Stabilization of Networked Control Systems with Variable Delays and Saturating Inputs

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Abstract: In this paper, improved conditions for the synthesis of static state-feedback controller are derived to stabilize networked control systems (NCSs) subject to actuator saturation. Both of the data packet latency and dropout which deteriorate the performance of the closed-loop system are considered in the NCS model via variable delays. Two different techniques are employed to incorporate actuator saturation in the system description. Utilizing Lyapunov-Krasovskii Theorem, delay-dependent conditions are obtained in terms of linear matrix inequalities (LMIs) to determine the static feedback gain. Moreover, an optimization problem is formulated in order to find the less conservative estimate for the region of attraction corresponding to different maximum allowable delays. Numerical examples are introduced to demonstrate the effectiveness and advantages of the proposed schemes.

Keywords: Networked Control Systems, Variable Delay, Input Saturation, Linear Matrix Inequality (LMI).

I. INTRODUCTION

A Networked Control System (NCS) is a feedback control structure wherein the control loop is closed through a communication network. The advantages of NCSs such as low cost and simple installation and maintenance make them more and more popular in many real-world applications including distributed industrial control, cluster of unmanned air vehicles and multi-agent systems. However, the presence of communication network in the control loop makes the analysis and design of the control system very complicated. Main issues are the network-induced delays and packet dropouts which occur when sensor, actuator and controller exchange data across the network. The design of NCSs with emphasis on the data packet delay and dropout problem has been studied by many researchers [1]-[5]. On the other hand, physical constraints, especially actuator saturation are encountered in practical control systems. The analysis and synthesis of time delay systems with input saturation have attracted attention in the past few years [8]-[13].

In [6], stabilizing controller was designed for networked control systems with actuator saturation and sampling period variation. A continuous functional whose values at the sampling instants coincides with a discrete-time quadratic Lyapunov function is utilized to derive sufficient condition in terms of LMI was to determine stabilizing state feedback controller. In [7], stabilization problem of networked stochastic systems subject to actuator saturation was studied. The nonlinearity of actuator saturation was modeled as a convex polytope of linear systems. The asymptotic stabilization of a constrained time-delay system was studied in [8]. The problem of designing linear state feedback stabilizing law and enlarging the domain of attraction is formulated as an optimization problem with LMI constraints. In [9], simple sufficient LMI conditions are derived for stabilization of systems with polytopic type uncertainty for regional stabilization of systems with sampled-data saturated state feedback via descriptor approach. The regional stabilization and H_∞ control problem have been studied in [10] by combining the descriptor model transformation and Moon's inequality which is used to get a less conservative bound for the cross terms. Using novel Lyapunov-Krasovskii functional and generalized sector condition, improved LMIs formulated in [12] for stabilizing controller design, aiming at enlarging the estimate of the region of attraction of the closed-loop system and maximizing the bounds on the sampling period. Stabilization problem of neutral delay systems in the presence of control saturation is solved in [13] based on the descriptor approach and the use of a modified sector condition. Moreover, improved results for stabilization and stability analysis for time-delay systems have been acquired in the recent years [14]-[16] and [18].

This paper presents less conservative procedures to synthesis asymptotically stabilizing state feedback controller for networked control systems subject to actuator saturation. The NCS model developed in [1] is adopted and actuator saturation nonlinearity is treated in two ways: In the first approach, the saturation is represented by a convex polytope of linear systems. In the second scheme, generalized sector condition (decentralized deadzone nonlinearity) is used to model saturation effects. Utilizing an appropriate Lyapunov-Krasovskii functional, a simple sufficient condition which ensures the local stability of the closed loop system is derived in terms of Linear Matrix Inequalities.

A key issue in control synthesis for nonlinear systems is designing a controller to stabilize the plant with a large domain of attraction, i.e. enlarging the set of initial states for which the asymptotic convergence of the corresponding trajectories to the origin is ensured. Thus, in this note to find an initial set with less conservative estimate of the domain of attraction, an optimization problem with LMI constraints is formulated.

The paper is organized as follows: In section II, the networked control system model is described. Section III presents the main results of this paper wherein the procedures to determine controller gain, maximum allowable delay and domain of attraction is derived. In section IV, numerical example is given to illustrate the effectiveness of the proposed method and then the obtained results are compared to the literature. Finally, a brief concluding remark is given in Section V.

Notations: Throughout the paper $\Re^{n}$ denotes the n dimensional Euclidean space with vector norm $|\ |$and $\Re^{n\times m}$ is the set of all $n\times m$ real matrices. The notation $\mathbf{P}>0$ ($\mathbf{P}\geq 0$) means that $\mathbf{P}$ is symmetric and positive definite (positive semi definite).The subscript^{T} stands for matrix transposition. $Co\{\ \}$symbolizes the convex hull. $diag\ \{\ \}$is used as an ellipse for block-diagonal matrix. $\bar{\sigma}()$ denotes the largest singular value of the matrix. The symbol * shows the symmetric entry in a symmetric matrix. Finally, the space of continuously differentiable vector function over $[-\eta,0]$ is represented by $C^{1}[-\eta,0]$.

II. Problem Statement

A typical networked control system is shown in Figure 1, wherein the controller, sensor and the actuator are assumed to be separated and connected through a communication network. The controlled system is linear and time invariant, sensor is time-driven and controller and actuator are event-driven. In the considered network, all the data are lumped together into one packet and transmitted at the same time (single packet transmission) and thesent packets are time stamped. The controller and actuator always use the new data packets and discard the old ones. When an old data packet arrives, it is dealt with as a packet loss. A zero-order-hold is placed in the input of the plant and the input is zero before the first controller packet arrives.

