1. Introduction

At design and exploitation of radar systems, the problem of signal detection from the moving target on the clutter background invariably remains the one of the most important, relevant, and difficult problems. The passive interference (clutter) in the form of undesirable reflections from fixed or slowly moved objects: the local objects, dry land or sea surfaces, the hydrometeors (clouds, rain, hail, snow) and metal reflectors dropping for target masking (so-called, chaff) essentially destroy the normal operation of radar systems of various purposes [1, 2]. Clutter intensity may significantly exceed the level of the proper receiver noises that leads to overload of the reception section (radar “blindness”) and, as a consequence, to useful signal missing. Nevertheless, even at overload absence, the useful signal can be lost or not detected at all, on the background of intensive undesirable interference. The main processing operation for received data is the rejection of the interference spectral components at extraction the signals from the moving targets on the background of the received clutter. As the effectiveness criterion, we consider the averaged (over the Doppler signal phase shift) improvement coefficient for a signal-to-clutter ratio (SCR). On the base of extreme properties of the characteristic numbers (eigen-numbers) of the matrices, the optimal vector (according to this criterion maximum) is defined as the eigenvector of the clutter correlation matrix corresponding to its minimal eigenvalue. The general type of the vector of optimal ARF weighting coefficients is determined and specific adaptive algorithms depending upon the ARF order are obtained, which in the specific cases can be reduced to the known algorithms confirming its authenticity. The comparative analysis of the synthesized and known algorithms is performed. Significant benefits are established in clutter rejection effectiveness by the offered processing algorithms compared to the known processing algorithms.

Abstract: In this paper, the algorithms for adaptive rejection of a radar clutter are synthesized for the case of a priori unknown spectral-correlation characteristics at wobbulation of a repetition period of the radar signal. The synthesis of algorithms for the non-recursive adaptive rejection filter (ARF) of a given order is reduced to determination of the vector of weighting coefficients, which realizes the best effectiveness index for radar signal extraction from the moving targets on the background of the received clutter. As the effectiveness criterion, we consider the averaged (over the Doppler signal phase shift) improvement coefficient for a signal-to-clutter ratio (SCR). On the base of extreme properties of the characteristic numbers (eigen-numbers) of the matrices, the optimal vector (according to this criterion maximum) is defined as the eigenvector of the clutter correlation matrix corresponding to its minimal eigenvalue. The general type of the vector of optimal ARF weighting coefficients is determined and specific adaptive algorithms depending upon the ARF order are obtained, which in the specific cases can be reduced to the known algorithms confirming its authenticity. The comparative analysis of the synthesized and known algorithms is performed. Significant benefits are established in clutter rejection effectiveness by the offered processing algorithms compared to the known processing algorithms.

Keywords: Adaptation, Rejection Algorithms, Wobbulation of Repetition Period, Clutter.
2. Description of radar wobbled sequence

At repetition period wobbulation, the sequence of processed samples forms the wob-bulation core consisting of \( p \) non-equidistant or non-equal time intervals \( T_1, T_2, \ldots, T_p \) and repeating with a constant period \( T_w \), which is called the wobulation period [1]:

\[
T_w = \sum_{i=1}^{p} T_i .
\]

As a rule, variation of inter-pulse intervals within the limit of wobulation core is performed in accordance with a some law, which is called the wobulation law and can be represented via variation of the repetition period with regard to the minimal period \( T_{\text{min}} \) by the value multiple to some time discrete value

\[
\Delta T = T_{\text{min}} / M ,
\]

where \( M \) is a integer scale multiplier.

