

# Least Squares Techniques for GPS Receivers Positioning Filter using Pseudo-range and Carrier Phase Measurements

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**Abstract:** In present study, using Least Squares (LS) method, we determine the position smoothing in GPS single-frequency receiver by means of pseudo-range and carrier phase measurements. The application of pseudo-range or carrier phase measurements in GPS receiver positioning separately can lead to defects. By means of pseudo-range data, we have position with less precision and more distortion. By use of carrier phase data, we do not have absolute position and just dislocation is available, but the accuracy is high. In present research, we have combined pseudo-range and carrier phase data using LS method in order to determine GPS receiver's position smoothing. The results of comparison by LS method show less RMS error, less calculation volume and more smoother in using carrier phase-pseudo-range data together relative to pseudo-range data in isolation.

**Keywords:** Carrier Phase, GPS, Least Squares, Position Smoothing, Pseudo-range.

## 1 Introduction

Global Positioning System (GPS) is a satellite tracking and strategic system. Their services are available in full time in any climate situation and in any area of the terrestrial globe. The GPS constellation consists of at least 24 satellites [1]. As GPS satellites moving in the space, they send two signals. L1 signal is modulated with the Precise code (P-code) and the Clear/Acquisition code (C/A code) while L2 signal is modulated with the only P-code [2].

The basic step in GPS is knowing distance rate from satellite. The precision of determined position by GPS depend upon two factors: 1) the selected satellite compound (depend on how many satellites are placed above observer's sight horizon) and 2) measurement errors which can be divided in to three categories: satellite errors, receiver errors and observation errors. Satellite errors involve error in satellite orbit and error in satellite clock. These errors include natural and intentional errors. Receiver error is as a result of receiver clock, constants length limitation, electrical interference, low number of channels of some of receivers and so on. Observation errors involve errors related to signal diffusion and errors related to observation method [3].

The most important error in GPS pseudo-range is ionospheric delay error. In ionospheric layer, free electrons are released as a result of gas molecules stimulation by sun rays. GPS signals passing through this layer can result in signal passing rate change and its dispersion in this layer. This ionospheric delay depend on receiver type, the error is in the range of 50m to almost 150m [4,5].

Troposphere is defined as atmosphere's low levels which involve water steam particles. GPS signals by moving in these levels suffer from delay tropospheric that is a function of height, pressure, temperature and steam quantity in this layer. The error of this delay is almost 2m to 20m and depends upon satellite position and a troposphere layer characteristic is variable. Considering that GPS signals delay is independent of frequency in this environment, we can not eliminate this error by integrating two-frequency observations. In order to decline error effect, we should use of mathematical modeling in this layer. This is important to note that this model influences on both code and carrier. Differential positioning is used to decline and eliminates troposphere error [4,5].

We need a very precise clock for measurement using GPS. GPS satellites send specific coded message diffusion beginning time. Receiver measures any signal receiving precise time and calculates its distance to satellite. Of course, this distance is calculated by regard to complete a synchronized clock is calculated. Any difference in time is multiplied by velocity of light and result in receiver error restricted area [3].

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At the present time, we can calculate satellite position with almost 20m precision using orbital data that is provided by satellite message that by current errors can reach to 80m, but in determining partial position satellites orbit error effects by means of differential observations of two or more stations of a satellite simultaneously and out slanderingly is decreased or eliminated [6].

Any error in receiver clock results in imprecision in measuring distance. Because atomic clocks are very expensive and heavy and temperature and environmental situation around them should be controlled. Equipping receivers with atomic clocks is not possible.

Multi-path error is created as a result of receiving GPS signals by receiver from an indirect path, following reflection of objects surface and reflective surfaces around receiver. This error is declined by selecting suitable receiver antenna designation and station location. The major error source is multi-path effect for GPS precise positioning near distance difference position [7].

