

# A New Model for Assessment and Optimization of Number of Spare Transformers and Their Locations in Distribution Systems

M. Sedaghati\* and R. Dashti\*(C.A.)

**Abstract:** In this paper, a new model has been presented to determine the number of spare transformers and their locations for distribution stations. The number of spare transformers must be so that they need minimum investment. Furthermore, they must be sufficient for replacing with transformers that have been damaged. For this reason, in this paper a new purpose function has been presented to maximize profit in distribution company's budgeting and planning. For determining the number of spares that must be available in a stock room, this paper considers the number of spares and transformer's fault at the same time. The number of spare transformers is determined so that at least one spare transformer will be available for replacing with the failed transformers. This paper considers time required for purchasing or repairing a failed transformer to determine the number of required spare transformers. Furthermore, whatever the number of spare equipment are increased, cost of maintenance will be increased, so an economic comparison must be done between reduced costs from reducing of outage time and increased costs from spare transformers existence.

**Keywords:** Maintenance, Mobile Transformer, Replacement, Spare Transformer.

## 1 Introduction

In the power systems, transformers have a critical role in reliability and availability and need a huge investment. Moreover, the risk involved in running the system without proper attention to assets integrity in service is quite high. Additionally, the probability of losing any equipment vital to the transmission and distribution system, such as power and distribution transformers, is increasing especially with the aging of power system's assets [1]. Therefore, investment risk and reliability of this equipment must be considered efficiently. Maintenance, repairing, replacement are some of the options that are used in the asset management for increasing reliability and investment. Moreover, most of the distribution substations are operated radially, and their transformer failures cause outages. Transformer's failures and fatigues have an important role in customer outages, because, repairing, purchasing and installation time of this equipment are long. For this reason, for reducing of customer outage

time, having a stock room for storing an appropriate number of spare transformers are necessary. For reducing access time to spare equipment and minimizing of transportation cost and reducing outage time, stock room's location must be close to where the density of transformer's failure is high.

On the other hand, failure of transformers in distribution substations causes customer's outages and cost. Recent cost depends on customer type and outage duration. In this situation, time for recovering of failed unit, by repairing and purchasing a new unit may last several months that it depends on the type of failure. Outage durations may be reduced considerably by using of spare transformers. So the number of spare transformers must be selected to supply sufficient transformers in the stock room with high reliability. If the numbers of spare transformers are increased excessively, cost of purchasing and maintenance will increase. On the other hand, whatever, number of spares are increased, total cost will increase and if the number of spares be less than a critical value, we must pay much costs for customer's outages.

Researches and papers are presented in designing, planning and reliability of distribution systems [2]. Ref. [3] presented a probabilistic method based on binomial distribution for determining number of and timing of the spare unit in distribution substations. Ref. [4] presented

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\* The Authors are with the School of Electrical Engineering, Iran University of Science and Technology (IUST), Tehran, Iran.

E-mails: sedaghatimilad@elec.iust.ac.ir and rezadashti83@yahoo.com.

probabilistic models based on Poisson distribution and total cost concept (cost of spares and cost of outages) for determining spare transformer of distribution substations. Ref. [5] presented a probabilistic model based on binomial distribution with considering repair time or replacement time for a one distribution transformer and total cost concept for determining optimum number of spare transformers. Mobile transformers have not been considered in references [3, 5]. Ref. [6] presented two probabilistic methods for determining the optimal number of spare transformers. First method used a simple Markov model and second one used Monte Carlo simulation model for determining optimum number of spare transformers. The presented model given in [6] has considered only critical failures and has not considered required time for installation of a spare transformer. Furthermore, Ref. [6] have not used mobile transformers. Ref. [7] used a probabilistic model based on Markov model for determining optimal number of required spare transformers for high-voltage substation so that acceptable level of reliability and the lowest cost are obtained. Ref. [8] has applied a cost/benefit analysis based on Markov model to determine optimal number of spare transformers for high voltage auto-transformers. Ref. [9] used a probabilistic model based on Markov model to determine benefit of using mobile unit substation in high voltage substation. Ref. [6] also used three methods to determine optimum number of spare transformers. First and second method are based on reliability and third method is based on minimum cost. In the first method, system performing states have been described using Markov process. In the second method, Monte Carlo simulation model was used to assess system reliability. Ref. [10] described a probabilistic model to determine number of spare transformers and mobile unit substations for a group of distribution transformers with a target of achieve a desired level of availability. Ref. [11] presented a probabilistic method using Markov model to assess the number of spare transformers and movable units for a group of distribution transformers.

Ref. [12] proposed evaluation approaches to multi-item base-stock inventory policies where unidirectional substitutions are allowed. The problems in this paper are in the context of spare parts management and identifies two substation cases: substation upon demand arrivals and substation upon order deliveries. Ref. [13] have given an overview of the research, models and literature about optimization approaches to the problem of optimally locating one or more new facilities in an environment where competing facilities are already established. Ref. [14] is devoted to the establishment of reasonable levels of critical power equipment inventory using available historical data and strict legal requirements for reliable electrical power supply. Several models, including an Extreme Value Theory and a Homogeneous Poisson model, are used with

results compared to provide a probabilistic formulation of suggested inventory. Ref. [15] presented a decision support system based on a multi-attribute p-median model. And in their paper back-up transformer locations have been indicated and analyzed.

