

Time-Varying Frequency Fading Channel Tracking In OFDM-PLNC System, Using Kalman Filter

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Abstract: Physical-layer network coding (PLNC) has the ability to drastically improve the throughput of multi-source wireless communication systems. In this paper, we focus on the problem of channel tracking in a Decode-and-Forward (DF) OFDM PLNC system. We proposed a Kalman Filter-based algorithm for tracking the frequency/time fading channel in this system. Tracking of the channel is performed in the time domain while data detection is implemented in the frequency domain. As an important advantage, this approach does not need for training of some subcarriers in every OFDM symbols and this, results in higher throughput, compared to other methods. High accuracy, no phase ambiguity, and stability in fast fading conditions are some other advantages of this approach.

Keywords: Physical Layer Network Coding; Decode-and-forward; Orthogonal Frequency Division Multiplexing (OFDM); channel tracking; Kalman Filter.

1 Introduction

Recently, the research community has shown growing interest in PLNC (Physical Layer Network Coding) as a way to exploit the network coding operation that occurs naturally in superimposed electromagnetic (EM) waves [1], [2]. In many wireless communication networks today, interference is treated as a destructive phenomenon. When multiple transmitters in the same environment send radio waves to their respective receivers, each receiver gets signals from its dedicated transmitter as well as from other transmitters. The radio waves, coming from other transmitters, are often treated as interference that corrupts the intended signal. Now, PLNC by exploiting the network coding operation, performed by nature, has changed the situation [3].

In a TWRC (two-way relay channel), for example, by allowing the two end nodes (n_1, n_2) to transmit simultaneously to the relay and not treating this as collision, PLNC can boost the system throughput by 100% [1]. This throughput advantage of PLNC, however, is predicated on accurate estimation of the channels between the end nodes and the relay. This estimation is typically done using known training

symbols and/or pilots embedded in the packets. Channel estimation in PLNC systems is an active area of research since unlike the point to point communication systems, at least two channel gains need to be estimated based on the simultaneously received signals from multi sources.

Broadband wireless communications, on the other hand, experience frequency/time fading which distorts the transmitted signal. Orthogonal frequency division multiplexing (OFDM) is a well-known transmission technique which is robust against the frequency selectivity of the channel. In addition, OFDM provides a natural way to deal with relative symbol offset between nodes 1 and 2 in PLNC. In particular, any time-domain symbol offset (less than cyclic prefix length) will be treated as a phase term in the channel gain in the frequency domain, which can simplify the system. Because of these advantageous potentials, OFDM PLNC is now, a popular system under investigation by many researchers [4]-[11]. Most of these papers assume perfect knowledge of channel at the receiver side [7], [8], [11] while the lack of channel state information (CSI) has a significant impact on system performance. On the other hand, estimation of the channel gains on the subcarriers of an OFDM-PLNC system is a new challenge that has not addressed by the traditional point to point OFDM researches. As an example, the training symbols and pilots need to be redesigned for OFDM PLNC.

In general, two types of a training sequence for channel estimation (CE) in OFDM systems were proposed; block-based and pilot-tone-based (PT)

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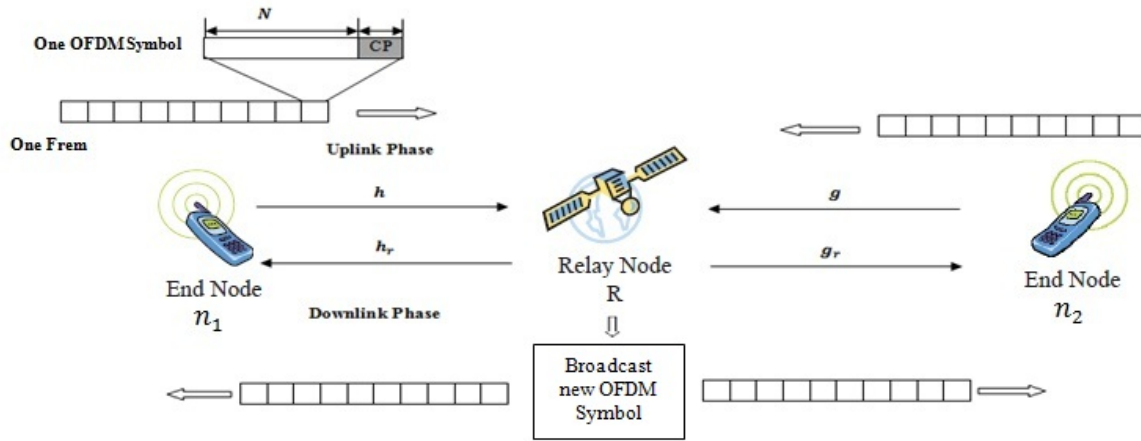


Fig. 1: PLNC-based two-way relay system. Two source nodes n_1 and n_2 , and the relay node R

training [4]. In block-based training method, one complete OFDM block is dedicated to training. However, the channel estimation obtained by this method remains static between successive training blocks, while the channel may be changing, continuously. In pilot-tone (PT) training, some carriers in every OFDM block are selected as pilots to track the channel. Clearly, this could greatly reduce the system throughput.

In this paper, we propound the problem of estimation and tracking of the time-varying channel in an OFDM-PLNC system, where, the channel with both frequency and time selectivity are considered. With the help of the state space model of the communication channel, we offer a method for the channel estimation and tracking in the time domain, and data detection in the frequency domain. Channel tracking in the time domain is more stable for estimation errors, owing to the very good extraction of the frequency correlation of the channel coefficients. In addition, the estimation errors are extended on the entire transmission frequency band, and it will not centralize only on a specific set of subcarriers. Channel tracking in the frequency domain, without considering the state-space model for it, the channel tracking is done separately on any subcarrier. Thus, the error in channel estimation on each sub-carrier directly affects the information estimation of that sub-carrier, and this error will continuously affect tracking the channel and data related to that sub-carrier. Tracking of the channel in time-domain and data detection in the frequency domain is well known in the traditional point to point OFDM systems [12]-[16]. Here, we propose a time-domain algorithm for an OFDM PLNC channel tracing derived from Kalman Filter (KF). Joint decode-and-forward (DF) PLNC scheme [17] is considered, where the relay decodes both packets from n_1 and n_2 simultaneously, and re-encodes by modulo-2 summation, before broadcasting. This method removes the adverse effect of fading and noise in the source-relay channels. For this purpose, we have to jointly estimate the source-relay channels and jointly decode

the superposed packets. Also, using this proposed KF algorithm, we don't need PTs for training which occupy at least $L_1 + L_2 + 2$ subcarriers in each OFDM block [4] (where L_j , $j = 1, 2$ represents the delay spread of the corresponding discrete channel model).

