

# A New Method for Merchandizing Surplus Allocation

I. Ehsani\*, A. Akbari Foroud<sup>\*(C.A.)</sup> and A. Soofiabadi\*

**Abstract:** Locational Marginal Pricing (LMP) is a method for energy pricing in deregulated power systems. Loss and congestion cause different prices at different buses. In this pricing method there is a different between payments of customers and revenue of generators which is called Merchandizing Surplus (MS). The Independent System Operator (ISO) receives MS and generally renders it to Transmission Company (Transco). It is rational that MS be allocated among power market participants fairly instead of granting whole MS to Transco. In this paper a novel method is proposed to allocate MS among market participants according to their role in the congestion of system. In the presented method by decomposing LMP and identifying congestion part of LMP, the part of generators' revenue and customers' payments which caused by congestion are calculated. Then MS is allocated among market participants as the payment of customers to be equal to revenue of generators. The proposed method has been tested on a five bus test system. Results indicate effectiveness of the proposed method to allocate MS between power market participants.

**Keywords:** Locational Marginal Price, Merchandizing Surplus, Transco Revenue.

## 1 Introduction

MS is mainly a result of congestion and is equal to customers' payment minus revenue of generators in LMP method [1, 2]. Although MS can be a result of loss and DCOPF settings too [3] but generally MS is considered as a result of congestion since other factors don't affect MS greatly. So in this paper it is assumed that only creation factor of MS is congestion, so in the following of the paper, congestion surplus term is substituted with merchandizing surplus term. Generally ISO collects MS [4] and considers it as a rent of transmission lines and sometimes considers it as a part of Transco revenue [5]. So Transco try to increase MS of system to increase its revenue.

Therefore rendering congestion surplus to Transco does not seem rational. Allocation of congestion surplus based on fair criteria among market participants is a plan that ISO can apply to remove misuse of this money.

But in real power markets, ISO faces with a serious question. How we can determine the role of power market participants in congestion of transmission lines and how to allocate MS among them fairly?

This issue is a significant subject in deregulated

systems but according to some mentioned methodologies, usually the share of market participants in congestion of system haven't been considered, thus these methodologies can be inequitable for some participants [6].

In this paper instead of granting MS to Transco, a novel method is presented to allocate MS fairly among market participant. The basis of this method is to determine the market participant revenue from congestion of the system. So LMP of each bus is decomposed to LMP of energy and LMP of congestion. Then the share of each generator in each bus demand is determined too. These data provide significant data such as: The revenue of certain generator from supplying demand of certain bus and the payments of customer for this certain demand. The difference between these two parameters results in MS of this energy exchange between certain generator and certain bus. In the proposed method the MS of each energy exchange is obtained and considered as a base to allocate MS among market participants. The proposed method establishes that MS allocation among market participants according to their role in MS of each energy exchange. Also in the proposed method all payments of customers would be equal to revenues of generators.

In the following, section 2 illustrates the LMP decomposition. In section 3 the share of each Generator in demand of every bus is calculated. In section 4 the revenue of each generator from demand supplying of a certain bus and payment of each customer for its

Iranian Journal of Electrical & Electronic Engineering, 2015.

Paper first received 28 Jan. 2015 and in revised form 4 Apr. 2015.

\* The Authors are with the Department of Electrical Engineering, Semnan University, Semnan, Iran.

Emails: abtin.456@gmail.com, aakbari@semnan.ac.ir and soofiiali8@gmail.com.

demand is calculated. Section 5 includes the calculation of MS of each power exchange in the power system and solutions to allocate MS of system among market participants. In section 6 the proposed method is tested on a 5 bus test system and ultimately section 7 concludes the paper.

## 2 LMP Decomposition

DCOPF problem determines the optimal generation dispatch and LMPs subject to a set of constraints which represents the operational and physical limits of power system. Generators make offers to sell electricity as linear supply function and for the purpose of simplicity, no demand side bidding is considered and hence, loads are known constants for the dispatch.

It is assumed that the Generator' offers expressed by Eq. (1) that is a straight lines with intercept  $a_i$  and slope  $b_i$  [7]:

$$\rho_i(P_i) = a_i + b_i P_i \quad (1)$$

Generator can change their pricing strategies by adjusting the slope and intercept of the line in Eq. (1). In [8, 9] it have been assumed that generating units only manipulate the intercept  $a_i$  of the bid functions and their slope  $b_i$  is constant. Several reasons have been discussed for justification of this assumption in [8]. For instance, it has been stated that the slope of bid functions for individual generator is usually very slight and therefore very steep slopes, resulting from manipulation of  $b_i$  are not plausible. According to presented discussion, in this paper it is assumed that the units may change their strategies by only adjusting the intercept values  $a_i$  and therefore  $b_i$  remains constant.

Therefore, the DCOPF can be stated as a problem of minimizing the total generation cost of generators subject to physical limits in the network:

$$\min \sum_{i=1}^N a_i * P_i + (b_i / 2) * P_i^2 \quad (2)$$

subject to

$$\sum_{i=1}^N P_i = P_d \quad (3)$$

$$\alpha_l \leq \sum_{i=1}^N \gamma_{l,i} * P_i \leq \bar{\alpha}_l, (\Gamma_l^{\min}, \Gamma_l^{\max}), l = 1, \dots, L \quad (4)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}, (\mu_i^{\min}, \mu_i^{\max}), i = 1, \dots, N \quad (5)$$

Constraints (3)-(4) represent generation capacity constraint, transmission line constraint and load balance constraint respectively. By solving this optimization problem, ISO determines generation of every generator and LMPs which is the Lagrangian multiplier of constraint (3).

By running the DCOPF, generators are classified in three categories. The first category includes the generators with high generation cost which are restricted to their minimum limit.

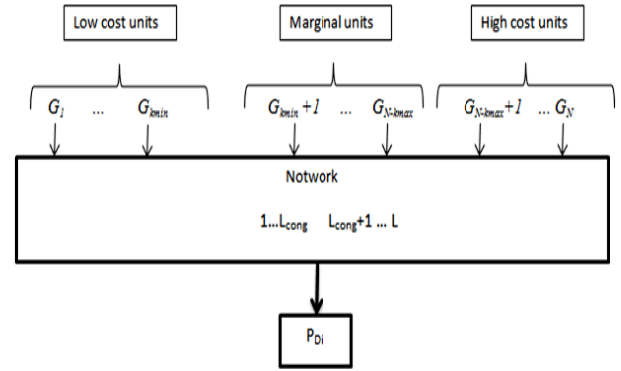


Fig. 1 The simple diagram of power system.

The second one includes the generator with marginal power generation and the third one comprises the generator with low generation cost which are restricted to their uppercase power limit. Figure 1 illustrates the stated classification.