Ref. [16] presented a brief description of the elements of general asset management model in the context of an electric utility and the implementation of the intelligent system for detection and diagnosis as well as the failure rate estimation model exemplified using data of measurements performed in real power transformers. A robust anomaly detection module using prediction models based on artificial intelligence techniques developed for top oil temperature monitoring and the use of decision trees as classifiers for the assessment of FRA2 measurements is also illustrated. For failure rate estimation, the use of a model based on hidden Markov chains presented using data of dissolved gas analysis tests.

Ref. [17] suggested a procedure for determining an optimal combination of activities and measures to minimize the expected total costs during the planned exploitation period of power transformers. Combinations of different types of preventive maintenance, spare parts keeping and installation of Condition monitoring systems of power transformer individual components have been taken into consideration in [17].

Ref. [18] suggested a method for evaluation of justification of certain activities and measures aimed at improving power transformer availability and reducing expected cost during a planned exploitation period. The model in [18] is based on the assumption that the failure renewal time is not a deterministic parameter, but is generally distributed. All calculations in [18] were made for the case when failure renewal time is Weibull distributed.

In reference [19], a new probabilistic method based on Monte Carlo simulation for determining the optimum number of spare transformers has been presented. With the method presented in that paper, modeling of aging process and load growth are allowed whereas these are impossible via traditional methods like poison and Markov processes. Combination of suggested method in Ref. [19] with a heuristic algorithm, the optimum number of spare transformers are determined over a pre-established planning horizon.

The innovation that there is in this paper than another papers is that the optimal number of spare transformers is determined so that there is at least a spare transformer in the stock room for replacing with damaged transformer. With this method, the customer would not get long outages and customer's cost will be minimized. Furthermore, the optimal location of the spare transformer is considered and it is determined with Genetic algorithm so that the distance between spare transformer and failed locations is minimized. With this method replacing of failed equipment are done

with faster acceleration and also most of parameters like installation time, installation cost and other parameters that are effective in having spare equipment have been considered. Moreover, mobile transformers are considered so that customer outage time is decreased. Additionally, asset management activities like spare transformer maintenance, repairing and replacements of failed transformers are considered.

In this paper, section 2 has presented a new model to find the number and location of spare transformers with considering effective factors on cost function and relations between them. In section 3 numerical analysis on inputs and outputs and sensitivity analysis are performed.

## 2 Proposed Model for Finding the Optimal Number and Location of Spare Transformers

Transformer failures in distribution systems are not preventable. For this reason, distribution system must be planned, maintained and operated so that failures of such equipment will be minimized. The issue that how many number of spare transformers and mobile transformers in a distribution network should be used and also how their location should be determined so that reliability will be in an acceptable rating are the matters discussed in this paper. This issue focused on determining the number of spare transformers that must be used. For achieving this goal the number of transformer failures that can be repaired and cannot be repaired and required time for repairing and purchasing of transformers are considered.

Locating of equipment is an issue that it has been improved for equipment that they have a low mobility. Determining the best location of the spare transformer is an example of these problems.

In proposed model, spare transformer's location is obtained by means of the lowest distance from failed transformer. Technically, it is a function of distances between substations or population. Indeed, if the population of area is more, distances between substations will be less and their failures will be more. If spare transformer location be near to failed transformer location, transportation cost, outage cost and not distributed energy will be less. These issues that how many number of spare transformers and where they must be located are the main purposes of the proposed model.

In assessment of spare equipment, two factors can be considered:

1. Achieving to an acceptable level of reliability.
2. Achieving to the lowest cost (such as not distributed energy and cost of spare transformers and etc.).

Each two above cases can be considered in the following main function:

$$F = INV + IMC \quad (1)$$

In the Eq. (1), 'F' is the inserted cost to the system. Technically, that is the purpose function of the proposed

model. Furthermore, Eq. (1) includes 'INV' investment cost & 'IMC' Imposed cost. If the system reliability is reduced less than a predetermined level, costs or penalties will be increased. Moreover, optimal number and the best location of the spare transformer should minimize the inserted costs (F).

In the proposed model, it is assumed that mobile transformer is used when a transformer is failed. With this method, outage duration will be decreased. The mobile transformer remains instead of the failed transformer until the spare transformer arrives at the location of failure. And because they are in the circuit for a short time, they are completely reliable, and failure rate of this equipment can be assumed zero. Transformer replacement is done when the mobile transformer is in the circuit, therefore spare transformer installation time does not have an effect on outage duration. With assuming that there are enough spare transformers in the stock room for replacing with failed transformers, numbers of mobile transformers are selected so that they support the maximum number of failures in a month. With this method minimum outage duration is achieved. Otherwise, for preventing from purchasing and maintenance costs, number of them are selected so that each of them can be used in one month. In this situation, numbers of them may not be equal to maximum number of failures that occur in one month, because in that specific month, enough spare transformers may not exist for replacing with failed transformers. Therefore, finding optimum number of spare transformers is crucial for providing high reliability with minimum costs.