The rest of the paper is organized as follows. In the next section, the system and signal models are presented. Proposed KF-CE (Kalman Filter-Channel Estimation) algorithm for decode-and-forward TWRC system in a time/frequency selective fading channel is introduced in Section 3. Some computer simulation results, as well as related discussions, come in Section 4. Finally, Section 5 concludes the paper.

2 System Model

We consider a PLNC-based two-way relay system, with two source nodes, n_1 and n_2 , and the relay node R as shown in Fig. 1. The source nodes and the relay communicate using a two-phase PLNC transmission consist of uplink and downlink phase.

2.1 Uplink Phase

In the uplink phase, n_1 and n_2 transmit their packets to R simultaneously. The data-modulated symbol of each node is represented by $\{X_j(n) = \mathcal{M}(b_j(n)), n \sim 0: N - 1, j = 1, 2, r\}$, where $b_j(n)$ is the binary data of each node, and $E[|X_j(n)|^2] = 1$. In each node, the symbol sequences $(\mathbf{X}_j^{(k)} = [X_j^{(k)}(1), X_j^{(k)}(2), \dots, X_j^{(k)}(N)]^T)$ in time instant k are given to an N -point normalized Inverse Fast Fourier Transform (IFFT) to generate an OFDM signal waveform as $\mathbf{x}_j^{(k)}$ in the k^{th} OFDM symbol time. Then a cyclic prefix (CP) vector with the length of N_{CP} , is inserted and signal from each node is transmitted over a time varying frequency selective fading channel. The baseband channel responses between n_1 and R, and n_2 and R at symbol time k , are denoted by $\mathbf{h}^{(k)} = [h_0^{(k)}, h_1^{(k)}, \dots, h_{L_1-1}^{(k)}]^T$, $\mathbf{g}^{(k)} = [g_0^{(k)}, g_1^{(k)}, \dots, g_{L_2-1}^{(k)}]^T$

respectively, where L_j , $j = 1, 2$ represents the delay spread of the corresponding discrete channel model in sample. The elements of \mathbf{h} and \mathbf{g} are assumed as zero-mean circularly symmetric complex Gaussian random variables and are independent from each other.

Relay Operation: the normalized Discrete Fourier Transform (DFT) of the k^{th} OFDM block, received at R (after removing the CP), can be expressed as:

$$\mathbf{r}^{(k)} = [r_0^{(k)}, r_1^{(k)}, \dots, r_{N-1}^{(k)}]^T = \mathbf{H}^{(k)}\mathbf{X}_1^{(k)} + \mathbf{G}^{(k)}\mathbf{X}_2^{(k)} + \mathbf{n}^{(k)}, \quad (1)$$

where, $\mathbf{H}^{(k)}$ and $\mathbf{G}^{(k)}$ are $N \times N$ diagonal matrices with diagonal elements given by N -point DFT of $\mathbf{h}^{(k)}$ and $\mathbf{g}^{(k)}$, respectively. $\mathbf{X}_j^{(k)}$, $j=1, 2$ is an $N \times 1$ OFDM block transmitted at time k from n_j to the relay R, and $\mathbf{n}^{(k)}$ is an $N \times 1$ zero-mean additive white Gaussian noise (AWGN) vector, each of its elements with variance σ_n^2 , i.e. $\mathbf{n}^{(k)} \sim \mathcal{N}_c(\mathbf{0}, \sigma_n^2 \mathbf{I})$. In time instant k , relay estimates $\mathbf{H}^{(k)}$, $\mathbf{G}^{(k)}$, $\mathbf{X}_1^{(k)}$ and $\mathbf{X}_2^{(k)}$ using the KF-CE algorithm, proposed in section 3.

2.2 Downlink Phase

For network coding the relay performs estimation with respect to the XOR of the binary information of the nodes n_1 and n_2 . $\mathbf{b}_r^{(k)} = \hat{\mathbf{b}}_1^{(k)} \oplus \hat{\mathbf{b}}_2^{(k)}$ based on $\mathbf{r}^{(k)}$. This estimated relay codeword is again modulated ($\mathbf{X}_r^{(k)} = \mathcal{M}(\mathbf{b}_r^{(k)})$) and the relay broadcasts OFDM modulation of $\mathbf{X}_r^{(k)}$ to both nodes n_1, n_2 . In fact the system in the downlink phase is like a SIMO (Single Input Multi Output) system (Look at downlink phase in Fig. 1) and the received signal in each end node experiences a simple point to point communications from the relay to the end node. Without loss of generality we focus on the process at n_1 . The k^{th} OFDM demodulated block received at n_1 , is represented as:

$$\mathbf{y}^{(k)} = [y_0^{(k)}, y_1^{(k)}, \dots, y_{N-1}^{(k)}]^T = \mathbf{H}_r^{(k)}\mathbf{X}_r^{(k)} + \mathbf{n}_r^{(k)}, \quad (2)$$

$$\mathbf{n}_r^{(k)} \sim \mathcal{N}_c(\mathbf{0}, \sigma_{n_r}^2 \mathbf{I}),$$

where, $\mathbf{H}_r^{(k)}$ is an $N \times N$ diagonal matrix with the diagonal elements given by N -point normalized DFT of $\mathbf{h}_r^{(k)}$. Here, the vector $\mathbf{h}_r^{(k)} = [h_{r_0}^{(k)}, h_{r_1}^{(k)}, \dots, h_{r_{L_r-1}}^{(k)}]^T$ shows the baseband downlink channel between R and n_1 at time k where L_r represents the delay spread of the

corresponding channel in sample. The elements of \mathbf{h}_r are assumed as zero-mean circularly symmetric complex Gaussian random variables and \mathbf{h}_r is independent from uplink channel \mathbf{h} (However, identical channels are often assumed in both up and downlink [4] which can make the process much simpler). $\mathbf{n}_r^{(k)}$ is the zero-mean AWGN, each element with variance $\sigma_{n_r}^2$. In this stage, n_1 can also estimate $\mathbf{h}_r^{(k)}$ and $\mathbf{b}_r^{(k)}$ by using KF-CE algorithm (details come in the next section), and estimates $\mathbf{b}_2^{(k)}$, using its known information $\mathbf{b}_1^{(k)}$, by applying:

$$\hat{\mathbf{b}}_r^{(k)} \oplus \mathbf{b}_1^{(k)} = (\hat{\mathbf{b}}_1^{(k)} \oplus \hat{\mathbf{b}}_2^{(k)}) \oplus \mathbf{b}_1^{(k)} = \hat{\mathbf{b}}_2^{(k)} \quad (3)$$