The corresponding Lagrangian formulation for the minimization the problem (2) can be stated as Eq. (6).

$$l(P_1, \dots, P_N, \lambda, \mu_1^{\min}, \mu_1^{\max}, \dots, \mu_N^{\min}, \mu_N^{\max}, \Gamma_1^{\min}, \Gamma_1^{\max}, \dots, \Gamma_L^{\min}, \Gamma_L^{\max}) = \sum_{i=1}^N (a_i * P_i + \frac{b_i}{2} * P_i^2) + \lambda (P_d - \sum_{i=1}^N P_i) + \sum_{i=1}^N \mu_i^{\min} (P_i^{\min} - P_i) + \sum_{i=1}^N \mu_i^{\max} (P_i - P_i^{\max}) + \sum_{l=1}^L (\Gamma_l^{\min} (\alpha_l - \sum_{i=1}^N \gamma_{l,i} P_i)) + \sum_{l=1}^L (\Gamma_l^{\max} (\sum_{i=1}^N \gamma_{l,i} P_i - \bar{\alpha}_l)) \quad (6)$$

By solving the Kuhn-Tucker conditions for the above Lagrange equation, it has been proved in [7] that nodal prices are as Eq. (7).

$$LMP_n = \frac{P_d}{C_1} - \sum_{i=1}^{K_{\min}} \frac{P_i^{\min}}{C_1} - \sum_{i=N-K_{\max}+1}^N \frac{P_i^{\max}}{C_1} + \sum_{i=N-K_{\max}+1}^{N-K_{\max}} \gamma_{1,i} / b_i \sum_{i=N-K_{\max}+1}^{N-K_{\max}} \left( \frac{a_i}{C_1 + b_i} \right) + \sum_{l=1}^{l_{cong}} \left( \frac{\gamma_{1,i}}{C_1} - \gamma_{1,i} \right) \Gamma_l^{\max} \quad (7)$$

where

$$\Gamma_k^{\max} = \frac{1}{\sum_{j=1}^{k_{\min}} \gamma_{1,j} P_j^{\min} - \sum_{k=1}^{l_{cong}} \left( \sum_{i=k_{\min}+1}^{N-k_{\max}} \frac{\gamma_{1,i} \gamma_{k,j}}{C_1 * b_i} - \frac{\gamma_{1,i} \gamma_{k,i}}{b_i} \right)} \quad (8)$$

$$\sum_{j=k_{\min}+1}^{N-k_{\max}} \gamma_{1,j} \frac{P_d - \sum_{i=1}^{k_{\min}} P_i^{\min}}{C_1 * b_j} - \sum_{j=N-k_{\max}+1}^N \left( \gamma_{1,j} - \frac{\sum_{i=k_{\min}+1}^{N-k_{\max}} \gamma_{1,i} / b_i}{C_1} \right) - \sum_{j=k_{\min}+1}^{N-k_{\max}} \left( \frac{\sum_{i=k_{\min}+1}^{N-k_{\max}} \gamma_{1,i} / b_i}{C_1 * b_j} - \frac{\gamma_{1,j}}{b_i} \right) a_j$$

Substituting Eq. (6) into Eq. (4) yields:

$$\sum_{i=1}^N P_i = P_d \longrightarrow P_d - \sum_{i=1}^{K_{\min}} P_i^{\min} - \sum_{i=N-K_{\max}+1}^N P_i^{\max} = \sum_{i=K_{\min}+1}^{N-K_{\max}} P_i \quad (9)$$

By ignoring loss (Eq. (9)) the LMP formulation can be stated as Eq. (10):

$$LMP_n = \sum_{i=K_{\min}+1}^{N-K_{\max}} P_i / C_1 + \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{a_i}{C_1 * b_i} \right) + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max} \quad (10)$$

By some calculations, Eq. (11) is obtained from Eq. (10) as below:

$$LMP_n = \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{P_i}{C_1} + \frac{a_i}{C_1 * b_i} \right) + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max} \quad (11)$$

$$= \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{P_i b_i + a_i}{C_1 * b_i} \right) + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max}$$

Substituting Eq. (1) into Eq. (11) yields:

$$LMP_n = \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{\rho_i}{C_1 * b_i} \right) + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max} \quad (12)$$

The obtained formula for LMP includes two terms. The first term (left part in Eq. (12)) is a common term in all buses price formula and depends on marginal units bids. If there is no congestion in the network, all buses of the system have uniform price that is equal to the first term of Eq. (12) (common part of all buses price) and second term is omitted. If congestion exists in the system, the second part of formula causes the advent of different price in different bus. The second term depends on each generator share in lines flow of the system.

The first part of Eq. (12) is called  $lmp_n^{energy}$  that is related to marginal unities bids and the second part of formula is called  $lmp_n^{cong}$  that causes different price in different buses. So Eq. (12) can be stated as below:

$$lmp_n = lmp_n^{energy} + lmp_n^{cong} \quad (13)$$

$$lmp_n^{cong} = \frac{\sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right)}{\sum_{k=1}^{l_{cong}} \left( \sum_{j=K_{\min}+1}^{N-K_{\max}} \frac{\gamma_{l,i} \gamma_{k,j} / b_j}{C_1 * b_i} - \frac{\gamma_{l,i} \gamma_{k,i}}{b_i} \right)} \times \left[ \bar{\alpha}_l - \sum_{j=1}^{K_{\min}} \gamma_{l,j} P_j^{\min} - \sum_{j=K_{\min}+1}^{N-K_{\max}} \gamma_{l,j} \frac{P_d - \sum_{i=1}^{K_{\min}} P_i^{\min}}{C_1 * b_j} - \sum_{j=N-K_{\max}+1}^N \left( \gamma_{l,j} - \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} \right) - \sum_{j=K_{\min}+1}^{N-K_{\max}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1 * b_j} - \frac{\gamma_{l,j}}{b_i} \right) a_j \right] \quad (14)$$

$$LMP_n^{energy} = \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{\rho_i}{C_1 * b_i} \right) \quad (15)$$

### 3 Nodal Supplying of a Generator

By running DCOPT, the generation vector of generator is obtained ( $pg$ ). To calculate the delivered power of a generator to each bus as Ref [10] the below equations are applied to the generation vector  $pg$ . According to Figure 2 which represents the inflow power, outflow power, load ( $Pd_n$ ) and generation ( $pg_n$ ) at bus n of system, the below equation can be obtained. According to Fig. 2, the below equation can be obtained.

$$PI_n = \sum_{k \in N_S} pl_{kn} + pg_n \quad (16)$$

where  $PI_n$  is total power inflow into the bus n. The  $pl_{kn}$  denotes the inflow power from bus k to bus n and the number of inflows to bus n is  $N_s$ . The above equation can be written as below for N node system.

$$PI_n - \sum_{k \in N_S} \left( \frac{pl_{kn}}{PI_k} * PI_k \right) = pg_n, n = 1, 2, \dots, N \quad (17)$$

The matrix form of above equation can be written as:

$$M * PI = pg \quad (18)$$

where  $PI$  denotes the vector of nodal supplying power,  $Pg$  is the vector of nodal generations, and  $M$  is the distribution matrix with element  $m_{kn}$ .