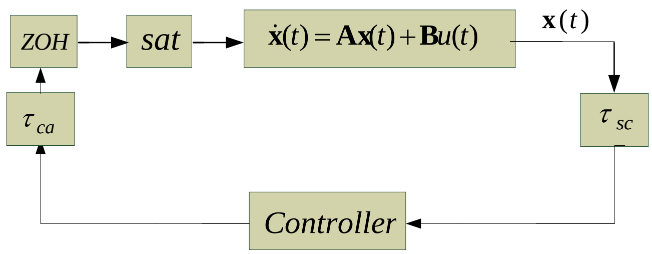

Fig. 1. Schematic Diagram of the Saturated NCS

Regarding the above assumption on the NCS, the following equations can describe the closed-loop system behavior:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \tag{1}$$

$$\mathbf{u}(t) = \mathrm{sat}(\mathbf{K}\,\mathbf{x}(t - \tau_{i_k})), \qquad t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}) \tag{2}$$

where $\mathbf{x}(t) \in \Re^n$ and $\mathbf{u}(t) \in \Re^m$ are the state vector and the control input vector respectively. $\mathbf{A}$ and $\mathbf{B}$ are two constant matrices with appropriate dimensions. $\mathbf{K}$ is the state feedback gain matrix. Function $\mathrm{sat}(\cdot) : \Re^m \to \Re^m$ is the standard saturation function, in which $\mathrm{sat}(u_i) = \mathrm{sign}(u_i) \min(\bar{u}_i, |u_i|)$ and $\bar{u}_i = \max(u_i)$. h is the sampling period, $k = 1,2,3,\ldots$ is the number of the controls which act on the system, i_k is an integer denoting the sampling instant of the state feedback corresponding to the kth effective control, τ_{i_k} denotes the network-induced time delay from the instant $i_k h$, when the sensor samples the state of the system to the instant when the actuator imposes the related control on the system. Transmission delay induced by the network is composed of two parts: sensor-to-controller delay τ_{sc} and controller-to-actuator delay τ_{ca}. Since the controller is static these two delays can be lumped together $(\tau_{i_k} = \tau_{sc} + \tau_{ca})$.

The closed loop system model (1)-(2) can be represented as $\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\ \mathrm{sat}(\mathbf{K}\,\mathbf{x}(i_k h))$ for

$t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$. Now, by definition of $\tau(t) = t - i_k h$, $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$, this relation can be rewritten as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\,\mathrm{sat}(\mathbf{K}\mathbf{x}(t-\tau(t))) \tag{3}$$

which is a continuous-time system with delayed input. Note that:

$$0 \le \tau_{i_k} \le \tau(t) \le (i_{k+1} - i_k)h + \tau_{i_{k+1}} \le \eta \tag{4}$$

Furthermore, the initial condition of system (3) is a continuous differentiable function which is shown as the following:

$$\mathbf{x_0} = \varphi(\theta), \theta \in [-\eta, 0]. \tag{5}$$

Consequently, the NCS was modeled as the time-delay system (3) where both of data packet dropout and latency in the network is considered as variable delay characterized in (4). The problem of interest is to determine the state feedback gain matrix $\mathbf{K}$ such that the controller (2) asymptotically stabilizes the closed loop system (1)-(2) and moreover to obtain an estimate of its domain of attraction.

III. MAIN RESULT

In the next subsection, some useful facts which will be employed to formulate actuator saturation in the stabilization problem are reviewed.

A. Preliminaries:

Definition 1: Let $\mathbf{k}_i$ be the i th row of the matrix $\mathbf{K}$, the polyhedron region $L(\mathbf{K})$ in the state space is defined as follows:

$$L(\mathbf{K}) = \left\{ \mathbf{x} \in \Re^n : \left|\mathbf{k}_i \mathbf{x}\right| \le \bar{u}_i ,\ \ i = 1, 2, \cdots, m \right\}.$$

Furthermore, the ellipsoid E in the state space is defined as follows:

$E(\mathbf{P},1)=\{\mathbf{x}\in\Re^n:\ \mathbf{x}^T\mathbf{P}\mathbf{x}\le 1\}$

wherein, $\mathbf{P}\in\Re^{n\times n}$ is a positive definite matrix.

Definition 2: The set of all $m\times m$ diagonal matrices whose diagonal elements are either 1 or 0 is shown by ν; then there are 2^m elements in ν. Let each D_j, $j=1,2,\cdots,2^m$ be an element of ν and denote $D_j^-=I-D_j$. It is clear that D_j^- is an element of ν if $D_j\in\nu$.

Lemma 1 [17]: Let $\mathbf{K},\mathbf{H}\in\Re^{m\times n}$ are given; for all n dimensional vector $\mathbf{x}\in L(\mathbf{H})$, the following holds:

$\mathrm{sat}(\mathbf{K}\,\mathbf{x})\in Co\{\,D_j\,\mathbf{K}\,\mathbf{x}+D_j^-\,\mathbf{H}\,\mathbf{x}\,,\quad j=1,\cdots,2^m\,\}$

Hence, $\mathrm{sat}(\mathbf{K}\,\mathbf{x})$ can be expressed as follows:

$$\mathrm{sat}(\mathbf{K}\,\mathbf{x})=\sum_{j=1}^{2^m}\lambda_j(D_j\mathbf{K}+D_j^-\mathbf{H})\mathbf{x} \tag{6}$$

in which, $\sum_{j=1}^{2^m}\lambda_j=1$ and $\lambda_j\ge 0$. ■

Definition 3: The vector function ψ is defined as follows:

$$\psi(\mathbf{Kx})=\mathbf{Kx}-sat(\mathbf{Kx}) \tag{7}$$

It's clear that $\psi(\mathbf{Kx})$ is the decentralized dead-zone nonlinearity.

Lemma2 [18]: Consider the function $\psi(\mathbf{Kx})$ defined in (7). For $\mathbf{x}\in\Re^n$ if $\mathbf{x}\in L(\mathbf{K}-\mathbf{H})$, the following is hold:

$$\psi^T(\mathbf{Kx})\mathbf{U}(\psi(\mathbf{Kx})-\mathbf{Hx})\le 0 \tag{8}$$

for any diagonal positive definite matrix $\mathbf{U}\in\Re^{m\times m}$. ■

The result in Lemma 2 is known as a generalized sector condition which will allow obtaining stability conditions directly in an LMI form.

