	18.40	73.99	73.70 [13]
1	6	108.01	25.90	63.03	62.71 [13]
1.025	8.875	159.54	26.31	51.45	50.00 [8]
3	10	356.22	45.97	26.05	25.47 [12]
10	6	579.87	108.40	13.30	13.34 [13]
10	9.5	912.91	108.40	10.60	10.57 [13]

**Table 3** An open single-layer microstrip coupled transmission lines (Example 2).

$w/h$	$s/h$	$C_{\text{even}}$ [pF/m] Eq. (13)	$C_{\text{even}}$ [pF/m] [9]	$C_{\text{odd}}$ [pF/m] Eq. (13)	$C_{\text{odd}}$ [pF/m] [9]
0.1	0.1	9.48	9.59	26.38	27.74
0.1	1	12.75	12.95	15.08	15.56
1	0.1	26.58	26.33	51.68	54.58
1	1	31.01	31.09	35.86	35.73

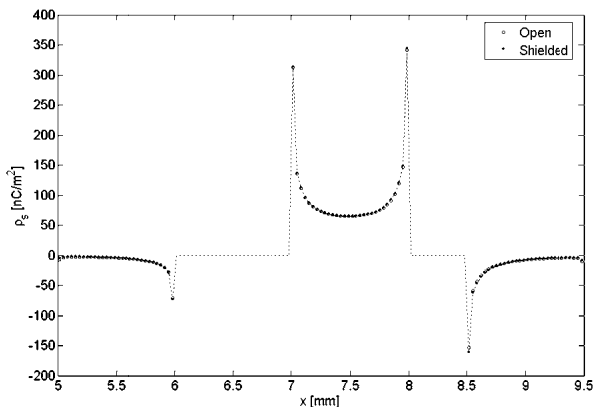
**Table 4** A shielded two-layer microstrip coupled transmission lines (Example 3).

	Eq. (18)	[13]
$C_{11}$ [pF/m]	49.33	49.51
$C_{22}$ [pF/m]	77.08	77.12
$C_{21}$ [pF/m]	-1.826	-1.835

**Table 5** A shielded single-layer microstrip coupled transmission lines in [pF/m] (Example 4).

	$C_{11}$	$C_{12}$	$C_{13}$	$C_{14}$	$C_{15}$	$C_{16}$	$C_{17}$	$C_{18}$	$C_{22}$	$C_{23}$
Eq. (22)	126.64	-57.60	-12.98	-5.711	-3.108	-1.895	-1.286	-1.218	153.52	-51.71
[9]	127.77	-58.45	-13.02	-5.720	-3.108	-1.892	-1.280	-1.211	155.23	-52.52
	$C_{24}$	$C_{25}$	$C_{26}$	$C_{27}$	$C_{33}$	$C_{34}$	$C_{35}$	$C_{36}$	$C_{44}$	$C_{45}$
Eq. (22)	-10.42	-4.348	-2.315	-1.445	154.83	-51.15	-10.14	-4.212	155.06	-51.05
[9]	-10.45	-4.349	-2.312	-1.440	156.55	-51.95	-10.17	-4.213	156.78	-51.86

It is noteworthy that the values of green's functions of the open and shielded structures are close together when the distance between the strip and lateral walls are large relative to height of strip. In this case, one can use the green's functions of shielded structures for open structures and vice versa. Fig. 8 shows the surface charge on the dielectric of two single-layer structures. The structures have three identical strips of width  $w = 1$  on a dielectric of  $\epsilon_r = 5$  and  $h = 1$ . The strips lie at points  $x = 5.5, 7.5$  and  $9$  and are connected to potentials  $0, 1$  and  $0$  volt, respectively. The height of the structures is  $b = 11$  and the width of the shielded structure is  $a = 14.5$ . One can see that the charge distribution of two structures are almost the same, whereas the first and third strips have a distance of only  $5h$  from two lateral walls.



**Fig. 8** The surface charge on the dielectric of two single-layer open and shielded structures.

## 6. Conclusions

Three types of green's functions are presented for a narrow strip line (not a thin wire) inside or on a homogeneous dielectric. One closed form green's function for open single-layer microstrip structure and two series form green's functions for both multi-layer and single-layer homogeneous microstrip structures are introduced. These functions have no singularity and have a high accuracy. The validity and accuracy of three presented green's functions are verified by some examples. Also, it was shown that the green's functions of open and shielded structures may be used in place of each other at some simple conditions.

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**Mohammad Khalaj-Amirhosseini** was born in Tehran, Iran in 1969. He received his B.Sc., M.Sc. and Ph.D. degrees from Iran University of Science and Technology (IUST) in 1992, 1994 and 1998 respectively, all in Electrical Engineering. He is currently a Professor at College of Electrical Engineering of IUST. His scientific fields of interest are electromagnetic direct and inverse problems including microwaves, antennas and electromagnetic compatibility.