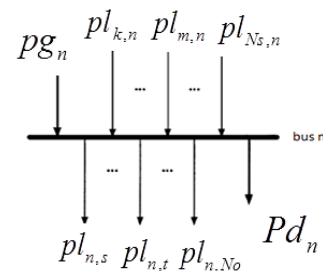


Fig. 2 A general node n in the system.

$$m_{kn} = \begin{cases} 1 \rightarrow \text{if } : k = n \\ -\frac{Pl_{kn}}{PI_n} \rightarrow \text{if } : k \in N_s \\ 0 \rightarrow \text{otherwise} \end{cases} \quad (19)$$

$$PI = M^{-1} * pg \quad (20)$$

Each generator contribution in load of bus n can be written as below:

$$P_{i,n} = \frac{P_{d,n}}{PI_n} [M^{-1}]_{i,n} * P_i \quad (21)$$

where  $P_{d,n}$  denotes bus n load and  $P_i$  is total generations of generator  $i$ . By applying Eq. (21) the share of each generator in each bus demand of system is obtained. The generator share in each demand bus of system is the basis of calculating the congestion revenue of generator as well as the congestion payment of customers as stated in the following.

#### 4 The Revenue of Each Generator from Demand Supplying of a Certain Bus and Payment of Each Customer for that Demand

##### 4.1 Revenue of Each Generator from Each Energy Exchange

As it was stated in section 2, LMP is decomposed to LMP of energy and LMP of congestion. Also generator share in bus load of system was determined in section 3. Now the nodal revenue of generator  $i$  from bus  $k$  can be stated as below:

$$d_{i,n} = P_{i,n} * LMP_k \quad (22)$$

$LMP_k$  denotes the LMP of bus  $k$  which generator  $i$  is connected it. As it was stated previously in Eq. (16), LMP consists of two parts (energy and congestion), so replacing Eq. (16) into Eq. (22) yields:

$$d_{i,n} = lmp_k^{energy} * P_{i,n} + lmp_k^{cong} * P_{i,n} \quad (23)$$

Substituting Eqs. (14) and (15) into Eq. (22) yields:

$$d_{i,n} = \sum_{i=K_{min}+1}^{N-K_{max}} \left( \frac{\rho_i}{C_1 * b_i} \right) * P_{i,n} + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{min}+1}^{N-K_{max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,k} \right) \Gamma_l^{max} P_{i,n} \quad (24)$$

Finally by replacing Eq. (21) into Eq. (24), revenue of generator  $i$  (which is connected to bus  $k$ ) from selling energy to bus  $n$  can be stated as:

$$d_{i,n} = \sum_{i=K_{min}+1}^{N-K_{max}} \left( \frac{\rho_i}{C_1 * b_i} \right) * \frac{P_{d,n}}{PI_n} [M^{-1}]_{i,n} P_i + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{min}+1}^{N-K_{max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,k} \right) \Gamma_l^{max} \frac{P_{d,n}}{PI_n} [M^{-1}]_{i,n} P_i \quad (25)$$

According to Eq. (25), revenue of generator  $i$  from selling electricity to bus  $n$ , can be divided to two parts. The first part (left side of Eq. (25)) is common among all generators which is related to marginal unit bid. But the second part denotes the increase or decrease of generator  $i$  revenue from energy sale to bus  $n$  which is related to congestion and more precisely to structure of system. So the revenue of generator  $i$  can be stated as below:

$$d_{i,n} = d_{i,n}^{energy} + d_{i,n}^{cong} \quad (26)$$

Eq. (26) denotes the revenue of generator  $i$  from energy sale to bus  $n$  which is decomposed to two separable parts. Eq. (27) denotes the common revenue of generator from bus  $n$ , while Eq. (28) denotes congestion revenue of generator from bus  $n$ . So the increase or decrease in generator  $i$  revenue depends on congested lines.

$$d_{i,n}^{energy} = \sum_{i=K_{min}+1}^{N-K_{max}} \left( \frac{\rho_i}{C_1 * b_i} \right) * \frac{P_{d,n}}{PI_n} [M^{-1}]_{i,n} P_i \quad (27)$$

$$d_{i,n}^{cong} = \frac{\sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{min}+1}^{N-K_{max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,k} \right)}{\sum_{k=1}^{l_{cong}} \left( \frac{\sum_{j=K_{min}+1}^{N-K_{max}} \gamma_{l,i} \gamma_{k,j} / b_j}{C_1 * b_i} - \frac{\gamma_{l,i} \gamma_{k,i}}{b_i} \right)} \times \quad (28)$$

$$\left[ \bar{\alpha}_i - \sum_{j=1}^{k_{min}} \gamma_{l,j} P_j^{min} - \sum_{j=k_{min}+1}^{N-k_{max}} \gamma_{l,j} \frac{P_d - \sum_{i=1}^{k_{min}} P_i^{min}}{C_1 * b_j} - \sum_{j=N-k_{max}+1}^N \left( \gamma_{l,j} - \frac{\sum_{i=K_{min}+1}^{N-K_{max}} \gamma_{l,i} / b_i}{C_1} \right) - \sum_{j=k_{min}+1}^{N-k_{max}} \left( \frac{\sum_{i=K_{min}+1}^{N-K_{max}} \gamma_{l,i} / b_i}{C_1 * b_j} - \frac{\gamma_{l,j}}{b_i} \right) a_j \right] * \frac{P_{d,n}}{PI_n} [M^{-1}]_{i,n} P_i$$

##### 4.2 Customer Payment for Each Energy Exchange

Customer payment at bus  $n$  to buy from generator  $i$  (that is connect to bus  $k$ ) can be stated as:

$$s_{i,n} = P_{i,n} * LMP_n \quad (29)$$

The customer's payment can be divided into two parts like generator revenue (Eqs. (30) and (31)). First part is corresponding to consumed energy and second part denotes the increase or decrease in payments of

customers which caused by lines congestion.

$$s_{i,n} = \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{\rho_i}{C_1 * b_i} \right) * P_{i,n} + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max} P_{i,n} \quad (30)$$

$$s_{i,n} = s_{i,n}^{energy} + s_{i,n}^{cong} \quad (31)$$

Since  $lmp^{energy}$  is equal in all buses, we can result that energy parts of customer payment and generator revenue are equal as below:

$$s_{i,n}^{energy} = d_{i,n}^{energy} \quad (32)$$

## 5 Congestion Surplus Determination for Each Energy Exchange and its Allocation

### 5.1 Congestion Surplus Determination for Each Energy Exchange

MS of an energy exchange between generator  $i$  (at bus  $k$ ) and demand at bus  $n$  is the difference between payment of customer at bus  $n$  to buy energy from generator  $i$  and revenue of generator  $i$  from selling energy to bus  $n$ . So MS of each energy exchange can be formulated as below:

$$MS_{i,n} = s_{i,n} - d_{i,n} \xrightarrow{s_{i,n}^{energy} = d_{i,n}^{energy}} \quad (33)$$

$$MS_{i,n} = s_{i,n}^{cong} - d_{i,n}^{cong} = P_{i,n} (LMP_n^{cong} - LMP_k^{cong})$$

Substituting Eq. (14) into Eq. (33) yields:

$$MS_{i,n} = \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max} P_{i,n} - \quad (34)$$

$$\sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,k} \right) \Gamma_l^{\max} P_{i,n}$$

$$MS_{i,n} = \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} - \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} + \gamma_{l,k} \right) \Gamma_l^{\max} P_{i,n} \quad (35)$$

$$MS_{i,n} = \sum_{l=1}^{l_{cong}} (\gamma_{l,k} - \gamma_{l,n}) \Gamma_l^{\max} P_{i,n} \quad (36)$$

In fact  $MS_{i,n}$  is related to difference between payment of customers of bus  $n$  for  $P_{i,n}$  and revenue that generator  $i$  receive for generation of that energy.

