Design of Endlessly Single Mode Photonic Crystal Fibers with Desirable Properties using HC-EDA Algorithm

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Abstract: In this article, Hill Climbing (HC) and Estimation of Distribution Algorithm (EDA) are integrated to produce a hybrid intelligent algorithm for design of endlessly Single Mode Photonic Crystal Fibers (SMPCFs) structure with desired properties over the C communication band. In order to analyzing the fiber components, Finite Difference Frequency Domain (FDFD) solver is applied. In addition, a special cost function which simultaneously includes the confinement loss, dispersion and its slope is considered in the proposed optimization algorithm. The results revealed that the proposed method is a powerful tool for solving this optimization problem. The optimized PCF exhibits an ultra-low confinement loss and low dispersion at 1.55 µm wavelength with a nearly zero dispersion slope over the C communication band.

Keywords: Confinement Loss, Dispersion, Hill Climbing, Estimation of Distribution Algorithm, Photonic Crystal Fiber.

1 Introduction
With the advent of the photonic crystal materials, a new concept in fiber optics called Photonic Crystal Fiber (PCF) has come to forefront in fiber research. One of the most promising applications of photonic crystals is the possibility of creating compact integrated optical devices with photons as the carriers of information, and then the speed and bandwidth of advanced communication systems can be increased dramatically.

Usually PCF are all pure silica fibers with a regular array of air-holes running along the length of the fiber acting as the cladding. A defect in the periodical structure acts as a core. PCFs possess dispersion properties significantly different from those of conventional fibers, for example: endless single-mode, ultra-flattened dispersion, and super-continuum generation [1-3].

Control of dispersion, dispersion slope and losses in PCFs is very important problem for realistic applications of optical fiber communications. Several designs for the PCF have been proposed to achieve the ultra-flattened dispersion properties. By varying different parameters of the photonic crystal fibers, such as the pitch ($\Lambda$) of the periodic array, the holes radius ($r$), the number of air-hole rings ($N$) and the refractive index ($n$), one can engineer the electromagnetic modes supported by the photonic crystal fibers and explore suitable properties for many practical applications.

The optimization of PCF design is often difficult due to the fact that the optical properties do not usually vary in a simple way with the fiber geometry parameters. The optimization problem of PCF gets more and more difficult as the numbers of variables \{\$\Lambda$, $r$, $N$, $n$\} and the number of fiber properties that should be considered (chromatic dispersion, slope of this dispersion, confinement loss, etc…) are increased. The design optimization is usually performed by trial and test approach. However, this is a time consuming approach, both for the computer and the designer. In recent works, heuristic optimization algorithms have been shown to offer a convenient platform for the solution of the optimization problems [4-9].

Hill Climbing (HC) is a technique for certain classes of optimization problems. The idea is to start with a sub-optimal solution to a problem (i.e., start at the base of a hill) and then repeatedly improve the solution (walk up the hill) until some condition is maximized (the top of the hill is reached) [10]. On the other hand the Estimation of Distribution Algorithm (EDA) offers another technique in which a probability model characterizing the distribution of excellent solutions [11]. This paper proposes a combination of HC and EDA (HC/EDA algorithm) to solve the optimization problem and to determine the parameters of PCF structure.

Consider that the chromatic dispersion is a key parameter for many applications, this study is focused
on the determination of the PCF structure that can lead to the minimum dispersion and nearly zero dispersion slope over C communication band. In addition, the other significant parameter, the confinement loss is optimized simultaneously. Therefore, two dimensional finite difference frequency domain (2D-FDFD) method is applied to determine the effective index of propagation of mode which then enables to ascertain the dispersion properties of PCFs structure [12-15].

This paper is organized as follows: In the next section, the fiber geometry structures and problems associated with the optimizing fiber structure are stated. This is followed by section 3, in which the principles of HC/EDA algorithm will be described. Section 4 will focus on the simulation results, analysing and making comparisons with similar works carried out in this field. The paper sets out its conclusion in section 5 and finally the trend for future research works will be pointed out in the last section.

2 Fiber Design and Optimization

At present, the design and optimization of Photonic Crystal fibers is still an area of active research [16]. As shown in Fig. 1, all the air-holes in the section of typical PCFs are arrayed according to triangle regularity with identical pitch $\Lambda$, spacing of the neighbouring air-holes. The scale of the air-holes is denoted by $r$ of its radius. Background is pure silica. Because the effective refractive index of the core region is higher than the cladding region, total internal reflective (TIR) can occur in the interface between the core and cladding.

