

# Analysis of Planar Diodes in Temperature Limited Region

M. Khalaj Amirhosseini<sup>\*(C.A.)</sup>

**Abstract:** The ideal analysis of planar diodes in Temperature Limited Region is presented. Two types of relations are obtained for electric potential and electric field distributions; one accurate but implicit and the other almost accurate but explicit.

**Keywords:** Electron Gun, Planar Diode, Temperature Limited Region.

## 1 Introduction

An electron gun is one of the basic components of an electron beam devices such as microwave tubes. The first step to introduce electron guns is to analyze a planar diode. This is because a planar diode has the simplest geometry suitable to demonstrate the performance of complicated diodes and electron guns. All types of diodes, including planar ones, have two operational regions, Space Charge Limited (SCL) and Temperature Limited (TL) [1]-[4]. In literature, the SCL region has been exhaustively explained but the TL region explanation is seemingly absent. In this article, the ideal analysis of planar diodes in TL region is presented. Two types of relations are obtained for electric potential and electric field distributions. One type is accurate but implicit and the other type is almost accurate but explicit.

## 2 Related Works

Planar diodes have planar and parallel cathode and anode, which are large compared with the spacing between them. The scheme of a planar diode of distance  $d$  between cathode and anode is depicted in Fig. 1. The anode is connected to voltage  $V_a$  and its current density is  $J_a$ . In SCL region, the current density  $J_a$  is equal to  $J_0$  which is proportional to anode voltage of power 3/2.

In TL region, the current density  $J_a$  is equal to  $J_{sat}$  which is independent of anode voltage and is dependent to temperature as follows [1]-[2].

$$J_{sat} = AT^2 \exp\left(-\frac{\phi e}{kT}\right) \quad (1)$$

where  $A = \frac{4\pi m e k^2}{h^3}$ ,  $T$  is the absolute temperature of the cathode and  $\phi$  is the work function of the cathode.

In TL region, we can define a normalized current density  $\bar{J} = \frac{J_{sat}}{J_0}$  which is less than one and varies by changing anode voltage or temperature. Fig. 2 shows a typical variation of current density versus anode voltage introducing two SCL and TL regions.

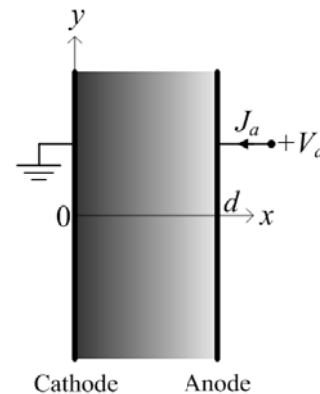


Fig. 1 The scheme of a planar diode

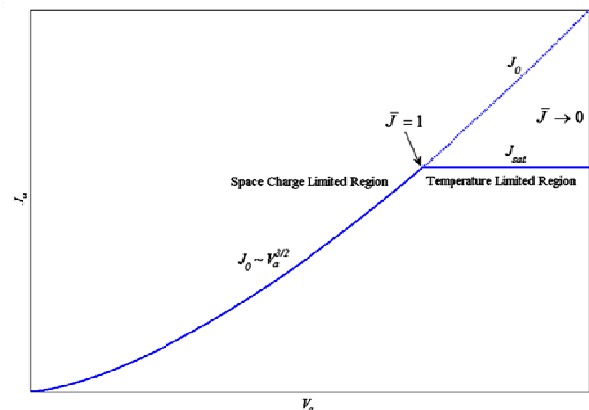


Fig. 2 A typical plot of current density versus anode voltage for a planar diode

Iranian Journal of Electrical & Electronic Engineering, 2015.  
 Paper first received 16 Sep. 2015 and in revised form 25 Oct. 2015.  
 \* The Author is with the School of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran.  
 E-mail: khalaja@iust.ac.ir.

### 3. Space Charge Limited Region

In SCL region, the main equation for electric potential is the following differential equation which is the combination of poison's equation, energy conservation and continuity equation.

$$\frac{d^2V(x)}{dx^2} = \frac{J_0}{\epsilon_0 \sqrt{2\eta V(x)}} \quad (2)$$

where  $\eta = \frac{e}{m}$  and  $J_0$  is the same current density  $J_a$  in

SCL region which is unknown.

$$V(0) = 0 \quad (3)$$

$$V(d) = V_a \quad (4)$$

$$E_c = E(0) = \left. \frac{dV}{dx} \right|_{x=0} = 0 \quad (5)$$

It is well known that the solution of Eq. (2) together with Eqs. (3)-(5), is obtained as follows [1]-[4].

$$V(x) = V_a \left( \frac{x}{d} \right)^{4/3} \quad (6)$$

$$E(x) = -\frac{4V_a}{3d} \left( \frac{x}{d} \right)^{1/3} \quad (7)$$

Also, the current density is obtained by the following relation known as Child-Langmuir equation.