Definition 4: Let $\phi(t,\mathbf{x_0})$ be the state trajectory of the system (3), starting from the initial function $\mathbf{x_0}\in C^1[-\eta,0]$; the domain of attraction of the origin is defined as in the following:

$$S=\{\ \mathbf{x_0}\in C^1[-\eta,0]:\ \lim_{t\to\infty}\varphi(t,\mathbf{x_0})=0\ \},$$

an estimate of the domain of attraction $\mathrm{X}_{DOA}\subset S$ can be found as the follows:

$$\mathrm{X}_{DOA}=\left\{\mathbf{x_0}\in C^1[-\eta,0]:\ \max|\mathbf{x_0}|\le\delta_1,\ \ \max|\dot{\mathbf{x}}_\mathbf{0}|\le\delta_2\right\} \tag{9}$$

by maximizing positive scalars $\delta_i\ (i=1,2)$.

B. Design Method

In this subsection, the sufficient conditions are derived to determine the static state-feedback gain to asymptotically stabilize the close-loop system (3). A simple condition to obtain the mentioned matrices and feedback gain is given in the Theorem 1.

Theorem 1: Given scalars $\eta>0$ and $p_i,\ i=2,3,4$ the system (1) with the networked memoryless state-feedback controller (2) is asymptotically stable if there exist matrices $\tilde{\mathbf{P}}=\tilde{\mathbf{P}}^T>0$, $\tilde{\mathbf{Q}}=\tilde{\mathbf{Q}}^T>0$, $\tilde{\mathbf{R}}=\tilde{\mathbf{R}}^T>0$, $\mathbf{G}$, $\mathbf{Y}$ and nonsingular matrix $\mathbf{X}$ of appropriate dimensions such that the following matrix inequalities hold:

$$\tilde{\mathbf{\Phi}} < 0, \quad j = 1,2,\ldots,2^m \tag{10}$$

$$\begin{bmatrix} \bar{u}_s & \mathbf{g}_s \\ * & \bar{u}_s \tilde{\mathbf{P}} \end{bmatrix} \geq 0, \quad s = 1,2,\ldots,m \tag{11}$$

where $\tilde{\mathbf{\Phi}} = \tilde{\mathbf{\Omega}}_1 + \tilde{\mathbf{\Omega}}_2 + \tilde{\mathbf{\Omega}}_2^T$.

$$\tilde{\mathbf{\Omega}}_1 = \begin{bmatrix} \tilde{\mathbf{Q}} - \tilde{\mathbf{R}} & \tilde{\mathbf{R}} & \tilde{\mathbf{P}} & \mathbf{0} \\ * & -2\tilde{\mathbf{R}} & \mathbf{0} & \tilde{\mathbf{R}} \\ * & * & \eta^2 \tilde{\mathbf{R}} & \mathbf{0} \\ * & * & * & -\tilde{\mathbf{Q}} - \tilde{\mathbf{R}} \end{bmatrix} \tag{12}$$

$$\tilde{\mathbf{\Omega}}_2 = \begin{bmatrix} -\mathbf{A}\mathbf{X}^T & -\mathbf{B}(D_j \mathbf{Y} + D_j^- \mathbf{G}) & \mathbf{X}^T & \mathbf{0} \\ -p_2 \mathbf{A}\mathbf{X}^T & -p_2 \mathbf{B}(D_j \mathbf{Y} + D_j^- \mathbf{G}) & p_2 \mathbf{X}^T & \mathbf{0} \\ -p_3 \mathbf{A}\mathbf{X}^T & -p_3 \mathbf{B}(D_j \mathbf{Y} + D_j^- \mathbf{G}) & p_3 \mathbf{X}^T & \mathbf{0} \\ -p_4 \mathbf{A}\mathbf{X}^T & -p_4 \mathbf{B}(D_j \mathbf{Y} + D_j^- \mathbf{G}) & p_4 \mathbf{X}^T & \mathbf{0} \end{bmatrix} \tag{13}$$

and $\mathbf{g}_s$ is the s th row of $\mathbf{G}$; Furthermore, $\mathbf{K} = \mathbf{Y}\mathbf{X}^{-T}$ and $\mathbf{H} = \mathbf{G}\mathbf{X}^{-T}$. An estimate of the domain of attraction is in the form of (9) with δ_1 and δ_2 satisfying:

$$\delta_1^2 (\bar{\sigma}(\mathbf{X}^{-1} \tilde{\mathbf{P}} \mathbf{X}^{-T}) + \eta \bar{\sigma}(\mathbf{X}^{-1} \tilde{\mathbf{Q}} \mathbf{X}^{-T})) + \frac{\eta^3}{2} \delta_2^2 \bar{\sigma}(\mathbf{X}^{-1} \tilde{\mathbf{R}} \mathbf{X}^{-T}) \leq 1 \tag{14}$$

Proof: Lyapunov-Krasovskii functional candidate is considered as follows:

$$V(t) = \mathbf{x}^T(t)\mathbf{P}\mathbf{x}(t) + \int_{t-\eta}^{t} \mathbf{x}^T(s)\mathbf{Q}\mathbf{x}(s)ds + \eta \int_{-\eta}^{0} \int_{t+\theta}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)dsd\theta$$

in which, the matrices $\mathbf{P} = \mathbf{P}^T > 0$, $\mathbf{Q} = \mathbf{Q}^T > 0$ and $\mathbf{R} = \mathbf{R}^T > 0$ are to be determined. Calculating the time derivative of $V(t)$ along the trajectories of the system (3) for $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$ yields:

$$\dot{V}(t) = 2\mathbf{x}^T(t)\mathbf{P}\dot{\mathbf{x}}(t) + \mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) - \mathbf{x}^T(t-\eta)\mathbf{Q}\mathbf{x}(t-\eta) + \eta^2 \dot{\mathbf{x}}^T(t)\mathbf{R}\dot{\mathbf{x}}(t) - \eta \int_{t-\eta}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)ds$$

Moreover, the following relation is true:

$$-\eta \int_{t-\eta}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\,\dot{\mathbf{x}}(s)ds = -\eta \int_{t-\eta}^{i_k h} \dot{\mathbf{x}}^T(s)\mathbf{R}\,\dot{\mathbf{x}}(s)ds - \eta \int_{i_k h}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\,\dot{\mathbf{x}}(s)ds \tag{15}$$