### 2.1 INV Model

INV comprises the following costs as Eq. (2).

$$INV = C_T + C_{TM} + C_S + C_{MS} + C_{MSM} + C_{TT} \quad (2)$$

where

$$C_T = nC_{TC} \quad (3)$$

$$C_{TM} = \sum_{j=1}^{12} N_j C_{TMC} + nC_{TPM} \quad (4)$$

$$C_S = nC_{SC} + kC_{MSST} \quad (5)$$

$$C_{MS} = kC_{MST} \quad (6)$$

$$C_{MSM} = kC_{MSMT}T \quad (7)$$

$$C_{TT} = \sum_{i=1}^{12} \sum_{j=1}^{m_i + A_i} ((D_j(C_{TTC} + C_{MST1}) + (C_{TTV} + C_{MSI}))/1000) \quad (8)$$

### 2.2 IMC Model

$$IMC = C_E + C_H + C_{TD} + C_{pn} \quad (9)$$

$$C_E = (ENS \times MC_{dist})/1000 \quad (10)$$

$C_E$  is revenue that has been lost for energy that has not been supplied. If the outage did not occur, the distribution company could earn these revenues.

$$ENS = \sum_{i=1}^{12} \sum_{j=1}^{m_i + A_i} \frac{r_{ij}}{60} L_{Eij} \quad (11)$$

ENS is the total energy not supplied in a year in kWh and  $r_{ij}$  is the  $j^{\text{th}}$  outage duration in  $i^{\text{th}}$  month in minute and  $L_{Eij}$  is the lost load at  $j^{\text{th}}$  outage in  $i^{\text{th}}$  month in kW.

$$C_H = r \times C_{hc} \times 3G \quad (12)$$

$C_H$  is the total annual cost of repairing groups. In above equation  $r$  is total outage duration in minute in a year that is calculated with (13):

$$r = \sum_{i=1}^{12} \sum_{j=1}^{m_i+A_{g_i}} r_{ij} \quad (13)$$

$$C_{TD} = (m - A_g)C_{TDC} + A_g \times C_{TC} \quad (14)$$

$$C_{pn} = (m + A_g)C_{pnc} + r \times C_{ptc} \quad (15)$$

$$m = \sum_{i=1}^{12} m_i \quad (16)$$

$$A_g = \sum_{i=1}^{12} A_{g_i} \quad (17)$$

Fig. 1 illustrates Eqs. 9-17.

### 2.3 Outage Duration Model

$$r_{ij} = r_f + D_{ij} \times r_m + r_a \quad (18)$$

$$r_{TT} = \sum_{j=1}^{m+A_g} r_{TTj} = \sum_{i=1}^{12} \sum_{j=1}^{m_i+A_{g_i}} (D_{ij} \times r_m + r_a) \quad (19)$$

Equations (18), (19) can be modeled as spare transformer replacement process durations that is shown in Fig. 2. In Fig. 2 system base speed is equal to management operation time.

### 2.4 NST Model

A spare transformer replacement process in  $i^{th}$  month includes below procedure:

Firstly, the number of failed and fatigued transformers are determined and then these information are given to the stock room management. Stock room

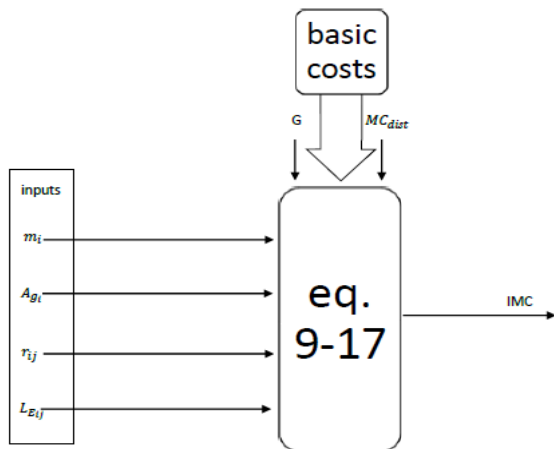


Fig. 1 Summarized relations between  $IMC$ 's inputs.

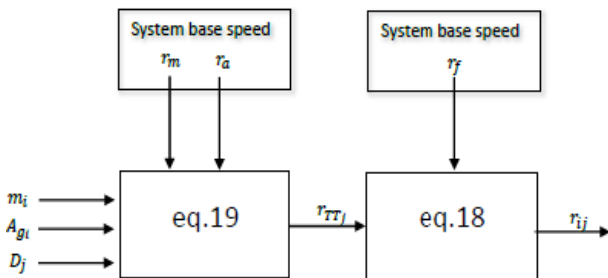


Fig. 2 Outage duration model.

management determines the number of transformers that must be repaired, purchased and allocated to the network. And this process must be repeated every month. In other words, number of spare transformers that are available in stock room in each month, is determined with the number of total spare transformers, number of failed and fatigued transformers in each month and their repair or purchase time span. Flowchart of this process has been shown in Fig. 3.