$$J_0 = \frac{4}{9} \epsilon_0 \sqrt{2\eta} \frac{V_a^{3/2}}{d^2} \quad (8)$$

### 4. Temperature Limited Region

In TL region, the main equation for electric potential is the following differential equation which is the combination of poison's equation, energy conservation and continuity equation.

$$\frac{d^2V(x)}{dx^2} = \frac{J_{sat}}{\epsilon_0 \sqrt{2\eta V(x)}} \quad (9)$$

in which  $J_{sat}$  is the constant current density and is known in contrast with SCL region in which  $J_0$  is unknown. Instead, the electric field at cathode is unknown in TL region in contrast with SCL region as in Eq. (5). Therefore, there are only two boundary conditions (3)-(4) in TL region.

To solve Eq. (9), we first multiply it by  $\frac{dV}{dx}$  and

then integrate to reach this relation.

$$\left( \frac{dV}{dx} \right)^2 = \frac{4J_{max}}{\epsilon_0 \sqrt{2\eta}} \sqrt{V} + c_1^2 \quad (10)$$

or equivalently

$$\frac{dV}{\sqrt{\frac{4J_{max}}{\epsilon_0 \sqrt{2\eta}} \sqrt{V} + c_1^2}} = dx \quad (11)$$

in which  $c_1$  is an unknown constant. Integrating two sides of Eq. (11), gives us

$$\left( \frac{4J_{max}}{\epsilon_0 \sqrt{2\eta}} \sqrt{V(x)} - 2c_1^2 \right) \left( \frac{4J_{max}}{\epsilon_0 \sqrt{2\eta}} \sqrt{V(x)} + c_1^2 \right)^{1/2} = \frac{6J_{max}^2}{\epsilon_0^2 \eta} (x + c_2) \quad (12)$$

in which  $c_2$  is another unknown constant.

Two constants  $c_1$  and  $c_2$  are related to each other through boundary condition (3), thus

$$c_2 = \frac{-\epsilon_0^2 \eta}{3J_{max}^2} c_1^3 \quad (13)$$

The boundary condition (4) along with relation (13), give us the following relation for the constant  $c_1$ .

$$c_1^3 - \frac{V_a}{d} c_1^2 + \frac{16}{27} \left( \frac{V_a}{d} \right)^3 (\bar{J} - \bar{J}^2) = 0 \quad (14)$$

After some mathematical manipulation, the value of constant  $c_1$  is obtained as follows.

$$c_1 = \frac{1V_a}{3d} (1 + 2 \cos \varphi) \quad (15)$$

in which

$$\varphi = \begin{cases} \frac{1}{3} \tan^{-1} \left( \frac{(\bar{J} - 0.5) \sqrt{\bar{J}(1-\bar{J})}}{|\bar{J}^2 - \bar{J} + 0.125|} \right); & 0 < \bar{J} \leq 0.146 \\ \frac{\pi}{3} + \frac{1}{3} \tan^{-1} \left( \frac{(\bar{J} - 0.5) \sqrt{\bar{J}(1-\bar{J})}}{|\bar{J}^2 - \bar{J} + 0.125|} \right); & 0.146 \leq \bar{J} \leq 0.854 \\ -\frac{2\pi}{3} + \frac{1}{3} \tan^{-1} \left( \frac{(\bar{J} - 0.5) \sqrt{\bar{J}(1-\bar{J})}}{|\bar{J}^2 - \bar{J} + 0.125|} \right); & 0.854 < \bar{J} \leq 1 \end{cases} \quad (16)$$

Substituting (13) and (15) in (12), gives finally us the following relation for electric potential function.

$$\left( 8\bar{J} \sqrt{\frac{V(x)}{V_a}} - (1 + 2 \cos \varphi)^2 \right) \times \left( 16\bar{J} \sqrt{\frac{V(x)}{V_a}} + (1 + 2 \cos \varphi)^2 \right)^{1/2} + (1 + 2 \cos \varphi)^3 = 32\bar{J}^2 \frac{x}{d} \quad (17)$$

Also, differentiating Eq. (17) yields electric field as follows.

$$E(x) = -\frac{V_0}{d} \sqrt{\frac{4}{3} \bar{J} \sqrt{\frac{V(x)}{V_a}} + \frac{1}{9} (1 + 2 \cos \varphi)^2} \quad (18)$$

Figures 3 and 4 illustrate electric potential and electric field, respectively, versus  $x$ , for  $\bar{J}$  between zeros to one with equal distance. It is seen that as the normalized current  $\bar{J}$  approaches zero, the distributions of electric potential and electric field tend to those of electron-free space. Electric potential distribution tends to become linear and electric field distribution tend to become constant versus the distance from cathode towards anode.