3 KF-CE Algorithm for OFDM-PLNC System

3.1 KF-CE Algorithm

In mobile communications, the channels are time varying and need to be tracked over the time. Equation (1) (that is in frequency domain) can then be written in time domain as:

$$\tilde{\mathbf{r}}^{(k)} = [\tilde{\mathbf{X}}_1^{(k)} \quad \tilde{\mathbf{X}}_2^{(k)}][\mathbf{h}^{(k)} \quad \mathbf{g}^{(k)}]^T + \tilde{\mathbf{n}}^{(k)} = \tilde{\mathbf{X}}^{(k)}\mathbf{C}^{(k)} + \tilde{\mathbf{n}}^{(k)} \quad (4)$$

where, $\tilde{\mathbf{X}}_j^{(k)}$, $j = 1, 2$ is a $N \times L_j$ circulant matrix (i.e. its columns are circularly shifted versions of each other), formed with the modulated block $\mathbf{x}_j^{(k)} = [x_j^{(k)}(0), x_j^{(k)}(1), \dots, x_j^{(k)}(N-1)]^T$ at time instant k , and $\mathbf{C}^{(k)} = [\mathbf{h}^{(k)} \quad \mathbf{g}^{(k)}]^T$ is the $(L_1 + L_2) \times 1$ channel state vector obtained from channel vectors $\mathbf{h}^{(k)}$ and $\mathbf{g}^{(k)}$ which introduced earlier. In fact, Eq. (4) shows the convolution process by a matrix product which makes the notation much easier. Also, we defined the new parameter \mathbf{C} to express the equation according to appropriate structure for Kalman Filter (Eq. (A1) in Appendix).

To model the time variations of wireless channel, on the other hand, we note that according to the Bello model [18], the fading process from the transmitter to receiver can be modeled as a complex Gaussian process. A widely accepted model, to represents the local behavior of the time-varying wireless channel is an auto-regressive (AR) model [19]. Therefore, using the first-order assumption, the channel state at time k can be evaluated, using a first-order autoregressive process of the form

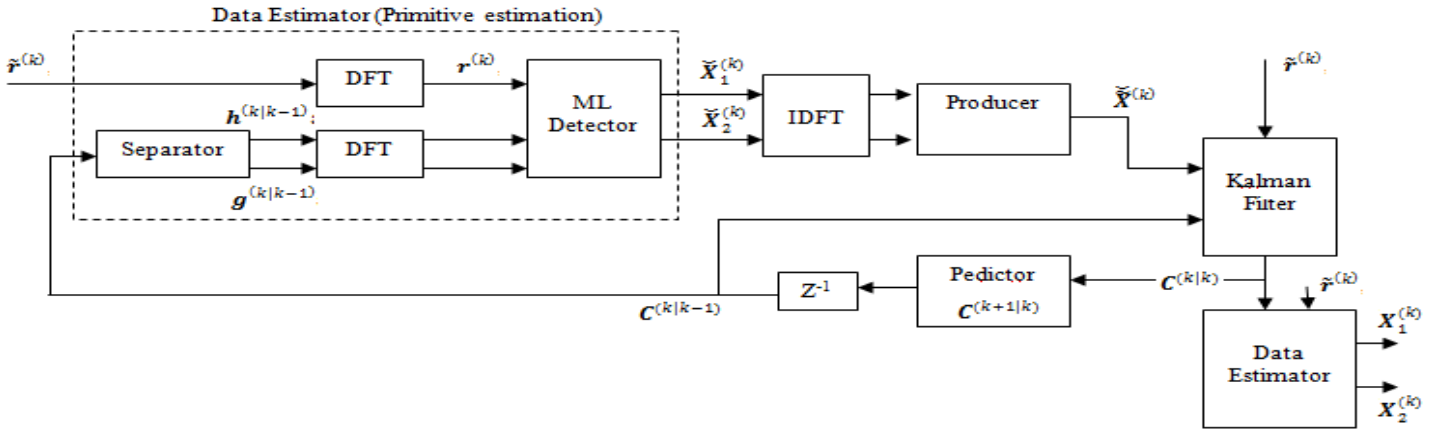


Fig. 2 Overall structure of relay operation

$$\mathbf{C}^{(k)} = \beta \mathbf{C}^{(k-1)} + \mathbf{V}^{(k)} \quad (5-a)$$

where, β is the static AR coefficient, and $\mathbf{V}^{(k)} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma_v^2 \mathbf{I})$ is the complex state noise of the model. Given the requirement to track the channel state as accurately as possible, there are different ways to determine values of β and σ_v^2 , [20], [21]. In our paper, due to the frequency fading and time varying nature of the wireless channel, we used the dynamic AR channel model proposed in [20], which is robust for realistic wireless situations. This paper rewrites the state-equation (5-a) as $\mathbf{C}^{(k)} = \mathbf{C}^{(k-1)} + (\beta - 1)\mathbf{C}^{(k-1)} + \mathbf{V}^{(k)}$. Then, it defines a new term $\boldsymbol{\mu}^{(k)} = (\beta - 1)\mathbf{C}^{(k-1)}$ and a primitive equation $\boldsymbol{\mu}^{(k)} = \boldsymbol{\mu}^{(k-1)} + \mathbf{w}^{(k)}$, where $\mathbf{w}^{(k)} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma_w^2 \mathbf{I})$, $\boldsymbol{\mu}^{(k)} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{m}^{(k)}, \mathbf{P}^{(k)})$. Then, using a Kalman filter formulation, the estimate of $\boldsymbol{\mu}^{(k)} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{m}^{(k)}, \mathbf{P}^{(k)})$ can be drawn corresponding to the change in the wireless channel estimate (see Table I). We refer the reader to [20] for details. Equations (4) and (5-a) or its modified version,

$$\mathbf{C}^{(k)} = \mathbf{C}^{(k-1)} + \boldsymbol{\mu}^{(k)} + \mathbf{V}^{(k)} \quad (5-b)$$

form the state space model to describe the system in uplink phase (equivalent to Eqs. (A1) and (A2) in Appendix). These equations are our signal model for running KF algorithm [22]. Whereas the system model is linear and the channel noise is Gaussian, we can design a Kalman Filter algorithm to continuously estimate the state vector \mathbf{C} . The overall structure of relay operation is illustrated in Fig. 2. As this figure shows, the relay operation is divided to two basic parts; Data estimation and Channel tracking. Data estimation is done in frequency domain, while channel tracking is done in time domain. Discrete Fourier Transform (DFT) and Inverse DFT (IDFT) blocks are used to switch from time to frequency domain and vice versa, respectively. According to Fig. 2 the following steps are realized:

- **Step 1:** At k^{th} time instant, the superimposed signal $\tilde{\mathbf{r}}^{(k)}$ is received at the relay, the prediction of the channel $\mathbf{C}^{(k|k-1)}$ is also propagated from symbol time $k-1$ to k .
- **Step 2:** $\mathbf{h}^{(k|k-1)}$ and $\mathbf{g}^{(k|k-1)}$ are extracted from $\mathbf{C}^{(k|k-1)}$ using separator block, easily. The normalized DFT is used to transfer signals to frequency domain (computing

$\mathbf{r}^{(k)}, \mathbf{H}^{(k|k-1)}, \mathbf{G}^{(k|k-1)}$). Then, using Eq. (1) and Maximum Likelihood (ML) detection, the primitive estimation of $\mathbf{X}_j^{(k)}$, say $\tilde{\mathbf{X}}_j^{(k)}$, $j = 1, 2$ is obtained. This step is done with Data Estimator block, surrounded part by the dotted in Fig. 2.

- **Step 3:** Building $\tilde{\mathbf{X}}^{(k)}$, a primitive estimation of time domain circulant matrix $\tilde{\mathbf{X}}^{(k)}$ presented in Eq. (4), by taking the normalized IDFT of $\tilde{\mathbf{X}}_j^{(k)}$. Now, $\tilde{\mathbf{X}}^{(k)}$ and $\mathbf{C}^{(k|k-1)}$ run the KF-CE Algorithm to obtain an updated channel estimation $\mathbf{C}^{(k|k)}$, and a new prediction $\mathbf{C}^{(k+1|k)}$ respectively.
- **Step 4:** Another Data Estimator block, the same as the first one in step 2, produces a new estimation of $\mathbf{X}_j^{(k)}$ using $\mathbf{H}^{(k|k)}$ and $\mathbf{G}^{(k|k)}$ extracted from $\mathbf{C}^{(k|k)}$. The inputs of this Data Estimator block are the updated channels estimation, therefore the output is a good estimation of data, $\mathbf{X}_j^{(k)}$.

The implemented Kalman filter process is illustrated in Table I, in details, using the standard notations. The reader is referred to Appendix for reminder the general structure of Kalman Filter.

Table I: KF-CE Algorithm

Initialization: Getting: $\sum^{0|k-1}, \sigma_n^2, \sigma_v^2, \hat{\mathbf{C}}^0, \mathbf{P}^0, \mathbf{m}^0, \delta$

For k^{th} OFDM symbol,

1. Input: $\tilde{\mathbf{r}}^{(k)}, \mathbf{C}^{(k|k-1)}, \tilde{\mathbf{X}}^{(k)}, \mathbf{P}^{(k-1)}, \mathbf{m}^{(k-1)}$
2. Compute the Kalman gain $\mathbf{K}^{(k)}$

$$\mathbf{K}^{(k)} = \sum^{k|k-1} \tilde{\mathbf{X}}^{(k)H} [\tilde{\mathbf{X}}^{(k)} \sum^{k|k-1} \tilde{\mathbf{X}}^{(k)H} + \sigma_n^2 \mathbf{I}]^{-1}$$
3. Estimate channel state $\mathbf{C}^{(k|k)}$,

$$\hat{\mathbf{C}}^{(k|k)} = \hat{\mathbf{C}}^{(k|k-1)} + \mathbf{K}^{(k)} [\tilde{\mathbf{r}}^{(k)} - \tilde{\mathbf{X}}^{(k)} \mathbf{C}^{(k|k-1)}]$$
4. $\sum^{k|k} = \sum^{k|k-1} - \mathbf{K}^{(k)} \tilde{\mathbf{X}}^{(k)} \sum^{k|k-1}$
5. Calculate primitive posterior estimate $\boldsymbol{\mu}^{(k)} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{m}^{(k)}, \mathbf{P}^{(k)})$, [20]

$$\sigma_w^2 \mathbf{I} = \frac{1 - \delta}{\delta} \mathbf{P}^{(k-1)}$$

$$\begin{aligned} \mathbf{Z}^{(k)} &= \mathbf{P}^{(k-1)} + \sigma_w^2 \mathbf{I} \\ \mathbf{Q}^{(k)} &= \mathbf{Z}^{(k)} + \sigma_v^2 \mathbf{I} \\ \mathbf{m}^{(k)} &= \mathbf{m}^{(k-1)} + \mathbf{Z}^{(k)} \mathbf{Q}^{(k)-1} \left[\hat{\mathbf{C}}^{(k|k)} \right. \\ &\quad \left. - (\hat{\mathbf{C}}^{(k|k-1)} + \mathbf{m}^{(k-1)}) \right] \end{aligned}$$

$$\mathbf{P}^{(k)} = \sigma_v^2 \mathbf{Z}^{(k)} \mathbf{Q}^{(k)-1}$$

6. Prediction for next iteration

$$\begin{aligned} \sum^{k+1|k} &= \sum^{k|k} + \mathbf{P}^{(k)} + \sigma_v^2 \mathbf{I} \\ = \mathbb{E} \left\{ \left[\mathbf{C}^{k+1} - \hat{\mathbf{C}}^{(k+1|k)} \right] \left[\mathbf{C}^{k+1} - \hat{\mathbf{C}}^{(k+1|k)} \right]^H \middle| \tilde{\mathbf{r}}^{(k)} \right\}, \\ \hat{\mathbf{C}}^{(k+1|k)} &= \hat{\mathbf{C}}^{(k|k-1)} + \mathbf{m}^{(k)}, \\ \hat{\mathbf{C}}^{(0|-1)} &= \hat{\mathbf{C}}^0 \end{aligned}$$

Steps 1 to 4 and step 6 in Table I are the relevant Kalman Filter steps for our proposed structure that can be written comparing Eqs. (4), (5-b) with (A1) and (A2) in Appendix. Step 5 has been also written considering the proposed dynamic model in [20], as we explained earlier. In Table I, $\sum^{(k|k-1)}$ is the covariance matrix of the prediction error, $\sum^{0|-1}$ is considered to be an identity matrix $\mathbf{I}_{(L_1+L_2) \times (L_1+L_2)}$. $\sigma_w^2 \mathbf{I}$ shows the covariance matrix of the measurement noise $\mathbf{n}^{(k)}$, $\sigma_v^2 \mathbf{I}$ is the covariance matrix of the state noise $\mathbf{V}^{(k)}$, and $\hat{\mathbf{C}}^0$ is the initial estimation of the channel state information $\mathbf{C}^{(k)}$. The term $\hat{\mathbf{C}}^0$ is obtained from training sequence as is explained in the next section. The initial distribution for the primitive equation was set to, $\mathbf{m}^0 = \mathbf{0}$, $\mathbf{P}^0 = 10^{-4} \mathbf{I}$. According to the [20], in all the experiments performed in this paper, at a SNR of 5dB, the discount factor δ was set to 0.999. Then as the SNR increases, δ is steadily decreased to 0.89 at 25dB. Experimentally, the discount factor in this range was found to produce the best performance results.