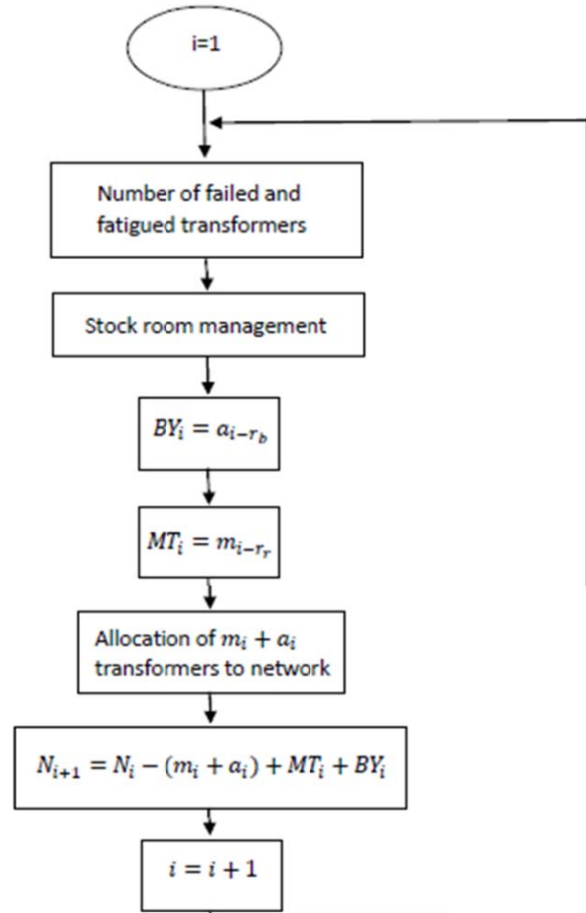


Fig. 3 Spare transformer replacement process.

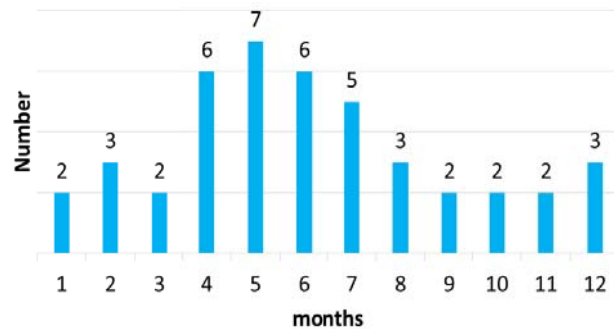


Fig. 4 Number of failed transformer that can be repaired.

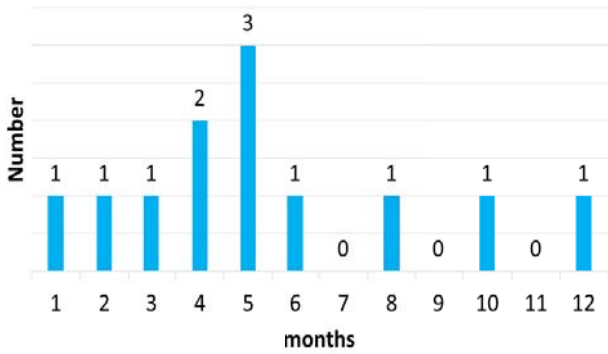


Fig. 5 Number of failed transformers that cannot be repaired.

For example, if  $r_r = 1, r_b = 2$  then  $N_1 = n$ ,  $N_2 = n - m_1 - a_1$ ,  $N_3 = n - m_2 - a_1 - a_2$ , for  $i = 4, 5, 6, 7, 8, 9, 10, 11, 12$ ,  $N_i = n - m_{i-1} - a_{i-1} - a_{i-2}$ .

### 3 Numerical Studies

#### 3.1 System Inputs

Numbers of failed transformers that can be repaired and cannot be repaired are assumed as shown in Figs. 4 and 5.

Failed transformer repairing duration is assumed 2 months and spare transformer purchasing duration is assumed 4 months. In other words  $r_r = 2, r_b = 4$ . Number of reparable transformers and non-reparable transformers, transformers that must be repaired and purchased and number of transformers that will be available in the stock room and are pertained to stock room management could be obtained easily.

