

# Modifying Nodal Pricing Method Considering Market Participants Optimality and Reliability

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**Abstract:** This paper develops a method for nodal pricing and market clearing mechanism considering reliability of power system. The impacts of power system component reliability on electricity price, market participants' profit and system social welfare is considered in this method. This paper considers reliability both for evaluation of market participant's optimality as well as for fair pricing and market clearing mechanism. To achieve fair pricing, nodal price is obtained through a two stage optimization problem and to achieve fair market clearing mechanism, comprehensive criteria are introduced for optimality evaluation of market participant. Social welfare of the system and system efficiency are increased under proposed modified nodal pricing method.

**Keywords:** Cost Allocation, Locational Marginal Pricing (LMP), Market Clearing, Market Participant Evaluation, Nodal Pricing.

## 1 Introduction

Pricing and market clearing mechanism are challenging issues in power market articles. Yet nodal pricing or Locational Marginal Pricing (LMP) is applied in some energy & ancillary service markets and even for transmission cost allocation and system planning [1-4]. LMP depends on line flows, generation and customer location in the network, lines losses and ... hence this dependencies cause sometimes unfair nodal prices. In the following, some of the LMP defects are discussed more in details.

Based on LMP mechanism the Transco revenue doesn't relate to the extent of generators and customers gain from transmission network [1]. For instance consider a two bus system with a load and generator at each bus. When generation in power transmitter bus increases, naturally the price in this bus increases. Despite the load of this bus decreases the line congestion between two buses, this load should pay more to gain this line caused by price increase in this bus and this is irrational. From another aspect, in this pricing method part-loaded generator determines the bus price so when a generation of a part-loaded generator increases, the bus price and therefor generator revenue increase too, without considering the efficiency of generators. The defects of LMP are discussed more in detail in [5]. Ultimately, LMP appears to be necessary, but it (in conventional format) is not certainly fair pricing method in competitive electricity markets.

Some literature have been tried to address the defects of nodal pricing and related market clearing mechanism [5-18], but each of them has its own superiorities and defects. To address the LMP imperfections some papers modify the LMP through modifying OPF objective function and its constraints [5-8].

While the above researches tend to achieve fair pricing and market clearing mechanism, they don't consider the probabilistic nature of power system. Generation, transmission and loads can affect the system reliability and fail of every generation unit or transmission line can affect LMPs. Forced outage of every market participant can face the power market to new generation commitment and new line flows and hence new LMPs.

Customer's reliability level which is one of the key elements of improved power market has not been considered in the previous researches. The load interruption cost for each customer in a contingency state, should be modeled and considered in pricing method. From another aspect, the reliability level of each component of power system should affect its revenue. For instance, the Forced Outage Rate (FOR) of a generation unit or line should affect the revenue of the Genco and the Transco.

As a matter of fact, neglecting reliability of the system causes unfair pricing and then unfair market clearing mechanism. This paper considers the power system reliability not only for evaluation of power market participant optimality, but also for pricing and cost allocation. The effects of power system components reliability on electricity price, market participants' profit and system social welfare is considered in proposed nodal pricing method. This

Iranian Journal of Electrical & Electronic Engineering, 2015.

Paper first received 21 Sep. 2014 and in revised form 20 Jan. 2015.

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paper modifies nodal pricing method through a two stage optimization method considering reliability and optimality of each power market participant. At first, comprehensive benchmarks are introduced to evaluate whole system efficiency. To achieve fair pricing, nodal price has been obtained through a two stage optimization problem and to achieve fair market clearing mechanism, comprehensive criteria has been introduced for optimality evaluation of market participant. Social welfare and efficiency of power system are increased under proposed modified nodal pricing method.

In this paper in section 2 some basic relations is explained to introduce the proposed nodal pricing method. In section 3 the proposed nodal pricing method is introduced. Section 4 contains numerical result of this pricing method and also comparison of proposed method with other pricing methods and section 5 concludes the paper.

## 2 Basics of the Proposed Nodal Pricing Method

In proposed method at first the whole power system is divided to three parts: 1-whole system generators, 2-whole system loads, 3-Transco.