Congestion surplus for each energy exchange between demand of bus  $n$  and generation of bus  $k$  can be calculated through above equation. Eq. (30) is multiply of two parts: the first part is share of generator  $i$  (there is bus  $k$ ) in bus  $n$  power supply ( $P_{i,n}$ ) and the second part denotes the difference between the share of each generator bus in flow of congested line and the share of each customer bus in flow of congested line ( $\gamma_{l,k} - \gamma_{l,n}$ )

Consider line  $l$  that delivers the generated power of generator  $i$  to bus  $n$ , the MS of this line can be stated as below:

$$MS_{i,n,l} = P_{i,n} (\gamma_{l_{cong},k} - \gamma_{l_{cong},n}) \Gamma_l^{\max} \quad (37)$$

Total MS to supply the demand of bus  $n$  is equal to:

$$MS_n = s_n - \sum_{i=1}^{N_g} P_{i,n} * LMP_i \quad (38)$$

$$= LMP_n * P_{d,n} - \sum_{i=1}^{N_g} P_{i,n} * LMP_i$$

$$MS_n = \sum_{i=1}^{N_g} MS_{i,n} = \sum_{i=1}^{N_g} P_{i,n} \sum_{l=1}^{l_{cong}} (\gamma_{l,i} - \gamma_{l,n}) \Gamma_l^{\max} \quad (39)$$

So total MS of system can be stated as:

$$MS = \sum_{n=1}^N MS_n = \sum_{i=1}^{N_g} s_i - \sum_{i=1}^{N_g} d_i \quad (40)$$

### 5.2 Congestion Surplus Allocation among Market Participant

Now by obtaining the share of each energy exchange in MS of system, the MS is allocated among market participant as all payments of customers to be equal to revenue of generators. As it was stated previously, MS of energy exchange between generator  $i$  (at bus  $k$ ) and customer of bus  $n$  is equal to:

$$MS_{i,n} = P_{i,n} (lmp_n^{cong} - lmp_k^{cong}) \quad (41)$$

Generator  $i$  revenue from selling energy to bus  $n$  can be stated as:

$$d_{i,n} = P_{i,n} (lmp_k^{energy} + lmp_k^{cong}) \quad (42)$$

Also payment of customers of bus  $n$  to buy energy from generator  $i$  (at bus  $k$ ) is as below:

$$s_{i,n} = P_{i,n} (lmp_n^{energy} + lmp_n^{cong}) \quad (43)$$

Now it is enough that MS allocation among

generator and customers which is called  $s_{i,n}$  and  $d_{i,n}$  to be equal. So  $S_{i,n}$  and  $d_{i,n}$  can be written as:

$$d_{i,n}^{new} = d_{i,n} + \Delta d_{i,n} \quad (44)$$

$$s_{i,n}^{new} = s_{i,n} + \Delta s_{i,n} \quad (45)$$

$$MS_{i,n} = \Delta d_{i,n} + \Delta s_{i,n} \quad (46)$$

$\Delta d_{i,n}$  denotes the revenue variation of generator  $i$  that is due to the MS allocation. In another word, the new revenue of generator  $i$  after MS allocation is equal to the revenue of generator  $i$  before MS allocation plus MS share of generator  $i$ .  $\Delta s_{i,n}$  denotes the payment variation of customer  $n$  that is due to the MS allocation. In another word, the new payment of customer  $n$  after MS allocation is equal to the payment of customer  $n$  before MS allocation plus MS share of customer  $n$ .

By allocating MS among power market participants, four cases occur depending on the amount of  $lmp_n^{cong}$  at generator and demand bus.

**First form:**  $lmp_n^{cong} < lmp_k^{cong}$

In this case MS of energy exchange is negative between bus  $n$  and generator  $k$ . In another word, the money that ISO receives from customers is less than the generator revenue thus ISO faces under budget for this energy exchange. Although it seems irrational that we buy a commodity from a place with higher price and sell it to another place with lower price, but it is the reality of electricity market in this case. This is related to transmission network topology and limits. So to equalize the payments of customers and revenue of generator, the terms  $s_{i,n}^{new}$  and  $d_{i,n}^{new}$  can be stated as:

$$s_{i,n}^{new} = s_{i,n}^{old} = P_{i,n} * (lmp_n^{energy} + lmp_n^{cong}) \quad (47)$$

$$d_{i,n}^{new} = d_{i,n}^{old} + \Delta d_{i,n} \quad (48)$$

$$\Delta s_{i,n} = P_{i,n} \sum_{l=1}^{l_{cong}} (\gamma_{l,k} - \gamma_{l,n}) \Gamma_l^{max} = \frac{P_{i,n} \sum_{l=1}^{l_{cong}} (\gamma_{l,k} - \gamma_{l,n})}{\sum_{k=1}^{l_{cong}} \left( \sum_{i=k^{min}+1}^{N-k^{max}} \frac{\gamma_{l,i} \gamma_{k,j} / b_j}{C_1 * b_i} - \frac{\gamma_{l,i} \gamma_{k,i}}{b_i} \right)} \left[ \bar{\alpha}_l - \sum_{j=1}^{k^{min}} \gamma_{l,j} P_j^{min} - \sum_{j=N-k^{max}+1}^N \gamma_{l,j} \frac{P_d - \sum_{i=1}^{k^{min}} P_i^{min}}{C_1 * b_j} - \sum_{j=N-k^{max}+1}^N \left( \gamma_{l,j} - \frac{\sum_{i=k^{min}+1}^{N-k^{max}} \gamma_{l,i} / b_i}{C_1} \right) - \sum_{j=k^{min}+1}^{N-k^{max}} \left( \frac{\sum_{i=k^{min}+1}^{N-k^{max}} \gamma_{l,i} / b_i}{C_1 * b_j} - \frac{\gamma_{l,j}}{b_i} \right) a_j \right] \quad (52)$$

$$s_{i,n}^{new} = s_{i,n}^{old} + \Delta s_{i,n} \xrightarrow{\Delta s_{i,n} = -P_{i,n} * (lmp_n^{cong} - lmp_k^{cong})} s_{i,n}^{new} = P_{i,n} (lmp_n^{energy} + lmp_n^{cong}) - P_{i,n} * (lmp_n^{cong} - lmp_k^{cong}) \quad (53)$$