On the other hand, the following inequalities hold:

$$-\eta \int_{i_k h}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)ds \le -[\mathbf{x}(t)-\mathbf{x}(i_k h)]^T\,\mathbf{R}[\mathbf{x}(t)-\mathbf{x}(i_k h)] \tag{16}$$

$$-\eta \int_{t-\eta}^{i_k h} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)ds \le -[\mathbf{x}(i_k h)-\mathbf{x}(t-\eta)]^T\,\mathbf{R}[\mathbf{x}(i_k h)-\mathbf{x}(t-\eta)] \tag{17}$$

So, substituting (16) and (17) in (15) results in:

$$-\eta \int_{t-\eta}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\,\dot{\mathbf{x}}(s)ds \ \le -[\mathbf{x}(i_k h)-\mathbf{x}(t-\eta)]^T\,\mathbf{R}[\mathbf{x}(i_k h)-\mathbf{x}(t-\eta)] - [\mathbf{x}(t)-\mathbf{x}(i_k h)]^T\,\mathbf{R}[\mathbf{x}(t)-\mathbf{x}(i_k h)]$$

Then, the following upper bound is obtained for $\dot{V}(t)$:

$$\begin{aligned}\dot{V}(t) \le{}& 2\mathbf{x}^T(t)\mathbf{P}\dot{\mathbf{x}}(t) + \mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) - \mathbf{x}^T(t-\eta)\mathbf{Q}\mathbf{x}(t-\eta) + \eta^2\dot{\mathbf{x}}^T(t)\mathbf{R}\dot{\mathbf{x}}(t) \\ & -[\mathbf{x}(i_k h)-\mathbf{x}(t-\eta)]^T\,\mathbf{R}[\mathbf{x}(i_k h)-\mathbf{x}(t-\eta) - [\mathbf{x}(t)-\mathbf{x}(i_k h)]^T\,\mathbf{R}[\mathbf{x}(t)-\mathbf{x}(i_k h)]\end{aligned} \tag{18}$$

On the other hand, utilizing the Lemma 1, If $\mathbf{x}(i_k h) \in L(\mathbf{H})$ the closed-loop system equation can be represented as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \sum_{j=1}^{2^m} \lambda_j\,\mathbf{B}(D_j\,\mathbf{K} + D_j^-\,\mathbf{H})\mathbf{x}(i_k h) \tag{19}$$

where $0 \le \lambda_j \le 1$ and $\sum_{j=1}^{2^m} \lambda_j = 1$. Hence, the system equation in vertex j is as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{A}_j\,\mathbf{x}(i_k h) \tag{20}$$

where $\mathbf{A}_j = \mathbf{B}(D_j\mathbf{K} + D_j^-\mathbf{H})$ for $j = 1, \cdots, 2^m$. Let $\boldsymbol{\xi}(t) = \left[\mathbf{x}(t),\ \mathbf{x}(i_k h),\ \dot{\mathbf{x}}(t),\ \mathbf{x}(t-\eta)\right]^T$, for any matrix $\mathbf{M}$, the following relation is true:

$$2\boldsymbol{\xi}^T(t)\mathbf{M}[\dot{\mathbf{x}}(t) - \mathbf{A}\mathbf{x}(t) - \mathbf{B}(D_j\mathbf{K} + D_j^-\mathbf{H})\mathbf{x}(i_k h)] = 0 \tag{21}$$

Adding equation (21) to (18) yields to:

$$\dot{V}(t) \le \xi^T(t)\mathbf{\Phi}\,\xi(t) \tag{22}$$

where $\mathbf{\Phi} = \mathbf{\Omega}_1 + \mathbf{\Omega}_2 + \mathbf{\Omega}_2^T$ and

$$\mathbf{\Omega}_1 = \begin{bmatrix} \mathbf{Q}-\mathbf{R} & \mathbf{R} & \mathbf{P} & \mathbf{0} \\ * & -2\mathbf{R} & \mathbf{0} & \mathbf{R} \\ * & * & \eta^2\mathbf{R} & \mathbf{0} \\ * & * & * & -\mathbf{Q}-\mathbf{R} \end{bmatrix}$$

$$\mathbf{\Omega}_2 = \begin{bmatrix} -\mathbf{MA} & -\mathbf{MB}(D_j\,\mathbf{K} + D_j^-\,\mathbf{H}) & \mathbf{M} & \mathbf{0} \end{bmatrix}.$$

Provided $\mathbf{\Phi} < 0$, the Lyapunov-Krasovskii Theorem ensures that the system (20) is asymptotically stable. The inequality condition $\mathbf{\Phi} < 0$ is nonlinear matrix inequality; thus by changing variables**,** it is transformed to LMI. The matrix $\mathbf{M}$ is partitioned as follows:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 \\ \mathbf{M}_2 \\ \mathbf{M}_3 \\ \mathbf{M}_4 \end{bmatrix}$$

Let $\mathbf{M}_1 = \mathbf{M}_0$, $\mathbf{M}_2 = p_2\mathbf{M}_0$, $\mathbf{M}_3 = p_3\mathbf{M}_0$, $\mathbf{M}_4 = p_4\mathbf{M}_0$, $\mathbf{X} = \mathbf{M}_0^{-1}$ and $\mathbf{Z} = \mathrm{diag}(\mathbf{X}, \mathbf{X}, \mathbf{X}, \mathbf{X})$. Defining $\tilde{\mathbf{\Phi}} = \mathbf{Z}\,\mathbf{\Phi}\,\mathbf{Z}^T = \tilde{\mathbf{\Omega}}_1 + \tilde{\mathbf{\Omega}}_2 + \tilde{\mathbf{\Omega}}_2^T$ with $\tilde{\mathbf{\Omega}}_i = \mathbf{Z}\mathbf{\Omega}_i\mathbf{Z}^T$, $i = 1, 2$, $\tilde{\mathbf{P}} = \mathbf{X}\,\mathbf{P}\,\mathbf{X}^T$, $\tilde{\mathbf{Q}} = \mathbf{X}\,\mathbf{Q}\,\mathbf{X}^T$, $\tilde{\mathbf{R}} = \mathbf{X}\,\mathbf{R}\,\mathbf{X}^T$, $\mathbf{Y} = \mathbf{K}\,\mathbf{X}^T$, $\mathbf{G} = \mathbf{H}\,\mathbf{X}^T$, the inequality $\tilde{\mathbf{\Phi}} < 0$ implies that $\mathbf{\Phi} < 0$.