Each of these three parts has its own effects on the whole system efficiency. A criterion is introduced to evaluate the role of every part in the whole system efficiency. This comprehensive criterion is called Total Social Welfare (SWt). It is equal to the summation of whole system generators' profit (*Gprofit*), whole system loads' profit (*Lprofit*) and Transmission Company's profit (*Nprofit*) as Eq. (1). After substituting the profits with revenue minus cost, the SWt is equal to the load revenue minus generation cost and instruction cost of lines as Eq. (1). This equation can be a comprehensive benchmark to evaluate the whole system efficiency [5].

$$\begin{aligned} SWt &= Gprofit + Lprofit + Nprofit \\ &= \underline{GR} - GC + LR - \underline{LC} + NR - IC \\ &= -\underline{NR} - GC + LR + NR - IC \\ &= LR - GC - IC (\$/h) \end{aligned} \quad (1)$$

where *R* stands for revenue, *C* stands for cost, *G* stands for generations, *L* stands for loads and *N* stands for transmission network. In the following, the share of every group from total social welfare is determined according to its Profit Share (*PS*) of total social welfare.

By running an OPF (as below) the generation vector  $P_g$  and demand vector  $P_l$  are determined and then SWt can be calculated using Eq. (1).

### OPF problem:

$$\begin{aligned} GC(P_g) &= a \times P_g + b \times P_g^2 \\ \min_{P_g, \delta} & \{ \sum (GC(P_g)) \} \\ \text{subject to: } & P_{gi} - P_{li} - P(V, \theta) = 0, \quad Q_{gi} - Q_{li} - Q(V, \theta) = 0, \quad F_k \leq F_{kmax}, \\ & P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max}, \quad Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max} \quad \text{and} \\ & |V_i|^{\min} \leq |V_i| \leq |V_i|^{\max}. \end{aligned} \quad (2)$$

Allocating SWt between power market participants comes about in 2 steps. In first step it is divided to 3 parts between generation companies, Transco and customers according to their role in the optimality of whole system. Eq. (3) signifies a criterion for assessing the optimality of act for each participant group with respect to all participant groups best act. For this purpose, the present performance of each group, their optimal performance and the situation of eliminating them from the system, should be investigated. In the following a criterion for optimality evaluation of each group can be defined as Eq. (3). This equation will be utilized to settle the participant groups' profit share. The term "co" in Eq. (3) alludes to every participants group.  $SWt_{presence,Co}$ ,  $SWt_{absence,Co}$ , and  $SWt_{best,Co}$  represent the Total Social Welfare for the present system, the system without the entity, and the system with its best possible behavior, respectively. Details about these quantities are described as follows.

$$PS_{Co} = \frac{SWt_{presence,Co} - SWt_{absence,Co}}{\sum_{Co} (SWt_{best,Co} - SWt_{absence,Co})} \quad (3)$$

### 2.1 Profit Share of Transco

To obtain the PS of Transco through Eq. (3) at first the  $SWt_{presence}$  is calculated through Eq. (1) after running OPF (Eq. 3). The best state of a transmission company is called Reference Transmission Network (RTN) as described in detail in [1] so the  $SWt_{best}$  for Transco is the total social welfare when network lines capacity are same as to RTN lines capacity. The  $SWt_{absence}$  for Transco denotes the SWt in the case of removing all network lines, so just local loads & generations benefits are considered in Eq. (1). After calculating the  $SWt_{presence}$ ,  $SWt_{absence}$  and  $SWt_{best}$  and substituting them in Eq. (3), the PS of Transco is obtained. Now the profit share of Transco from the SWt is determined through Eq. (4).

$$Nprofit = \frac{PS_{TransCo}}{\sum_{Co} PS_{Co}} \cdot SWt = \frac{PS_{TransCo}}{PS_{Customers} + PS_{Producers} + PS_{TransCo}} \cdot SWt \quad (4)$$

### 2.2 Profit Share of Whole System Generators

To obtain the PS of whole system generators through Eq. (3) at first the  $SWt_{presence}$  is calculated through Eq. (1) after running OPF (Eq. (2)). To calculate the  $SWt_{best}$  for generation companies, at first the best state of generation companies should be defined. The best state of whole system generation is obtained through the OPF problem without considering of the generation upper limit. The  $SWt_{absence}$  for Generation companies is zero since no supply and demand exists in the system without generation units. After calculating the  $SWt_{presence}$ ,  $SWt_{absence}$  and  $SWt_{best}$  and substituting them in Eq. (3) the PS of whole system generation is obtained. Now the profit share of Generation companies from the SWt is determined through Eq. (5).

$$Gprofit = \frac{PS_{Producers}}{\sum_{Co} PS_{Co}} .SWt = \frac{PS_{Producers}}{PS_{Customers} + PS_{Producers} + PS_{TransCo}} .SWt \quad (5)$$