Two major issues of the PCFs designing are chromatic dispersion and confinement loss which are explained in the followings.

PCFs possess the attractive property of great controllability in chromatic dispersion. Controllability of chromatic dispersion in PCFs is a very important problem for practical applications to optical communication systems, dispersion compensation, and nonlinear optics. So far, various PCFs with remarkable dispersion properties have been investigated numerically.

A mode of a PCF is characterized by the mode’s field pattern and its effective indices $n_{\text{eff}} = \beta/k_0$, where $\beta$ is its propagation constant and $k_0 = 2\pi/\lambda$ is the free space wave number. Because of the finite transverse extent of the confining structure, the effective index is a complex value.

The chromatic dispersion $D$ of a PCF is easily calculated from the effective index of the fundamental mode $n_{\text{eff}}$ versus the wavelength using

$$D = \frac{\lambda^2}{c} \frac{d^2 n_{\text{eff}}}{d\lambda^2}$$

in which, $c$ is the speed of light in vacuum and $n_{\text{eff}}$ is a function of wavelength and material dispersion $(n_m(\lambda))$, so that;

$$n_{\text{eff}} = \beta(n_m(\lambda))/k_0$$

where $n_m(\lambda)$ can be estimated by using the Sellmeier’s formula [13, 17]. The background material is Corning Silica whose refractive index can be estimated using the following Sellmeier’s equation:

$$n^2(\lambda) = 1 + \frac{0.6837\lambda^2}{\lambda^2 - 0.00460353 \lambda^2 - 0.133969} + \frac{0.420324\lambda^2}{\lambda^2 - 64.4933} + \frac{0.585027\lambda^2}{\lambda^2 - 64.4933}$$

When the holes diameter to pitch ratio ($d/\Lambda$) is very small and the pitch is large, the dispersion curve is close to the material dispersion of pure silica. As the air-hole diameter is increased, the influence of waveguide dispersion becomes stronger [18-19].

Confinement loss is an additional form of loss that occurs in single-material fibers and it reflects the light confinement ability within the core region. When the optical mode propagates in the core region, one must take into consideration that the number of layers of air-holes is finite and leaking light from the core to the exterior material occurs through the bridges between air-holes, resulting in confinement loss. Confinement loss $L$, in units of dB/km, is calculated as follows:

$$L = \frac{2\pi}{\ln 10} \left( \frac{20}{\ln 10} \frac{\text{Im}(n_{\text{eff}})}{\lambda} \right) = 8.686 \frac{2\pi}{\lambda} \text{Im}(n_{\text{eff}})$$

where $\text{Im}(n_{\text{eff}})$ is the imaginary part of the refractive index [20]. This confinement loss can be reduced exponentially by increasing the number of rings of air holes that surround the solid core, and is determined by the geometry of the structure.

As mentioned in the previous section, FDFD method combined with HC/EDA algorithm is used to optimize the fiber’s profile as well as to accurately determine its modal properties. The simulation study was carried out with the database consisting of 530 individuals. Every individual has 4 features which are fiber parameters including pitch ($\Lambda$), number of air-hole rings ($N$), refractive index ($n$), and air-holes radius ($r$).
The logical constraints are considered in the optimization process. The refractive index range of silica is $1.44 \leq n \leq 1.46$. In the structure, the air-hole diameter changes between 0.25 $\mu$m and 0.45 $\mu$m. In fact, unlike conventional fibers, triangular PCFs can be designed to be endlessly single mode (ESM) that is to support only the propagation of the fundamental mode whatever the wavelength and the pitch value. From the previous works [8-9, 21], $d/\Lambda$ is chosen less than 0.406 to guarantee single mode operation of PCF design. Furthermore, the lattice constant or pitch might be set to any value (microns). The value of lattice constant limits the value of the radius of particular air-hole. The radius should be lower than the half lattice constant as, mathematically, the diameter cannot be greater than the pitch and neither can it be equal to this value because the silica would cease to be continual. Here, the pitch varies in the range of 1.5 $\mu$m to 3 $\mu$m. Also the number of air-holes rings is selected between 5 and 9.

The characteristics of the individuals chosen here are dispersion ($D$) and its slope ($S$) in the wavelength range from 1.53 $\mu$m to 1.565 $\mu$m (C communication band). These characteristics are calculated using the set of parameters $\{A, r, N, n\}$. The optimization problem is now considered as:

$$\min f(x)$$

where $f(x)$ is a real-value function which has to be minimized to find the best solution. So, it is needed to define the preferred cost function for the proposed algorithm. Here are three kinds of cost functions:

$$f_1(x) = \sum_{\lambda} |D|$$

(6)

$$f_2(x) = \sum_{\lambda} |D|\times\sum_{\lambda} |S|$$

(7)

$$f_3(x) = \sum_{\lambda} |D|\times\sum_{\lambda} |S|\times\sum_{\lambda} |L|$$

(8)

As it can be seen in the first case, dispersion is minimized and in the second case, both dispersion and dispersion slope are minimized while in the third one, dispersion, dispersion slope and confinement loss are minimized simultaneously.

