Analysis and Design of a Permanent-Magnet Outer-Rotor Synchronous Generator for a Direct-Drive Vertical-Axis Wind Turbine

H. A. Lari*, A. Kiyoomarsi*(C.A.), B. Mirzaeian Dehkordi*, A. Darijani* and S. M. Madani*

Abstract: In Permanent-Magnet Synchronous Generators (PMSGs) the reduction of cogging torque is one of the most important problems in their performance and evaluation. In this paper, at first, a direct-drive vertical-axis wind turbine is chosen. According to its nominal value operational point, necessary parameters for the generator is extracted. Due to an analytical method, four generators with different pole-slot combinations are designed. Average torque, torque ripple and cogging torque are evaluated based on finite element method. The combination with best performance is chosen and with the analysis of variation of effective parameters on cogging torque, and introducing a useful method, an improved design of the PMSG with lowest cogging torque and maximum average torque is obtained. The results show a proper performance and a correctness of the proposed method.

Keywords: Cogging Torque, Finite Element Method, PMSG, Vertical-Axis Wind Turbine.

1 Introduction

PMSGs are one of the important parts in wind energy conversion systems. With using PMSG in the direct-drive wind power generation systems, it is possible to extract wind energy at wider range of wind speed [1]. PMSG purveys a high performance, compact size, light weight, and low noise high thrust, and the ease of maintenance. Most WECSs (wind energy conversion system) at low wind speed usually use PMSGs. These generators have advantages of high efficiency and reliability, since there is no need of external excitation and loss of drivers are removed from the rotor [2]. Also there is no need to have gearboxes.

Most of the time, wind speed is small [3]. Hence In order to extract as much power as possible from the wind and reducing the payback period of investment, it is urgent that the turbine can start and run even at a small wind speed. So it can be concluded that, the cut-in speed of wind turbine affects considerably on commercial aspects of wind power generation systems. In fact, cogging torque is one of the inherent features of PMSG and it can be affected the cut-in wind speed [4, 5]. Therefore in order to have the system more commercially, it is needed to remove this torque as much as possible.

In this paper, a 20 kW wind turbine is selected and by direct-drive coupling, the essential parameters for designing a PMSG is derived. Using an analytical method based on electrical machine theory, four electrical generators with different pole-slot combinations are firstly designed. The designed models are evaluated according to torque versus tip speed ratio of the wind turbine. Using finite element method and the definition of most effective parameters on the characteristics of the PMSGs, an improved design of generators is also derived.

2 Characteristics of Wind Turbine

Vertical-axis wind turbines (VAWTs) usually have characteristics such as independence of wind direction, reducing noise and power fluctuations, control system more simple and inexpensive, and more compatible with the environment [6]. Hence these turbines have received more attention in recent years. So that several studies in order to complete and improve the various parts of VAWT technology have been done [7, 8]. Among the different types of VAWTs, H-type wind turbine due to its features such as simplicity and robustness housing, flexibility in design in order to access the high wind speeds, and low maintenance costs are more economical and have received more attention [9]. On this basis, and according to the studies, in this paper, a VAWT, H-type with the specifications included in Table 1 is preliminary considered.
After identifying the characteristics of the turbine, the next step should be the determination of the necessary parameters to design the generators. Therefore, the design of generator based on nominal values will be carried out. Since the gearless structure of the turbine is taken into account, the rated speed of rotation of the blades will be considered as the synchronous speed. In addition, according to the basic design of generator, input torque should also be specified. To this end, transmitted torque in direct-drive wind turbine is expressed as:

\[ T_T = D \omega + T_g \]  

where \( T_T \) is the wind turbine torque, \( T_g \) is the input torque generator, \( \omega \) is the angular velocity of rotation, \( D \) is the mechanical damping and \( J \) is the moment of inertia constant of both the rotor of the generator and the hub of the wind turbine. Because the design is done in a steady-state condition, variations of speed considered zero and the above equation will be modified as follows:

\[ T_T = D \omega + T_g \]  

### 3.1 PM Dimensions

In order to determine the geometrical dimensions of the permanent magnets, the effective air gap (magnetic air-gap) should be assessed. It should be noted that the large airgap produces a sinusoidal flux density with low harmonic content. But in this case, by increasing the size of the permanent magnets, the weight and cost of the generator will increase. In permanent-magnet machines, the effective length of the airgap will be determined as follows:

\[ L_{ge} = k_c k_s \left( L_g + \frac{h_m}{\mu_{rm}} \right) \]  

where \( L_{ge} \) is actual airgap, \( k_c \) is the Carter Factor, \( k_s \) is a coefficient representing the saturation level of iron in the stator magnetic core, \( h_m \) is magnet height and \( \mu_{rm} \) is the relative recoil permeability of PMs.