The next stage is to guarantee the condition $\mathbf{x}(i_k h) \in L(\mathbf{H})$ which is necessary to use the result of Lemma 1 in (19). Let the ellipsoid $E(\mathbf{P},1)$ is a subset of the set $L(\mathbf{H})$, so the following inequality is satisfied:

$$2\left|\mathbf{h}_j\,\mathbf{x}(i_k h)\right| \le \bar{u}_j\,\left(1 + \mathbf{x}(i_k h)^T\,\mathbf{P}\,\mathbf{x}(i_k h)\right) \le 2\,\bar{u}_j\,, \quad j = 1, \cdots, m$$

Since

$$2\left|\mathbf{h}_i\,\mathbf{x}(i_k h)\right| \le \bar{u}_i\,(1+\mathbf{x}(i_k h)^T\,\mathbf{P}\,\mathbf{x}(i_k h)) = \begin{bmatrix}1 & \pm\mathbf{x}(i_k h)\end{bmatrix}\begin{bmatrix}\bar{u}_i & \mathbf{h}_i \\ * & \bar{u}_i\mathbf{P}\end{bmatrix}\begin{bmatrix}1 \\ \pm\mathbf{x}(i_k h)\end{bmatrix} \ge 0 \ ,$$

the following holds:

$$\begin{bmatrix}\bar{u}_i & \mathbf{h}_i \\ * & \bar{u}_i\mathbf{P}\end{bmatrix} \ge 0 \tag{23}$$

If both sides of (23) pre and post multiplied simultaneously with $diag(\mathbf{I},\,\mathbf{X})$ and its transpose respectively, the inequality (11) is obtained where $\mathbf{g}_i = \mathbf{h}_i\,\mathbf{X}^T$.

From $\dot{V}(t) < 0$, it follows that $V(\mathbf{x}_t) < V(\mathbf{x}_0)$ and therefore for $t > 0$:

$$\mathbf{x}(t)^T\,\mathbf{P}\;\mathbf{x}(t) < V(\mathbf{x}_t) < V(\mathbf{x}_0)$$

Regarding (9), the following inequality holds:

$$V(\mathbf{x}_0) \le \max_{\theta\in[-\eta,0]}\left|\phi(\theta)\right|^2(\bar{\sigma}(\mathbf{P}) + \eta\bar{\sigma}(\mathbf{Q})) + \max_{\theta\in[-\eta,0]}\left|\dot{\phi}(\theta)\right|^2\frac{\eta^3}{2}\bar{\sigma}(\mathbf{R}) \le \delta_1^2(\bar{\sigma}(\mathbf{P}) + \eta\bar{\sigma}(\mathbf{Q})) + \delta_2^2\frac{\eta^3}{2}\bar{\sigma}(\mathbf{R})$$

So, if the following holds:

$$\delta_1^2\,(\bar{\sigma}(\mathbf{P}) + \eta\,\bar{\sigma}(\mathbf{Q})) + \frac{\eta^3}{2}\delta_2^2\,\bar{\sigma}(\mathbf{R}) \le 1$$

then, for all the initial functions belong to X_{DOA} in (9), the trajectories of the closed-loop system remain in the ellipsoid $E(\mathbf{P},1) \subset L(\mathbf{H})$ and the polyhedron representation of saturation function is valid. Finally, note that if the subsystems (20) are stable for $j = 1,\cdots,2^m$, then the overall system is stable.■

Corollary 1: Let $\mathbf{K},\mathbf{H} \in \Re^{m\times n}$ be given. The closed-loop system (3) is asymptotically stable if there exist matrices $\mathbf{P} > 0, \mathbf{Q} > 0, \mathbf{R} > 0$ and $\mathbf{M}$ such that the following LMIs hold:

$$\mathbf{\Phi} < 0, \quad j = 1,2,...,2^m \tag{24}$$

$$\begin{bmatrix} \bar{u}_s & \mathbf{h}_s \\ * & \bar{u}_s \mathbf{P} \end{bmatrix} \geq 0, \quad s = 1, 2, \ldots, m \quad (25)$$

where $\mathbf{\Phi} = \mathbf{\Omega}_1 + \mathbf{\Omega}_2 + \mathbf{\Omega}_2^T$.

$$\mathbf{\Omega}_1 = \begin{bmatrix} \mathbf{Q}-\mathbf{R} & \mathbf{R} & \mathbf{P} & \mathbf{0} \\ * & -2\mathbf{R} & \mathbf{0} & \mathbf{R} \\ * & * & \eta^2 \mathbf{R} & \mathbf{0} \\ * & * & * & -\mathbf{Q}-\mathbf{R} \end{bmatrix}$$

$$\mathbf{\Omega}_2 = [\, -\mathbf{MA} \quad -\mathbf{MB}(D_j\,\mathbf{K} + D_j^-\,\mathbf{H}) \quad \mathbf{M} \quad \mathbf{0}\,]. \blacksquare$$

Based on the result of Corollary 1, an optimization problem with LMI constraints is formulated to obtain a large estimate of the domain of attraction. To simplify the procedure we select $\delta_1 = \delta_2 = \delta_{\max}$ and parameters $w_i > 0, i = 1,2,3$ are used to bound the matrices $\mathbf{P},\mathbf{Q}$ and $\mathbf{R}$ for getting a less conservative estimate of the domain of attraction. The matrices $\mathbf{K}$ and $\mathbf{H}$ are computed utilizing Theorem1, then the following optimization problem is solved using YALMIP [19] to obtain a maximal estimate of domain of attraction:

$$\begin{aligned} & \min \ \gamma \\ & s.t. \\ & corollary 1 \\ & w_1\mathbf{I} - \mathbf{P} \geq 0 \\ & w_2\mathbf{I} - \mathbf{Q} \geq 0 \\ & w_3\mathbf{I} - \mathbf{R} \geq 0 \end{aligned} \qquad (25)$$

where $\gamma = w_1 + \eta\, w_2 + \frac{1}{2}\eta^3 w_3$. Thus a maximal estimate of domain of attraction can be obtained by:

$$\left|\delta_{\max}\right| \leq \frac{1}{\sqrt{\bar{\sigma}(\mathbf{P}) + \eta\bar{\sigma}(\mathbf{Q}) + \frac{\eta^3}{2}\bar{\sigma}(\mathbf{R})}} \qquad (26)$$

In the Theorem 2, based on decentralized dead zone nonlinearity property presented in Lemma 2, sufficient conditions are derived to obtain static state-feedback gain.

