A Novel MRAS Based Estimator for Speed-Sensorless Induction Motor Drive

S. M. Mousavi Gazafroodi* and A. Dashti*

Abstract: In this paper, a novel stator current based Model Reference Adaptive System (MRAS) estimator for speed estimation in the speed-sensorless vector controlled induction motor drives is presented. In the proposed MRAS estimator, measured stator current of the induction motor is considered as a reference model. The estimated stator current is produced in an adjustable model to compare with the measured stator current, where rotor flux identification is needed for stator current estimation. In the available stator current based MRAS estimator, rotor flux is estimated by the use of measured stator current, where the adjustable model and reference model depend on each other since measured stator current is employed in both of them. To improve the performance of the MRAS speed estimator, both the stator current and rotor flux are estimated in the adjustable model by using the state space equations of the induction motor, adjusted with the rotor speed calculated by an adaptation mechanism. The stability of the proposed MRAS estimator is studied through a small signal analysis. Sensorless induction motor drive along with the proposed MRAS speed estimator is verified through computer simulations. In addition, performance of the proposed MRAS is compared with the available stator current based MRAS speed estimator.

Keywords: Induction Motor, Model Reference Adaptive System, Rotor Flux Identification, Speed Estimation.

1 Introduction

The induction motors are the most popular AC motors used in various industrial applications such as wind energy conversion system, loads in power system, and electric vehicles due to their reliable construction. To achieve the high performance of induction motor drive, vector control is used [1, 2]. Accurate speed identification is required for all high performance vectors controlled induction motor drives. The speed identification of induction motor can be performed by a shaft speed encoder; however, compared to speed estimation, shaft speed encoder has several disadvantages such as an increase in cost, size, complexity, and maintenance requirements and a decrease in the reliability and robustness [3-5].

The main issue of the sensorless induction motor drive is speed identification. Several techniques are proposed in the literature to estimate the speed in a sensorless induction motor drive. They are broadly classified as: signal injection based methods and fundamental model based methods [3, 6]. The fundamental model based methods are more common because of their simplicity and less associated problems with the signal injection based methods like torque ripple and additional losses [5, 7]. The measured stator voltages and currents are used to estimate the speed of induction motor in the fundamental model based methods, which can be classified as open loop speed calculators [3], Adaptive Flux Observers [8, 9], Kalman Filters [10-12], Sliding Mode Observers [13, 14], Model Reference Adaptive Systems (MRAS) [15-21] and Artificial Intelligence Techniques [22, 23].

Among these techniques, the MRAS estimators are the most conventional method because of their simple structure and less computation requirement compared with the other methods. The basic structure of the MRAS consists of a reference model, an adjustable model and an adaption mechanism as shown in Fig. 1. The adaptation mechanism uses the error between the reference model and adjustable model to generate the estimated speed used in the adjustable model. There are several methods for speed estimation of the induction motor based on MRAS.
2 Methods of Rotor Flux Identification

There are two methods for rotor flux identification, which is necessary for estimation of stator current. In the first method, the measured stator current is used in Eq. (1) to estimate the rotor flux. Subsequently, the stator current can be estimated by Eq. (2). Therefore, estimation of rotor flux and stator current directly depends on the measured stator current. Hence, the first method is called “dependent method” in this paper.

\[
\frac{d}{dt} \begin{bmatrix}
\psi_{\text{est}(\text{dep})} \\
\psi_{\text{est}(\text{dep})}
\end{bmatrix} = A_1 \begin{bmatrix}
i_{\text{st}(\text{mes})} \\
i_{\text{st}(\text{mes})}
\end{bmatrix} (1)
\]

\[
\frac{d}{dt} \begin{bmatrix}
i_{\text{st}} \\
i_{\text{est}}
\end{bmatrix} = A_2 \begin{bmatrix}
i_{\text{st}} \\
i_{\text{est}}
\end{bmatrix} + \frac{1}{\sigma L_{\text{m}}} \begin{bmatrix}
V_{\text{st}(\text{mes})} \\
V_{\text{st}(\text{mes})}
\end{bmatrix} (2)
\]