Therefore, in each time instant k , the relay receives composite signal $\tilde{\mathbf{r}}^{(k)}$, updates its estimation of the two uplink channels, de/modulates and decodes data information from both nodes, re-encodes and forwards the data to end nodes using the concept of PLNC.

As mentioned in the previous section, in downlink phase, the received signal at destination n_1 can be presented by Eq. (2). Equation (2) (that is in frequency domain) can also be written in time domain as:

$$\tilde{\mathbf{y}}^{(k)} = \left[\tilde{\mathbf{X}}_r^{(k)} \right] \left[\mathbf{h}_r^{(k)} \right]^T + \tilde{\mathbf{n}}_r^{(k)}, \quad (6)$$

where $\tilde{\mathbf{X}}_r^{(k)}$, the normalized IDFT of $\mathbf{X}_r^{(k)}$ in Eq. (2), is a $N \times L_r$ circulant matrix formed with the modulated block $\mathbf{x}_r^{(k)} = [\mathbf{x}_r^{(k)}(0), \mathbf{x}_r^{(k)}(1), \dots, \mathbf{x}_r^{(k)}(N-1)]^T$ (expressing the convolution process by a matrix product which makes the notation much easier). $\tilde{\mathbf{n}}_r^{(k)} \sim \mathcal{N}_c(\mathbf{0}, \sigma_{nr}^2 \mathbf{I})$ and $\mathbf{V}_r^{(k)} \sim \mathcal{N}_c(\mathbf{0}, \sigma_{vr}^2 \mathbf{I})$ are the measurement and the state noise models, respectively, and $\boldsymbol{\mu}_r^{(k)} \sim \mathcal{N}_c(\mathbf{m}_r^{(k)}, \mathbf{P}_r^{(k)})$.

Also the dynamic model for downlink channel from the relay to node n_1 is expressed as,

$$\mathbf{h}_r^{(k)} = \mathbf{h}_r^{(k-1)} + \boldsymbol{\mu}_r^{(k)} + \mathbf{V}_r^{(k)} \quad (7)$$

Therefore, Eqs. (6), (7) make the state space equations for the downlink phase equivalent with (A1), (A2) in Appendix. Now, with the same procedure the relevant KF-CE algorithm can be written for downlink channel estimation.

As a result, we can see that using the KF-CE algorithm channel tracking is performed with high accuracy, no phase ambiguity (the problem which some methods [4] suffer from it), and stability. By this method, it is enough to send a training OFDM block at the start as well as after every N_T transmission, to update primitive estimation of channels information, \mathbf{C}^0 . This has an important advantage that we don't have to send training pilots in each OFDM block to track the channels variations. Comparing with other methods which need some pilots (at least $(L_1 + 1) + (L_2 + 1)$, [4]), in each OFDM block, our proposed algorithm needs much less training data, and so achieves much more throughput. A special training sequence which can be used in our system will be proposed in the next part.

3.2 Proposed Training Sequence

To avoid the problem stemming from overlapping of the channel impulse responses from two users throughout training phase, at the first training time, both nodes n_1 and n_2 transmit the same training OFDM block with uniform scaling on all the sub-carriers, say

$$\begin{aligned} X_2^{(0)}(n) &= X_1^{(0)}(n) = a, \quad n = 1, 2, \dots, N-1, \\ |a|^2 &= 1. \end{aligned}$$

And at the second training time they transmit

$$X_2^{(1)}(n) = -X_1^{(1)}(n) = a.$$

Assuming fading channel gains are constant over two consecutive OFDM blocks (we need this assumption just for training phase), according to Eq. (1), two consecutive OFDM block (after CP removal and N -point normalized DFT) received at the relay can be express, as:

$$\mathbf{r}^{(0)} = a\mathbf{H}^{(0)} + a\mathbf{G}^{(0)} + \mathbf{n}^{(0)} \quad (8)$$

$$\mathbf{r}^{(1)} = a\mathbf{H}^{(0)} - a\mathbf{G}^{(0)} + \mathbf{n}^{(1)} \quad (9)$$

Using (8) and (9), the least-square channel estimate, $\hat{\mathbf{H}}^{(0)} \approx \mathbf{H}^{(0)}$ and $\hat{\mathbf{G}}^{(0)} \approx \mathbf{G}^{(0)}$, can be easily obtained from the following equations:

$$\hat{\mathbf{H}}^{(0)} = \frac{1}{2a} (\mathbf{r}^{(0)} + \mathbf{r}^{(1)}) = \mathbf{H}^{(0)} + \mathbf{n}_{ch} \quad (10)$$

$$\hat{\mathbf{G}}^{(0)} = \frac{1}{2a} (\mathbf{r}^{(0)} - \mathbf{r}^{(1)}) = \mathbf{G}^{(0)} + \mathbf{n}_{ch} \quad (11)$$

where $\mathbf{n}_{ch} = \frac{1}{2a} (\mathbf{n}^{(0)} \pm \mathbf{n}^{(1)}) \sim \mathcal{N}_c(\mathbf{0}, \frac{\sigma_n^2}{2} \mathbf{I})$.

Consequently, the relay is able to estimate the CSIs from both users in the training phase, which is used to calculate the first estimation of \mathbf{C}^0 , and updates it every N_T OFDM blocks. Between two training phase, the relay uses KF-CE algorithm to track the CSIs from both users. According to this method for training, we use 2 *