$$s_{i,n}^{new} = P_{i,n} (lmp_n^{energy} + lmp_k^{cong})$$

$$\Delta d_{i,n} = -P_{i,n} \left| \Delta lmp_{k,n} \right| = P_{i,n} \sum_{l=1}^{l_{cong}} (\gamma_{l,k} - \gamma_{l,n}) \Gamma_l^{max} = \frac{P_{i,n} \sum_{l=1}^{l_{cong}} (\gamma_{l,k} - \gamma_{l,n})}{\sum_{k=1}^{l_{cong}} \left( \sum_{i=k^{min}+1}^{N-k^{max}} \frac{\gamma_{l,i} \gamma_{k,j} / b_j}{C_1 * b_i} - \frac{\gamma_{l,i} \gamma_{k,i}}{b_i} \right)} \left[ \bar{\alpha}_l - \sum_{j=1}^{k^{min}} \gamma_{l,j} P_j^{min} - \sum_{j=k^{min}+1}^{N-k^{max}} \gamma_{l,j} \frac{P_d - \sum_{i=1}^{k^{min}} P_i^{min}}{C_1 * b_j} - \sum_{j=N-k^{max}+1}^N \left( \gamma_{l,j} - \frac{\sum_{i=k^{min}+1}^{N-k^{max}} \gamma_{l,i} / b_i}{C_1} \right) - \sum_{j=k^{min}+1}^{N-k^{max}} \left( \frac{\sum_{i=k^{min}+1}^{N-k^{max}} \gamma_{l,i} / b_i}{C_1 * b_j} - \frac{\gamma_{l,j}}{b_i} \right) a_j \right] \quad (49)$$

$$d_{i,n}^{new} = d_{i,n} + \Delta d_{i,n} \xrightarrow{\Delta d_{i,n} = P_{i,n} * (lmp_n^{cong} - lmp_k^{cong})} d_{i,n}^{new} = P_{i,n} * (lmp_k^{cong} + lmp_k^{energy}) + P_{i,n} * (lmp_n^{cong} - lmp_k^{cong}) = P_{i,n} * (lmp_k^{energy} + lmp_n^{cong}) \quad (50)$$

In this case the lines congestion cause that generators revenue increase. So the under budget of ISO is received from generator. Equation (40) denotes the decrease of generators revenue to supply the ISO under budget. In this case the ISO under budget is recompensed as the payments of customers to be equal to revenue of generator.

**Second form:**  $lmp_n^{cong} > lmp_k^{cong}$  and  $lmp_k^{cong} > 0$

In this case the MS of system is allocated among market participants according to below formulation.

$$d_{i,n}^{new} = d_{i,n}^{old} = P_{i,n} (lmp_k^{energy} + lmp_k^{cong}) \quad (51)$$

$$\Delta d_{i,n} = P_{i,n} * | \text{Imp}_k^{\text{cong}} | = P_{i,n} * \left[ \frac{\sum_{l=1}^{l_{\text{cong}}} \left( \frac{\sum_{i=k_{\text{min}}+1}^{N-k_{\text{max}}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,k} \right)}{\sum_{k=1}^{l_{\text{cong}}} \left( \sum_{i=k_{\text{min}}+1}^{N-k_{\text{max}}} \frac{\gamma_{l,i} \gamma_{k,j} / b_j}{C_1 * b_i} - \frac{\gamma_{l,i} \gamma_{k,i}}{b_i} \right)} \left[ \bar{\alpha}_l - \sum_{j=1}^{k_{\text{min}}} \gamma_{l,j} P_j^{\text{min}} \right. \right. \quad (54)$$

$$\left. \left. - \sum_{j=k_{\text{min}}+1}^{N-k_{\text{max}}} \gamma_{l,j} \frac{P_d - \sum_{i=1}^{k_{\text{min}}} P_i^{\text{min}}}{C_1 * b_j} - \sum_{j=N-k_{\text{max}}+1}^N (\gamma_{l,j} - \frac{\sum_{i=k_{\text{min}}+1}^{N-k_{\text{max}}} \gamma_{l,i} / b_i}{C_1}) - \sum_{j=k_{\text{min}}+1}^{N-k_{\text{max}}} \left( \frac{\sum_{i=k_{\text{min}}+1}^{N-k_{\text{max}}} \gamma_{l,i} / b_i}{C_1 * b_j} - \frac{\gamma_{l,j}}{b_i} \right) a_j \right] \right]$$

$$\Delta s_{i,n} = -P_{i,n} * | \text{Imp}_n^{\text{cong}} | = -P_{i,n} * \left[ \frac{\sum_{l=1}^{l_{\text{cong}}} \left( \frac{\sum_{i=k_{\text{min}}+1}^{N-k_{\text{max}}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right)}{\sum_{k=1}^{l_{\text{cong}}} \left( \sum_{i=k_{\text{min}}+1}^{N-k_{\text{max}}} \frac{\gamma_{l,i} \gamma_{k,j} / b_j}{C_1 * b_i} - \frac{\gamma_{l,i} \gamma_{k,i}}{b_i} \right)} \left[ \bar{\alpha}_l - \sum_{j=1}^{k_{\text{min}}} \gamma_{l,j} P_j^{\text{min}} - \right. \quad (55)$$

$$\left. \sum_{j=k_{\text{min}}+1}^{N-k_{\text{max}}} \gamma_{l,j} \frac{P_d - \sum_{i=1}^{k_{\text{min}}} P_i^{\text{min}}}{C_1 * b_j} - \sum_{j=N-k_{\text{max}}+1}^N (\gamma_{l,j} - \frac{\sum_{i=k_{\text{min}}+1}^{N-k_{\text{max}}} \gamma_{l,i} / b_i}{C_1}) - \sum_{j=k_{\text{min}}+1}^{N-k_{\text{max}}} \left( \frac{\sum_{i=k_{\text{min}}+1}^{N-k_{\text{max}}} \gamma_{l,i} / b_i}{C_1 * b_j} - \frac{\gamma_{l,j}}{b_i} \right) a_j \right] \right]$$

The negative sign of  $\Delta s_{i,n}$  denotes the decrease in the customers payment of bus n which is the result of MS allocation method. Since  $\text{Imp}_n^{\text{energy}} = \text{Imp}_k^{\text{energy}}$ , MS payment to customers causes that the payments of customers to be equal to revenue of generator.

Since in this paper the revenue of Transco derived from different resources except MS of system, so it is rational that the extra payment of customers (due to the lines congestion) to be paid back to them.

**Third form:**  $\text{Imp}_n^{\text{cong}} > \text{Imp}_k^{\text{cong}}$  and  $\text{Imp}_k^{\text{cong}} < 0$  and  $\text{Imp}_n^{\text{cong}} > 0$ .