Receiver calculates its distance to any satellite with available error. Any distance which computed in this method, has high error, but is coordinated. In pseudo-range, we have receiver position, but precision is low. The pseudo-range is equal to the sum of the code phase and an unknown integer number of C/A code epochs multiplied by the speed of light [8]. In carrier phase, we have not location and dislocation is computed. The use of frequency concept carrier phase in GPS receiver operate in this way that system first begins to sending Pseudo Random Code (PRC) and for measuring and correspondence of cods it uses of carrier phase frequency. The carrier phase signals frequencies are very more than PRC codes frequency and their pulses are very closer to each other and thus they are more precise for correspondence. So the error in pseudo-range is more than error in code. Receiver error in pseudo-range is a bout 25m and in carrier phase is about 1m. The measurements of carrier phase can lead to more efficiency compared to code measurement [9]. Table 1 shows GPS pseudo-range error source.

For measuring pseudo-range and carrier phase in receiver positioning, we can use of Least Squares (LS) method by which we can access the minimum error. There are different kinds of LS methods: simple LS [10,11], Auto-covariance Least Squares (ALS) [12], Parallel Least Squares (PLS) [13], Non-linear Least Squares (NLS) [14], Consecutive Least Squares (CLS) [15], Weight Least Squares (WLS) [16], Moving Least Squares (MLS) [16] and Iterative Least Squares (ILS) [17].

In present study, we are going to compound carrier phase and pseudo-range data and express a new algorithm by LS method in which receiver can receive both phase and code data and simultaneously uses them. In the second section, we refer to how positioning

receiver by LS method using pseudo-range data, first order difference of pseudo-range data, first order difference of carrier phase data, second order difference of carrier phase data, and by use of proposed method in this article i.e., first order difference between pseudo-range and carrier phase data. In section of five, we refer to any method and amount of Root Mean Square (RMS), assessment results of proposed method are presented and comparison between five referred methods is carried out. At the end, we present ultimate conclusion.

## 2 GPS Pseudo-range Measurement

Let the time of transmission as marked by the satellite is  $t^j$ , time of reception as determined by the receiver is  $t_i^j$ , error clock satellite is  $dt^j$ , and error clock receiver is  $dt_i^j$ . The pseudo-range equation is determined as:

$$r_i^j = ((t_i^j + dt_i^j) - (t^j + dt^j))c + e_r = (t_i^j - t^j)c + (dt_i^j - dt^j)c + e_r \quad (1)$$

Where  $e_r$  is unmodelled errors. The ionospheric and tropospheric delays are introduced, since the propagation velocity is not always  $c$ . Multi-path errors and satellite orbital errors should also consider for every satellite as follows:

$$P_i^j = r + dt_i^j c - dt^j c + T_r + I_r + M_r + e^j \quad (2)$$

Satellite orbital errors are transmitted in the navigation message, but multi-path errors depend on the location of the user receiver. The pseudo-range equation without these errors can be written as:

**Table 1** GPS pseudo-range error source.

Error source	Value [m]
Satellite clock and navigation	3
Satellite perturbations	1
Satellite ephemeris predication model	4.2
Receiver noise	7.5
Ionospheric noise	5-10
Tropospheric delay	2
Multi-path	1.2
Other	1.2
User estimated range error	10.8-13.9

$$P_i^j = r + dt_i^c - dt_i^j c + T_r + I_r \quad (3)$$

Where  $r$  represents the true range of the satellite from the user receiver the quantity which we would ideally like to measure. Let the  $x_i = (x_i, y_i, z_i)$  be receiver position and  $x^j = (x^j, y^j, z^j)$  be satellite position,  $r$  in Eq. (3) is calculated as Eq. (4):

$$r^j = \sqrt{(x^j - x_i)^2 + (y^j - y_i)^2 + (z^j - z_i)^2} \quad (4)$$

### 3 GPS Carrier Phase Measurement

A measurement much more precise than that of code phase is the phase of the carrier received from a satellite. The carrier phase measurement is the difference between the phases of the receiver generated carrier signal and the carrier received from a satellite at the instant of the measurement. The carrier phase data will be used to estimate the difference between the receiver and satellite clock. Received signal phase at each moment is fitted with phase in transmission time in term of the transit time of the signal [18,19].

The carrier phase measurement is ambiguous and indirect measurement of the signal transit time. The carrier phase measurement would remain fixed at a fraction of a cycle and the distance between a satellite and the receiver would be an unknown number of whole cycles plus a measured fractional cycle [20].