In particular, at the linear law, the consequent variation of the \( T_i \) interval occurs from a pulse to the next by the value \( \Delta T \) according to a rule

\[
T_i = T_{\text{min}} + (i - 1) \Delta T , \quad i = 1, \ldots, p .
\]

The cross law can be obtained as follows:

\[
T_{2i-1} = T_{\text{min}} + (i - 1) \Delta T , \\
T_{2i} = T_{\text{max}} - (i - 1) \Delta T , \quad i = 1, \ldots, p/2 ,
\]

where the maximal repetition period within the limits of wobulation core is

\[
T_{\text{max}} = T_{\text{min}} + (p - 1) \Delta T .
\]

RF characteristics essentially depends on the choice of quantities \( P, \Delta T \) (or \( M \)) and the wobulation law. With a growth of wobulation range for repetition period, we must expect improvement of RF velocity characteristics; however, from a point of view of radar problems under solution, it is necessary to impose the following limitations:

- The minimal period \( T_{\text{min}} \) must meet the condition of unambiguous determination of a given range;
- The maximal period \( T_{\text{max}} \) must satisfy the condition

\[
T_{\text{max}} \leq 1.5 T_{\text{min}}
\]

from energy considerations.

The depth or wobulation index is the one of the main quantitative index of the wobbled sequence

\[
\text{mod} = \left( \frac{T_{\text{max}}}{T_{\text{min}}} - 1 \right) \times 100\% .
\]

Taking into consideration a non-stationarity of \( (T_1, T_2, \ldots, T_p) \) intervals within the limits of the wobulation period \( T_w \), the interference properties is expedient to describe by the \( p \) correlation sub-matrices \( R_l \), which elements are equal to

\[
R_{jk}^{(l)} = \rho_{jk}^{(l)} \exp(i \psi_{jk}^{(l)}) ,
\]

where \( \rho_{jk}^{(l)} = \rho(t_{j-l} - t_{l-k}) \) are coefficients on the inter-period correlation, \( \psi_{jk}^{(l)} = \psi(t_{j-l} - t_{l-k}) \) are the Doppler phase shifts, \( j, k = 0, m, l = (m+1), (m+p) \), \( m \) is the RF order. Here and further, boundaries of \( l \) value variations are displaced by \( m \), to not to take into account an effect of the transient in the filter.

3. The adaptation Criterion and the Algorithm Synthesis

The synthesis of required algorithms for the non-recursive adaptive rejection filter (ARF) of the given order consists in determination of the weighting coefficients vector \( G = \{G_k\}, \ k = 0, m \), which realizes the best effectiveness for extraction of moving target signals on the background of the received clutter. As the effectiveness criterion, we consider the \( \mu \) coefficient of signal-to-clutter ratio (SCR) improvement [5], which is averaged over the Doppler signal phase shift. Then, during RF adaptation process, in general case, the \( G \) vector realizing the maximum value of \( \mu \) should be formed. And, at repetition period wobulation during RF adaptation, for each from \( p \) correlation sub-matrices \( R_l \) of the clutter, the own optimal sub-vector of weighting coefficients should be formed. Taking this into consideration, the above-mentioned criterion can be written as

\[
\mu_l = \max (G_l^{(l)} G_l / G_l^{(l)*} R_l G_l) , \quad l = (m+1), (m+p) ,
\]

where symbols ‘*’ and ‘\( \cdot \)’ designate a complex conjugation and a transposition.

On the base of extreme properties of the matrix eigenval-
uses α, the optimal vector $\mathbf{G}_l$ is determined as the matrix eigenvector $\mathbf{R}_l$ [6] corresponding to its minimal eigenvalue $\alpha_{\text{min}}^{(l)}$ from the following matrix equation:

$$ (\mathbf{R}_l - \alpha_{\text{min}}^{(l)} \mathbf{I}) \mathbf{G}_l = 0, $$

where $\mathbf{I}$ is the unitary matrix, and the value $\alpha_{\text{min}}^{(l)}$ is the least root of the characteristic equation

$$ \det(\mathbf{R}_l - \alpha \mathbf{I}) = 0. $$

Overcoming of a priori uncertainty of interference parameters is based on the adaptive Bayesian approach [7] according to which the unknown correlation sub-matrices $\mathbf{R}_l$ of interference are replaced by their estimation values $\hat{\mathbf{R}}_l$.