### 2.3..Profit Share of Whole System Loads

To obtain the PS of whole system loads the  $SWt_{presence}$  has been calculated through Eq. (1). Load revenue is calculated via equation  $\alpha P_1^2 + \beta P_1$  [5]. The  $SWt_{absence}$  of loads is equated to zero since without system loads, no generation exists and so the  $SWt_{absence}$  for loads is equated to zero. The best state of whole system load doesn't make any sense since the loads don't carry out any operation or task in the system to have the best state. In another word the loads don't have participation in any system expansion planning, neither transmission expansion plans nor generation expansion plans. So loads don't have the best state to determine, hence the  $SWt_{best}$  for loads is equal to their  $SWt_{presence}$ . Ultimately by calculating the  $SWt_{presence}$ ,  $SWt_{absence}$  and  $SWt_{best}$  and substituting them in Eq. (3) the PS of whole system loads is calculated and the profit share of the whole system loads can be calculated through Eq. (6).

$$Lprofit = \frac{PS_{Customers}}{\sum_{Co} PS_{Co}} .SWt = \frac{PS_{Customers}}{PS_{Customers} + PS_{Producers} + PS_{TransCo}} .SWt \quad (6)$$

Totally, according to above descriptions, each group in power system should benefit according to its effect on system efficiency. The second step of SWt allocation is allocating Lprofit, Gprofit and Nprofit between loads, Gencos and Transco respectively, which comes about via running the second stage of optimization as describe in detail in section 3-2.

## 3 The Proposed Nodal Pricing Method

### 3.1 Optimality Evaluation of Market Participants to Allocate Lprofit, Gprofit and Nprofit Between Loads, Gencos and Transco Respectively

#### 3.1.1 Gencos Optimality Evaluation

To allocate Gprofit between Gencos, at first a criterion should be introduced to evaluate each Genco efficiency. As regards the outage of a generation unit can increase cost of the system and also electricity price. So the force outage rate of every generator or in another word, the reliability of a Genco should affect its revenue, so the generation efficiency vector is defined as the vector containing the ratios of the producers' expected mean revenue to their expected costs, according to Eq. (7). The symbol  $\lambda^*$  and  $\bar{\lambda}^*$  denotes to the nodal price and the average electricity price in the system respectively as defined in Eq. (7).

$$\eta_G = \bar{\lambda}^* Q_G^* (1-FOR) / GC(Q_G^* (1-FOR)) \quad (7)$$

FOR is the generator unavailability possibility and the term 1-FOR denotes the generator availability possibility, therefore the term  $Q_G^* (1-FOR)$  denotes the available power of a generator. In another word, the power generation of a generator is  $Q_G$  and the revenue of a generator is  $\lambda^* Q_G$  if the generator is available.

#### 3.1.2 Loads Optimality Evaluation

As expressed previously, the optimality evaluation of loads doesn't make any sense hence the gain of a load from transmission network is considered as a criterion to allocate the Lprofit between system loads. The usage of loads from transmission network ( $U_D$ ) is directly related to the net power ( $p_i$ ) of the bus  $i$  as Eq. (8). The denominator of the fraction is the summation of whole inflow and outflow power at bus  $i$ . If the demand and generation are equal at bus  $i$  the net power  $p_i$  and therefore  $U_D$  will be zero for the load at this bus. If the net power of bus  $i$  ( $p_i$ ) is positive the  $U_D$  is equated to zero since this bus injects power to the system and loads at this bus have no usage from transmission network and if the net power of bus  $i$  ( $p_i$ ) is negative represents that this bus is receiving power from the system.

$$U_D = \{u_{iD}\} = \begin{cases} -p_i / \sum_{j=1}^{N_B} |p_j|, & \text{if } p_i < 0 \\ 0, & \text{if } p_i \geq 0 \end{cases}, i=1,2,\dots,N_B \quad (8)$$

By obtaining  $U_D$  through above equation and multiplying it to the instruction cost of network lines (IC), the share of each load from network instruction cost is obtained. In another word, the share of each load from the instruction cost of the lines relates to the usage of each load from transmission network ( $U_D$ ). Then after subtracting the load revenue from network usage cost of load ( $IC * U_D$ ), the efficiency of load can be formulated like generator efficiency as below. The dominator of this fraction denotes the load payment or load cost. As much as this cost decrease, load efficiency increase.

$$\eta_D = ((LR(Q_D) - IC * U_D) / (\lambda^* Q_D)) \quad (9)$$

#### 3.1.3 Transco Optimality Evaluation

In this paper Transco is modeled as a unique company and doesn't have any rival in the system. Therefore Nprofit is paid to this unique company and the efficiency of Transco is just evaluated in the first stage of SWt allocation (section 2-A).