The carrier phase measurement contains no information regarding the number of whole cycles (integer ambiguity), but the carrier phase measurement accuracy is more accurate than the pseudo-range [21-23]. In the absence of clock biases and measurement

error, the carrier phase measurement  $f_i^j$  in unit of cycles can be written as [24,25]:

$$f_i^j(t) = f_i^j(t) - f_i^j(t-t) + N_i^j \quad (5)$$

So carrier phase can be expressed as:

$$lf_i^j(t) = lf_i^j(t) - lf_i^j(t-t) + lN_i^j \Rightarrow \quad (6)$$

$$\Phi_i^j = r + dt_i^c - dt_i^j c + lN_i^j$$

The carrier phase equation in meters, with all the biases included, is written as Eq. (7):

$$\Phi_i^j = r + dt_i^c - dt_i^j c + T_r - I_r + M_r + lN_i^j + e_f \quad (7)$$

Where  $r$  is distance between receiver and satellite,  $N_i^j$  is the integer ambiguity and  $l$  is the wavelength of the carrier signal (L1 is equal to 19cm and L2 is equal to 24cm).

### 4 Position Estimation using LS

Follow up determining receiver position using LS method by means of pseudo-range and carrier phase data are assessed.

#### 4.1 Position Estimation using LS by means of GPS Pseudo-range Data

Assuming that the initial co-ordinates of the receiver ( $x_0, y_0, z_0$ ) (assumed as the center of earth) are known, then the actual receiver co-ordination can be expressed as Eq. (8):

$$x_i = x_0 + \Delta x_i, \quad y_i = y_0 + \Delta y_i, \quad z_i = z_0 + \Delta z_i \quad (8)$$

In above equation, ( $\Delta x_i, \Delta y_i, \Delta z_i$ ) are new unknowns.

( $x_i, y_i, z_i$ ) can be written in Eq. (9):

$$f(x_i, y_i, z_i) = f(x_0 + \Delta x_i, y_0 + \Delta y_i, z_0 + \Delta z_i) \quad (9)$$

Using Taylor's series  $f(x_0 + \Delta x_i, y_0 + \Delta y_i, z_0 + \Delta z_i)$  can be expanded as:

$$f(x_i, y_i, z_i) = f(x_0, y_0, z_0) + \frac{\partial f(x_0, y_0, z_0)}{\partial x_0} \Delta x_i + \quad (10)$$

$$\frac{\partial f(x_0, y_0, z_0)}{\partial y_0} \Delta y_i + \frac{\partial f(x_0, y_0, z_0)}{\partial z_0} \Delta z_i + \frac{1}{2!} \frac{\partial^2 f}{\partial x^2} + L$$

We can ignore Taylor's expansion after linear expressions. Linear expression coefficients are obtained by Eq. (11):

$$\frac{\partial f(x_0, y_0, z_0)}{\partial x_0} = -\frac{x_j - x_0}{r_0^j}, \quad \frac{\partial f(x_0, y_0, z_0)}{\partial y_0} = -\frac{y_j - y_0}{r_0^j}, \quad (11)$$

$$\frac{\partial f(x_0, y_0, z_0)}{\partial z_0} = -\frac{z_j - z_0}{r_0^j}$$

We can rewrite pseudo-range equation irrespective of unmodeled errors in from of Eq. (12):

$$P^j = r_0^j - \frac{x_j - x_0}{r_0^j} \Delta x_i - \frac{y_j - y_0}{r_0^j} \Delta y_i - \frac{z_j - z_0}{r_0^j} \Delta z_i + dt_i^c \quad (12)$$

Eq. (12) can be simplified as:

$$l^j = a_{x_i}^j \Delta x_i + a_{y_i}^j \Delta y_i + a_{z_i}^j \Delta z_i + c \Delta t_i \quad (13)$$

Where:

$$l^j = P^j - r_0^j, \quad a_{x_i}^j = -\frac{x_j - x_0}{r_0^j}, \quad (14)$$

$$a_{y_i}^j = -\frac{y_j - y_0}{r_0^j}, \quad a_{z_i}^j = -\frac{z_j - z_0}{r_0^j}$$