In the most situations  $\text{Imp}_k^{\text{cong}}$  is negative in generation bus whereas in the most demand bus  $\text{Imp}_k^{\text{cong}}$  is positive. The reason is that when congestion occurs in the system, at generation bus the supply is more than demand which result a decrease in generation bus price whereas in load bus demand is more than supply which result in increase in demand bus price. So in this case the LMP of generation bus can be lower than its bid at this bus which seems irrational. Here a method is proposed to solve this issue. In the proposed method MS of system is allocated among market participant as the revenue of generators to be equal to customers' payment. Proposed solution is presented from Eqs. (53)-(57).

In this condition:

$$\Delta d_{i,n} + \Delta s_{i,n} = P_{i,n} * \text{Imp}_k^{\text{cong}} - P_{i,n} * \text{Imp}_n^{\text{cong}} = P_{i,n} * (\text{Imp}_k^{\text{cong}} - \text{Imp}_n^{\text{cong}}) = -MS_{i,n} \quad (56)$$

According to above equations the ISO payments to a certain generator and customer is exactly equal to MS of the energy exchange between them.

$$d_{i,n}^{\text{new}} = d_{i,n} + \Delta d_{i,n} = P_{i,n} * (\text{Imp}_k^{\text{energy}} + \text{Imp}_k^{\text{cong}}) + P_{i,n} * | \text{Imp}_k^{\text{cong}} | \xrightarrow{\text{Imp}_k^{\text{cong}} < 0} d_{i,n}^{\text{new}} = \text{Imp}_k^{\text{energy}} * P_{i,n} \quad (57)$$

$$s_{i,n}^{\text{new}} = s_{i,n} + \Delta s_{i,n} = P_{i,n} (\text{Imp}_n^{\text{energy}} + \text{Imp}_n^{\text{cong}}) - P_{i,n} * | \text{Imp}_n^{\text{cong}} | \xrightarrow{\text{Imp}_n^{\text{cong}} > 0} s_{i,n}^{\text{new}} = \text{Imp}_n^{\text{energy}} * P_{i,n} \quad (58)$$

According to above equation the revenue of generator i from electricity sale to bus n is increased up to  $P_{i,n} * \text{Imp}_k^{\text{cong}}$  whereas the customers payment of bus n decreases up to  $\text{Imp}_k^{\text{energy}}$ . So we can claim that MS allocation is caused that revenue of generator and payment of customers is equal.

More precisely, although congestion exists in the system, the revenue of generator i from selling energy to bus n is equal to the payments of bus n customers from buying energy from generator i.

**Forth form:**  $lmp_n^{cong} > lmp_k^{cong}$  and  $lmp_k^{cong} < 0$  and  $lmp_n^{cong} < 0$ .

The analysis of forth case is like the second case.

### 6 Case Study

The selected test case to study is PJM 5 bus test system. Fig. 3 demonstrates the diagram of this test case and Tables 1 and 2 depict its lines and generation data. Here there are 4 Gencos and 3 loads (Genco A has 2 generator Alta and Park city). Table 3 depicts load data.

The system may be roughly divided into two areas, a generation center consisting of Buses A and E including three low-cost generation units and a load center consisting of Buses B, C, and D including two high-cost generation units. The result of decomposing LMP as described previously in section 2 depicted in Table 4.

Table 5 contains share of each generator in load supplying of each bus. It shows all exchanges between system's buses ( $P_{i,n}$ ).

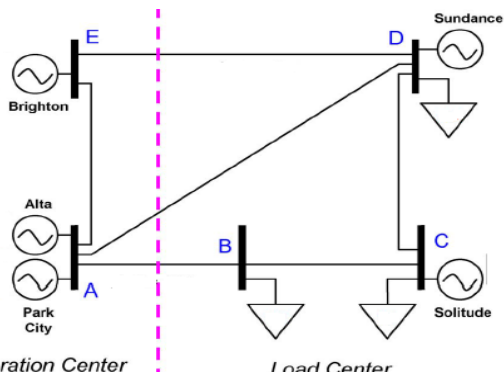
Table 6 contains MS of energy exchanges in the system. According to this table, MS of energy exchange between customers at bus C and generator D is negative.

**Table 1** Lines' impedance and flow limits.

Line	ED	EA	AB	AD	DC	CB
Limit(MW)	240	700	400	800	900	900
X(%)	2.97	0.64	2.81	3.04	2.97	1.08

**Table 2** Generation limits of generators and their bid coefficients.

GEN	$a_i$ (\$/MWh)	$b_i$ (\$/MW <sup>2</sup> h)	$P^{max}$ (MW)
Brighton	10	0	600
Alta	14	0.00559	40
Park city	15	0.02148	170
Sundance	35	0.365	200
Solitude	30	0.37937	520



**Fig. 3** Diagram of modified PJM five-bus.

**Table 3** Buses' demand data.

BUS	$P_{d,n}$ (MW)
BUS A	0
BUS B	200
BUS C	300
BUS D	300
BUS E	0

**Table 4** LMP,  $lmp^{energy}$  and  $lmp^{cong}$  at each bus.

	Lmp (\$/MWh)	$lmp^{energy}$ (\$/MWh)	$lmp^{cong}$ (\$/MWh)
BUS A	19.4368	18.4368	+1
BUS B	26.9368	18.4368	+8.5
BUS C	30.4368	18.4368	+12
BUS D	38.4368	18.4368	+20
BUS E	13.4368	18.4368	-5

**Table 5** The share of each generator in load supplying of each bus.

Nodal supplying power of	Generator A (MW)	Generator C (MW)	Generator D (MW)	Generator E (MW)	SUM M (MW)
BUS B	81.45	0	0	118.5	200
BUS C	67.19	0	25.24	207.55	300
BUS D	63.22	0	36.25	200.51	300

**Table 6** Congestion surplus from each energy exchange ( $MS_{i,n}$ ).

$MS_{i,n}$ (\$)	Generator			
	A	C	D	E
BUS B	610.875	0	0	1599.75
BUS C	739.09	0	-201.92	3528
BUS D	1201.18	0	0	5012.75

It denotes that ISO is faced to under budget for this energy exchange. To solve this under budget, the first form of formulation of Eq. (39) is applied. By applying this method the revenue of generator and payments of customers will be equal. The energy exchanges between generator E and customers at buses B and C are correspond to third form of congestion surplus allocation (that  $lmp_{B,C}^{cong} > lmp_E^{cong}$  and  $lmp_E^{cong} < 0$  and  $lmp_{B,C}^{cong} > 0$ ). By applying Eq. (41) the revenue of generator and payments of customers will be equal.



**Table 7** Revenue of generators and cost of customers before and after MS allocation and revenue and payment variations of market participants.