The subsequent relation is defined as flux conservation equation [11], i.e.,

\[ B_m \omega_p + B_g \tau_p \approx 0 \]  

where \( B_m \) is the flux density of the magnet, \( \tau_p \) is pole pitch, \( \omega_m \) is width of the permanent-magnet and \( B_g \) is the average flux density of in the air gap. Then the magnet recoil line equation becomes:

\[ \frac{\tau_p}{\omega_m} B_g = \mu_0 \mu_m H_m + B_{rm} \]  

in which \( H_m \) is magnetic field and \( B_{rm} \) is residual magnetic flux density of permanent-magnet. Therefore, the Ampere's circuit law reforms as follows:

\[ B_g = \frac{k_c k_s L_g}{h_m} \]  

Using Eqs. (5-6) and remove \( H_m \), the thickness of the permanent-magnet will be calculated by the following equation:

\[ h_m = \frac{B_{rm} \omega_m}{B_g} - \frac{\tau_p}{\omega_m} \]  

### 3.2 Stator and Rotor Dimensions

In the analytical method, the coefficient of tangential stress is used to determine the principle generator dimensions. This factor connects the volume of an area with a diameter \( D_{ag} \) to generator’s input torque.

\[ T_g = \sigma_{Fam} \pi \frac{D^2}{2} L_{ag} = 2 \sigma_{Fam} V_{ag} \]  

where \( \sigma_{Fam} \) is tangential stress, \( D_{ag} \) is airgap diameter and \( L_{ag} \) is generator length. The length to diameter ratio of the PMSG will be defined as follows:

\[ \chi = \frac{\pi \sqrt{P}}{4P} \]  

where \( P \) is the number of pole-pairs. With using Eqs. (8) and (9), the diameter airgap and length of the generator will be calculated as the following equations.
Another important parameter, the thickness of the stator yoke, should be determined by the maximum flux density allowed in the stator:

$$h_{sy} = \frac{B_{m} \omega_{m}}{2k_{j} B_{sy}}$$  \hspace{1cm} (12)

where $B_{sy}$ is the maximum allowable flux density in the stator and $k_{j}$ is the stacking factor of the iron lamination. Usually in design of PM machines, maximum flux density allowed of the stator and the rotor is assumed to be almost identical. Hence, the rotor yoke thickness is considered equal to the thickness of the stator yoke.

### 3.3 Stator Windings

In low-speed applications, PMSM with distributed winding is not recommended. Because of the high number of slots in the stator, distributed winding causes complexity of the manufacturing processes; and machine dimensions will be larger [12]. Therefore, in order to achieve the minimum size, machines with high pole numbers, in addition to use of concentrated winding, the number of slots is chosen close to poles number [13]. In fact, with the use of fractional-slot concentrated winding, the short end windings, a low cogging torque, a good fault-tolerant capability and a high constant power speed range can be achieved [14].

### 4 Inductances

Synchronous inductance plays a vital role in the assessment of torque and performance in PM machines. In fact, in synchronous machines, the direct- and quadrature-axis magnetizing inductances and the stator leakage, form together the synchronous inductance, which can be used in the evaluation of the machine torque production in analytical methods. Hence, in this section, the inductance components, in concentrated winding permanent-magnet synchronous machine, is introduced and analytical relations are derived.