And its matrix presentation can be expressed as Eq. (15):

$$\underline{L} = \underline{A} \underline{X} \quad (15)$$

or:

$$\begin{bmatrix} l^1 \\ l^2 \\ l^3 \\ \mathbf{M} \\ l^J \end{bmatrix} = \begin{bmatrix} a_{x_i}^1 & a_{y_i}^1 & a_{z_i}^1 & c \\ a_{x_i}^2 & a_{y_i}^2 & a_{z_i}^2 & c \\ a_{x_i}^3 & a_{y_i}^3 & a_{z_i}^3 & c \\ \mathbf{M} & \mathbf{M} & \mathbf{M} & \mathbf{M} \\ a_{x_i}^J & a_{y_i}^J & a_{z_i}^J & c \end{bmatrix} \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ \Delta t_i \end{bmatrix} \quad (16)$$

Where  $\mathbf{L}$  is a vector of  $n$  observations (the least numbers of available satellite should be four),  $\mathbf{X}$  is a vector of four unknowns and  $A$  is a design matrix with  $n \times 4$  dimensions. The solution of Eq. (16) by means of LS method can be obtained in Eq. (17):

$$\mathbf{X} = (A^T A)^{-1} A^T \mathbf{L} \quad (17)$$

Which  $(\Delta x_i, \Delta y_i, \Delta z_i, \Delta t_i)$  are found. The receiver coordinations are obtained by Eq. (18):

$$x_i = x_0 + \Delta x_i, \quad y_i = y_0 + \Delta y_i, \quad z_i = z_0 + \Delta z_i \quad (18)$$

#### 4.2 Position Estimation using LS by means of First Order Difference of GPS Pseudo-range Data

We can define a linear relationship with regard to receiver and satellite position by means of pseudo-range data first order difference between satellites  $i$  and  $j$ . By considering first order differential, we have:

$$\Delta P^{ij} = \Delta P^j - \Delta P^i \quad (19)$$

$$A = \left[ \begin{array}{cc} \frac{\Delta x^i}{r_0^i} - \frac{\Delta x^j}{r_0^j} & \frac{\Delta y^i}{r_0^i} - \frac{\Delta y^j}{r_0^j} & \frac{\Delta z^i}{r_0^i} - \frac{\Delta z^j}{r_0^j} \\ \frac{\Delta x^j}{r_0^j} - \frac{\Delta x^i}{r_0^i} & \frac{\Delta y^j}{r_0^j} - \frac{\Delta y^i}{r_0^i} & \frac{\Delta z^j}{r_0^j} - \frac{\Delta z^i}{r_0^i} \end{array} \right]$$

Where  $\Delta x^i = x_i - x_0$  imply difference between  $X$ -th components of  $i$ -th satellite and receiver and has similar expressions for another differential elements.

#### 4.3 Position Estimation using LS by means of First Order Difference of GPS Carrier Phase Data

The multi-path error and GPS receiver noise of the carrier phase signal are much smaller than those of the pseudo-range. Carrier phase measurement is much more accurate than the code measurement. The pseudo-ranges provide coarse position estimations and the relative carrier phase measurements provide precise position change estimations [26-28]. The observation equation for carrier phase with out the environmental errors is equal to:

$$\Phi_i^j = r_i^j + dt_i^j c - dt^j c + IN_i^j \quad (20)$$

The carrier phase observation equation by means of Taylor's series in a simple way can be written as:

$$\Phi_i^j = r_0^j - \frac{x_j - x_0}{r_0^j} \Delta x_i - \frac{y_j - y_0}{r_0^j} \Delta y_i - \frac{z_j - z_0}{r_0^j} \Delta z_i \quad (21)$$

$$+ \Delta t_i^j c + IN_i^j$$

Eq. (21) can be simplified as:

$$l^j = a_{x_i}^j \Delta x_i + a_{y_i}^j \Delta y_i + a_{z_i}^j \Delta z_i + c \Delta t_i^j + IN_i^j \quad (22)$$

Where:

$$l^j = \Phi^j - r_0^j, a_{x_i}^j = -\frac{x_j - x_0}{r_0^j}, a_{y_i}^j = -\frac{y_j - y_0}{r_0^j}, a_{z_i}^j = -\frac{z_j - z_0}{r_0^j} \quad (23)$$