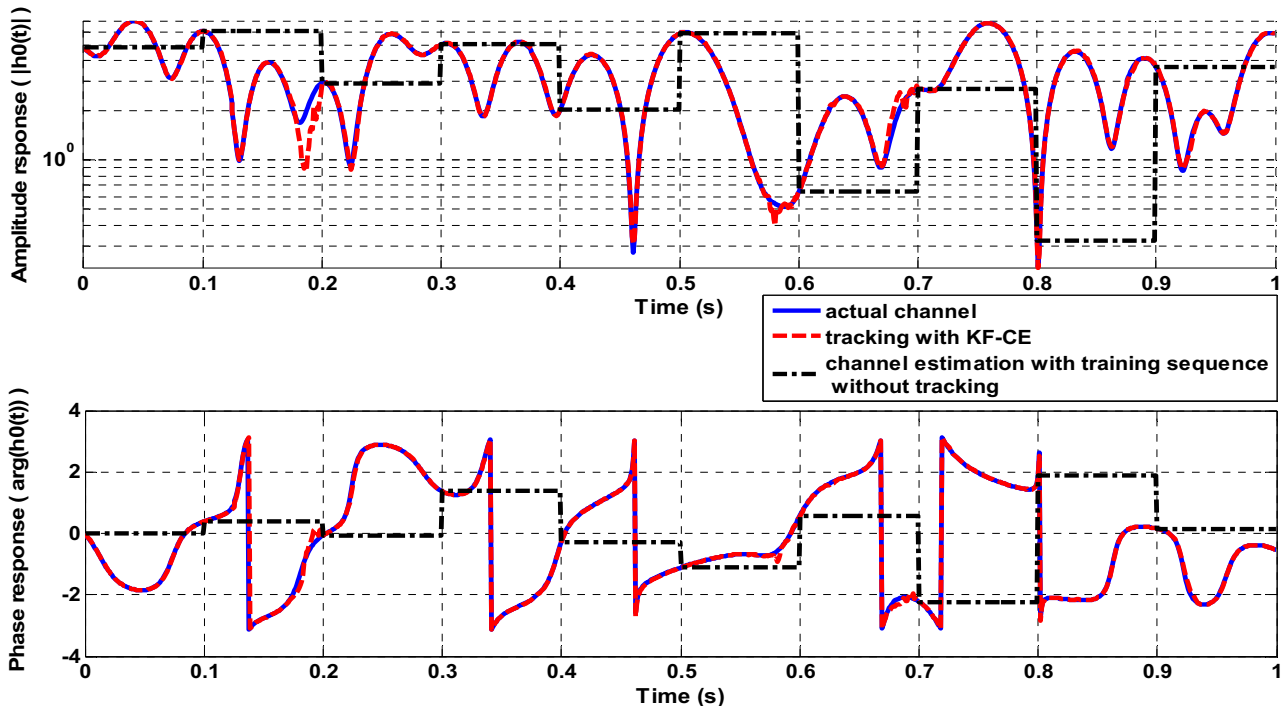


Fig. 3 Time-domain Channel tracking for a typical Doppler faded channel (SNR=15 dB)

N modulated symbols per N_T OFDM symbols compared with $N_T * (L_1 + L_2 + 2)$ ones in pilot based training [4]. For example if you consider $N_T = N$ (although, usually $N_T > N$) and $L_1 = L_2 = 2$, bandwidth efficiency in our proposed method is three times more than [4], and if $L_1 = L_2 = 5$ (as we assumed in our simulations), here bandwidth efficiency is six times more than [4].

Note that the assumption of the constant channel between two consecutive OFDM blocks is a reasonable assumption. However if it does not hold, the primary channel estimate can be done in other ways like what was proposed in [23].

4 Computer Results and Discussions

This section represents the simulation results, with the use of our proposed approach of channel estimation and tracking algorithm in time-domain for an OFDM-PLNC system.

Channels from both n_j 's to R are assumed to have five taps, each presented by a symmetric complex Gaussian random process. The Doppler spectrum is Jake's and $\sigma_{v_r}^2 = \sigma_v^2 = 1 - (J_0(2\pi f_D T_s))^2$, where f_D denotes the Doppler frequency between moving transmitter and receiver, and $f_s = \frac{1}{T_s}$ is the rate of sampling. For example, if the normalized considered fading rate is $f_D T_s = 0.01$ (an example of fading rate for fast fading channel), the noise variances of the model are considered as $\sigma_{v_r}^2 = \sigma_v^2 = 1.972 \times 10^{-3}$, according to [18], [20]. We assumed that there is no frequency offset between transmitters and channels are assumed constant during one OFDM symbol. We always fix $P_1 = P_2 = P_r$ (the average transmission

power of n_1 , n_2 and R) and define SNR as P_1/σ_n^2 . Number of subcarriers, following IEEE 802.11a, was set as $N = 64$, and the length of CP is $L_{CP} = 16$ samples. QPSK symbol modulation is employed and Re-training symbols are sent repeatedly every $N_T = 100$ OFDM symbols in our simulations. No channel coding was considered.

Monte Carlo simulations over 10^4 runs are performed for averaging and MSE figure of merit was used which is defined as;

$$MSE = \frac{1}{5 \times 10^4} \sum_{k=1}^{10^4} \|\hat{\mathbf{h}}^{(k)} - \mathbf{h}^{(k)}\|^2 \quad (12)$$

where $\hat{\mathbf{h}}^{(k)}$ is the k^{th} estimation of $\mathbf{h}^{(k)}$, and 5 in denominator is just for scaling and refer to the number of channel taps.

At first, the performance of proposed method is studied in the time domain for tracking the amplitude and phase impulse response of the channel. Magnitude and phase tracking of h_0 (the first tap of) is shown in Fig. 3, in the case of $f_D T_s = 0.01$ and at average SNR 15dB. Channel estimation only based on training sequence (without KF-CE tracking) is also illustrated. This figure shows that the time variations of channel taps are traced with precision, even in low instantaneous SNR.

Fig. 4 illustrates the average MSE of \mathbf{h} and \mathbf{g} at the relay. As two references curves, this figure also shows the average MSE of \mathbf{h} and \mathbf{g} at the relay obtained with the perfect knowledge of data (\mathbf{b}_1 and \mathbf{b}_2), and the other obtained with channel estimation only based on training sequence and without KF-CE tracking. Besides, the average MSE of the relevant downlink channel at node \mathbf{n}_1 is also shown in this figure. Note that the channel estimation at the relay (uplink channels estimation) is

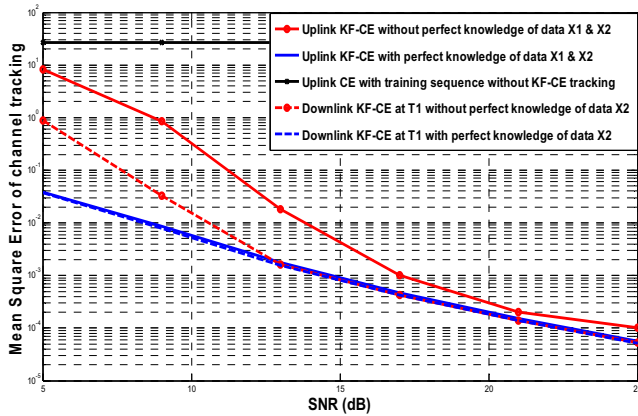


Fig. 4 The average MSE of the uplink channels estimation at the relay and downlink channel estimation at node n_1

done based on Eq. (1), estimating of \mathbf{h} and \mathbf{g} after receiving the superimposed signal $\mathbf{r}^{(k)}$. However, the estimation at the end nodes is done based on Eq. (2) which is in fact, similar to a channel estimation in point to point communications. In the uplink phase the estimation error of both \mathbf{b}_1 and \mathbf{b}_2 effects on the channel estimation error, while in the downlink phase, at node n_1 for example, only the estimation error of \mathbf{b}_2 is effective. For this reason, the average MSE of uplink channels estimation is more than the downlink channel, as Fig. 4 illustrates. It is seen in this figure the MSE of uplink and downlink channels estimation are nearly the same under the perfect knowledge of data consideration.