		GEN A	GEN C	GEN D	GEN E
<b>BUS B</b>	$d_{i,n}^{old}$	1583.12736	0	0	1592.2608
	$s_{i,n}^{old}$	2194.00236	0	0	3192.0108
	$\Delta d_{i,n}$	0	0	0	592.5
	$\Delta s_{i,n}$	-610.875	0	0	-1007.25
	$s_{i,n}^{new}$	1583.1273	0	0	2184.7608
	$d_{i,n}^{new}$	1583.1273	0	0	2184.7608
<b>BUS C</b>	$d_{i,n}^{old}$	1305.9585	0	970.144832	2788.80764
	$s_{i,n}^{old}$	2045.04859	0	768.224832	6317.15784
	$\Delta d_{i,n}$	0	0	-201.91	1037.75
	$\Delta s_{i,n}$	-739.09	0	0	2490.6
	$s_{i,n}^{new}$	1305.9585	0	768.224832	3826.557
	$d_{i,n}^{new}$	1305.9585	0	768.224832	3826.557
<b>BUS D</b>	$d_{i,n}^{old}$	1228.794	0	0	2694.21276
	$s_{i,n}^{old}$	2429.974	0	0	7706.96276
	$\Delta d_{i,n}$	0	0	0	1002.55
	$\Delta s_{i,n}$	-1201.18	0	0	-4010.2
	$s_{i,n}^{new}$	1228.794	0	0	3696.7627
	$d_{i,n}^{new}$	1228.792	0	0	3696.7627

Other energy exchanges are corresponding to second form of congestion surplus allocation. By applying Eq.

(40) the revenue of generators and payments of customers will be equal. By applying the proposed method to allocate MS among market participants, revenue of Generator, payments of different customers at different bus and the changes due to the MS allocation for each energy exchange is depicted in Table 7. By applying the proposed method, the payment and revenue in each energy exchange is equalized.

Tables 8 and 9 denote the MS allocation to each generator and each customer for all energy exchanges.

As stated previously, since  $S_{i,n}$  is defined as payments of bus  $n$  customers to supply its demand through generator  $i$ , so giving back a part of MS to customers of bus  $n$  means decrease in customers cost which result in negative sign of  $\Delta s$ .

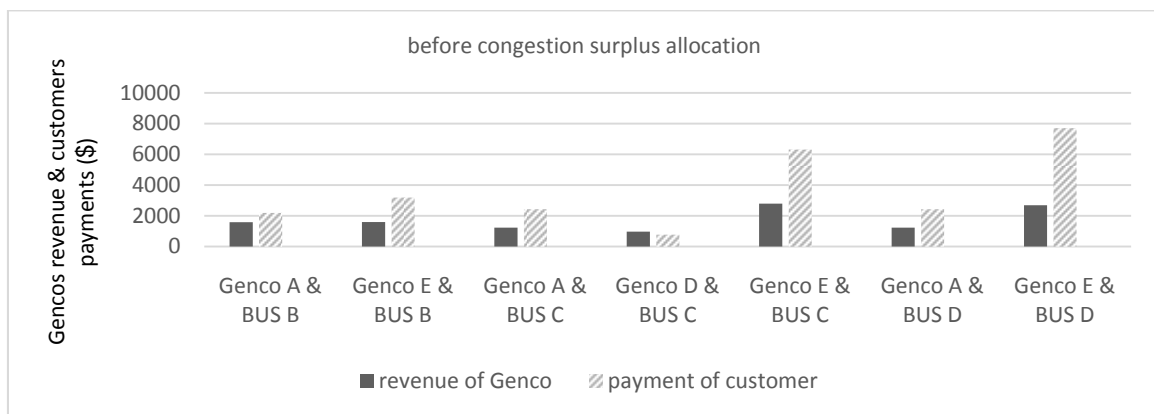
Figs. 4 and 5 depict Gencos' revenue & customers payments in each energy exchange before and after congestion surplus allocation respectively.

**Table 8** Allocated MS to each generator from all energy exchanges and revenue of each generator before and after MS allocation.

Genco	$d_i^{before}$ (\$)	$\Delta d_i$ (\$) (allocated MS)	$d_i^{after}$ (\$)
<b>GEN A</b>	4117.87986	0	4117.87986
<b>GEN C</b>	0	0	0
<b>GEN D</b>	970.144832	-201.92	768.224832
<b>GEN E</b>	7075.2812	2632.8	9708.0812
<b>SUM</b>	12163.30589	2430.88	14594.185

**Table 9** Allocated MS to each customer for all energy exchanges and customer payment for buying energy before and after MS allocation.

Customer	$s_n^{before}$ (\$)	$\Delta d_i$ (\$) (allocated MS)	$s_n^{after}$ (\$)
<b>BUS B</b>	5386.01316	-1618.125	3767.88816
<b>BUS C</b>	9130.43126	-3229.96	5900.47126
<b>BUS D</b>	10136.93676	-5211.38	4925.55676
<b>SUM</b>	15553.38118	-10059.40	14594.185



**Fig. 4** Gencos' revenue & customers payments in each energy exchange before congestion surplus allocation (\$).

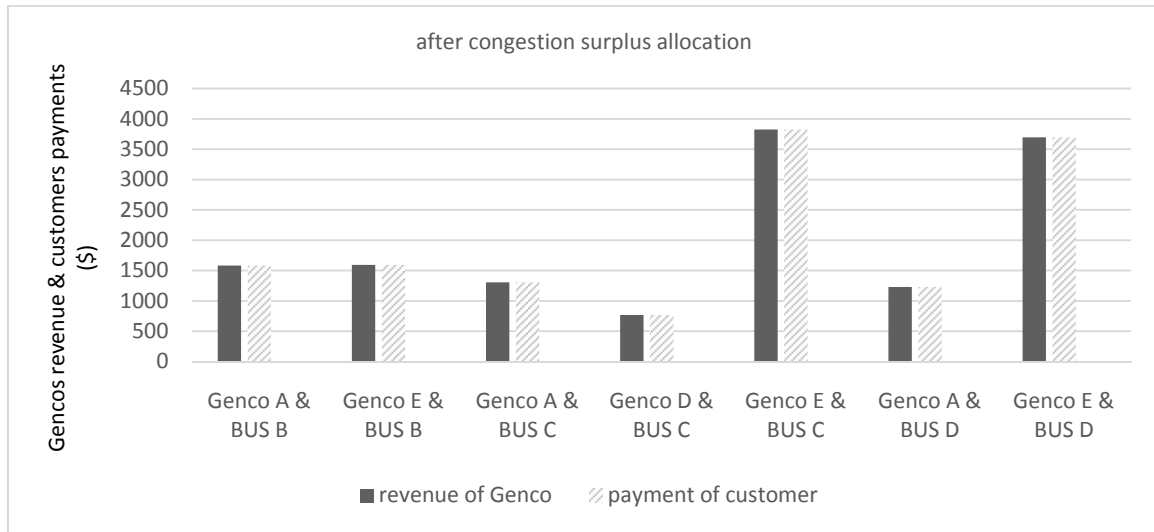


Fig. 5 Gencos' revenue & customers payments in each energy exchange after congestion surplus allocation (\$).

