

A Novel DOA Estimation Approach for Unknown Coherent Source Groups with Coherent Signals

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Abstract: In this paper, a new combination of Minimum Description Length (MDL) or Eigenvalue Gradient Method (EGM), Joint Approximate Diagonalization of Eigenmatrices (JADE) and Modified Forward-Backward Linear Prediction (MFBLP) algorithms is proposed which determines the number of non-coherent source groups and estimates the Direction Of Arrivals (DOAs) of coherent signals in each group. First, the MDL/EGM algorithm determines the number of non-coherent signal groups, and then the JADE algorithm estimates the generalized steering vectors considering white/colored Gaussian noise. Finally, the MFBLP algorithm is applied to estimate DOAs of coherent signals in each group. A new Normalized Root Mean Square Error (NRMSE) is also proposed that introduces a more realistic metric to compare the performance of DOA estimation methods. Simulation results show that the proposed algorithm can resolve sources regardless of QAM modulation size. In addition, simulations in white/colored Gaussian noises show that the proposed algorithm outperforms the JADE-MUSIC algorithm in a wide range of Signal to Noise Ratios (SNRs).

Keywords: DOA, EGM, JADE, MDL, MFBLP, RMSE.

1 Introduction

In wireless communications, electromagnetic waves experience fading and multipath phenomena which introduce coherent signals in receiver. Coherent signals leads to rank deficiency in the spatial covariance matrix. Therefore, some second-order-statistics-based subspace methods, such as Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT), fail to correctly resolve the signals, thus the estimation of the Direction Of Arrival (DOA) is not possible [1, 2].

To overcome this problem, many effective methods, such as spatial smoothing [3], Maximum Likelihood (ML) [4], deflation approach [5], method in [6] and method based on Matrix Pencil (MP) [7], are proposed. The spatial smoothing algorithm is based on a preprocessing scheme, which divides an original array into several overlapping subarrays, computes covariance

matrix for each subarray, and then applies high-resolution methods to the covariance matrices in order to resolve the signals [3]. In the deflation approach, uncorrelated signals are first estimated and then eliminated and finally coherent signals will be resolved [5]. A different technique presented in [6] utilizes symmetric configuration of Uniform Linear Array (ULA) and constructs non-Toeplitz matrix to resolve signals. The proposed method in [7] utilizes idea of MP on the spatial samples of the data. The MP method can find DOA of coherent signals without spatial smoothing. However, spatial smoothing requires more sensors for preprocessing and the ML approach is computationally intensive. The deflation approach has significant loss of array aperture, because it cannot utilize all the constructed Toeplitz matrices [5] and [8]. Also, the method in [6] suffers from burdensome computation and shortcoming of MP is its requirement for high Signal to Noise Ratio (SNR). Many other efforts based on Fourth Order Cumulants (FOC), such as Steering Vectors DOA (SV-DOA) [9], modified MUSIC [10], methods in [11-14], have been also made for DOA estimation of coherent signals. SV-DOA, first, estimates the steering vectors blindly and then, it utilizes Modified Forward-Backward Linear Prediction (MFBLP) to estimate DOA using the estimated vectors. Modified MUSIC in [10], utilizes switching matrix to construct a new covariance matrix. Method in [11], first,

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constructs the data vector which contains all DOA information. Then, it constructs rotation matrix and finally estimates signal DOAs. In [12], Joint Approximate Diagonalization of Eigenmatrices (JADE) and MUSIC were combined, as JADE-MUSIC, to estimate source groups containing coherent signals. Method represented in [13], first, estimates the independent signals, and then eliminates them. Finally, it constructs FOC matrix of coherent signals and also resolves them. Method in [14], combines two algorithms, JADE and MFBLP, to estimate signals. However, SV-DOA requires a large number of snapshots. Computational complexity of the proposed methods in [10] and [12] is relatively high due to spatial spectrum calculation and peak searching. Moreover, the ESPRIT method [11] needs special array geometry [11] and the proposed method in [13] requires a large number of snapshots.

All above mentioned DOA estimation algorithms suppose that the number of coherent and/or non-coherent signals is known or it is determined, approximately. In order to evaluate the number of sources, Wax and Kailath suggested the use of the Minimum Description Length (MDL) criterion [15]. Also, many other criteria such as Eigenvalue Gradient Method (EGM) [16] and Eigen Increment Threshold (EIT) [17] were proposed. The EGM criterion, first computes eigenvalues of auto-correlation matrix, then it determines the number of sources by checking the difference between neighbor eigenvalues. The EIT method utilizes a threshold to evaluate the eigenincrement between neighbor eigenvalues. According to the simulation results in [18] for MDL and EGM methods, the MDL is more appropriate for AWGN channel but EGM is suitable for colored noise.

In this paper, we extend the work in [14] for estimating signal groups. Unlike [14], the number of signal groups is considered to be unknown. In the proposed approach, first MDL/EGM is used to detect the number of coherent signal groups with considering white/colored Gaussian noise. Then, generalized steering vectors are estimated with the JADE algorithm. Finally, MFBLP algorithm is used to estimate DOAs. We also compare the performance of the proposed scheme with JADE-MUSIC equipped with MDL and EGM techniques to determine the number of signal groups.