Magnetizing inductance for integral-slot multi-phase machines can be determined as follows [10]:

$$L_{md} = \frac{2m \mu_{0} \tau_{s}}{\pi P \pi L_{ge}} L_{ac} \left(k_{w} \cdot N_{ph}\right)^2$$  \hspace{1cm} (13)

in which $m$ is the number of phases; $k_{w}$ is the fundamental component of the winding factor; and $N_{ph}$ is the number of windings per phase. Above equation is based on the idea that, along entire pole pitch, the flux distribution assumed to be sinusoidal. But it wills not the case in concentrated winding machines, where the flux is concentrated mainly on the tooth area in the airgap. Generally, in fractional-slot concentrated winding machines, area of airgap flux passing through it to produce torque, includes $Q/m$ windings and an area $\tau_{s} L_{ac}$. Hence, the magnetizing inductance for three-phase concentrated windings machine is proposed as follows:

$$L_{md} = \frac{2m \mu_{0} \tau_{s}}{\pi} \frac{Q}{m} \pi L_{ge} \left(k_{w} \cdot N_{ph}\right)^2$$  \hspace{1cm} (14)

The leakage inductance includes five components, which are airgap leakage inductance, slot leakage inductance, tooth tip leakage inductance and skew leakage inductance, and the last one is not considered in this study.

Airgap flux leakage is different from other types. This flux passes through the airgap [10]. Airgap leakage inductance could be determined as:

$$L_{leakage} = L_{md} \sigma_{l}$$  \hspace{1cm} (15)

where the factor $\sigma_{l}$ can be modified from the winding harmonic contents and can be calculated as follows:

$$\sigma_{l} = \sum_{v=1}^{\infty} \left(\frac{k_{w}}{v} \cdot \frac{k_{w}}{v}\right)^2$$  \hspace{1cm} (16)

where $v$ is harmonic order and $k_{w}$ is winding factor of the related space harmonic.

The slot leakage inductance can be defined as:

$$L_{slot} = \frac{4m q N_{ph} \mu_{r} L_{ac} \lambda_{u}}{Q_{s}}$$  \hspace{1cm} (17)

where the slot leakage factor $\lambda_{u}$ depends on the shape and dimensions of the slot and the winding construction. Different parts of a slot are defined in Fig. 2. In this case, for a two-layer winding, slot leakage factor can be calculated as follows:

$$\lambda_{u} = k_{1} \frac{h_{3} - h_{0}}{3b_{s2}} + k_{2} \frac{h_{2} - h_{1}}{b_{s2}} - h_{1} - b_{1} \frac{b_{1} - b_{0}}{b_{1} - b_{0}} + \frac{h_{0}}{4b_{s2}}$$  \hspace{1cm} (18)

Fig. 2 Geometric parameters of the generator slots.
where $h_0$ is the thickness of isolation between conductors and factors $k_1$ and $k_2$ can be calculated as functions of relative pitch winding to pole-pitch [10].

The end winding leakage is produced by the magnetic field surrounded a coil after it leaves one slot and before it enters another slot. The end winding inductance can be calculated as follows:

$$L_{\text{end winding}} = \frac{4mq N_2^2 L_{\text{end,avg}} \lambda_{\text{w}}}{Q_i} \quad (19)$$

where $L_{\text{end,avg}}$ is average length of the end winding and $\lambda_{\text{w}}$ is winding leakage factor that is defined as:

$$\lambda_{\text{w}} = \frac{2L_{\text{ew}} \lambda_{\text{ew}} + w_{\text{ew}} \lambda_{\text{ew}}}{L_{\text{end,avg}}} \quad (20)$$

where $\lambda_{\text{ew}}$ and $\lambda_{\text{ew}}$ are reactance factors; $L_{\text{ew}}$ is the height and $w_{\text{ew}}$ is the width of end winding. In fact, the end winding reactance factors depend on structure of the winding and the number of layers [10].