3 Design Strategy

Heuristic optimization algorithms, applied to inverse PCF design, involve a stochastic search for a globally optimal PCF structure provides the best performance of a PCF for a specific function. In this paper, an optimum design technique for PCF that utilizes an algorithm combining HC and EDA (HC/EDA) is proposed. Simulation results demonstrate that HC/EDA outperforms the other popular optimization algorithms. In this section, HC and EDA algorithms are explained briefly and then, the applied optimization algorithm, HC/EDA is explained in details.

3.1 Hill Climbing

Hill climbing is a greedy local search algorithm and can be used for optimization problems. Hill climbing algorithms can find reasonable solutions in large or infinite (continuous) state spaces for which systematic algorithms fail. Generally, when looking for a maximum of a function (optimization problem), the hill climbing algorithm works as follows:

Step1: Start at an arbitrary point.
Step2: Calculate values for neighbouring points.
Step3: Move to the point with increased value.
Step4: Terminate if no higher value could be found, otherwise continue at 1.

The standard problem with this algorithm is that it may not find the optimal solution (i.e. the global maximum), but only a local maximum. However, with an extension known as random-restart one can increase the probability to find a global maximum considerably. Starting the hill climbing algorithm over and over again each time with randomly chosen initial states and saving only the maximum of the new values improves the probability of finding the global maximum.

The hill-climbing algorithm is a local optimization algorithm. It can exploit the information about the current search point effectively. However, the search may be trapped at a local optimum [22].

3.2 Estimation of Distribution Algorithm

EDA is a new class of GAs. EDA directly extracts the global statistical information about the search space from the search so far and builds a probability model of promising solutions. New solutions are sampled from the model thus built. Let $Pop(t)$ be the population of solutions at generation $t$. EDAs work in the following iterative way: At first $M$ promising solutions is selected from population $Pop(t)$ to form the parent set $Q(t)$ by a selection. Afterward a probabilistic model $p(x)$, based on the statistical information extracted from the solutions in $Q(t)$ is built. In the next step, new solutions according to the constructed probabilistic model $p(x)$ are generated. Finally, new solutions fully or partly replace in the population $Pop(t)$ to form a new population $Pop(t+1)$.

One of the major issues in EDAs is how to select parents. A widely-used selection method in EDA is the truncation selection. In the truncation selection, individuals are sorted according to their objective function values and only the best individuals are selected as parents [10]. Another major issue in EDAs is how to build a probability distribution model $p(x)$. In EDAs for the global continuous optimization problem, the probabilistic model $p(x)$ can be a Gaussian distribution [23], a Gaussian mixture [10, 24], a histogram [25], or a Gaussian model with diagonal covariance matrix (GM/DCM) [24] which is utilized in this work.
In GM/DCM, the joint density function of the k-th generation is written as follows:

\[ p_k(x) = \prod_{i=1}^{N} N \left( x_i; \mu_i^k, \sigma_i^k \right) \]  

(9)

where

\[ N(x_i; \mu_i^k, \sigma_i^k) = \frac{1}{\sqrt{2\pi\sigma_i}} \exp \left( -\frac{1}{2} \left( \frac{x_i - \mu_i^k}{\sigma_i} \right)^2 \right) \]  

(10)

In Eq. (9), the n-dimensional joint probability distribution is factorized as a product of n univariate and independent normal distributions. There are two parameters for each variable required to be estimated in the k-th generation: the mean \( \mu_i^k \), and the standard deviation \( \sigma_i^k \). They can be estimated as follows:

\[ \hat{\mu}_i^k = \bar{x}_i^k = \frac{1}{M} \sum_{i=1}^{M} x_{j,i}^k, \quad \hat{\sigma}_i^k = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (x_{j,i}^k - \bar{x}_i^k)^2} \]  

(11)

where \( x_{j,i}^k, x_{j,2}^k, \ldots, x_{j,M}^k \) are values of the i-th variable of the selected M parent solutions in the k-th generation.

3.3 HC/EDA Algorithm

This hybrid algorithm combines a global search algorithm (EDA-Estimation of Distribution Algorithm) with a local search Algorithm (HC-Hill Climbing) in order to maintain a balance between the exploration and exploitation.