Here we use notations $(\cdot)^T$, $(\cdot)^*$, $E(\cdot)$, $(\cdot)^\#$ and $(\cdot)^H$ which denote transpose, conjugate, expectation value, pseudo inverse and conjugate transpose, respectively. Also, vectors are denoted by bold small letters and matrices are denoted by bold capital letters.

This paper is organized as follows. In Section 2, the signal model for coherent signals is described. Section 3 presents MDL and EGM criteria. In Section 4, steering vector estimation and estimating DOAs with MFBLP algorithm are summarized. In Section 5, simulation results are presented. These results include comparison

of DOA estimation performance of [12] and the proposed algorithm. Finally, conclusions are drawn in Section 6.

2 Signal Model

Let us assume that N signals; $s_i(k)$; $i=1, 2, \dots, N$, impinge on an M -element antenna array from directions θ_i . Then, the received signal is

$$\mathbf{X}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{n}(k), \quad k = 1, 2, \dots, N_s \quad (1)$$

where N_s is the number of snapshots and \mathbf{A} is the array response (manifold) matrix given by:

$$\mathbf{A} = [\mathbf{a}(\theta_1) \mathbf{a}(\theta_2) \dots \mathbf{a}(\theta_N)] \quad (2)$$

We suppose that a uniformly spaced linear sensor array is used. So, the steering vector, $\mathbf{a}(\theta_k)$, can be expressed as follows.

$$\mathbf{a}(\theta_k) = [1 e^{\frac{j2\pi d \sin \theta_k}{\lambda}} \dots e^{\frac{j2\pi d(M-1) \sin \theta_k}{\lambda}}]^T \quad (3)$$

where d and λ are sensor inter-element spacing and signal wavelength, respectively. In Eq. (1), \mathbf{s} is a vector of signals:

$$\mathbf{s}(k) = [s_1(k) s_2(k) \dots s_N(k)]^T \quad (4)$$

and \mathbf{n} is a vector of additive Gaussian noise.

$$\mathbf{n}(k) = [n_1(k) n_2(k) \dots n_M(k)]^T \quad (5)$$

Since the fourth-order cumulants of Gaussian signals are zero [19], we assume that signals are statistically independent and they are non-Gaussian zero-mean complex random processes with symmetric distribution. Also, we suppose that there are K statistically independent groups that each group contains G coherent signals. Hence, source vector is given by:

$$\mathbf{s}(k) = [s_1(k) s_2(k) \dots s_K(k)]^T \quad (6)$$

and

$$\mathbf{A} = [\mathbf{a}(\theta_{ef_1}) \mathbf{a}(\theta_{ef_2}) \dots \mathbf{a}(\theta_{ef_K})] \quad (7)$$

For instance

$$\mathbf{a}(\theta_{ef_1}) = \sum_{m=1}^G \alpha_m \cdot \mathbf{a}(\theta_m) \quad (8)$$

where, α_m is a complex scalar that describes the gain and phase associated to path in coherent signals. Therefore, G signals will be appeared as a single signal that it arrives from new θ_{ef} direction.

3 Determining the Number of Sources

An MDL criterion is used to determine the number of independent groups in AWGN scenario. The MDL criterion cannot determine the number of sources in colored noise scenario [18]. So, EGM criterion is used for this purpose in colored noise scenario.

3.1 MDL Criterion

Steps of MDL are presented as follows [15]:

- 1) Form the array covariance matrix as follows

$$\mathbf{R} = E\{\mathbf{X}(t)\mathbf{X}^H(t)\} \quad (9)$$

- 2) The number of groups is determined as the value of $k \in \{0, 1, 2, \dots, M\}$ that minimizes MDL

$$MDL(k) = -\log \left(\frac{\prod_{i=k+1}^M \lambda_i^{M-k}}{\frac{1}{M-k} \sum_{i=k+1}^M \lambda_i} \right)^{(M-k)N_s} + \frac{1}{2} k(2M-k) \log N_s \quad (10)$$

where $\lambda_1 > \lambda_2 > \dots > \lambda_M$ are eigenvalues of the covariance matrix \mathbf{R} .

3.2 EGM Criterion

Steps of EGM are presented as follows [16].

- 1) Find the average eigenvalues of the covariance matrix \mathbf{R} .

$$\Delta\lambda = \frac{\lambda_1 - \lambda_M}{M-1} \quad (11)$$

- 2) Calculate the gradients of all neighbor eigenvalues.

$$\Delta\lambda_i = \lambda_i - \lambda_{i+1}; \quad i = 1, \dots, M-1 \quad (12)$$

- 3) Find i_0 which is the smallest i that satisfies $\Delta\lambda_i \leq \Delta\lambda$. The number of sources is i_0-1 .

4 Steering Vector Estimation using JADE Algorithm

JADE algorithm is applied to estimate the generalized steering vectors. It is summarized as follows [12].