Theorem 2: Given scalars $\eta > 0$ and $p_i,\ i = 2,3,4$, the system (1) with the networked memoryless state-feedback controller (2) is asymptotically stable if there exist matrices $\tilde{\mathbf{P}} = \tilde{\mathbf{P}}^T > 0$, $\tilde{\mathbf{Q}} = \tilde{\mathbf{Q}}^T > 0$, $\tilde{\mathbf{R}} = \tilde{\mathbf{R}}^T > 0$, $\mathbf{G}, \mathbf{Y}$, diagonal positive definite $\tilde{\mathbf{U}}$ and nonsingular matrix $\mathbf{X}$ of appropriate dimensions such that the following matrix inequalities hold:

$$\tilde{\mathbf{\Phi}} < 0, \tag{27}$$

$$\begin{bmatrix} \bar{u}_s & \mathbf{y}_s - \mathbf{g}_s \\ * & \bar{u}_s \mathbf{P} \end{bmatrix} \geq 0, \quad s = 1, 2, \dots, m \tag{28}$$

where $\tilde{\mathbf{\Phi}} = \tilde{\mathbf{\Omega}}_1 + \tilde{\mathbf{\Omega}}_2 + \tilde{\mathbf{\Omega}}_2^T$.

$$\tilde{\mathbf{\Omega}}_1 = \begin{bmatrix} \tilde{\mathbf{Q}} - \tilde{\mathbf{R}} & \tilde{\mathbf{R}} & \tilde{\mathbf{P}} & \mathbf{0} & \mathbf{0} \\ * & -2\tilde{\mathbf{R}} & \mathbf{0} & \tilde{\mathbf{R}} & \mathbf{G}^T \\ * & * & \eta^2 \tilde{\mathbf{R}} & 0 & 0 \\ * & * & * & \tilde{\mathbf{Q}} - \tilde{\mathbf{R}} & 0 \\ * & * & * & * & -2\tilde{\mathbf{U}} \end{bmatrix}$$

$$\tilde{\mathbf{\Omega}}_2 = \begin{bmatrix} -\mathbf{A}\mathbf{X}^T & -\mathbf{B}\mathbf{Y} & \mathbf{X}^T & \mathbf{0} & \mathbf{B}\tilde{\mathbf{U}} \\ -p_2\mathbf{A}\mathbf{X}^T & -p_2\mathbf{B}\mathbf{Y} & p_2\mathbf{X}^T & \mathbf{0} & p_2\mathbf{B}\tilde{\mathbf{U}} \\ -p_3\mathbf{A}\mathbf{X}^T & -p_3\mathbf{B}\mathbf{Y} & p_3\mathbf{X}^T & \mathbf{0} & p_3\mathbf{B}\tilde{\mathbf{U}} \\ -p_4\mathbf{A}\mathbf{X}^T & -p_4\mathbf{B}\mathbf{Y} & p_4\mathbf{X}^T & \mathbf{0} & p_4\mathbf{B}\tilde{\mathbf{U}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$\mathbf{y}_s$ and $\mathbf{g}_s$ are the s th row of $\mathbf{Y}$ and $\mathbf{G}$, respectively. Moreover, $\mathbf{K} = \mathbf{Y}\mathbf{X}^{-T}$ and $\mathbf{H} = \mathbf{G}\mathbf{X}^{-T}$. An estimate of the domain of attraction is in the form of (9) with δ_1 and δ_2 satisfying:

$$\delta_1^2(\bar{\sigma}(\mathbf{X}^{-1}\tilde{\mathbf{P}}\mathbf{X}^{-T}) + \eta\bar{\sigma}(\mathbf{X}^{-1}\tilde{\mathbf{Q}}\mathbf{X}^{-T})) + \frac{\eta^3}{2}\delta_2^2\bar{\sigma}(\mathbf{X}^{-1}\tilde{\mathbf{R}}\mathbf{X}^{-T}) \leq 1$$

Proof: The sketch of proof is similar to the Theorem 1. But here, regarding Definition 3, the closed-loop system is represented as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{K}\mathbf{x}(i_k h) - \mathbf{B}\psi(\mathbf{K}\mathbf{x}(i_k h)) \tag{29}$$

For $\boldsymbol{\xi}(t) = \left[\mathbf{x}(t),\ \mathbf{x}(i_k h),\ \dot{\mathbf{x}}(t),\ \mathbf{x}(t-\eta), \psi(\mathbf{K}\mathbf{x}(i_k h))\right]^T$ and any matrix $\mathbf{M}$ of the form:

$$\mathbf{M}=\begin{bmatrix}\mathbf{M}_1\\ \mathbf{M}_2\\ \mathbf{M}_3\\ \mathbf{M}_4\\ \mathbf{0}\end{bmatrix}$$

the following equation holds:

$$2\xi^T\mathbf{M}[\dot{\mathbf{x}}(t)-\mathbf{A}\mathbf{x}(t)-\mathbf{B}\mathbf{K}(x(i_kh))+\mathbf{B}\psi(\mathbf{K}\mathbf{x}(i_kh))]=0 \tag{30}$$

Utilizing Lemma2, for $\mathbf{x}(i_k h)\in L(\mathbf{K}-\mathbf{H})$, the following relation is true :

$$-\psi^T(\mathbf{Kx})\mathbf{U}(\psi(\mathbf{Kx})-\mathbf{Hx})\geq \mathbf{0} \tag{31}$$