Other inputs are: Cost of a transformer is 5 thousand dollars, cost of maintenance and service of each spare transformer is 0.05 (\$1000), cost of annual spare transformer repairing is 0.2 (\$1000), cost of spare transformer maintenance location is 1 (\$1000), cost of mobile transformer maintenance location is 2 (\$1000), cost of each mobile transformer is 10 (\$1000), cost of maintenance and service of each mobile transformer in each service period is 0.1 (\$1000), number of mobile transformer service is 4 in one year, cost of carrying of spare transformer in one kilometer is 25 (\$), cost of loading and depletion and installation of spare transformer is 150 (\$), cost of carrying each mobile transformer in one kilometer is 20 (\$), cost of mobile transformer installation is 100 (\$).

The locations of failed transformers in coordinate plane are assumed according to Fig. 6.

Number of below of each point shows the number of failures and number of above of each point shows lost load in each outage in kilowatt. Other inputs assumed as following:

$$\begin{aligned}
 r_a &= 20, r_m = 10, r_f = 30, MC_{dist} = 0.3 \\
 C_{hc} &= 1, G = 3, C_{tdc} = 0.5, C_{ptc} = 0.075, \\
 C_{pnc} &= 1, T_m = 2, T_b = 4
 \end{aligned}
 \tag{18}$$

### 3.2 Outputs (Simulation Results)

$L_s$  is the optimum location of the spare transformer that is obtained with optimum toolbox in MATLAB software. In this toolbox, the objective function for finding the spare transformer's location is defined so that, the total distances among spare transformer's location and failed transformer's locations gets minimum. Some of the main GA parameters and settings for optimization are as follows: Population size=20, selection function: Stochastic uniform, Mutation function: Constraint dependent, Migration direction: Forward, Migration fraction=0.2, Migration interval=20, and other required parameters like Mutation function, Crossover function, ... are assumed default.

$$L_s = [ 3.93 \ 5.53 ] \tag{19}$$

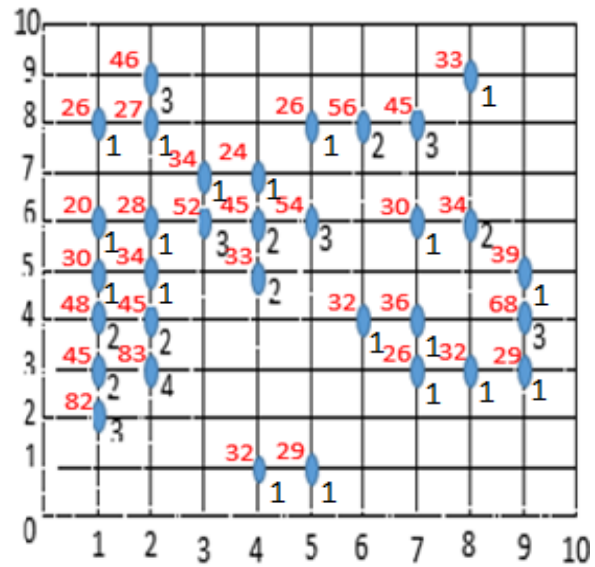


Fig. 6 Location of failed transformers.

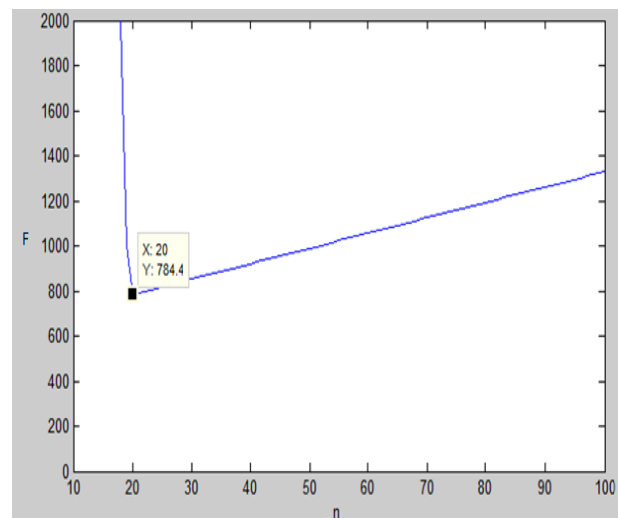


Fig. 7 Cost curve at number of spare transformers

**Table 1** outputs of the proposed model.

$C_{TT} = 21.6084$	$C_T=100$
$C_E = 1.05$	$C_h = 40.4667$
$C_{TD} = 75.500$	$C_{pn} = 393.89$
$C_{TM}=10$	$C_s=40$

Cost curve at number of spare transformers obtained as Fig. 7. As it can be seen from Fig. 7, with 20 spare transformer, the objective function will be the lowest. So, the optimum number of spare transformer for optimizing the objective function is 20. In the following figures "n" is the number of spare transformer.

Then optimal number of spare transformers is 20. For this number of spare transformers other outputs are:  
 $r = 4.49 \times 10^3$   
 $N_{min} = 20$   
 $ENS = 3.43e+003$  (20)

Different costs has been depicted in Table 1. It is observed that penalty cost is bigger than the other costs and the lost revenue because of not distributed energy is the lowest. And the minimum of the invested and imposed costs are  $F_{min}=784.43$ .