- 1) Compute whitening matrix \mathbf{W} . Whitening process can be expressed as follows

$$\mathbf{Z}(t) = \mathbf{W}\mathbf{X}(t) \quad (13)$$

- 2) Form fourth-order cumulants of $\mathbf{Z}(t)$ as follows.

$$\begin{aligned} Cum(z_{k_1}, z_{k_2}, z_{l_1}, z_{l_2}) = \\ E\{z_{k_1} z_{k_2} z_{l_1}^* z_{l_2}^*\} - E\{z_{k_1} z_{l_1}^*\} E\{z_{k_2} z_{l_2}^*\} \\ - E\{z_{k_1} z_{l_2}^*\} E\{z_{k_2} z_{l_1}^*\} \end{aligned} \quad (14)$$

- 3) Jointly diagonalize the set $\{\lambda_{z_r}, M_{z_r}, r = 1, 2, \dots, D\}$ by a unitary matrix \mathbf{U} that eigenpairs $\{\lambda_{z_r}, M_{z_r}\}$ corresponds to the D largest eigenvalues.

- 4) Estimate the array response matrix

$$\mathbf{A}' = \mathbf{W}^\dagger \mathbf{U} \quad (15)$$

in which \mathbf{W}^\dagger is Moore-Penrose inverse of whitening matrix.

Eq. (3) can be expressed as follows.

$$\mathbf{a}(\theta_k) = [1 e^{i\omega_k} e^{i2\omega_k} \dots e^{i(M-1)\omega_k}]^T \quad (16)$$

with

$$\omega_k = \frac{2\pi f d \sin\theta_k}{c} \quad (17)$$

where f is the carrier frequency of signals and c is the speed of wave propagation.

JADE algorithm proposed by Cardoso is an ICA method which has been successfully applied to the beam-forming and DOA estimation. By using JADE

algorithm, the array response vectors can be estimated without any prior knowledge about the array manifold.

5 DOA Estimation Using MFBLP Algorithm

In this paper, MFBLP algorithm is used to estimate DOA of sources. MFBLP algorithm is summarized as follows [9].

- 1) Choose L for each $M \times 1$ estimated steering vector $\hat{\mathbf{a}}$ that L must satisfy Eq. (18). In each $\hat{\mathbf{a}}$, there are G coherent signals. Then, form matrix \mathbf{Q} and vector \mathbf{h} . \mathbf{Q} is $2(M-L) \times L$ matrix and \mathbf{h} is $2(M-L) \times 1$ vector which are shown below.

$$G \leq L \leq M - \frac{G}{2} \quad (18)$$

$$\mathbf{Q} = \begin{bmatrix} \hat{a}_L & \hat{a}_{L-1} & \dots & \hat{a}_1 \\ \hat{a}_{L+1} & \hat{a}_L & \dots & \hat{a}_2 \\ \hat{a}_{L+2} & \hat{a}_{L+1} & \dots & \hat{a}_3 \\ \vdots & \vdots & \dots & \vdots \\ \hat{a}_{M-1} & \hat{a}_{M-2} & \dots & \hat{a}_{M-L} \\ \hat{a}_2^* & \hat{a}_3^* & \dots & \hat{a}_{L+1}^* \\ \hat{a}_3^* & \hat{a}_4^* & \dots & \hat{a}_{L+2}^* \\ \hat{a}_4^* & \hat{a}_5^* & \dots & \hat{a}_{L+3}^* \\ \vdots & \vdots & \dots & \vdots \\ \hat{a}_{M-L+1}^* & \hat{a}_{M-L+2}^* & \dots & \hat{a}_M^* \end{bmatrix} \quad (19)$$

$$\mathbf{h} = [\hat{a}_{L+1} \hat{a}_{L+2} \dots \hat{a}_M \hat{a}_1^* \hat{a}_2^* \dots \hat{a}_{M-L}^*]^T \quad (20)$$

- 2) Compute a singular value decomposition of \mathbf{Q} as:

$$\mathbf{Q} = \mathbf{U}\mathbf{D}\mathbf{V}^H \quad (21)$$

Then, set $L-G$ smallest singular values on the diagonal of \mathbf{D} to 0 and name new matrix $\mathbf{\Sigma}$. Dimension of $\mathbf{\Sigma}$ is the same as the dimension of \mathbf{D} .

- 3) Compute \mathbf{g} as follows

$$\mathbf{g} = [g_1 g_2 \dots g_L]^T = -\mathbf{V}\mathbf{\Sigma}^{\#}\mathbf{U}^H \mathbf{h} \quad (22)$$

Then, find roots of the polynomial as

$$H(o) = 1 + g_1 o^{-1} + g_2 o^{-2} + \dots + g_L o^{-L} \quad (23)$$

$H(o)$ has L zeros, $o_i = be^{j\omega_k}$; $i = 1, 2, \dots, L$. G zeros of $H(o)$ lie on unit circle, i.e., $b=1$. So, G unknown frequencies ω_k are determined. Finally, θ_k can be computed by using Eq. (17).

6 Proposed Algorithm

A novel approach to jointly determine the number and DOA estimation of coherent signals in non-coherent groups is proposed as a combination of MDL/EGM, JADE and MFBLP. In the proposed algorithm, MDL and EGM methods are utilized to determine the number of non-coherent groups for white noise and colored noise, respectively. Therefore, a decision stage is utilized to determine the kind of noise. If the power of noise at all frequencies is constant then noise is white, otherwise, noise is nonwhite or colored. In other word, the auto-correlation function of a white noise process with a variance of σ^2 is a delta function. The steps of the proposed algorithm are illustrated in Table 1.