In Eqs. (1) and (2), \(i_s\), \(V_s\), and \(\psi_r\) are stator current, stator voltage, and rotor flux, respectively. The “dep”, “mes”, and “est” are abbreviation of “dependent”, “measured”, and “estimated”, respectively. Also, \(d\) and \(q\) component in stationary reference frame are defined with \(\alpha\) and \(\beta\), respectively. In addition, matrices \(A_1\) and \(A_2\) are defined by following equations:

\[
A_1 = \begin{bmatrix}
a_4 & 0 & -a_3 & i_{\text{st}(\text{mes})} \\
0 & a_4 & \omega_{\text{st}(\text{mes})} & -a_5
\end{bmatrix} (3)
\]

\[
A_2 = \begin{bmatrix}
-a_1 & 0 & a_2 & a_3 \omega_{\text{st}(\text{mes})} \\
0 & -a_1 & -a_5 \omega_{\text{st}(\text{mes})} & a_2
\end{bmatrix} (4)
\]

where \(\omega_{\text{st}}\) is rotor speed and:

\[
\sigma = 1 - \frac{L^2_{\text{m}}}{L_{\text{m}} L_{\text{r}}} (5)
\]

\[
a_1 = \frac{1}{\sigma L_{\text{m}}} \left( L_{\text{r}} - \frac{L^2_{\text{m}}}{L_{\text{r}}} \right) (6)
\]

\[
a_2 = \frac{1}{\sigma L_{\text{m}}} \left( L_{\text{m}} \right) (7)
\]

\[
a_3 = \frac{1}{\sigma L_{\text{r}}} \left( L_{\text{m}} \right) (8)
\]

\[
a_4 = \frac{L_{\text{m}}}{\tau_{\text{r}}} (9)
\]

\[
a_5 = \frac{1}{\tau_{\text{r}}} (10)
\]

In the second method, called “independent method”, the stator current and rotor flux are estimated by Eq. (11). The estimated stator current is then used to estimate the rotor flux instead of using measured stator current. It should be noted that Eqs. (11) and (12) express the state space equations of induction motor in
stationary reference frame. The “indep” is the abbreviation of “independent”.

\[
\begin{bmatrix}
\dot{i}_{a_{\text{est}}} \\
\dot{i}_{\beta_{\text{est}}} \\
\psi_{m_{\text{est (indep)}}} \\
\psi_{\beta_{\text{est (indep)}}}
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22} \\
A_{31} & A_{32} \\
A_{41} & A_{42}
\end{bmatrix}
\begin{bmatrix}
i_{a_{\text{est}}} \\
i_{\beta_{\text{est}}} \\
\psi_{m_{\text{est (indep)}}} \\
\psi_{\beta_{\text{est (indep)}}}
\end{bmatrix} + \begin{bmatrix}
\frac{1}{\sigma L_a} \\
0 \\
0 \\
0
\end{bmatrix} V_{a_{\beta}}
\]

(11)

In the Eq. (11), matrix \(A\) is defined as

\[
A = \begin{bmatrix}
-a_1 & 0 & a_2 & a_3 & \omega_{r_{\text{est}}} \\
0 & -a_1 & -a_2 & -a_3 & -\omega_{r_{\text{est}}} \\
a_4 & 0 & 0 & -a_5 & -\omega_{r_{\text{est}}} \\
0 & a_4 & a_5 & -a_6 & -\omega_{r_{\text{est}}}
\end{bmatrix}
\]

(13)

3 Mathematical Model of Proposed Speed Estimator

The structure of the conventional stator current based MRAS is shown in Fig. 2. In the conventional stator current based MRAS, the measured stator current of the induction motor in the stationary reference frame is considered as the reference model. The stator current and rotor flux are then estimated in the adjustable model. Using the adaptation mechanism, the error between the measured stator current and estimated stator current is multiplied to the estimated rotor flux vector to make the error signal defined by Eq. (14). Then, through a PI controller, the error signal is converted to estimated speed expressed with Eq. (15).

\[
\epsilon = (i_{a_{\text{est}}} - i_{a_{\text{mest}}}) \psi_{\beta_{\text{est}}} + (i_{\beta_{\text{est}}} - i_{\beta_{\text{mest}}}) \psi_{m_{\text{est}}}
\]

(14)

\[
\omega_{r_{\text{est}}} = \left( K_{\text{p-MRAS}} + \frac{K_{\text{l-MRAS}}}{s} \right) \epsilon
\]

(15)