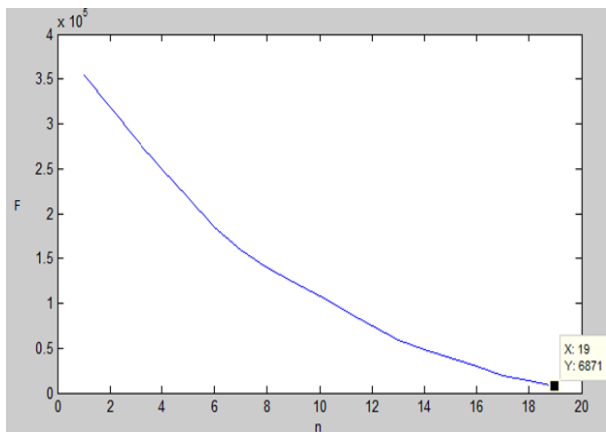
If the number of spare transformers be so that the stock room gets empty, and therefore the spare transformer is not existence for replacement with failed transformer, in this situation customers get a long-time outage and cost of outage will be extremely high.

For more clearness, for states that customers get a long-time outage, cost curve is according to Fig. 8. It is observed that if even one transformer be less than the required number, total cost will be 6871 million dollars.

### 3.3 Sensitivity Analysis

#### 3.3.1 Sensitivity to System Speed

If the required time for repairing has been reduced one month and the required time for purchasing has been reduced to two months, in this situation, number of required spare transformers will be reduced and cost of spare transformers and maintenance will be reduced.



**Fig. 8** Cost curve for state that customers get long time outage.

But repairing is done faster, for this reason its cost will be increased. Furthermore,  $C_{TDC}, C_{Tc}$  are increased and  $C_T, C_{TM}, C_s$  are decreased and other parameters are not changed. In this state, the cost curve will be according to Fig. 9.

For more clearness, in a state that the number of spare transformer be so that customers get a long outage duration, cost curve will be such as Fig. 10.

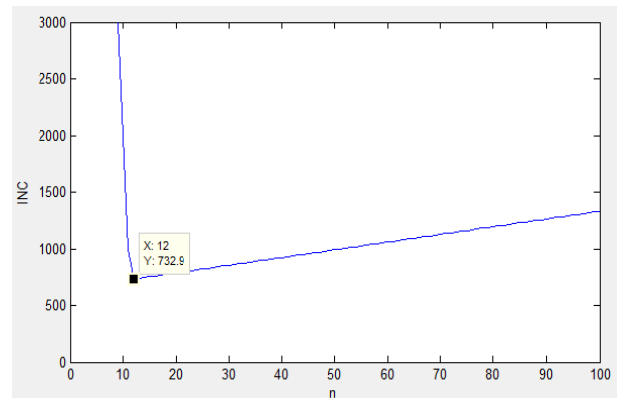
As it is observed from Figs. 9 and 10, total cost for these situations will decrease.

If repairing and purchasing times are increased, for example, repairing time is increased to 3 months and purchasing time is increased to 6 months, in this state, output will be according to Fig. 11.

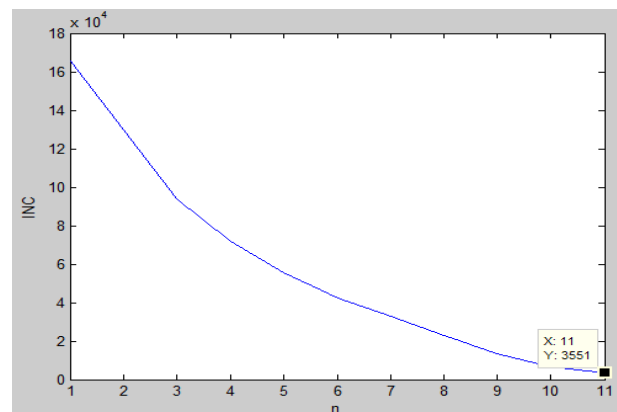
For more clearness, in a state that the number of spare transformer be so that customers get a long outage duration, cost curve will be such as Fig. 12.

As it is observed from Figs. 11 and 12, total cost for these situations will increase.

If the required time for purchasing transformers be two months and be assumed fixed, but the required time for repairing changes from one to five months, number of required spare transformers will be according to Fig. 13. So, for different repairing time, different number of spare transformers will be needed.



**Fig. 9** Cost curve for state  $r_r = 1, r_b = 2$ .



**Fig. 10** Cost curve for state that customers get a long time outage duration with  $r_r = 1, r_b = 2$ .



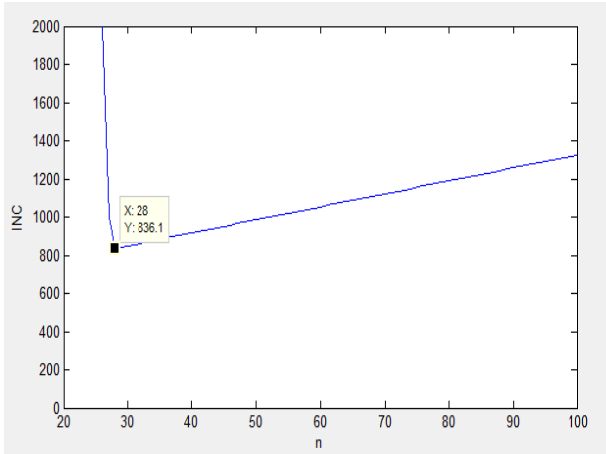


Fig. 11 Cost curve for state  $r_r = 3, r_b = 6$ .