Table 1 Summary of the proposed DOA estimation method.

Given	A received signal \mathbf{X}
Step 1.	Apply MDL/EGM method to \mathbf{X} and determine the number of non-coherent source groups.
Step 2.	Estimate generalized steering vectors with JADE algorithm.
Step 3.	Use MFBLP algorithm to estimate DOAs.

7 Simulation Results

In this section, several sets of simulation results are provided to demonstrate the performance of the proposed algorithm. JADE-MUSIC algorithm is selected to be a comparative method. In all simulations, a uniform linear array with relative inter-element spacing ($d/\lambda=0.5$) is considered and 12 signals impinge on the array is considered. Three source groups of coherent signals are considered which their multipath fading coefficients are $[1, -0.6426+0.7266j, 0.8677+0.0632j, 0.7319-0.1639j]$, $[1, -0.8262+0.4690j, 0.1897-0.8593j, 0.2049-0.7630]$ and $[1, -0.1681-0.9045j, -0.7293-0.1750j, 0.6102+0.1565j]$, respectively [12]. Simulations are organized in three sections. In section A, we consider five examples. The results of these examples are summarized in Table 2. In sections B and C, the new approach is compared with JADE-MUSIC algorithm in two scenarios of AWGN and colored noise.

These algorithms are simulated in MATLAB 7.11 software. A PC with an Intel Core i5-2400, 3.1 GHz CPU, and 4 GB RAM is used to run computer codes. Computational time of the JADE-MUSIC algorithm for estimation of DOAs of three groups of coherent signals in SNR=10 dB is 968.906828 seconds. However, this time for the proposed algorithm is 95.567817 seconds. So, computation time of our algorithm is about 10 times less than computation time of the JADE-MUSIC algorithm in similar conditions.

7.1 Five Examples

In this Section, 5 examples show the effectiveness of the proposed method. Two metrics, mean value and standard deviation of DOA estimation for each group are defined as:

$$Mean = \frac{1}{L} \sum_{k=1}^L \theta_{iest}(k) \quad (24)$$

$$Standard\ deviation = \sqrt{\frac{1}{LN} \sum_{k=1}^L \sum_{i=1}^N (\theta_{iest}(k) - Mean)^2} \quad (25)$$

where θ_i contains G signal of one group (i.e., N signals of all groups) and $\theta_{iest}(k)$ is the estimate of θ_i for the k-th Monte Carlo trial and L is the number of Monte Carlo trials.

In examples 1-4, we consider three groups of coherent signals from $[10^\circ, 20^\circ, 28^\circ, 45^\circ]$, $[5^\circ, 25^\circ, 35^\circ, 55^\circ]$, $[40^\circ, 60^\circ, 15^\circ, 30^\circ]$ and AWGN noise, $N_s=3000$, SNR=10 dB and M=10. Also, 1000 Monte-Carlo runs are made and modulation type is changed for each group. Examples 1-4 show that the standard deviation of each group depends on its QAM modulation size.

In example 5, it is assumed that three groups of coherent signals coming from $[10^\circ, 20^\circ, 28^\circ, 45^\circ]$, $[5^\circ, 20^\circ, 35^\circ, 55^\circ]$ and $[30^\circ, 10^\circ, 20^\circ, 40^\circ]$. In other word, three independent signals approach the array from exactly the same direction (i.e., 20°) and two independent signals approach from direction 10° . This algorithm can estimate DOAs relatively well because coherent signals in each group are combined to form a steering vector. It means, three steering vectors are distinct from each other.

7.2 Comparison of Proposed Method and JADE-MUSIC Considering AWGN

In this section, it is assumed that three groups of coherent signals approach the array of sensors at $[10^\circ, 20^\circ, 28^\circ, 45^\circ]$, $[5^\circ, 20^\circ, 35^\circ, 55^\circ]$ and $[30^\circ, 10^\circ, 20^\circ, 40^\circ]$, respectively. Modulation size of each coherent signal group is 4-QAM and SNR=10 dB.

The MUSIC spectrums for each group by JADE-MUSIC algorithm are shown in Figs. 1-3. As depicted in Fig. 1, JADE-MUSIC can resolve the first coherent signal groups. But, Figs. 2 and 3 show that JADE-MUSIC algorithm cannot resolve one and two signals of the second and third groups, respectively. However, estimated DOAs by the proposed algorithm is more accurate with respect to JADE-MUSIC algorithm.