Tooth tip Leakage inductance is created, by leakage flux penetrating via the airgap to the next tooth and can be calculated as follows:

$$L_{\text{tooth}} = \frac{4mq N_2^2 L_{\text{tooth}} \lambda_{\text{d}}}{Q_i} \quad (21)$$

where $\lambda_{\text{d}}$ is leakage inductance factor and is a function of airgap length and slot opening width calculated as follows:

$$\lambda_{\text{d}} = \frac{5k \frac{L_s}{b_{\text{so}}} + \frac{L_s}{b_{\text{so}}}}{5 + 4 \frac{L_s}{b_{\text{so}}}} \quad (22)$$

After determination of the leakage inductance of the parts, total leakage inductance, as their sum, will be achieved as follows:

$$L_{\text{leak}} = L_{\text{tooth}} + L_{\text{air gap}} + L_{\text{end winding}} + L_{\text{slot}} \quad (23)$$

Thus, when the inductances are known the torque can be predicted. In fact, the torque developed by a surface-mounted PM machine is:

$$T = \frac{m P}{\omega_s} \left( \frac{E_{\text{pm}} U}{L_d} \sin(\delta) \right) \quad (24)$$

where $E_{\text{pm}}$ is the induced back EMF, $\omega_s$ is angular speed of stator field and $\delta$ is load angle. The torque curve as a function of load angle for the designed PM generators.

5 Torque Characteristics

In PMSGs, pole-slot combination will effect on parameters such as fundamental winding factor, ripple and cogging torque, noise and vibration, rotor losses and machine inductances [15]. Hence, after determining the number of poles, by synchronous speed and frequency of the stator currents, the number of slots should be carefully selected. In fact, in a PMSG, the number of periods of cogging torque per mechanical revolution (CPMR) determines by number of poles and slots as follows [16]:

$$\text{CPMR} = \text{LCM} (Q_s, 2P) \quad (25)$$

where $Q_s$ is the slot number, $P$ is pole-pair number and LCM stands for least common multiple. Also the number of cogging cycles per slot and per pole pair are defined as $N_s$ and $N_p$, respectively and calculated as the following equations:

$$N_s = \text{CPMR}/Q_s \quad (26)$$

$$N_p = \text{CPMR}/P \quad (27)$$

It should be noted that higher values of $N_s$ are preferable since the pole-slot combinations with high CPMR have low cogging torque amplitude. On the other hand, the ripple of electromagnetic torque is mostly due to interaction of magnet field and slotting. So the number of torque ripples per electrical cycle is defined as:

$$N_r = \text{CPMR}/2P \quad (28)$$

Cogging torque is the torque due to the interaction between the permanent magnets of the rotor and the stator slots of a permanent magnet machine. This torque is position dependent and its periodicity per revolution depends on the number of magnetic poles and the number of teeth on the stator; but, the torque ripple in electrical machines is caused by many factors such as cogging torque, the interaction between the MMF and the airgap flux harmonics, or mechanical imbalances.
In Fig. 4, electromagnetic torque of the designed PM generators with different pole slot combination and, same \( q \), is shown and in Tables 2 and 3, the characteristic of generator’s torque and main parameters of designed generators are respectively compared.

As can be seen in Fig. 4, with increasing the number of poles for a same \( q \) in outer-rotor PMSG, average value of the electromagnetic torque boosts and torque ripple and cogging torque are reduced. One reason for the increase in the average electromagnetic torque is because of the reduction of copper losses with same \( q \) at the higher poles [17]. On the other hand, with fixed \( q \), increasing the number of poles will decrease the generator weight. In fact, it can be concluded that the 60-pole generator is better than other designs in terms of weight, and performance.

6 Influence of Design Parameters

Magnet pole arc, slot opening, skewing of rotor magnets or stator, step-skew of magnets, creating an artificial gap in the teeth and artificial slots, the slot wedge, magnet shifting and airgap variation are some effective techniques for reducing cogging torque and torque ripple [18, 19]. As was mentioned, the right choice of pole-slot combination is important for the design of PMSG with low cogging torque. After that, the other design parameters can be optimized for minimizing cogging torque. Among these parameters, optimization of slot opening width and permanent magnet arc play an important role in reducing the cogging torque. In fact, by design optimization of these parameters, cogging torque can be significantly reduced without making any difficulty for manufacturers and without increasing the cost in methods such as in skewed magnets, skewing stator teeth and airgap variations [20]. To this end, in this section, the effect of slot opening and magnet arc on cogging torque and average torque are investigated. An efficient method for optimum selection of these parameters in order to minimize the cogging torque in PM machines with fractional-slots and concentrated winding will be presented.