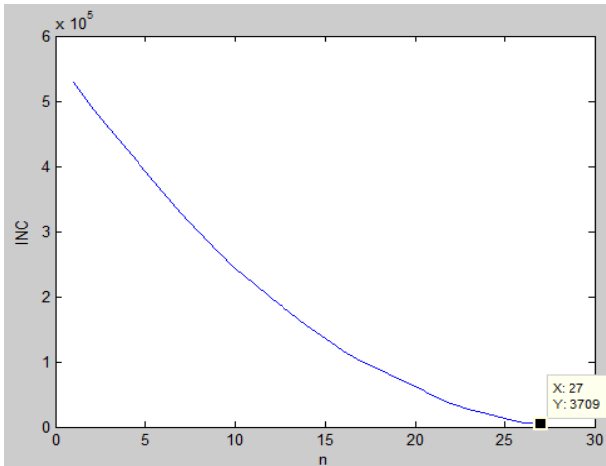


Fig. 12 Cost curve for state that customers get a long time outage duration with  $r_r = 3, r_b = 6$ .

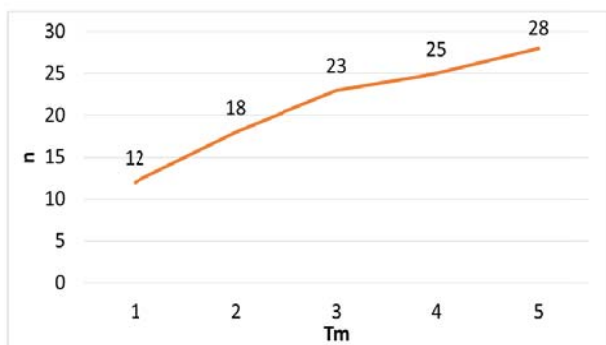


Fig. 13 Number of required spare transformers at  $r_r$ .

### 3.3.2 Sensitivity to Costs Sensitivity of *INC* to Transformer Cost

For considered example with  $r_r = 2, r_b = 4$  and other inputs are similar to first simulation, if cost of transformers change from 3 to 7 thousand dollars, cost curves will be according to Fig. 14.

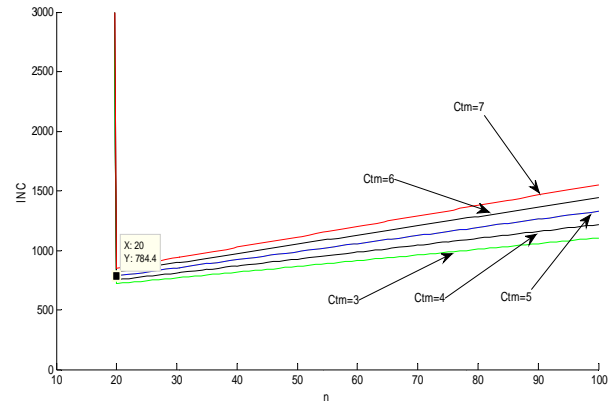


Fig. 14 Cost curve sensitivity to price of transformer.

In Fig.14, curves have different slopes because *INC* for  $n \geq 20$  has a direct relationship with  $n$ . Outage duration cost for state  $n \leq 20$  is very high, for this reason, it is also observed that transformer's price does not have a certain effect in this state.

## 4 Discussion

It has been observed that with the considered assumption, the penalty cost is bigger than the other costs and the lost revenue because of not distributed energy is the lowest. If the number of spare transformers be so that the stock room gets empty, and therefore the spare transformer is not existence for replacement with failed transformer, in this situation customers get a long-time outage and cost of outage will be extremely high.

From the sensitivity analysis of *INC* curves, it can be deduced that with decreasing of required time for repairing and purchasing, we will need fewer spare transformers and their maintenance costs will decrease. For different price of transformers, *INC* curves will have different slopes. If the number of spare transformers available in the stock room be so that customers do not get a very long outage duration, or  $r$  be constant, in this situation *INC* curves for different  $C_{ptc}$  will be parallel together. Otherwise, *INC* curves will have different slopes and are not parallel with together. Also, for different repairing time, different number of spare transformers will be needed.