To better compare the two algorithms, Root Mean Square Error (RMSE) is utilized as the performance metric. RMSE is defined as follows:

$$RMSE = \sqrt{\frac{1}{LN} \sum_{k=1}^L \sum_{i=1}^N (\theta_{iest}(k) - \theta_i)^2} \quad (26)$$

where $\theta_{iest}(k)$ is the estimate of θ_i for the k-th Monte Carlo trial, N is the total number of signals, and L is the number of Monte Carlo trials. To calculate RMSE, according to previous works [1, 3, 6, 9, 12], it is considered that one original DOA is estimated two times when the number of estimated DOAs is less than that of original DOAs. However, the algorithm cannot resolve those DOAs, so RMSE is not an accurate performance measure; especially, when two original DOAs are close to each other. For example, when two sources coming from 20° and 25° and the algorithm could not resolve 25° , then it is considered that algorithm estimates 20° two times. It means, a new metric is needed to accurately show the performance of DOA estimation algorithms. We propose a new performance metric referring it as Shirvani-Ebadi Normalized RMSE (SE-NRMSE), in this paper.

Table 2 Mean value and standard deviation of estimated DOAs for different QAM modulation sizes ($N_s=3000$, $SNR=10$ dB, $L=1000$, $M=10$).

Example	Original DOAs			Mean value of estimated DOAs			Standard deviation of estimated DOAs			QAM sizes		
	Group			Group			Group			Group		
	1	2	3	1	2	3	1	2	3	1	2	3
1	10	5	40	9.9602	4.9959	40.0356	0.6451	0.0640	0.3040	4	4	4
	20	25	60	19.5954	24.9967	60.0072						
	28	35	15	27.7200	35.0150	14.9507						
	45	55	30	45.1492	55.0040	29.8760						
2	10	5	40	9.9989	4.9984	40.0084	0.2341	0.1439	0.3724	16	4	4
	20	25	60	20.0110	24.9964	59.9959						
	28	35	15	28.0414	34.9918	14.9911						
	45	55	30	44.9913	54.9985	29.9084						
3	10	5	40	10.0023	4.9998	39.9589	0.3142	0.0583	1.5370	16	16	4
	20	25	60	20.0617	25.0060	59.9792						
	28	35	15	28.1217	34.9959	14.9041						
	45	55	30	44.9772	55.0033	29.7219						
4	10	5	40	9.9486	4.9960	40.0351	0.8234	0.0678	0.4101	16	16	16
	20	25	60	19.6705	24.9958	60.0196						
	28	35	15	27.9217	35.0098	14.9537						
	45	55	30	45.1371	54.9964	29.9010						
5	10	5	30	9.9724	4.9922	30.0244	0.3604	0.0922	0.4341	4	4	4
	20	20	10	19.6927	19.9723	10.0076						
	28	35	20	27.6218	34.9877	19.8944						
	45	55	40	45.1076	55.0101	40.0127						

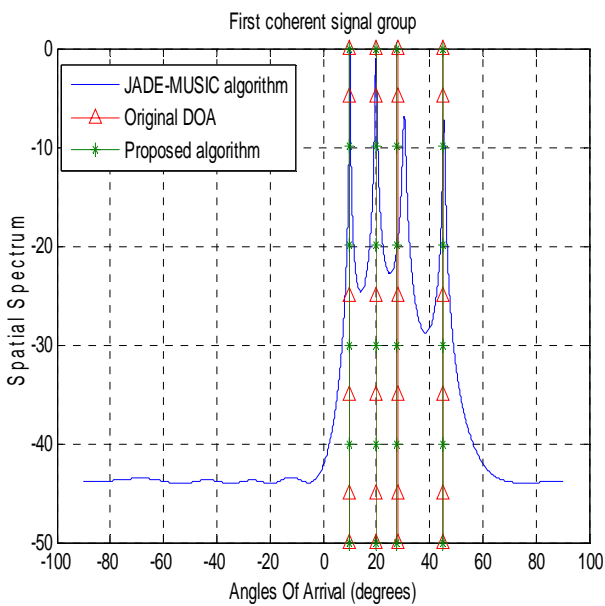


Fig. 1 MUSIC spectrum for the first coherent signal group.

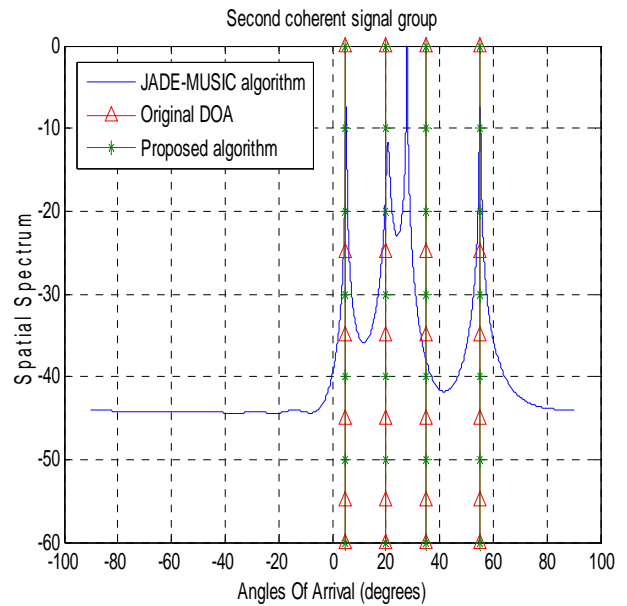


Fig. 2 MUSIC spectrum for the second coherent signal group.