The structure of the proposed MRAS is Similar to the conventional stator current based MRAS, as shown in Fig. 3. The only difference between the proposed MRAS and conventional stator current based MRAS is in their adjustable model. In the conventional stator current based MRAS, rotor flux are first estimated by using measured stator currents (Eq. (1)), then stator current is estimated by Eq. (2). On the other hand, in the proposed MRAS both stator current and rotor flux are estimated in the adjustable model by Eq. (11) (state space equation of induction motor), which is independent of measured stator current. The sensorless indirect field oriented control with the proposed speed estimator is shown in Fig. 4.
4 Stability Analysis of the Proposed Method

The state space equation of induction motor in synchronous rotating (ωe) reference frame is expressed with Eq. (16).

\[ x = Ax + Bu \]

\[ y = Cx \]  \hspace{1cm} (16)

where

\[ x = \begin{bmatrix} i_{sd} & i_{sq} & \psi_{sd} & \psi_{sq} \end{bmatrix}^T \]  \hspace{1cm} (17)

\[ u = \begin{bmatrix} V_{sd} & V_{sq} \end{bmatrix}^T \]  \hspace{1cm} (18)

\[ y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \]  \hspace{1cm} (19)

\[ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \]  \hspace{1cm} (20)

\[ B = \frac{1}{\sigma L_s} C^T \]  \hspace{1cm} (21)

The matrix A is:

\[ A = \begin{bmatrix} -a_1 & \omega_x & a_2 & a_1 \omega_y \\ -\omega_x & -a_1 & -a_2 \omega_x & a_2 \\ a_4 & 0 & -a_3 & (\omega_x - \omega_y) \\ 0 & a_4 & -(\omega_x - \omega_y) & -a_4 \end{bmatrix} \]  \hspace{1cm} (22)

The state space equations are changed to Eq. (23) by using the small signal analysis:

\[ \Delta x = A \Delta x + \Delta Ax_0 \]  \hspace{1cm} (23)

\[ \Delta y = C(sI - A)^{-1} \Delta Ax_0 \]

where \( x_0 \) (the operation point) and \( \Delta A \) are represented with following equations:

\[ x_0 = \begin{bmatrix} i_{sd0} & i_{sq0} & \psi_{sd0} & \psi_{sq0} \end{bmatrix}^T \]  \hspace{1cm} (24)

\[ \Delta A = \begin{bmatrix} 0 & 0 & a_3 \\ 0 & 0 & -a_3 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Delta \omega_{r} \]  \hspace{1cm} (25)

From Eq. (23), \( \Delta y \) can be expressed as:

\[ \Delta y = C(sI - A)^{-1} \Delta Ax_0 \]  \hspace{1cm} (26)

By using Eq. (26), \( \frac{\Delta i_{sd}}{\Delta \omega_{r}} \) and \( \frac{\Delta i_{sq}}{\Delta \omega_{r}} \) can be expressed as:

\[ \frac{\Delta i_{sd}}{\Delta \omega_{r}} = \frac{c_{14} - a_3 c_{12}}{\Delta \omega_{r}} \psi_{sd0} \]  \hspace{1cm} (27)

\[ \frac{\Delta i_{sq}}{\Delta \omega_{r}} = \frac{c_{24} - a_3 c_{22}}{\Delta \omega_{r}} \psi_{sd0} \]  \hspace{1cm} (28)

where

\[ (sI - A)^{-1} = \frac{\text{adj}(sI - A)}{|sI - A|} \]  \hspace{1cm} (29)

and

\[ \text{adj}(sI - A) = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \]  \hspace{1cm} (30)

The error equation is:

\[ e = \psi_{sd0} \Delta i_{sd} - \psi_{sq0} \Delta i_{sq} \]  \hspace{1cm} (31)

where

\[ \Delta i_{sd} = i_{sd} - i_{sd\text{est}} \]

\[ \Delta i_{sq} = i_{sq} - i_{sq\text{est}} \]  \hspace{1cm} (32)

The closed loop block diagram of the MRAS is shown in Fig. 5. The transfer function \( G \) can be expressed as:

\[ G = \frac{e}{\psi_{sd0} \Delta i_{sd} - \psi_{sq0} \Delta i_{sq}} \]  \hspace{1cm} (33)

The closed loop transfer function of the MRAS can be expressed as:

\[ \frac{\omega_r}{1 + G(s) K_{p-MRAS} + K_{I-MRAS}} \]  \hspace{1cm} (34)

If all the poles of the closed loop transfer function lie in the left half of the s-plane, the system will be stable in the operating point. It is worth mentioning that the closed loop transfer function of every operating point is different from the other operating points because matrix ‘A’ is changed by altering the speed. According to this method, the stability loss was not observed in the motoring region. The root locus of proposed MRAS at 20 rad/sec and 10 Nm (load torque) is shown in Fig. 6.