In reviewing of explained sections graphically, the two steps of SWt allocation are depicted in the Fig. 1. According to this figure the first step of SWt allocation occurs between Gencos, Transco and whole system Loads as explained in section 2. After performing the first step of SWt allocation, Lprofit, Gprofit and Nprofit are determined. Then the second step of SWt allocation allocates Lprofit, Gprofit and Nprofit between loads, Gencos and Transco, respectively.

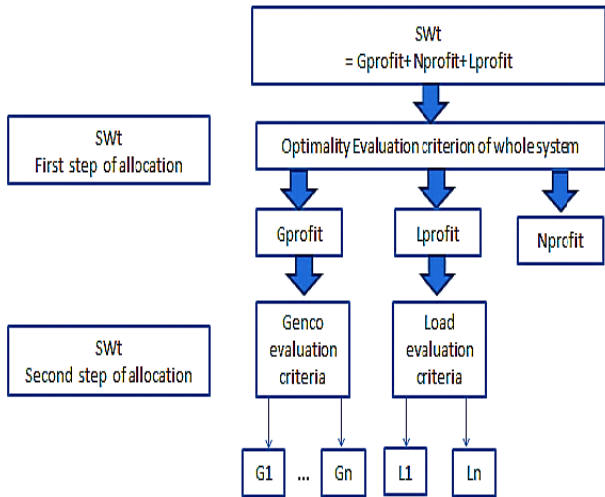


Fig. 1 The two step of SWt allocation between power market participants.

The second step of SWt allocation comes about via solving the two stage optimization problem presented. In section 3, in the following basis of two stage optimization problem to modify nodal price is explained.

### 3.2 Expected Load (EL) & Expected Generation (EG) Calculation through the First Stage of Optimization Problem

To model the demand side reliability, the interruption cost of load should be considered in the first stage of optimization problem. It is obvious that customer will reduce consumption when the price of electricity is higher than the customer marginal cost. This means when a contingency occurs in the power system, customer response to the variation of prices can be indirectly measured by the customer interruption cost, which expresses the importance of electricity service for customers. Customers classified in 5 groups from the viewpoint of electricity value for them. These groups are: large user, industrial, commercial, agriculture, residential categories. Ref. [19] has been estimated the customer interruption costs of each group to give sector Customer Damage Functions CDFs for each group, which is depicted in Fig. 2. In the power system with  $N_c$  independent component, the reliability parameters for the contingency state  $j$  with  $b$  failed component, can be calculated by applying Eqs. (10-12):

$$pr_j = \prod_{c=1}^b U_c \times \prod_{c=b+1}^{N_c} A_c \quad (10)$$

$$D_j = \sum_{c=1}^b \mu_c + \sum_{c=b+1}^{N_c} \lambda_c \quad (11)$$

$$d_j = 1 / D_j \quad (12)$$

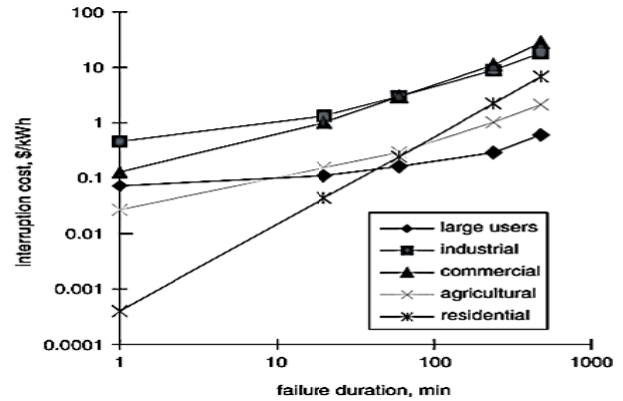


Fig. 2 Customer damage functions for different groups of customers.

The objective function of the first stage of optimization problem is to minimize generation cost and load curtailment cost (Eq. (13)) in every contingency state  $j$ , subject to Eqs. (14-20).

Constraint (14) represents power balance at every bus. Constraint (15) determines load curtailment cost according to customers' damage function in every contingency state  $j$ . Constraints (16)-(20) depict units capacity limits, units ramp rates, lines flow limits, maximum permissible load curtailment and buses' voltage limits respectively. The optimization problem is coded in Matpower in Matlab software.