Since data detection is carried in the frequency domain, hence the accuracy of estimating the channel

frequency responses in the frequency of sub-carriers is also needed to be evaluated. Fig. 5 shows amplitude and phase responses, for the actual channel and its estimation, at a given OFDM block time. As it is seen, channel transfer functions are also estimated accurately, in both amplitude and phase.

The average BER of $\mathbf{b}_1, \mathbf{b}_2$ at the relay, as well as, BER of \mathbf{b}_2 at node n_1 (end to end BER), are also shown in Fig. 6.

In Fig. 7 we show the average MSE of tracking for different receiver types as the channel dispersion increases. The signal-to-noise ratio of the channel is considered to be 15 dB in this figure. For small normalized Doppler rate (< 0.001), the channel is almost constant between tandem training symbols. However, by increasing the Doppler rate, the dispersive nature of the rises and tracking the channel becomes more difficult. As you can see in this figure, the overall MSE of the downlink channel is lower than the uplink channels. This is because only one channel in the downlink direction is required to estimate, but the relay estimates two uplink channels at the same time in the uplink phase. As it is seen, all these figures predict a very good performance of the proposed algorithm in a time-varying frequency selective fading channel conditions.

5 Conclusion

In this paper, we proposed a time varying frequency fading channel tracking algorithm for OFDM-PLNC system, where, two users exchange their information

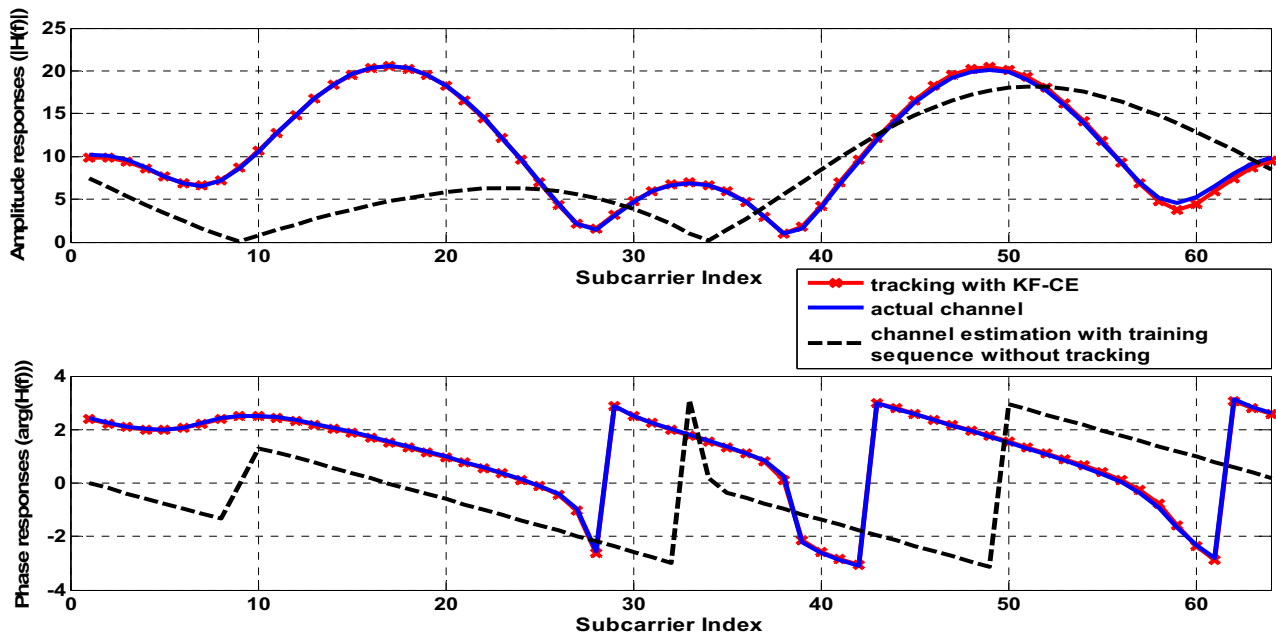


Fig. 5 Frequency-domain Channel tracking for a typical Doppler faded channel (SNR=5 dB, $t=60(s)$)

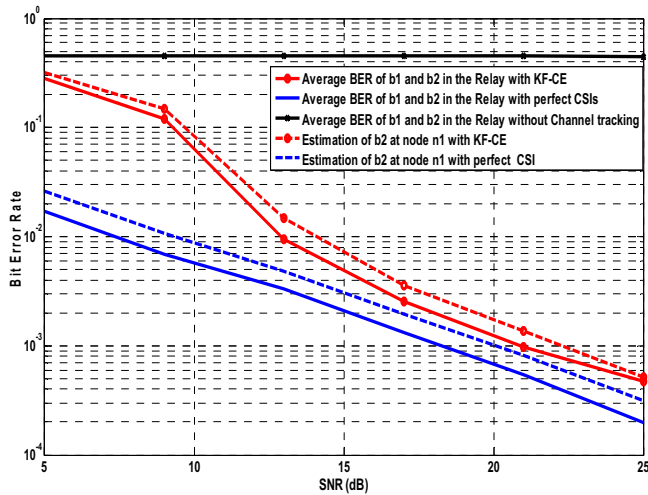


Fig. 6 The average BER of \mathbf{b}_1 , \mathbf{b}_2 at the relay, as well as, BER of \mathbf{b}_2 at node \mathbf{n}_1 (with and without channel state information)