Matrix presentation of Eq. (22) is in the form of Eq. (24):

$$\begin{bmatrix} l^1 \\ l^2 \\ l^3 \\ \mathbf{M} \\ l^J \end{bmatrix} = \begin{bmatrix} a_{x_i}^1 & a_{y_i}^1 & a_{z_i}^1 & c & l \\ a_{x_i}^2 & a_{y_i}^2 & a_{z_i}^2 & c & l \\ a_{x_i}^3 & a_{y_i}^3 & a_{z_i}^3 & c & l \\ \mathbf{M} & \mathbf{M} & \mathbf{M} & \mathbf{M} & \mathbf{M} \\ a_{x_i}^J & a_{y_i}^J & a_{z_i}^J & c & l \end{bmatrix} \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ \Delta t_i \end{bmatrix} \quad (24)$$

Where the number of unknowns are  $J + 4$ , comparing of  $J$  integer ambiguities  $N_i^j = 1, \mathbf{L}, J$ ,  $\Delta x_i, \Delta y_i, \Delta z_i$  and  $dt_i$ .

In order to estimate phase ambiguity number from accounts, for a single satellite visible in two consecutive time epochs, phase ambiguity number can be eliminated completely by carrier phase differences.

$$\Phi^j(t_1) - \Phi^j(t_2) = r_0^j(t_1) - r_0^j(t_2) + \quad (25)$$

$$[dt_i(t_1) - dt_i(t_2)]c + I(N_i^j - N_i^j)$$

The above matrix for the difference in carrier phase for two epochs can be written as:

$$\mathbf{L} = A\mathbf{X} \quad (26)$$

Where:

$$\mathbf{r} = \begin{bmatrix} l^1(t_1) - l^1(t_2) \\ l^2(t_1) - l^2(t_2) \\ l^3(t_1) - l^3(t_2) \\ \mathbf{M} \\ l^J(t_1) - l^J(t_2) \end{bmatrix} \quad (27)$$

$$A = \begin{bmatrix} a_{x_i}^1(t_1) - a_{x_i}^1(t_2) & a_{y_i}^1(t_1) - a_{y_i}^1(t_2) & a_{z_i}^1(t_1) - a_{z_i}^1(t_2) & c \\ a_{x_i}^2(t_1) - a_{x_i}^2(t_2) & a_{y_i}^2(t_1) - a_{y_i}^2(t_2) & a_{z_i}^2(t_1) - a_{z_i}^2(t_2) & c \\ a_{x_i}^3(t_1) - a_{x_i}^3(t_2) & a_{y_i}^3(t_1) - a_{y_i}^3(t_2) & a_{z_i}^3(t_1) - a_{z_i}^3(t_2) & c \\ \mathbf{M} & \mathbf{M} & \mathbf{M} & \mathbf{M} \\ a_{x_i}^J(t_1) - a_{x_i}^J(t_2) & a_{y_i}^J(t_1) - a_{y_i}^J(t_2) & a_{z_i}^J(t_1) - a_{z_i}^J(t_2) & c \end{bmatrix} \quad (28)$$

$$\mathbf{r} = \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ dt_i(t_1) - dt_i(t_2) \end{bmatrix} \quad (29)$$

$\Delta x_i, \Delta y_i, \Delta z_i$  and  $dt_i(t_1) - dt_i(t_2)$  in parameters of above matrix are accounted.

Delta-phase measurements can be used to estimate average velocity because the delta phase is clearly treated as a position difference observable. The problem with this method is that initial coordinates of the receiver must be available.

#### 4.4 Position Estimation using LS by means of Second Order Difference of GPS Carrier Phase

By means of carrier phase second order difference which is differentiated two times relative to time and satellite, change in produced receiver clock can be estimated. According to the observed equity in relation to phase is:

$$\Delta \nabla J_{t1t2}^{ij} = \Delta J_{t1t2}^j - \Delta J_{t1t2}^i \quad (30)$$