#### First stage of optimization problem:

$$\begin{aligned} \text{Min } f_j = & \sum_{i \in NG} \sum_{g \in NG_i^j} C_{ig}^j (P_{ig}^j) + \\ & \sum_{i \in NL} \sum_{s \in NL_i^j} OC_{is}^j (LCP_{is}^j) \end{aligned} \quad (13)$$

Subject to:

$$\sum_{g \in NG_i^j} P_{ig}^j - \sum_{s \in NL_i^j} (P_{is}^{j0} - LCP_{is}^j) = \sum_{k=1}^N |V_i^j| |V_k^j| |Y_{ik}^j| \cos(\theta_i^j - \theta_k^j - \delta_{ik}^j) \quad (14)$$

$$OC_{is}^j (LCP_{is}^j) = LCP_{is}^j \times CDF_s(d_j) \quad (15)$$

$$P_{ig}^{j, \min} \leq P_{ig}^j \leq P_{ig}^{j, \max} \quad (16)$$

$$\Delta P_{ig}^{j, \text{low}} \leq P_{ig}^j - P_{ig}^{j0} \leq \Delta P_{ig}^{j, \text{upp}} \quad (17)$$

$$|S_{ik}^j|^{\min} \leq S_{ik}^{j, \max} \quad (18)$$

$$0 \leq LCP_{is}^j \leq LCP_{is}^{j, \max} \quad (19)$$

$$|V_i^j|^{\min} \leq |V_i^j| \leq |V_i^j|^{\max} \quad (20)$$

The output of this stage of optimization is  $P^j$  (generation vector) and  $LCP^j$  (load curtailment of active power vector) at each contingency which are the basis of calculating Expected Load (EL) and Expected Generation (EG) at each bus.

Following the first stage of optimization problem, here two parameters are introduced to consider the contingency nature of power system at each bus.  $EL$  and expected  $EG$  at each bus are parameters which represent the contingency nature of power system at each bus. These parameters are calculated after running the first stage of optimization as explained below.

At each bus the LCP is load curtailment of active power which can be calculated as below:

$$LCP = \sum_1^{SN} LCP_j * pr_j \quad (21)$$

In the above formula  $pr_j$  denotes the probability of contingency  $j$ .  $SN$  denotes the number of total contingencies. Then  $EL$  supplied at each bus is equal to normal situation load minus LCP.

$$EL = D - LCP \quad (22)$$

For each generator,  $p^*$  is the generator's active power variation in contingency states with respect to normal state generation ( $p_0$ ), which can be calculated as below:

$$p^* = \sum_1^{SN} (p - p_0) * pr_j \quad (23)$$

The  $p^*$  could be positive or negative according to the result of optimization problem. Then  $EG$  for each generator is defined equal to normal state generation plus  $p^*$ .

$$EG = p_0 + p^* \quad (24)$$

Fig. 3 is graphical summary expresses the relationship between input and outputs of the first and second stage of optimization problem. As Fig. 3 depicts, by running the first step of the optimization problem,  $P$  &  $LCP$  are obtained and utilized to calculate  $EL$  &  $EG$  through Eqs. (21)-(24).

Then these parameters ( $LE$  &  $EG$ ) are applied to the second stage of optimization problem to calculate the modified nodal prices.

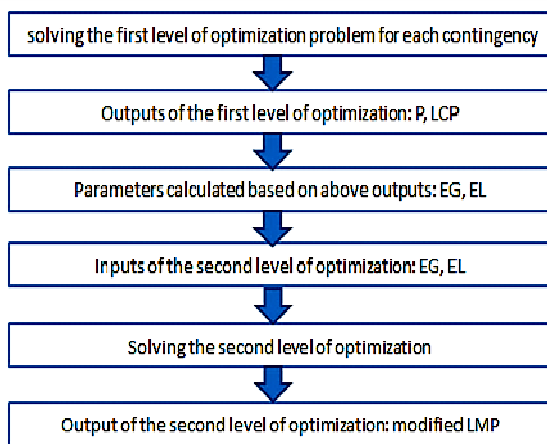


Fig. 3 Relationships between the stages of two stage optimization problem.

### 3.3 Basis of the Second Stage of Optimization Problem

In second stage of optimization, the nodal price ( $\lambda$ ) will be modified so that the expected generator revenue ( $\lambda * EG$ ) and expected load payments ( $\lambda * EL$ ) get close as much as possible to their rational values.

The rational revenue of a generator is equal to its rational profit plus its generation cost and the rational profit of a generator is directly related to its efficiency as expressed in the Eq. (25) ( $\eta_G$  has been calculated in section 3-1-1).

$$GR(Q_G) = GC(Q_G) + Gprofit * \eta_G / \sum \eta_G \quad (25)$$

Also for loads, the payment of a customer (or load cost) is equal to its revenue minus the rational load profit. The rational load profit is directly related to its efficiency as expressed in below equation ( $\eta_D$  is calculated in section 3-1-2).