6.1 Permanent-Magnet Width

Permanent-magnet width is one of the main parameters that affects cogging torque. This parameter is important because it directly affects the amount of air gap flux density and consequently the electromagnetic torque. Hence, in this section, to select the optimum width of the permanent-magnet, in addition to its effect in reducing cogging torque, the average value of the electromagnetic torque is also considered. If \( \omega \) is considered as the ratio of permanent-magnet width to pitch pole, for a fractional-slot PM machine, the appropriate value of this, to reduce the cogging torque will be calculated as follows:

\[
\omega = k \frac{2P}{Q} - N
\]

where \( N = 0, 1, 2, ..., 2P - 1 \) and \( k = 1, 2, ..., 2Q - 1 \). The values of \( \omega \) for different pole-slot combination are given in Table 4.

### Table 2 Characteristic torque of different generators.

<table>
<thead>
<tr>
<th>Poles number</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average torque (Nm)</td>
<td>2267</td>
<td>2328</td>
<td>2461</td>
<td>2564</td>
</tr>
<tr>
<td>Torque ripple</td>
<td>18.1%</td>
<td>12.3%</td>
<td>8.9%</td>
<td>7.7%</td>
</tr>
<tr>
<td>Cogging torque</td>
<td>0.96%</td>
<td>0.92%</td>
<td>0.8%</td>
<td>0.78%</td>
</tr>
</tbody>
</table>

### Table 3 The main parameters for designed generators.

<table>
<thead>
<tr>
<th>Pole number</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slot number</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
</tr>
<tr>
<td>Outer diameter (mm)</td>
<td>704</td>
<td>731</td>
<td>754</td>
<td>774</td>
</tr>
<tr>
<td>Inner diameter (mm)</td>
<td>496</td>
<td>528</td>
<td>554</td>
<td>574</td>
</tr>
<tr>
<td>Overall length (mm)</td>
<td>134</td>
<td>122</td>
<td>113</td>
<td>107</td>
</tr>
<tr>
<td>Active weight (kg)</td>
<td>192</td>
<td>177</td>
<td>153</td>
<td>134</td>
</tr>
</tbody>
</table>

In Fig. 4, electromagnetic torque of the designed PM generators with different pole slot combination and, same \( q \), is shown and in Tables 2 and 3, the characteristic of generator’s torque and main parameters of designed generators are respectively compared.

### Table 4 Optimal values of \( \omega \) and \( \beta \) in different pole slot combinations.

<table>
<thead>
<tr>
<th>( Q )</th>
<th>( 2P )</th>
<th>( Q )</th>
<th>( 2P )</th>
<th>( Q )</th>
<th>( 2P )</th>
<th>( Q )</th>
<th>( 2P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>72</td>
<td>78</td>
<td>72</td>
<td>60</td>
</tr>
<tr>
<td>( \omega )</td>
<td>( \beta )</td>
<td>( \omega )</td>
<td>( \beta )</td>
<td>( \omega )</td>
<td>( \beta )</td>
<td>( \omega )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>0.83</td>
<td>1</td>
<td>0.83</td>
<td>0.857</td>
<td>0.917</td>
<td>0.923</td>
<td>0.83</td>
<td>1</td>
</tr>
<tr>
<td>0.67</td>
<td>0.8</td>
<td>0.87</td>
<td>0.714</td>
<td>0.83</td>
<td>0.846</td>
<td>0.67</td>
<td>0.8</td>
</tr>
<tr>
<td>0.50</td>
<td>0.6</td>
<td>0.50</td>
<td>0.571</td>
<td>0.75</td>
<td>0.769</td>
<td>0.50</td>
<td>0.6</td>
</tr>
<tr>
<td>0.33</td>
<td>0.4</td>
<td>0.33</td>
<td>0.429</td>
<td>0.67</td>
<td>0.692</td>
<td>0.33</td>
<td>0.4</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>0.286</td>
<td>0.583</td>
<td>0.615</td>
<td>0.538</td>
<td>0.2</td>
<td>0.538</td>
</tr>
</tbody>
</table>
Using finite element analysis, the treatment of cogging torque of a 60-pole generator as a function of $\omega$ and for a fixed slot opening width, is shown in Fig. 5. As can be seen, the values of $\omega$ that obtained low cogging torque, are coincident on the optimal values derived from Eq. (17).