Matrix relationship for second order difference state between carrier phase relative to consequent time and satellite can be written as:

$$\dot{\mathbf{L}} = \mathbf{A} \dot{\mathbf{X}} \quad (31)$$

Where:

$$\mathbf{r} \dot{\mathbf{L}} = \begin{bmatrix} l_{t1t2}^2 - l_{t1t2}^1 \\ l_{t1t2}^3 - l_{t1t2}^2 \\ l_{t1t2}^4 - l_{t1t2}^3 \\ \mathbf{M} \\ l_{t1t2}^J - l_{t1t2}^{J-1} \end{bmatrix} \quad (32)$$

$$\mathbf{A} = \begin{bmatrix} [a_{x_i}^2(t1) - a_{x_i}^1(t1)] - [a_{x_i}^2(t2) - a_{x_i}^1(t2)] & \mathbf{M} & \mathbf{M} \\ [a_{x_i}^3(t1) - a_{x_i}^2(t1)] - [a_{x_i}^3(t2) - a_{x_i}^2(t2)] & \mathbf{M} & \mathbf{M} \\ \mathbf{M} & \mathbf{M} & \mathbf{M} \\ [a_{x_i}^J(t1) - a_{x_i}^{J-1}(t1)] - [a_{x_i}^J(t2) - a_{x_i}^{J-1}(t2)] & \mathbf{M} & \mathbf{M} \end{bmatrix} \quad (33)$$

$$\mathbf{r} \dot{\mathbf{X}} = \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{bmatrix} \quad (34)$$

In above method, we can consider one of the entrance satellites as main satellites (master) which has higher visual angle than others and we can use other satellite's (slave) carrier phase data difference in algorithm with regard to this satellite.

#### 4.5 Position Estimation using LS by means of First Order Difference of GPS Pseudo-range and Carrier Phase Data

In this method of first order difference carrier phase which discussed in above part and selected satellite pseudo-range are used simultaneously. Therefore, the carrier-phase and pseudo-range combination uses [29]. One of the advantages of this approach with regard to above method is low account volume and at the same time more precise in accounting receiver position.

For a single frequency receiver, the pseudo-range and the carrier phase equations can be described as follows. The tropospheric delay can be neglected for short baseline lengths. Let:

$$g = r + dt_i c - dt^j c \quad (35)$$

The pseudo-range and the carrier phase equations can be simplified as:

$$P = g + I_r \quad (36)$$

$$\Phi = g - I_r + IN \quad (37)$$

The change in the code and phase with time are given by:

$$\Delta P = \Delta g + \Delta I_r \quad (38)$$

$$\Delta \Phi = \Delta g - \Delta I_r \quad (39)$$

$\Delta I_r$  is small if the two measurement epochs are close

together, so the ionospheric delay can be disregarded for time differencing. An estimate of the new pseudo-range from carrier phase is:

$$P(t_1) = P(t_0) + (\Phi(t_1) - \Phi(t_0)) \quad (40)$$

$$\dot{\mathbf{L}} = \mathbf{A} \dot{\mathbf{X}} \quad (41)$$

Where:

$$\mathbf{r} \dot{\mathbf{L}} = \begin{bmatrix} l_{t1t2}^2 - l_{t1t2}^1 \\ l_{t1t2}^3 - l_{t1t2}^2 \\ l_{t1t2}^4 - l_{t1t2}^3 \\ \mathbf{M} \\ l_{t1t2}^J - l_{t1t2}^{J-1} \end{bmatrix} \quad (42)$$

$$P^J(t1) = [j^J(t1) - j^J(t2)] + P^J(t2)$$

$$L = P^J(t1) = [j^J(t1) - j^J(t2) - r_0^J(t1) + r_0^J(t2)] + (43)$$

$$[P^J(t2) - r_0^J(t2)] = \nabla j_{t1t2}^J - r_0^J(t1) + P^J(t2)$$

$$\mathbf{A} = \begin{bmatrix} a_{x_i}^1(t1) & a_{y_i}^1(t1) & a_{z_i}^1(t1) \\ a_{x_i}^2(t1) & a_{y_i}^2(t1) & a_{z_i}^2(t1) \\ \mathbf{M} & \mathbf{M} & \mathbf{M} \\ a_{x_i}^J(t1) & a_{y_i}^J(t1) & a_{z_i}^J(t1) \end{bmatrix} \quad (44)$$

$$\mathbf{r} \dot{\mathbf{X}} = \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{bmatrix} \quad (45)$$