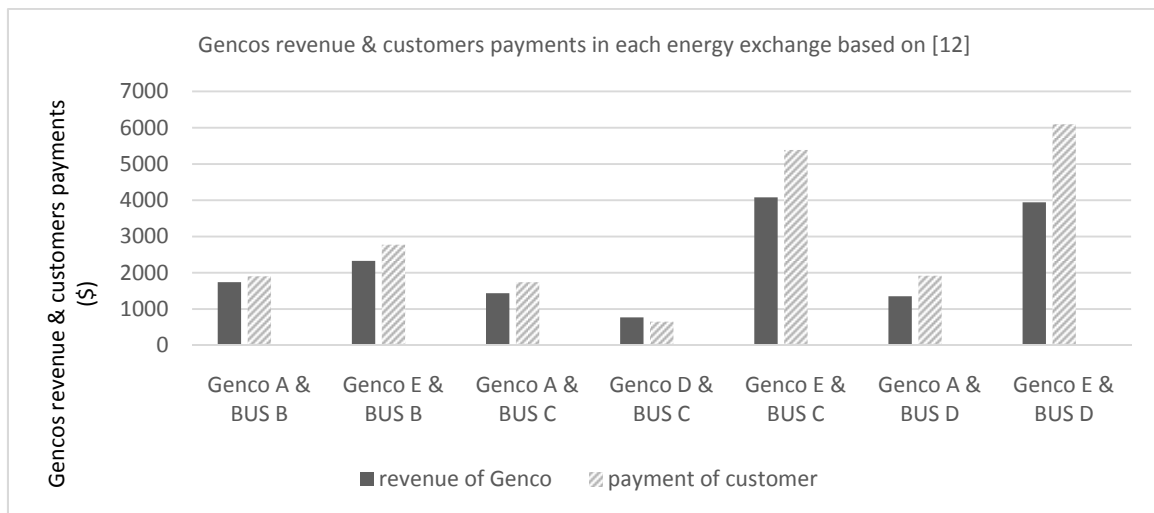


Fig. 6 Gencos' revenue & customers' payments in each energy exchange calculated through method of [12].

Results indicate that the proposed method presents a logical and fair method for MS allocation which causes that the ISO receptions and payments get equal (Fig. 5). By applying the proposed method, ISO is no more amazed to allocate MS of power market among market participants. Proposed MS allocation method is a significant step toward the fairness in power market and also removes some of LMP method defects.

In the followings first the proposed method is compared with researches that consider the MS of power market as a transmission rent ([3, 11]) and then the proposed method is compared with the researches that try to modifying LMPs to remove the LMP method defects and decreasing MS of power market.

In [3, 11] MS has been considered as transmission rent. Fig. 4 demonstrates the Gencos revenue and customers payments in each energy exchange based on [3, 11]. According to Fig. 4 in each energy exchange

there is a considerable difference between generator's revenue and customer's payment. Summation of these differences granted to Transco (MS of power market) and since share of every player of MS isn't proportional to his usage of transmission system, so the result is unfair allocation of MS. Also, in that method Transco tries to increase MS of system to increase its revenue. So allocation of MS as transmission rent to Transco does not seem rational. MS should be allocated among market participants according to their role in congestion of transmission lines which proposed in our paper.

Some researches express that LMP is not a fair method for pricing in power market and has proposed methods to modify LMP defects and decrease MS of power market [12]. In this paper the methods of [12] is applied on 5 bus test system and the results are depicted in Table 10 and Fig. 6. Table 10 demonstrates

the modified nodal price based on [12]. Fig. 6 demonstrates Gencos' revenue & customers' payments in each energy exchange based on [12].

Based on Fig. 6 by applying ref [12] pricing method the differences between Gencos' revenue & customers' payments in each energy exchange has been decreased, but yet there is a considerable difference between Gencos' revenue & customers' payments in each energy exchange which causes MS for each energy exchange. So again ISO is amazed to allocate this MS and there is no determined plan in that reference to allocate MS of system among market participants fairly.

Ultimately by applying the proposed method in this paper, ISO is no more amazed what to do with MS of power market and the ISO receptions and payments get equal.

**Table 10** Modified nodal price based on [12] in 5 bus test system.

Bus no	lmp (\$)
BUS A	21.387
BUS B	23.458
BUS C	25.987
BUS D	30.423
BUS E	19.657

## 7 Conclusion

In some pool markets which have LMP system, ISO receive MS and doesn't have any determined plan to spend it. Sometimes MS is considered as a transmission rent, sometimes as a part of Transco revenue and etc. The global consensus concludes that MS should allocate among market participants instead of granting it to Transco. But there is not a clear plan that determines how MS should allocate among market participant fairly. In this paper a novel method is proposed to allocate MS among market participants. This method by clarifying the role of each market participant in congestion of transmission lines, allocates MS among them. By applying this method, ISO is no more amazed what to do with MS of power market. ISO can identify the MS share of each market participant based on fair index and allocates MS among them fairly.

## Nomenclature

$N, L$	Number of generation units and transmission lines.
$LMP_n$	Electricity prices at bus n.
$\Gamma_l^{\min} \Gamma_l^{\max}$	Lagrangian multipliers of transmission line limit constraint of line l
$P_i, P_i^{\min} P_i^{\max}$	Generator i power generation and its lower and upper capacity limits

$\rho_i, a_i, b_i$	(MW). Bid function and its intercept and slope (\$/MWh), (\$/MWh), (\$/MWh).
$\mu_i^{\min} \mu_i^{\max}$	Lagrangian multipliers of generation limit constraints of generator i.
$S_{i,n}$	Payment of bus n customers to buying energy from generator i.
$\gamma_{l,i}$	Flow of line l due to the generation power of generator i.
$P_{d,n}$	Total load of bus n.
$l_{cong}$	Number of congested lines.
$P_i$	Total generations of generator i.
$P_D, P_d$	Vector of load, total load (MW).
$d_{i,n}$	The revenue of generator i from bus n.
$N_g$	Total number of generators.
$N$	total number of buses
$\lambda$	Lagrange multipliers of the equality constraint.
$\underline{\alpha}_l, \overline{\alpha}_l$	Lower and upper limits of line l (MW).
$MS_{i,n}$	Merchandizing surplus for energy exchange between generator i and customers at bus n.
$K_{min} K_{max}$	Number of units, bound by their minimum and maximum generation at the market equilibrium point.

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**Iman Ehsani** was born in Semnan, Iran in 1990. He received B.Sc. degree and pursuing M.Sc. degree in electrical engineering faculty from Semnan University, Semnan, Iran. His research interests include power market and reliability in power system



**Asghar Akbari Fraud** was born in Hamadan, Iran in 1972. He received B.Sc. degree from Tehran University and M.Sc. and Ph.D. degrees From Tarbiat-modares University, Tehran, Iran. He is now with Semnan University. His research interests include power system dynamics & operation and restructuring.



**Alireza Soofiabadi** was born in Rey, Iran in 1989. He received B.Sc. degree and pursuing M.Sc. degree in electrical engineering faculty from Semnan University, Semnan, Iran. His research interests include power market and reliability in power system.