## 5 Conclusion

In this paper, a comprehensive illustration asset management activities in relation with spare transformers have been presented and number and location of spare transformers have been optimized using with data of failed transformers. In the presented model for decreasing of outage time due to failed transformers, mobile transformers are used. Many of the required parameters for assessment of spare transformers, for example number of failed transformers and fatigued transformers in each month, cost of each

spare transformer and cost of mobile transformers, location cost of maintenance, transportation cost, required time for repairing and purchasing have been considered. Transformers have a critical role in reliability and need a huge investment, and maintenance, repairing, replacement are some of the options that are used in the asset management for increasing reliability and investment so for decreasing the number of required spare transformers and their maintenance cost, we must decrease repairing and purchasing times; These actions in turn increase cost. With the method that has been presented in this paper, some asset management activities that are required for determining the optimal number of spare transformers are considered and it is the advantage of this method rather than other methods.

### Nomenclature

$A_{gi}$	Number of obsolete transformers in $i^{\text{th}}$ month.
$a_i$	Number of fatigued transformers in $i^{\text{th}}$ month and must be bought.
$BY_i$	Number of transformers that have been bought in $i^{\text{th}}$ month.
$C_E$	Lost revenue because of not distributed energy (\$ 1000).
$C_H$	Cost of maintenance groups (\$ 1000).
$C_{hc}$	Cost of a serviceman (\$/minute).
$C_{MS}$	Total mobile transformers cost (\$ 1000).
$C_{MSI}$	Installation cost of a mobile transformer (\$).
$C_{MSM}$	Total mobile transformer maintenance cost (\$ 1000).
$C_{MSMT}$	Cost of a mobile transformer maintenance and service (\$ 1000/month).
$C_{MSST}$	Mobile transformer location cost (\$ 1000).
$C_{MST}$	Capital cost of a mobile transformer (\$ 1000).
$C_{MST1}$	Cost of carrying a mobile transformer (\$/kilometer).
$C_{pn}$	Fine of outages (\$ 1000).
$C_{pnc}$	Fine of each additional outage (\$ 1000).
$C_{ptc}$	Fine of each additional outage duration (\$ 1000)
$C_S$	location cost of spare transformers and mobile transformers (\$ 1000).
$C_{SC}$	Cost of location of a spare transformer (\$ 1000).
$C_T$	Total cost of spare transformers (\$ 1000).
$C_{TC}$	Capital cost of each spare transformer (\$ 1000).
$C_{TD}$	Cost of repairing and replacing of failed transformers (\$ 1000).
$C_{TDC}$	Cost of each transformer repairing (\$ 1000).
$C_{TM}$	Total maintenance cost and spare transformers service (\$ 1000).
$C_{TPM}$	Annually repairing cost of a spare transformer (\$ 1000).
$C_{TT}$	carrying and installation costs of mobile transformers and spare transformers (\$ 1000).

$C_{TTC}$	Cost of carrying each spare transformer (\$/kilometer).
$C_{TTV}$	Cost of loading, depletion and installation of a spare transformer (\$).
$D_{ij}$	Distance of a spare transformer location to $j^{\text{th}}$ failed transformer in $i^{\text{th}}$ month (km).
$ENS$	Energy not distributed (kWh).
$G$	Number of 3 serviceman groups.
$IMC$	Imposed Costs.
$INC$	Inserted Costs (\$ 1000).
$INV$	Investment cost.
$k$	Number of mobile transformers.
$L_{E_{ij}}$	Lost load in $j^{\text{th}}$ outage in $i^{\text{th}}$ month (kW).
$MC_{dist}$	Revenue of distribution company from each kilowatt-hour (\$).
$m_i$	Number of failed transformers in $i^{\text{th}}$ month and must be repaired.
$MT_i$	Number of transformers that have been repaired in $i^{\text{th}}$ month.
$N_i$	Number of available transformer in the stock room in $i^{\text{th}}$ month.
$n$	Total number of spare transformer.
$r$	Total outage durations (min).
$r_a$	Mobile transformer installation durations (min).
$r_b$	Spare transformer purchasing durations (month).
$r_f$	durations of failed transformer locating (min).
$r_{ij}$	$j^{\text{th}}$ Outage durations in $j^{\text{th}}$ month (min).
$r_m$	mobile transformer carrying durations (min/km).
$r_r$	Failed transformer Repairing durations (month).
$r_{TT}$	Carrying and installation of mobile transformer durations (minute).
$T$	Number of mobile transformer servicing in each year.

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**Milad Sedaghati** was born in 1989 in Rasht, Iran. He received the B.Sc. degree in electrical engineering from Tafresh University, in 2013. Since 2013, he is pursuing the M.Sc. degree in Iran University of Science and Technology. His research interests include spare equipment, electrical distribution systems, smart grid, and stochastic optimization.



**Reza Dashti** received his B.Sc. and M.Sc. degrees in Electrical Engineering in 2003 and 2005 and the Ph.D. degree in Electrical Engineering from Tehran University, in 2011. Currently, He is an Assistant Professor in Electrical Engineering Department of IUST, Tehran, Iran. His research interests are in Distribution System Asset Management and Security. He has published more than 10 Journal papers and 40 conference papers.