Combining equations (30) and (31) and the upper bound of $\dot{V}$ in (18) yields to:

$$\begin{aligned}\dot{V}(t) \;\leq\; & 2\mathbf{x}^T(t)\mathbf{P}\dot{\mathbf{x}}(t)+\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t)-\mathbf{x}^T(t-\eta)\mathbf{Q}\mathbf{x}(t-\eta)+\eta^2\dot{\mathbf{x}}^T(t)\mathbf{R}\dot{\mathbf{x}}(t)\\ & -[\mathbf{x}(i_kh)-\mathbf{x}(t-\eta)]^T\mathbf{R}[\mathbf{x}(i_kh)-\mathbf{x}(t-\eta)]-[\mathbf{x}(t)-\mathbf{x}(i_kh)]^T\mathbf{R}[\mathbf{x}(t)-\mathbf{x}(i_kh)]\\ & -2\psi^T(\mathbf{Kx}(i_kh)\mathbf{U}[\psi(\mathbf{Kx}(i_kh)-\mathbf{Hx}(i_kh)]+2\boldsymbol{\xi}^T\mathbf{M}[\dot{\mathbf{x}}(t)-\mathbf{A}\mathbf{x}(t)-\mathbf{BK}(x(i_kh))+\mathbf{B}\psi(\mathbf{Kx}(i_kh))]\end{aligned}$$

which can be rearranged as $\dot{V}(t)\leq\boldsymbol{\xi}^T(t)\boldsymbol{\Phi}\,\boldsymbol{\xi}(t)$; where $\boldsymbol{\Phi}=\boldsymbol{\Omega}_1+\boldsymbol{\Omega}_2+\boldsymbol{\Omega}_2^T$ and

$$\boldsymbol{\Omega}_1=\begin{bmatrix}\mathbf{Q}-\mathbf{R} & \mathbf{R} & \mathbf{P} & \mathbf{0} & \mathbf{0}\\ * & -2\mathbf{R} & \mathbf{0} & \mathbf{R} & \mathbf{H}^T\mathbf{U}\\ * & * & \eta^2\tilde{\mathbf{R}} & \mathbf{0} & \mathbf{0}\\ * & * & * & \mathbf{Q}-\mathbf{R} & \mathbf{0}\\ * & * & * & * & -2\mathbf{U}\end{bmatrix}$$

$$\boldsymbol{\Omega}_2=\begin{bmatrix}-\mathbf{MA} & -\mathbf{MBK} & \mathbf{M} & 0 & \mathbf{MB}\end{bmatrix}$$

If $\boldsymbol{\Phi}<0$, the Lyapunov-Krasovskii Theorem guarantees that the system (20) is asymptotically stable. The inequality condition $\boldsymbol{\Phi}<0$ is nonlinear; thus by the changing variables**,** it is transformed to LMI condition.

Let $\mathbf{M}_1=\mathbf{M}_0$, $\mathbf{M}_2=p_2\mathbf{M}_0$, $\mathbf{M}_3=p_3\mathbf{M}_0$, $\mathbf{M}_4=p_4\mathbf{M}_0$, $\mathbf{X}=\mathbf{M}_0^{-1}$ and $\mathbf{Z}=\mathrm{diag}(\mathbf{X},\mathbf{X},\mathbf{X},\mathbf{X})$. Defining $\tilde{\boldsymbol{\Phi}}=\mathbf{Z}\,\boldsymbol{\Phi}\,\mathbf{Z}^T=\tilde{\boldsymbol{\Omega}}_1+\tilde{\boldsymbol{\Omega}}_2+\tilde{\boldsymbol{\Omega}}_2^T$ with $\tilde{\boldsymbol{\Omega}}_i=\mathbf{Z}\boldsymbol{\Omega}_i\mathbf{Z}^T$, $i=1,2$, $\tilde{\mathbf{P}}=\mathbf{X}\,\mathbf{P}\,\mathbf{X}^T$, $\tilde{\mathbf{Q}}=\mathbf{X}\,\mathbf{Q}\,\mathbf{X}^T$, $\tilde{\mathbf{R}}=\mathbf{X}\,\mathbf{R}\,\mathbf{X}^T$, $\mathbf{Y}=\mathbf{K}\,\mathbf{X}^T$, $\mathbf{G}=\mathbf{H}\,\mathbf{X}^T$, $\tilde{\mathbf{U}}=\mathbf{U}^{-1}$, the condition $\tilde{\boldsymbol{\Phi}}<0$ implies that $\boldsymbol{\Phi}<0$. The reminder of proof is the same as the Theorem 1 and omitted for the sake of brevity. ■

Corollary 2: Let $\mathbf{K},\mathbf{H} \in \Re^{m\times n}$ be given. The closed-loop system (3) is asymptotically stable if there exist matrices $\mathbf{P}>0, \mathbf{Q}>0, \mathbf{R}>0$, diagonal $\mathbf{U}>0$ and $\mathbf{M}$ such that the following LMIs hold:

$$\mathbf{\Phi}<0 \tag{32}$$

$$\begin{bmatrix} \bar{u}_s & \mathbf{k}_s-\mathbf{h}_s \\ * & \bar{u}_s\mathbf{P} \end{bmatrix} \geq 0, \quad s=1,2,\ldots,m \tag{33}$$

where

$$\mathbf{\Phi}=\mathbf{\Omega}_1+\mathbf{\Omega}_2+\mathbf{\Omega}_2^T.$$

$$\mathbf{\Omega}_1=\begin{bmatrix} \mathbf{Q}-\mathbf{R} & \mathbf{R} & \mathbf{P} & \mathbf{0} & \mathbf{0} \\ * & -2\mathbf{R} & \mathbf{0} & \mathbf{R} & \mathbf{H}^T\mathbf{U} \\ * & * & \eta^2\mathbf{R} & \mathbf{0} & \mathbf{0} \\ * & * & * & \mathbf{Q}-\mathbf{R} & \mathbf{0} \\ * & * & * & * & -2\mathbf{U} \end{bmatrix}$$

$$\mathbf{\Omega}_2=\begin{bmatrix} -\mathbf{MA} & -\mathbf{MBK} & \mathbf{M} & 0 & \mathbf{MB} \end{bmatrix}$$

The optimization problem for domain of attraction is similar to (25).