The cogging torque for these values is shown in Fig. 6. In fact it can be concluded that permanent-magnet width, affects strongly on cogging torque. On the other hand, in order to achieve a high flux density in the airgap and thereby a high torque, the optimal magnet width should be selected as wide as possible. In Fig. 7 it is clear that for small values of $\omega$, the generator performance will be weakened.

### 6.2 Slot Opening

In PM machines, slot opening leading to the airgap flux density is inhomogeneous. In these machines, the radial flux density at the position of the front of slots with low permeability is lower than to the position of the front teeth with high permeability. This non-uniformity of the airgap flux density will result cogging torque. Radial flux density for different slot opening width is shown in Fig. 8. As seen with decreasing slot opening width, the air gap flux density distribution will be more uniform and it is expected that cogging torque should be lower. In this case, if $\beta$ is considered as the ratio of tooth width to slot pitch, for a fractional-slot PM machine, the appropriate values of this, in order to reduce the cogging torque, will be calculated as follows:

$$\beta = \frac{N Q s}{2P} - k$$  \hspace{1cm} (30)

where $k = 0, 1, 2, ..., Q_s - 1$ and $N = 1, 2, ..., 2P - 1$.

The values of $\beta$ for different pole-slot combinations are also given in Table 4. As can be seen in this Table, the optimal values for $\omega$ and $\beta$ for pole-slot combinations, with the similar $q$, are identical. The case $\beta = 1$, where the slots were closed, due to the rising costs of construction of machinery [20], is ignored.

The cogging torque for $\omega = 0.83$ and the optimal $\beta$ is obtained, using the above equations, and is shown in Fig. 9.

As can be seen for larger values of $\beta$ or lower values, the slot opening width becomes less than before, and cogging torque will be lower.
In Fig. 10 the average value of the electromagnetic torque as a function of the slot opening width is shown. It can be observed that this parameter affects on the average value of electromagnetic torque. So that if slot opening width is larger than 9 mm the reluctance is increasing and the flux linkage is also decreasing. Therefore, the torque due to interaction between magnet field and magneto motive force, is reduced.

7 Conclusions

In this paper, the analysis and design of an outer-rotor permanent-magnet synchronous generator for using in vertical-axis wind turbines are studied. Four PM synchronous generators (PMSGs) with different pole number and the same number of slot per pole per phase (q) are designed based on an analytical method. In this paper, based on the finite element analysis, it is showed that increasing pole numbers of PMSG, with the same number of slot per pole per phase, electromagnetic torque is increased and torque ripple magnitude is decreased. The influence of design parameters, such as permanent-magnet arc and slot opening width, on the cogging torque and average torque is discussed. Also an efficient method for selecting the optimal value of these parameters, for minimizing the cogging torque in concentrated-winding PM machines is presented. By comparing the results of the proposed method and the finite element analysis, the efficiency and accuracy of the method is confirmed.

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Appendix

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_m$</td>
<td>Flux density of the permanent-magnet</td>
</tr>
<tr>
<td>$D_{ag}$</td>
<td>Air gap diameter</td>
</tr>
<tr>
<td>$H_m$</td>
<td>The magnetic field</td>
</tr>
<tr>
<td>$h_m$</td>
<td>The magnet height</td>
</tr>
<tr>
<td>$h_{ys}$</td>
<td>Thickness of the stator yoke</td>
</tr>
<tr>
<td>$L_{ax}$</td>
<td>Generator effective length</td>
</tr>
<tr>
<td>$L_{ge}$</td>
<td>The actual (mechanical) airgap</td>
</tr>
<tr>
<td>$L_t$</td>
<td>The effective length of the airgap</td>
</tr>
<tr>
<td>$N_{ph}$</td>
<td>Number of windings per phase</td>
</tr>
<tr>
<td>$P$</td>
<td>Number of pole-pairs</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Ratio of teeth width to slot pitch</td>
</tr>
<tr>
<td>$\sigma_{tan}$</td>
<td>The tangential stress</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>Pole pitch</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Ratio of permanent-magnet width to pitch pole</td>
</tr>
<tr>
<td>$\omega_{pm}$</td>
<td>Width of the permanent-magnet</td>
</tr>
</tbody>
</table>

References


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