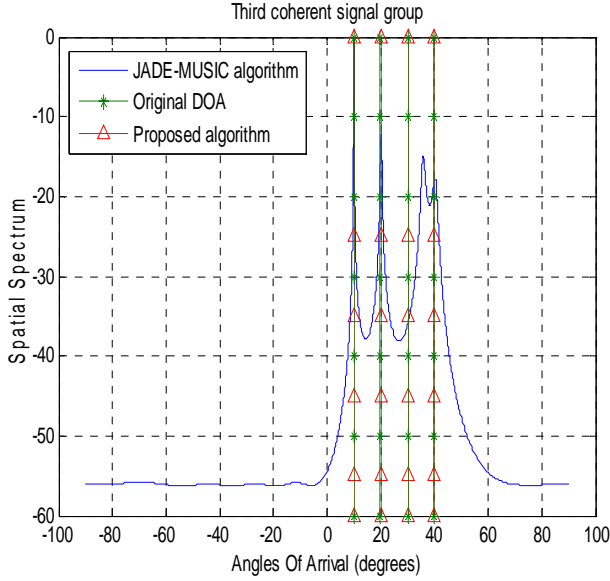


Fig. 3 MUSIC spectrum for the third coherent signal group

To define SE-NRMSE, consider that the algorithm resolves H sources, where $H < N$. It is assumed that if $|\theta_{\text{iest}} - \theta_{ij}| < Th_j$; $i=1, 2, \dots, G$, and $j=1, 2, \dots, K$, the algorithm can resolve the signals. It is also assumed that the total number of angles that satisfy $|\theta_{\text{iest}} - \theta_{ij}| < Th_j$ is not greater than H , θ_{ij} is the i -th θ in the j th group and Th_j is threshold for the j -th group of coherent signals. In other word, Th_j is defined as half of the difference between two nearest angles in the j th group. Thereby, $NMSE_{ij}$ is calculated by Eq. (27). Note that $0 \leq NMSE_{ij} \leq 1$. In Eq. (27), if $\theta_{\text{iest}} = \theta_{ij}$ then $NMSE_{ij} = 0$ and if $\theta_{\text{iest}} = \theta_{ij} \pm Th_j$ then $NMSE_{ij} = 1$.

$$NMSE_{ij} = \left(\frac{\theta_{\text{iest}(k)} - \theta_{ij}}{Th_j} \right)^2; i = 1, \dots, G, j = 1, \dots, K \quad (27)$$

$$\mathbf{nmse} = [NMSE_{ij}]_{1 \times F} \quad (28)$$

If $|\theta_{\text{iest}} - \theta_{ij}| \geq Th_j$; $i=1, 2, \dots, G$ and $j=1, 2, \dots, K$, so the algorithm could not resolve the signals. The total number of angles that satisfy $|\theta_{\text{iest}} - \theta_{ij}| \geq Th_j$ is $H-F$. Then $NRMSE_{ij}$ is given by Eq. (29).

$$NRMSE_{ij} = \frac{1}{N}; i = 1, \dots, G \quad (29)$$

DOA estimation algorithm also could not resolve $N-H$ signals, especially in low SNRs and/or small number of snapshots. So, $NRMSE_{ij}$ of these signals is calculated by Eq. (29) again. Thereby, $NRMSE$ for each Monte Carlo trial is calculated by Eq. (30).

$$NRMSE(l) = \frac{N-H}{N} + \frac{H-F}{N} + \frac{\sqrt{F}}{N} \sqrt{\sum_{f=1}^F NMSE(f)} \quad (30)$$

$$= 1 - \frac{F}{N} \left(1 - \sqrt{\frac{\sum_{f=1}^F NMSE(f)}{F}} \right); l = 1, 2, \dots, L$$

where $NMSE(f)$ is the f -th element of \mathbf{nmse} in Eq. (28). Finally, the total SE-NRMSE is defined as

$$SE - NRMSE = \frac{1}{L} \sum_{l=1}^L NRMSE(l) \quad (31)$$

RMSE and SE-NRMSE versus SNR are shown in Figs. 4 and 5, respectively. In these simulations, 200 Monte-Carlo runs with $N_s=2000$ are made. As shown in Fig. 4, when SNR increases from -2 dB, the proposed algorithm begins to demonstrate smaller RMSE than the JADE-MUSIC algorithm. When SNR is below -2 dB, RMSE of the proposed algorithm is a bit more than RMSE of the JADE-MUSIC algorithm. To clearly show the performance, SE-NRMSE is plotted for both algorithms in Fig. 5. It illustrates that SE-NRMSE of the proposed algorithm is less than SE-NRMSE of JADE-MUSIC algorithm in all SNR values.