IV. Illustrative Example

To illustrate the efficiency of the proposed method, a numerical example is presented and the result obtained from the proposed method is compared with the ones in the literature.

Example: Consider the system (3) with the following matrices [9]:

$$\mathbf{A}=\begin{bmatrix} 1.1 & -0.6 \\ 0.5 & -1 \end{bmatrix}, \ \mathbf{B}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

and $\bar{u}_1=\bar{u}_2=5$. The controller is designed and corresponding domain of attraction is obtained.

The results are summarized in Table I, wherein $\delta_{\max}$ stands for the radius of domain of attraction and $\eta_{\max}$ is the maximum attainable η which was defined in (4).

Table I. Stability ball radius and state feedback controller

	Theorem1		Theorem2		[9]	[12]
	$\eta_{\max}=1.052$	$\eta=0.75$	$\eta_{\max}=1.050$	$\eta=0.75$	$\eta_{\max}=0.75$	$\eta_{\max}=0.75$
$\delta_{\max}$	0.3248	1.9361	0.3852	1.9999	0.356	0.23
K	$\begin{bmatrix}-1.3822\\0.4262\end{bmatrix}^T$	$\begin{bmatrix}-1.4628\\0.4520\end{bmatrix}^T$	$\begin{bmatrix}-1.3821\\0.4262\end{bmatrix}^T$	$\begin{bmatrix}-1.4936\\0.4658\end{bmatrix}^T$	$\begin{bmatrix}-1.696\\0.533\end{bmatrix}^T$	$\begin{bmatrix}-1.7491\\0.5417\end{bmatrix}^T$

The method of [9], lead to the feedback gain $K=\begin{bmatrix}-1.696 & 0.533\end{bmatrix}$ to stabilize the closed-loop system for the sampling interval $\eta=0.75$ and the set of admissible initial conditions is given by an ellipsoid $\mathrm{E}(\mathbf{P},1)$ with

$$\mathbf{P}=\begin{bmatrix}0.9132 & -0.2816\\-0.2816 & 0.0868\end{bmatrix}.$$

The largest circle can be included in this ellipsoid is of radius 0.356 which is approximately six times smaller than the one obtained from Theorem 2 $(1.9999/0.356\approx 6)$.

Using the approach of [12], the feedback gain is obtained as $K=\begin{bmatrix}-1.7491 & 0.5417\end{bmatrix}$ with the sampling interval $\eta=0.75$ and the corresponding set of admissible initial conditions is given by an ellipsoid $\mathrm{E}(\mathbf{P},1)$ with

$$\mathbf{P}=\begin{bmatrix}0.4450 & 0.2307\\0.2307 & 21.0091\end{bmatrix}.$$

The largest circle can be included in this ellipsoid is of radius 0.23 which is approximately nine times smaller than the one obtained from Theorem 2 $(1.999/0.23 \approx 9)$.

Moreover, by the proposed methods maximum allowable $\eta_{\max}$ can be increased up to 1.052 which is considerably comparable with the $\eta_{\max}$ obtained from approaches in [9] and [12].

Figures 2 and 3 illustrate the convergence of state trajectories to the origin, by using the controller obtained from Theorem 2 for two different values of η. The inner ellipse in figure 2 shows the estimate of the domain of attractions. The outer ellipse in figure 3 shows the ellipsoid $\mathbf{x}^T \mathbf{P} \mathbf{x} \le \beta^{-1}$, as seen all state trajectories begin on the periphery of the inner ellipse never leave the outer ellipsoid and end up at the origin. Comparison of figures 2 and 3 together with the information in Table I clarify that there is inverse relation between η and $\delta_{\max}$.

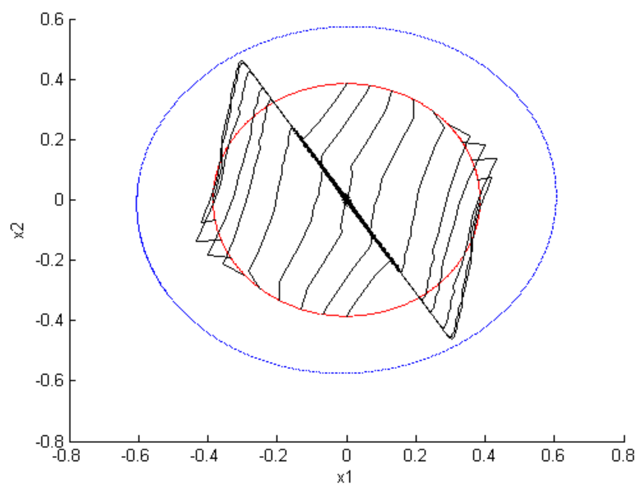

Figure 2. State trajectories and stability ball ($\eta = 1.05$)

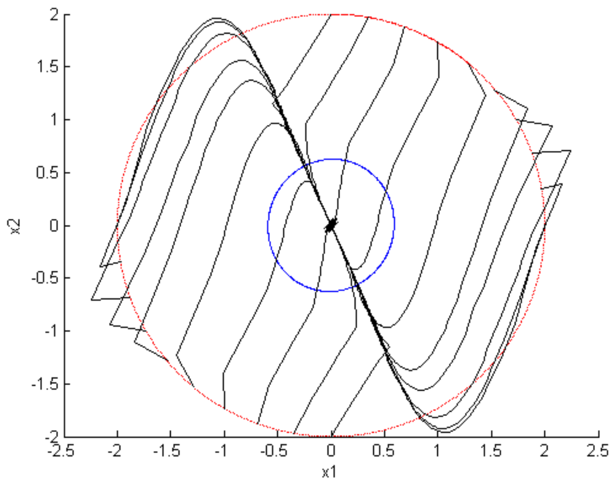

Figure 3. State trajectories and stability ball ($\eta = 0.75$)

V. CONCLUSION

In this paper two procedures were presented to design stabilizing controller for networked control system subject to input saturation. In the first method,the saturated linear system was represented with a set of linear systems embedded within a convex polytope and in the second method, actuator saturation was tackled via a generalized sector condition. Furthermore, an estimate of domain of attraction was obtained through the LMI optimization. Illustrative example demonstrated that the suggested methods leads to the less conservative result compared with the ones in the literature.

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