Fig. 5 Closed loop block diagram of the MRAS.

Fig. 6 Root locus plot of proposed MRAS at 20 rad/sec and \( T_L = 10 \) Nm.
Table 1  Induction motor parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated shaft power</td>
<td>1.3 kW</td>
</tr>
<tr>
<td>Line-to-line voltage</td>
<td>400 V</td>
</tr>
<tr>
<td>Pole pair</td>
<td>2</td>
</tr>
<tr>
<td>Rated speed</td>
<td>1430 rpm</td>
</tr>
<tr>
<td>Magnetizing inductance</td>
<td>670.5 mH</td>
</tr>
<tr>
<td>Stator and rotor Leakage inductance</td>
<td>14.3 mH</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>5.71 Ω</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>4.08 Ω</td>
</tr>
<tr>
<td>Motor inertia</td>
<td>0.087 kg.m²</td>
</tr>
</tbody>
</table>

5  Simulation of Proposed MRAS

The proposed MRAS speed estimator is simulated in MATLAB/Simulink and some of the simulation results are presented in this section. The parameters of the induction motor used in the simulation are presented in Table 1.

5.1  Step Increase in Reference Speed

Performance of the proposed MRAS for a step change in reference speed is illustrated in Fig. 7. Step changes of 20 rad/sec are applied at 5 and 10 s, respectively. Note that the load torque is considered proportional to the rotor speed since the load is applied to the IM through a DC generator. The actual and estimated speed are plotted in Figs. 7(a) and (b), respectively. It can be found that estimated speed is very close to the actual speed, according to the speed estimation error, which is shown in Fig. 7(c). The components of stator current are depicted in Figs. 7(d) and (e). This indicates that decoupling between flux and torque producing components of stator current is performed correctly since $i_{sd}$ is not affected significantly by a change in $i_{sq}$. It can be seen that the magnitude of rotor flux is kept constant (Fig. 7(f)) and flux orientation is well maintained (Fig. 7(g)).
5.2 Load Torque Variation

The loading and unloading performance of the sensorless drive is observed in Fig. 8. The induction motor is operated at 40 rad/sec. The applied load torque is suddenly increased from 5 to 20 N m at 5 s and returned to 5 N m at 12 s, as illustrated in Fig. 8(a). The profile of the produced electromagnetic torque is shown in Fig. 8(b). The overshoot and undershoot appear in actual speed during loading and unloading, respectively (Fig. 8(c)). Fig. 8(d) shows that the estimation is performed with good accuracy.

6 Instantaneous Reactive Power Based MRAS

The structure of the instantaneous reactive power based MRAS is shown in Figs. 9. The equivalent presentation of this estimator is shown in Fig. 10. The speed estimator is introduced in [18]. In this estimator, Eqs. (35) and (36) are used in the reference model and adjustable model, respectively.

\[ Q = V_{s\beta}i_{s\alpha} - V_{s\alpha}i_{s\beta} \]  
\[ Q_{est} = V_{s\beta}i_{s\alpha,est} - V_{s\alpha}i_{s\beta,est} \]  

The estimation of stator current is needed to construct the Eq. (36). The independent method (Eq. 11) is employed to estimate both stator current and rotor flux. Note that estimated current is used in the adjustable model, whereas measured current is employed in the reference model. The difference between the reference model and adjustable model makes the error signal (Eq. (37)). Through a PI

Fig. 9 Structure of instantaneous reactive power based MRAS estimator.