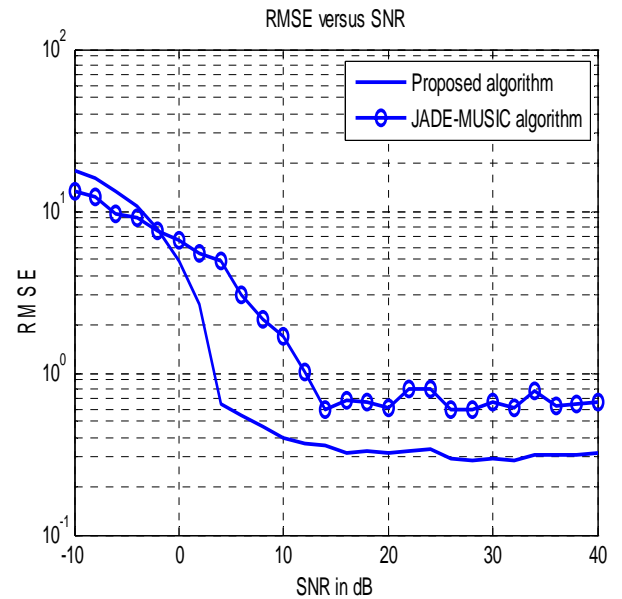


Fig. 4 RMSE versus SNR for AWGN scenario.

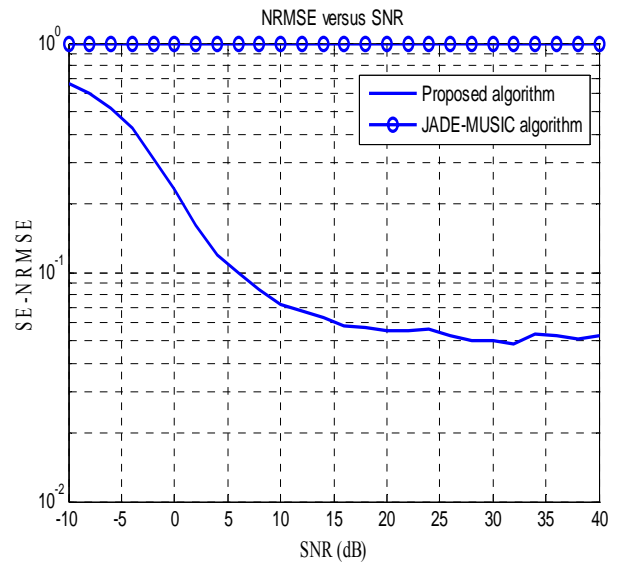


Fig. 5 SE-NRMSE versus SNR for AWGN scenario.

7.3 Comparison of the Proposed Method and JADE-MUSIC Considering Colored Noise

In this section, it is assumed that three groups of coherent signals approach the array of sensors at $[0^\circ, 10^\circ, 20^\circ, 28^\circ]$, $[25^\circ, 33^\circ, 10^\circ, 19^\circ]$ and $[30^\circ, 8^\circ, 19^\circ, 26^\circ]$, respectively. Modulation size of each coherent signal group is 4-QAM, SNR = 10 dB and noise is colored Gaussian. As we know, the power spectral density of the colored noise is not uniform across the entire frequency spectrum. Hence, the colored Gaussian additive noise is generated by passing the AWGN through a first-order autoregressive (AR1) filter with coefficient a_0 . Correlation coefficient α , is 0.7 and 200 Monte-Carlo runs with $N_s = 2000$ is made.

Fig. 6 shows that MDL criterion cannot detect the number of coherent signal groups in colored Gaussian noise scenario. It also shows that the detection probability of EGM criterion is 100 %, in SNR ≥ -2 dB. So, in SNR ≤ -2 dB, it is assumed that the number of coherent signal groups is known and in SNR ≥ -2 dB, the number of independent groups is detected by EGM, exactly. Hence, in colored noise scenario, EGM is selected as the determiner of the number of source groups.

As shown in Fig. 7, RMSE of the proposed algorithm in SNR > -6 dB is lower than JADE-MUSIC algorithm and vice versa. Fig. 8 shows that, in wide range of SNRs the proposed algorithm demonstrates a smaller SE-NRMSE than the JADE-MUSIC algorithm.

In Table 3, the effect of correlation coefficients on RMSE and SE-NRMSE of two algorithms is investigated. Three levels of SNR, low, moderate and high are equal to -10, 10 and 30 dB, respectively. For each SNR, α is 0.1, 0.3, 0.5 and 0.8. As reported in this table, in low and moderate SNRs, RMSE of the proposed algorithm is higher than RMSE of the JADE-MUSIC algorithm.

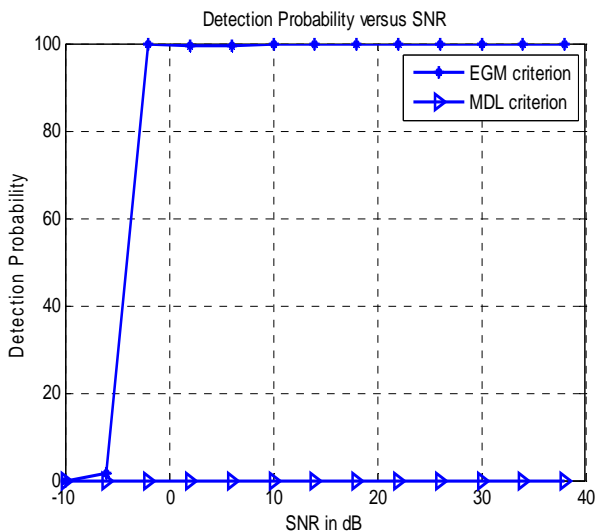


Fig. 6 Detection probability versus SNR for AWGN scenario.