$$LC(Q_D) = LR(Q_D) - Lprofit * \eta_D / \sum \eta_D \quad (26)$$

In this stage of optimization the difference between the expected revenue and rational revenue of generation companies and the difference between the expected payment and the rational payment of customers are minimized subject to satisfaction of following constraints (Eqs. (28)-(29)). The second stage of optimization is formulated as below:

$$\begin{aligned} \text{Min}_{\lambda} \left\{ \left( \lambda \cdot EG - [GC(EG) + Gprofit * \eta_G / \sum \eta_G] \right) \times GBuses \right. \\ \left. + \left( \lambda \cdot EL - [LR(EL) - Lprofit * \eta_D / \sum \eta_D] \right) \times DBuses \right\} \quad (27) \end{aligned}$$

Subject to the following constraints:

$$\lambda * EG - GC = Gprofit = \frac{PS_{producer}}{\sum COPS} SWT \quad (28)$$

$$LR - \lambda * EL = Lprofit = \frac{PS_{customers}}{\sum COPS} SWT \quad (29)$$

The first constraint causes the whole system generation companies profit be equal to  $Gprofit$  (calculated in section 2) and The second constraint causes the whole system loads profit be equal to  $Lprofit$  (calculated in section 2). Totally these constraints cause fair allocation of  $SWt$  between generation companies and customers according to their role in the efficiency of the system.

The  $EG$  and  $EL$  vector are constant in the second stage of optimization. Ultimately the result of second stage of optimization problem is modified nodal price ( $\lambda$ ) at each bus.

### 4 Numerical Results

The proposed nodal pricing method is applied on RBTS reliability test system [20]. In the first stage of optimization the first and second order contingency is considered.

**Table 1** Generating units' reliability data.

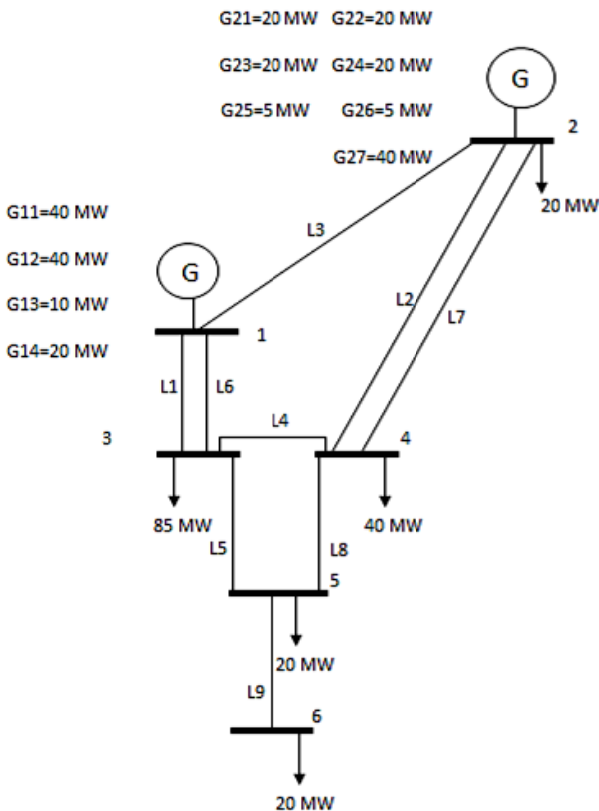
Unit capacity (MW)	Number of Units	FOR	MTTF (h)	MTTR (h)	Bus Number
5	2	0.01	4380	45	2
10	1	0.02	2190	45	1
20	4	0.015	3650	55	2
20	1	0.025	1752	45	1
40	1	0.02	2920	60	2
40	2	0.03	1460	45	1

**Table 2** The coefficients of demand revenue function.

Load Revenue (LR)	A	B
$LR(D)=A \cdot D^2+B$	-0.011	50

**Table 3** Expected Load & Expected Generation at each bus.

EL of Bus (MW)					
1	2	3	4	5	6
0	20	83.49223	39.86294	19.45538	19.5512
EG of Bus (MW)					
1	2	3	4	5	6
56.95	131.1	0	0	0	0



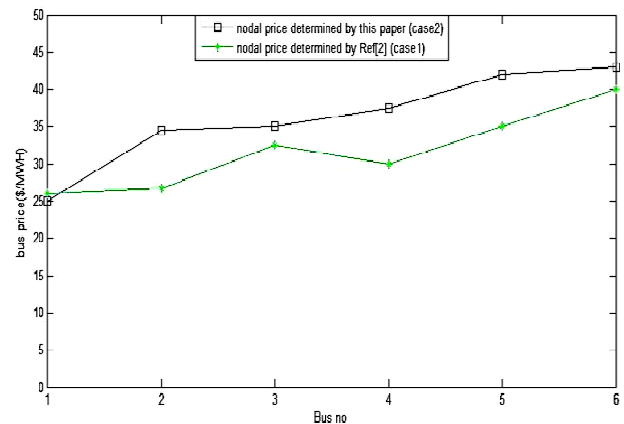
**Fig. 4** RBTS diagram with load & generation data.