Fig. 10 Equivalent presentation of instantaneous reactive power based MRAS estimator.
controller, the error signal is converted to estimated speed expressed with Eq. (38).

\[
\varepsilon = (K_{\text{PMRAS}} + \frac{K_{\text{LMRAS}}}{s})[Q - Q_{\text{est}}] 
\]

(37)

\[
\omega_{\text{est}} = (K_{\text{PMRAS}} + \frac{K_{\text{LMRAS}}}{s})\varepsilon 
\]

(38)

7 Rotor Flux Identification Methods Comparing

In this section, performance of the dependent method and independent method in the stator current based MRAS and instantaneous reactive power based MRAS estimator is compared by employing MATLAB/Simulink. The comparison is performed under the maximum variation of the rotor resistance and stator resistance. During the induction motor operation, the stator and rotor resistance can increase up to 50% and 100% of the startup value, respectively [2].

7.1 Stator Current Based MRAS

The performance of the stator current based MRAS estimators, which uses the dependent method (conventional stator current based MRAS) and independent method (proposed stator current based MRAS) is compared in this section. The test is performed for two different reference speeds and the nominal load torque is applied to the induction motor. At 10 sec, the stator and rotor resistance are suddenly increased by 50% and 100% of the startup value, respectively. The responses of estimators, when reference speed is 10% of the nominal speed, are shown in Fig. 11. Performance of conventional MRAS is shown in Fig. 11(a) where the steady state error is about 8% of reference speed (Fig. 11(b)). The response of proposed MRAS is illustrated in Fig. 11(c). It is observed that the error in speed estimation is about 3% of the reference speed at steady state (Fig. 11(d)).

Speed estimation errors of estimators when reference speed is 5% of the nominal speed is about 20% (Fig. 12(a)) and 7% (Fig. 12(b)) of reference speed in the conventional MRAS and proposed MRAS, respectively. These results indicate using independent method for rotor flux identification in the stator current based MRAS (proposed MRAS) significantly improves performance of the conventional stator current based MRAS estimator, especially at low speeds.

![Fig. 11](image1.png)

**Fig. 11** Response at 15 rad/sec: (a) actual and estimated speed of conventional MRAS, (b) error of conventional MRAS, (c) actual and estimated speed of proposed MRAS and (d) error of the proposed MRAS.

![Fig. 12](image2.png)

**Fig. 12** Response at 7.5 rad/sec: (a) error of the conventional MRAS and (b) error of the proposed MRAS.
7.2 Instantaneous Reactive Power Based MRAS

In this section, similar to the previous simulation, test is performed under the nominal load torque. Performance of the instantaneous reactive power based MRAS estimator, which uses dependent method, (Fig. 13) and independent method are respectively shown in Figs. 14(a) and (b), when the reference speed is 10% of the nominal speed. It is observed that the error in speed estimation is about 6% of the reference speed when the dependent method is used (Fig. 14(c)) and about 3% of the reference speed when the independent method is employed (Fig. 14(d)). At a reference speed equal to 5% of the nominal speed, the error in the speed estimation by the dependent method is about 9% of the reference speed (Fig. 15(a)) that can be reduced to about 4% by the use of the independent method (Fig. 15(b)).

These results confirm instantaneous reactive power based MRAS estimates speed with better accuracy when the independent method is employed instead of the dependent method for rotor flux identification.

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**Fig. 13** Structure of instantaneous reactive power based MRAS estimator with the dependent method.

**Fig. 14** Response at 15 rad/sec: (a) actual and estimated speed of conventional MRAS, (b) actual and estimated speed of proposed MRAS, (c) error of conventional MRAS, and (d) error of proposed MRAS.

**Fig. 15** Response at 7.5 rad/sec: (a) error of conventional MRAS and (b) error of proposed MRAS.
8 Conclusion
This paper has presented a novel stator current based MRAS method for speed estimation of an induction motor. The proposed method has been applied to an indirect field oriented sensorless induction motor drive. It has been found that the variations of rotor resistance and especially stator resistance deteriorate the performance of conventional stator current based MRAS method at low speeds. In the proposed MRAS, the independent method is used for estimation of rotor flux, whereas dependent method is employed in the conventional stator current based MRAS method. Tests confirmed that the use of the independent method instead of the dependent method improves the performance of stator current based estimator at low speeds and increases the accuracy of speed estimation. Comparing the dependent and independent method results in a similar trend for the performance of instantaneous reactive power based MRAS estimator. Therefore, it can be concluded that the independent method is superior to the dependent method for estimation of rotor flux and speed estimation in stator current based and instantaneous reactive power based MRAS methods.

References
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