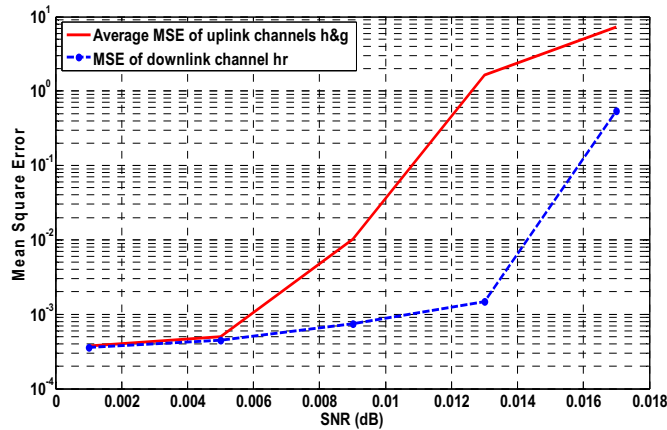


Fig. 7 Mean squared error tracking performance (averaged over all channels) vs. normalized Doppler rate.

through a relay using the decode-and-forward relaying scheme. A Kalman Filter Algorithm working in time domain is the base of this approach where the initial condition needed by KF, comes from a simple and linear precoding algorithm implemented over every N_T OFDM symbols. Accuracy, no phase ambiguity, and stability of this method in fast fading channels are its advantages, in comparison with other tracking algorithms running in frequency domain. Furthermore, our method uses less training symbols and gives much more throughput in comparison with other approaches that needs pilot insertion over different OFDM symbols in a frame. The simulation results, illustrate a very good performance. In this paper, equations are written assuming the same fading rate for uplink channels (\mathbf{h} , \mathbf{g}). This could be an inspiring direction for future work to extend KF-CE algorithm to the Generalized KF-CE that

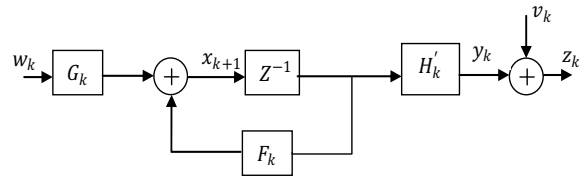


Fig. A1 Basic signal model

is able to track channels with different fading rates in a PLNC system.

Appendix

The Kalman filter is an efficient recursive filter that estimates the internal state of a linear dynamic system discretized in the time domain from a series of noisy measurements. A general model for such a system is depicted in Fig. A1, and described for $\mathbf{k} > \mathbf{0}$ by the following equations:

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \quad (\text{A1})$$

$$\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{G}_k \mathbf{w}_k \quad (\text{A2})$$

For the linear, finite dimensional, discrete-time system of (A1) and (A2) defined for $\mathbf{k} > \mathbf{0}$, suppose that \mathbf{v}_k and \mathbf{w}_k are independent, zero mean, Gaussian White processes with

$$\mathbf{E}[\mathbf{v}_k \mathbf{v}_k'] = \mathbf{R}_k \delta_k \quad \mathbf{E}[\mathbf{w}_k \mathbf{w}_k'] = \mathbf{Q}_k \delta_k \quad (\text{A3})$$

Suppose further that the initial state \mathbf{x}_0 is a Gaussian random variable with mean $\bar{\mathbf{x}}_0$ and covariance \mathbf{P}_0 , independent of \mathbf{v}_k and \mathbf{w}_k . Determine the estimates

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{E}[\mathbf{x}_k | \mathbf{Z}_{k-1}] \quad \hat{\mathbf{x}}_{k|k} = \mathbf{E}[\mathbf{x}_k | \mathbf{Z}_k] \quad (\text{A4})$$

and the associated error covariance matrix $\Sigma_{k|k-1}$ and $\Sigma_{k|k}$, where $\mathbf{Z}_{k-1} = \{\mathbf{z}_0, \mathbf{z}_1, \dots, \mathbf{z}_{k-1}\}$ and

$$\Sigma_{k|k-1} = \mathbf{E}\{[\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}][\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}]' | \mathbf{Z}_{k-1}\} \quad (\text{A5})$$

shows how good the estimate $\hat{\mathbf{x}}_{k|k-1}$ is.

The Kalman Filter comprises the system depicted in Fig. A2 and is described for $\mathbf{k} > \mathbf{0}$ by the following equations. The proof of these equations can be seen in [22].

$$\hat{\mathbf{x}}_{k|k-1} = [\mathbf{F}_k - \mathbf{K}_k \mathbf{H}_k'] \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{z}_k \quad (\text{A6})$$

With

$$\hat{\mathbf{x}}_{0|-1} = \bar{\mathbf{x}}_0 \quad (\text{A7})$$

The gain matrix \mathbf{K}_k is determined from the error covariance matrix by

$$\mathbf{K}_k = \mathbf{F}_k \Sigma_{k|k-1} \mathbf{H}_k' [\mathbf{H}_k' \Sigma_{k|k-1} \mathbf{H}_k + \mathbf{R}_k]^{-1} \quad (\text{A8})$$

Assuming the inverse exists, and the conditional error covariance matrix is given recursively by

$$\Sigma_{k+1|k} = \mathbf{F}_k [\Sigma_{k|k-1} - \Sigma_{k|k-1} \mathbf{H}_k' (\mathbf{H}_k' \Sigma_{k|k-1} \mathbf{H}_k + \mathbf{R}_k)^{-1} \mathbf{H}_k' \Sigma_{k|k-1}] \mathbf{F}_k' + \mathbf{G}_k \mathbf{Q}_k \mathbf{G}_k' \quad (\text{A9})$$

This equation is initialized by

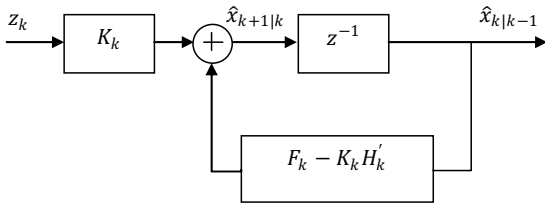


Fig. A2 Structure of Kalman Filter

$$\Sigma_{0|-1} = \mathbf{P}_0 \quad (\text{A10})$$

One obtains $\hat{\mathbf{x}}_{k|k}$ and $\Sigma_{k|k}$ as follows:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \Sigma_{k|k-1} \mathbf{H}_k' (\mathbf{H}_k' \Sigma_{k|k-1} \mathbf{H}_k + \mathbf{R}_k)^{-1} (\mathbf{z}_k - \mathbf{H}_k' \hat{\mathbf{x}}_{k|k-1}) \quad (\text{A11})$$

$$\Sigma_{k|k} = \Sigma_{k|k-1} - \Sigma_{k|k-1} \mathbf{H}_k' (\mathbf{H}_k' \Sigma_{k|k-1} \mathbf{H}_k + \mathbf{R}_k)^{-1} \mathbf{H}_k' \Sigma_{k|k-1} \quad (\text{A12})$$

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