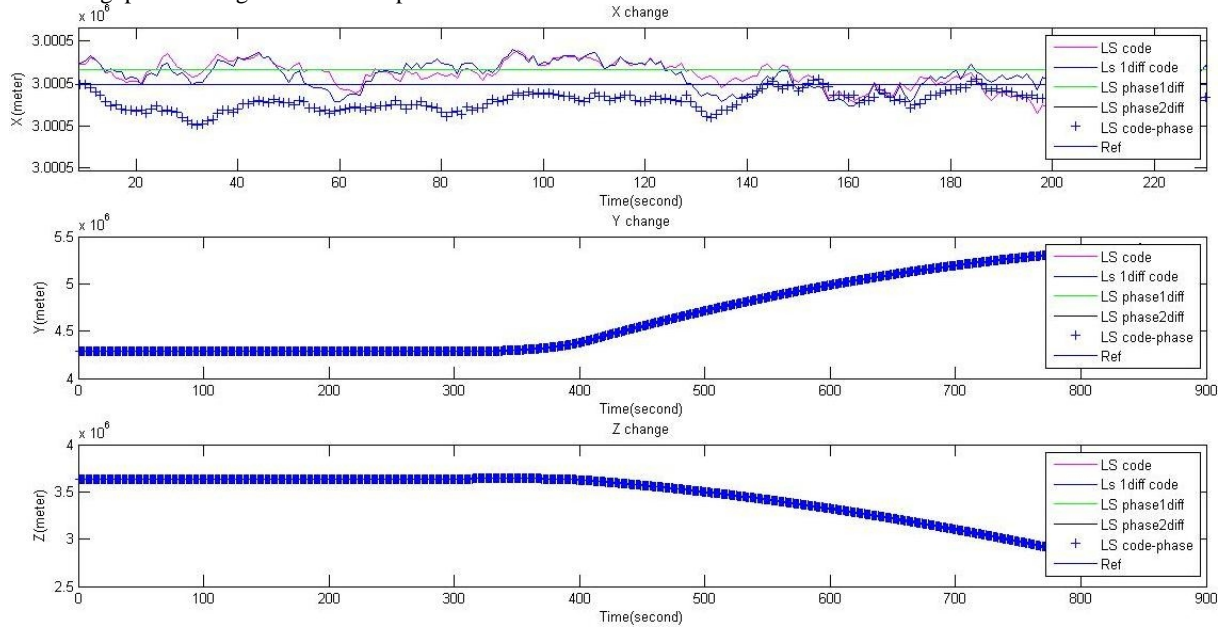
The advantage of phase smoothing in the positioning domain over phase smoothing in the range domain is that phase-smoothed pseudo-ranges require continuous tracking of a single observation to contribute effectively to the solution. The proposed method does not need to know the initial position and uninterrupted tracking of a single observation.