The diagram of 6 bus reliability test system with load & generation data is depicted in Fig. 4. The reliability data of generating units is depicted in Table 1. Also the coefficient of demand revenue function is depicted in Table 2 as Ref. [5]. It should be noted that choosing larger test case increases the calculation consumedly.

Result of Table 3 denotes that bus 3 has the maximum LCP among the other buses. This is due to existence of the major part of system load in bus 3 that causes congestion on lines connected to this bus in some contingencies and congestion in these lines are alleviated through load curtailment in bus 3. Hence the Expected Load (EL) in this bus decreases more than other buses. Reversely bus 2 has no LCP because of the high value load and also existence of generation units on that bus. Totally variations in load and generation of power system in comparison to normal state are not great but this slight changes (as explained in the following) have considerable effect on price and there for on profit of generation units and loads.

Fig. 5 depicts buses' prices in two cases. The first case is nodal prices resulted from Ref. [5] method without considering reliability of the system and the second case is nodal prices resulted from the proposed method in this article (considering reliability of system both for pricing and market participant evaluation). As depicted in this figure the nodal prices have considerable change with respect to Ref. [5] method and this causes great change in profits and revenues of market participants.

As Fig. 5 illustrates, the determined nodal price at bus 1 is lower than the price at bus 2 in both cases. This is due to more efficient generator at bus 2 in comparison to bus 1. According to Fig. 3 by considering FOR of generating unit (case 2), the nodal price at bus 1 decreases while at bus 2 increases due to small FOR of generating units at bus 2 in comparison to bus1. So in proposed method generating units' revenue at bus 2 increases while generating units' revenue at bus 1 decreases.



**Fig. 5** Nodal prices in cases 1 and 2.

Also at bus 3 in comparison to bus 4, in case 1 the nodal price decreases while in case 2 increases. That is due to low value load at bus 4 that comes to consideration in proposed method. In both cases the price at buses 4-6 increases due to more usage of transmission lines by the loads at these buses.

Figs. 6 and 7 illustrate the generation and load profit at buses in cases 1 and 2. As depicted in these figures the generation profit and load profit have been changed considerably when reliability of the system comes to consideration in case 2. The profit of generators at bus 2 is increased in case 2 caused by more efficient generator at bus 2 and their lower FOR; whereas the profit of generators at bus 1 decreases. Also load profit at bus 4 has a considerable change in comparison to case 1. Bus 4 has low value loads, so its profit decreases more in case 2. Also whole system loads profit decreases in case 2 due to high nodal price at buses in case 2.

In this test system the major part of the load is supplied through generators at bus 2 and also these generators are more efficient and have small FOR.

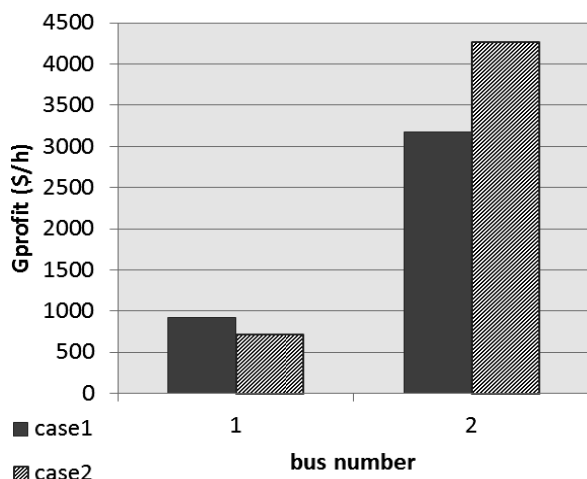


Fig. 6 Generation profit in cases 1 and 2 at buses 1 and 2.

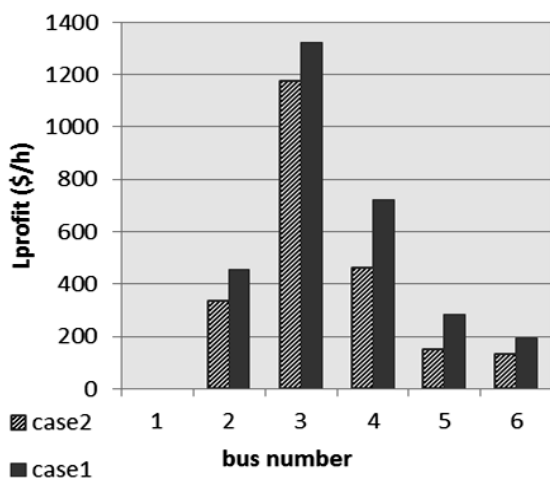


Fig. 7 Load profit in cases 1 and 2 at each bus.