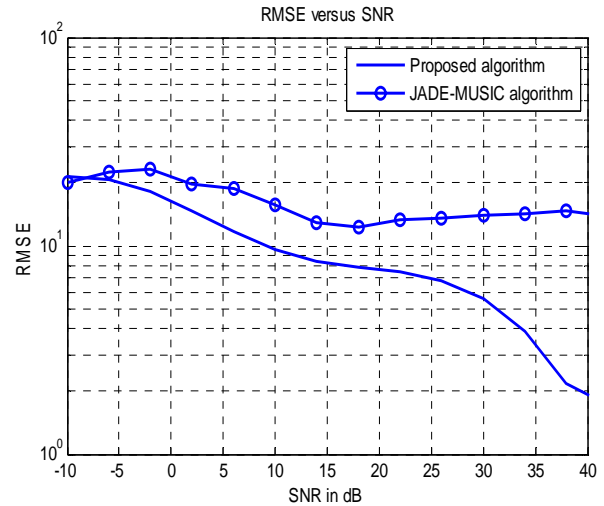


Fig. 7 RMSE versus SNR for colored noise scenario.

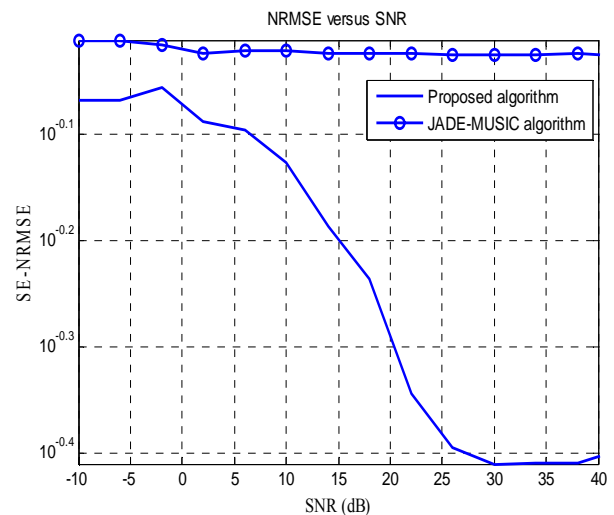


Fig. 8 SE-NRMSE versus SNR for colored noise scenario.

Table 3 RMSE and SE-NRMSE for JADE-MUSIC and proposed algorithms in different correlation coefficients ($N_s = 2000$, $M = 10$).

α	SNR (dB)	JADE-MUSIC		Proposed Algorithm	
		RMSE	SE-NRMSE	RMSE	SE-NRMSE
0.1	-10	5.5764	0.9324	7.4890	0.7810
	10	5.2157	0.9230	9.0826	0.5004
	30	5.5731	0.9228	3.8195	0.3972
0.3	-10	8.6095	0.9533	13.8726	0.8808
	10	4.963	0.9234	11.1608	0.5561
	30	5.3064	0.9223	3.8171	0.3911
0.5	-10	13.161	0.9654	22.3197	0.8491
	10	5.2537	0.9272	14.4367	0.6323
	30	5.3998	0.9221	3.4659	0.3997
0.8	-10	13.749	0.9746	23.8089	0.8824
	10	8.2846	0.9517	18.0275	0.8846
	30	5.3258	0.9241	5.6303	0.4214

In high SNRs, RMSE of the proposed algorithm is lower than that of the JADE-MUSIC algorithm. Also, Table 3 shows that SE-NRMSE of the proposed algorithm is lower than that of the JADE-MUSIC algorithm, in low, moderate and high SNRs regardless of α .

8 Conclusion

In this paper, we proposed a new algorithm for DOA estimation of source groups with coherent signals by combining MDL/EGM, JADE and MFBLP algorithms. MDL criterion was used to determine the number of independent groups in AWGN scenario, JADE algorithm was utilized to estimate the steering vectors, and finally MFBLP algorithm was applied to each steering vectors to estimate DOAs. It should be noted that instead of MDL, the EGM criterion was used to detect the number of groups in colored noise scenario.

Also, a new normalized RMSE performance metric was suggested which showed the performance of each algorithm more accurately. To evaluate the performance of the proposed algorithm and also compare the numerical results with conventional JADE-MUSIC algorithm, several Monte-Carlo simulations were run in both AWGN and colored noise channels. Simulation results show that, for different QAM modulation sizes of coherent signals, the proposed method can completely resolve sources. When AWGN is considered, in SNR larger than -2 dB RMSE of the proposed algorithm is less than RMSE of JADE-MUSIC algorithm and in wide range of SNRs, SE-NRMSE of the proposed algorithm is less than SE-NRMSE of JADE-MUSIC algorithm. Also, simulation results show the effect of colored Gaussian noise on RMSE and SE-NRMSE. In SNR larger than -6 dB, RMSE, and in wide range of SNRs, SE-NRMSE of new algorithm is less than JADE-MUSIC algorithm. It also shows that, in low, moderate and high SNRs, in all coefficient correlation ranges, the proposed algorithm shows lower SE-NRMSE. Besides above mentioned superiorities of the new proposed method, it needs 10 times lower computational complexity compared to the JADE-MUSIC algorithm.

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