## 5 Experimental Results

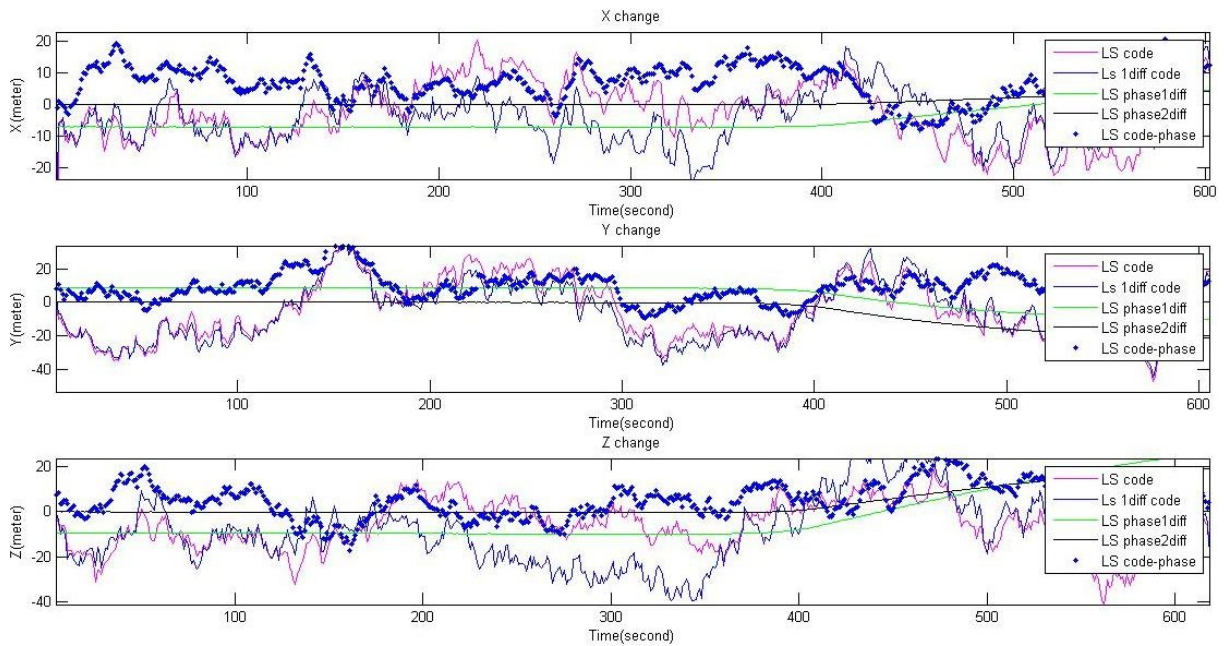
Receiver position estimate results by LS method using different data. In this part, we refer to real data test results. Rohde & Schwarz GNSS Simulator is used to generate raw GPS data such as pseudo-range, integrated carrier phase, Doppler shift and satellite ephemeris. The GNSS simulator in the R&S<sup>®</sup> SMBV100A includes the ability to simulate

realistic transmission conditions through the use of multipath signal generation and modeling of various atmospheric effects. The motion of an aircraft containing GNSS receiver, simulator is used to model effects that impact GNSS receiver performance, such as atmospheric conditions, multipath reflections, antenna characteristics and interference signals. Fig.1 indicates that using pseudo-range and carrier phase data at the

same time has more precision compared to when pseudo-range data or carrier phase data are used in isolation. In Fig.1, X component has been magnified in order to it can be seen more exactly. Also, Fig.2 shows methods error. They show performance and accuracy of the proposed method.



**Fig. 1** Receiver position estimate results by five proposed LS method ( X component has been larged).



**Fig. 2** Receiver position errors by five proposed LS method.

As seen, the position estimation diagram by pseudo-range data has distortion. For the performance evaluating of the proposed method, RMS error was used as:

$$RMS = \sqrt{\frac{1}{N} \sum_{i=1}^N [Desired(i) - Output(i)]^2} \quad (46)$$

Where  $N$  is number of tests and  $Desired(i)$  and  $Output(i)$  represent the true coordinates data and algorithms output data, respectively.

Error influential values in receiver position estimation by means of different methods can be seen in Table 2. The values presented in this table are in meter. By comparing the numbers in the table, we realize that RMS by compounding pseudo-range and carrier phase data has improved.

The next point to be mentioned is that the overall RMS error obtained by double-difference carrier phase data in a period of time in which the receiver is fixed is about 5m. Whereas the overall RMS error in this method in the whole time period is about 17m. However in pseudorange-carrier phase combined method, there is a more stability in overall RMS error in the whole time period which shows the optimal performance of this method in high velocities. This is in addition to the positioning capability of combined method (without needing to initial position) versus displacement calculation in double-difference carrier phase method, because double-difference carrier phase method can only calculate displacement and it's positioning firmly depends on initial conditions. As the initial position estimate gets farther from the actual location, therefore, to get a good position estimate with this method, one must have an accurate estimate of the initial location and this imposes a major limitation.

## 6 Conclusion

In this study, we have presented a method to reap the benefits of using pseudo-range and carrier phase data simultaneously and reduction of defects in determining receiver position by means of error LS method. In this method, we compounded pseudo-range and carrier phase and accessed to less RMS error and more precision in determining receiver position using error LS method. For future research, we can propose compounding pseudo-range and carrier phase using weight error LS or recursive LS and improve receiver positioning.

**Table 2** Error influential values in receiver position estimation by means of different methods.

Methods	LS-code	LS-code one diff.	LS-phase one diff.	LS-phase two diff.	LS-code- phase
RMS	23.43	21.69	20.83	17.51	12.87

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