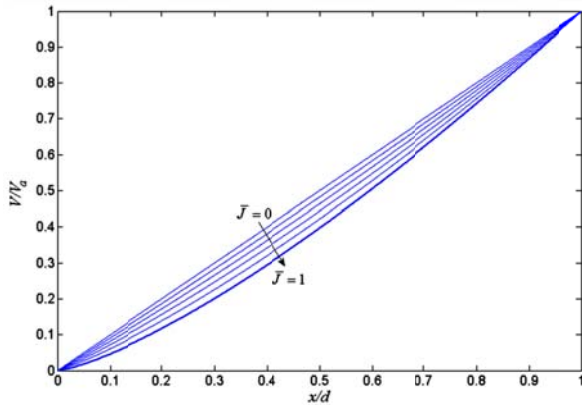


Fig. 3 Distribution of electric potential

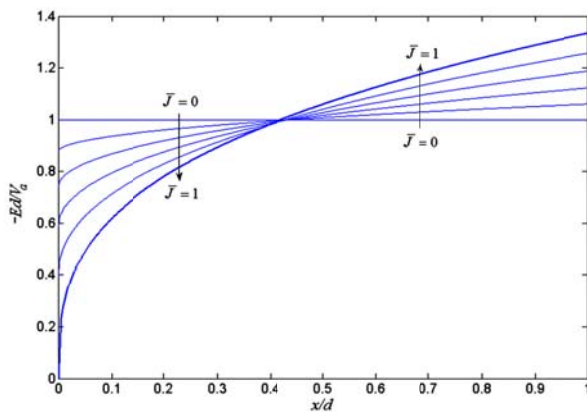


Fig. 4 Distribution of Electric field

To have explicit relations for electric potential and electric field, a very excellent approximation is obtained by curve fitting for electric field and then by integrating for electric potential.

$$E(x) \cong -\frac{V_a}{d} \left( a + b \left( \frac{x}{d} \right)^c \right) \quad (19)$$

$$V(x) = -\int_0^x E(x') dx' \quad (20)$$

$$= V_a \left( a \left( \frac{x}{d} \right) + \frac{b}{c+1} \left( \frac{x}{d} \right)^{c+1} \right)$$

In Eqs. (19) and (20),  $a$ ,  $b$  and  $c$  are three unknown coefficients which are obtained by curve fitting of  $E(x)$ , as follows.

$$a = (1 - \bar{J})^{0.621} \quad (21)$$

$$b = \frac{4}{3} (1 - (1 - \bar{J})^{0.683}) \quad (22)$$

$$c = \frac{1}{3} + \frac{1}{6} (1 - \bar{J})^{0.498} \quad (23)$$

The least mean square error of the above approximations is very low and less than 0.3% for  $V(x)$  and for all range of  $\bar{J}$ .

From Eqs. (18), (19) and (21), the electric field at cathode is obtained as follows.

$$E_c = E(0) = -\frac{V_a}{3d} (1 + 2 \cos \varphi) \quad (24)$$

$$\cong -(1 - \bar{J})^{0.621} \frac{V_a}{d}$$

One sees that the cathode electric field is not zero in TL region.

Also, the charge density will be such

$$\rho(x) = -\frac{J_{sat}}{\sqrt{2\eta V(x)}} \quad (20)$$

$$= -\frac{J_{sat}}{\sqrt{2\eta}} \left( a \left( \frac{x}{d} \right) + \frac{b}{c+1} \left( \frac{x}{d} \right)^{c+1} \right)^{-1/2}$$

## 5. Conclusion

The ideal analysis of planar diodes in Temperature Limited Region is presented. Two types of relations are obtained for electric potential and electric field distributions. One type is accurate but implicit and the other type is almost accurate but explicit. The least mean square error of the approximate relations is very low and less than 0.3%. It is seen that as the normalized current  $\bar{J}$  approaches zero, the distributions of electric potential and electric field tend to those of electron-free space.

## References

- [1] J. W. Gewartowski and H. A. Watson, *Principles of Electron Tubes*, D. Van Nostrand Company, Inc., New York, 1965.
- [2] M. Sedlacek, *Electron Physics of Vacuum and Gaseous Devices*, John Wiley & Sons, Inc., USA, 1996.
- [3] S. E. Tsimring, *Electron Beams and Microwave Vacuum Electronics*, John Wiley & Sons, Inc., Hoboken, New Jersey, USA, 2007.
- [4] Jr. S. Humphries, *Charged Particle Beams*, John Wiley & Sons, Inc., USA, 2002.



**Mohammad Khalaj Amirhosseini** was born in Tehran, Iran in 1969. He received his B.Sc., M.Sc. and Ph.D. degrees from Iran University of Science and Technology (IUST) in 1992, 1994 and 1998 respectively, all in Electrical Engineering. He is currently a professor in College of Electrical Engineering of IUST. His scientific fields of interest are electromagnetic direct and inverse problems including microwaves, antennas and propagation.