The most important operation in this algorithm is to generate new offspring. Each offspring is sampled from the model built by the probability model. In the other word, EDA tries to guide its search towards a promising area by sampling new solutions from a probability model. The EDA mechanism is incorporated into the HC algorithm in order to find solutions which are more promising than solutions generated by the EDA, and consequently, to explore the search space more effectively [23]. The pseudo-code of HC/EDA algorithm is given as follows:

**Step 1:** Initialize population: Randomly generate N solutions \( x_1^0, x_2^0, \ldots, x_N^0 \) from the feasible search space to form an initial population, set \( k=0 \).

**Step 2:** Perform Hill Climbing on the initial population.

**Step 3:** Count fitness value for the initial population.

**Step 4:** While Maximum Generation \( \geq k \) do:

- Select the best M solutions from the current population, construct a probability model as Eq. (9). Here M is equal to the half of the population size.
- Build a probabilistic model \( p(x) \) based on the statistical information extracted from the solutions in \( Q(t) \).
- Sample new solutions according to the constructed probabilistic model \( p(x) \).
- Fully or partly replace solutions in \( Pop(t) \) by the sampled new solutions.

- For \( i=1 \) to Number of solutions do: Perform Hill Climbing on New-Solutions Count fitness value for New-Solutions End for.
- Merge the new chromosomes with old chromosomes.
- Select fittest chromosomes from all the chromosomes as the next generation. End while.

The ability and performance of this algorithm for optimization of the PCFs structure is presented in details in the following section.

4 Implementation of HC-EDA Algorithm; Results and Discussion

In this section the results of HC/EDA method in order to optimize the PCFs properties are presented. Also, the performance of this method is compared with the results of DE (Differential Evolution), EDA and DE/EDA methods.

At the first generation, a population of “individuals” is randomly created, each individual being a possible solution to the problem. In the particular case of this paper, each individual corresponds to a particular design of PCF and has 4 parameters \( \{A, r, N, n\} \) which constitute the variables of the problem. The simulation study was carried out with the database consisting of 530 individuals. These following steps are performed 10 times:

First of all 100 individuals are selected randomly. Then HC/EDA algorithm is applied to this selected population. In order to calculate the cost function, one needs to determine the PCFs characteristics over the C communication band. As mentioned in the previous section, FDFD method is applied to analyze the dispersion and loss properties of the triangular PCF and it has been one of the major tools for the analysis and understanding of PCFs. This evolution process continues until the number of generations is equal to a given maximum value (It is 100 in this case). In the last step, the ten best individual are selected and they put in the pool as the new population. Finally, a new population is created with 100 individuals. Again the proposed algorithm is performed with this population and the best individual with the minimum cost function is selected as the solution. In this case, the number of generations is made equal to 100. In order to make a fair comparison, the process is repeated several times for each cost function of these algorithms.

4.1 Cost Function with Dispersion

The first cost function is the summation of absolute dispersion over all \( \lambda \) (wavelength) in the specified wavelength range of optimization (C band). The results of the optimization are summarized in Tables 1 and 2.
Table 1 shows the best individuals (solutions) achieved by the algorithms and Table 2 presents the corresponding dispersion characteristics. Also, the dispersion characteristics as a function of wavelength are depicted in Fig. 2. As it can be seen the minimum value of dispersion at 1.55 µm wavelength is for the PCF (best solution) achieved by the EDA method. Although the dispersion slope and the effective mode field area (EMF) are not intended by this cost function, but the best PCF found by DE algorithm has the minimum dispersion slope and the maximum effective mode field area. However, the hybrid algorithms; DE/EDA and HC/EDA yield approximately the same results.

4.2 Cost Function with Dispersion and Dispersion Slope

The second cost function is the summation of the absolute dispersions over all λ multiplied by the summation of absolute dispersion slope over all λ in the specified range of optimization. Tables 3 and 4 summarize the results of the optimization. Table 3 shows the best individuals achieved by the algorithms. The dispersion characteristics are depicted in Table 4 and also in Fig. 3. It is obvious that overall the DE/EDA outperforms the other algorithms with this cost function, but the solution found by HC/EDA has dispersion slope characteristics superior than that of DE/EDA and also has a large effective mode field area.
4.3 Cost Function with Dispersion, Dispersion Slope and Confinement Loss

In this case, the cost function is defined as Eq. (11). Tables 5 and 6 show the best individuals (best solutions) achieved by the algorithms and their characteristics respectively. The dispersion and confinement loss characteristics of the optimized PCFs are depicted in Fig. 4 and Fig. 5. It is obvious that overall the HC/EDA outperforms the other algorithms.