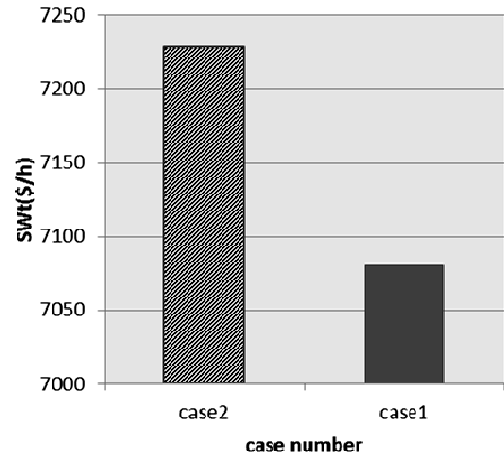


Fig. 8 the social welfare in case 1 and case 2.

In case 2 optimality of these generators affect the revenues of these generators and increases the revenue of generators at bus 2.

As the results demonstrates generators' benefits at bus 2 increases in case 2 (as shown in Fig. 6) and the total load profit decrease (as shown in Fig. 7). As depicted in Fig. 8 the social welfare of case 2 increases in comparison to case 1. This increase in SWt demonstrates that whole system efficiency is increases using the proposed pricing method.

Ultimately according to the numerical results, the proposed pricing method which considers the reliability of market participants in pricing and also considers reliability criteria for cost allocation is superior in comparison to the Ref. [5] pricing method.

## 5 Conclusions

This paper modifies nodal pricing method through a two stage optimization method considering reliability and optimality of each power market participant. At first, comprehensive benchmarks are introduced to evaluate whole system efficiency (SWt). In the first stage of optimization, two probabilistic parameters EL & EG are calculated for every bus which are applied to the second stage of optimization to modify nodal prices. Also in the second stage of optimization, reliability and optimality criteria causes that each market participant benefits vary according to its optimality. Results demonstrates that considering reliability in nodal pricing method has great effect on nodal prices and therefore on profits of market participants. Also total social welfare in market that is cleared with the proposed method is higher than the previous nodal pricing methods. Based on results, the proposed nodal pricing method increases system efficiency. Results demonstrate that reliability can affect nodal pricing considerably hence pricing method without considering the reliability is not a fair pricing. The ultimate result is fair pricing that is the result of considering optimality and reliability of market participant simultaneously.

## Nomenclature

Here are symbols which are used in this paper.

$N_g$ : Generation bus.

$N_l$ : Load bus.

$Q_G$ : Column vector of active power generation of generator (MW).

$Q_G^{UP}$ : Column vector of higher active power generation limit of generator (MW).

$Q_D$ : Column vector of demand of bus (MW).

$\delta$ : Column vector of voltage angle (rad).

$P(\delta)$ : Column vector of net active power injected to system buses (MW).

$f$ : Column vector of power flow in the system lines (MW).

$B$ : transmission network susceptance matrix ( $N_B * N_B$ ).

$H$ : sensitivity matrix  $f = H\delta_x$  ( $N_L * N_L$ ).

$SWt$ : Total Social welfare, i.e summation of all participant economic profit in the network after running OPF (\$/hour).

$SWLG$ : Loads & Generators Social Welfare in the network after running OPF.

$Gbenefit$ : Generators benefit in the network after running OPF (\$/hour).

$Lbenefit$ : Loads benefit in the network after running OPF (\$/hour).

$Nbenefit$ : Network benefit in the network after running OPF (\$/hour).

$NR$ : Network Revenue in the network after running OPF (\$/hour).

$IC$ : Instruction cost of lines in the network after running OPF (\$/hour).

$LR$ : Loads Revenue in the network after running OPF (\$/hour).

$LC$ : Loads Cost in the network after running OPF (\$/hour).

$GC$ : Generators Cost in the network after running OPF (\$/hour).

$GR$ : Generators Revenue in the network after running OPF (\$/hour).

$LCP$ : Load curtailment of active power.

$FOR$ : Forced outage rate.

$U$ : Unavailability of a component.

$A$ : Availability of a component.

$\lambda_c$ : Failure rate of a component.

$\mu_c$ : Repair rate of a component.

$d_j$ : The mean repair time of a failed component.

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