Table 5 The solution (PCFs parameters) found by 4 methods with the third cost function.

<table>
<thead>
<tr>
<th>Method</th>
<th>N</th>
<th>Λ (µm)</th>
<th>n</th>
<th>r (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE</td>
<td>9</td>
<td>2.989</td>
<td>1.46</td>
<td>0.6576</td>
</tr>
<tr>
<td>EDA</td>
<td>9</td>
<td>3</td>
<td>1.459</td>
<td>0.6407</td>
</tr>
<tr>
<td>DE/EDA</td>
<td>9</td>
<td>2.919</td>
<td>1.453</td>
<td>0.6345</td>
</tr>
<tr>
<td>HC/EDA</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6 The properties of PCFs found by 4 methods with the third cost function at 1.55 µm wavelength.

<table>
<thead>
<tr>
<th>Method</th>
<th>D (ps/nm/km)</th>
<th>S (ps/nm²/km)</th>
<th>EMF (µm²)</th>
<th>L (dB/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE</td>
<td>18.45</td>
<td>0.013428</td>
<td>25.07</td>
<td>2.1×10⁻⁷</td>
</tr>
<tr>
<td>EDA</td>
<td>20.86</td>
<td>0.01187</td>
<td>24.48</td>
<td>2×10⁻⁸</td>
</tr>
<tr>
<td>DE/EDA</td>
<td>18.85</td>
<td>0.01285</td>
<td>24.98</td>
<td>1.25×10⁻⁷</td>
</tr>
<tr>
<td>HC/EDA</td>
<td>18.96</td>
<td>0.01143</td>
<td>23.43</td>
<td>1.28×10⁻⁸</td>
</tr>
</tbody>
</table>

4.4 Discussion

To summarize, we have shown that for this specific optimization problem, HC/EDA algorithm outperforms the other algorithms such as DE, EDA and DE/EDA algorithms when the optimization problem is the fiber with desired dispersion, dispersion slope and confinement loss. Although, there are some reports on the design and optimization of photonic crystal fibers, but most of the designed fibers do not provide single mode operation over the C communication band [26-30]. A related work that has used the simple GA is done by Kerrinckx et al [4]. In this work each individual has two chromosomes {Λ, r} and the dispersion error is defined as cost function. The best solution reported by [4] is a 9 ring structure of PCF with the pitch Λ = 2.35 µm and the radius r = 0.33 µm. The dispersion and dispersion slope of this PCF are 2.5 ps/nm/km at 1.55 µm wavelength and 0.03575 ps/nm²/km respectively. In another similar work the chromatic dispersion of 0.8 ps/nm/km at 1.55 µm wavelength has been obtained for a 9 ring structure with the following parameters: Λ = 2.59 µm and r = 0.29 µm [31].

In the proposed approach, a PCF with the dispersion of 0.25 ps/nm/km at 1.55 µm wavelength and dispersion slope of 0.02511 ps/nm²/km over the C communication band has been designed. In addition, a special cost function which simultaneously includes the confinement loss, dispersion and its slope is used in the proposed design approach. The optimized PCF exhibits an ultra-low confinement loss in order of 10⁻⁸ and low dispersion at 1.55 µm wavelength with a dispersion slope of 0.011 ps/nm²/km over the C communication band. So, it is revealed that HC/EDA method is a powerful tool for the optimum design of PCFs.

5 Conclusion

In this paper, a novel design technique using HC/EDA to achieve a SMPCF with desirable properties
is presented. The simulation results demonstrate that HC/EDA is an excellent method in optimization problem of PCF structure. The optimized PCF exhibits an ultra-low confinement loss in order of $10^{-4}$ and low dispersion at 1.55 $\mu$m wavelength with a dispersion slope of 0.011 $ps/\mu$m$^2$/km over the C communication band. With further optimization of the structure and increasing the number of individuals' chromosomes in HC/EDA method, PCFs characteristics can be further improved.

In further work, we are going to use Particle Swarm Optimization (PSO) and Gravitational Search Algorithm (GSA) for tackling this optimization problem. Also we will try to present the cost function using combination of dispersion, dispersion slope, confinement loss, bending loss and effective mode field area characteristics in order to design an optimum photonic crystal fiber. Furthermore, we will attempt to demonstrate that the number of the individuals’ chromosomes (PCF parameters) can be increased to achieve the PCF structure with desired characteristics.

References


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