In the general case the equation (9) is satisfied by the vector of ARF optimal weight-coefficients, which can be represented as

$$ \hat{\mathbf{G}}_l = \left\{ g_k^{(l)} \right\} = \left\{ g_k^{(l)} \exp \left( \sum_{i=1}^{k} \psi_{l-i} \right) \right\}, \quad k = 0, m. $$

where $g_k^{(l)}$ are coefficients determined by estimations $\hat{\rho}_{jk}^{(l)}$ and $\hat{\rho}^{(l)}_{\text{min}}$ in accordance with specific (depending on RF order $m$) adaptive algorithms; $\psi_l$ is an estimate of the Doppler phase shift of the clutter over the period $T_l$ (evidentlyly, $T_{l+np}=T_l$, $n=0, 1, 2, ...$).

To determine coefficients $g_k^{(l)}$, as it follows from (9), it is necessary to solve the system of $(m+1)$ autonomous linear equations with $(m+1)$ unknown variables

$$ \sum_{j=0}^{m} (\hat{\rho}_{jk}^{(l)} - \hat{\rho}^{(l)}_{\text{min}}) \delta_{jk} g_k^{(l)} = 0, \quad k = 0, m. $$

This system has the following solutions, which differ from the trivial (zero) one [8]:

$$ g_k^{(l)} = C \hat{A}_k^{(l)} \text{Row}_k, \quad k = 0, m. $$

where $C$ is the arbitrary constant; $\hat{A}_k^{(l)} \text{Row}_k$ is the algebraic compliment of the appropriate element in the determinant $\det(\hat{\rho}_{jk}^{(l)} - \hat{\rho}^{(l)}_{\text{min}}) \delta_{jk} 1$, in which the expansion line with the number is chosen so that at least one of the compliment $\hat{A}_k^{(l)} \text{Row}_k$ differs from zero; $\delta_{jk}$ is the Kronecker symbol.

Under the limiting condition $g_0^{(l)} = g_0 = 1$, we have for the optimal weighting coefficients

$$ g_k^{(l)} = \hat{A}_k^{(l)} \text{Row}_k, \quad k = 0, m, $$

where the value of expansion element (a column) $\text{Col}$ corresponds to a number of weighting coefficient, which is equalled to unit, i.e., in this case, $\text{Col} = 0$.

Solutions of equation system (12) obtained at different values of a number of expansion row $\text{Row}$ are identical on the final result (the $\mu_k$ value). Supposing for distinctness $\text{Row}=1$, we finally obtain for optimal weighting coefficients

$$ g_k^{(l)} = \hat{A}_k^{(l)} \text{Col}_k, \quad k = 0, m. $$

The specific view of adaptive algorithms from (15) should be found out in the following specific cases:

for $m=1$ $\Rightarrow g_0^1=1$, $g_2^2=1$;

for $m=2$ $\Rightarrow g_0^2=1$, calculation values of $\hat{A}_0^{(l)}, \hat{A}_1^{(l)}$ and $\hat{A}_2^{(l)}$, we obtain

$$ g_0^1 = -\frac{(1 - \hat{\rho}^{(l)}_{\text{min}})^2 - (\hat{\rho}^{(l)}_{02})^2}{(1 - \hat{\rho}^{(l)}_{\text{min}})^2 - (\hat{\rho}^{(l)}_{01})^2 - (\hat{\rho}^{(l)}_{12})^2},
$$

$$ g_2^2 = \frac{(1 - \hat{\rho}^{(l)}_{\text{min}})^2 \hat{\rho}^{(l)}_{12} - \hat{\rho}^{(l)}_{01} \hat{\rho}^{(l)}_{02}}{(1 - \hat{\rho}^{(l)}_{\text{min}})^2 - \hat{\rho}^{(l)}_{01} \hat{\rho}^{(l)}_{12} - \hat{\rho}^{(l)}_{02} \hat{\rho}^{(l)}_{23}}, $$

for $m=3$ $\Rightarrow g_0 = 1$,

$$ \hat{A}_1^0 = -(1 - \hat{\rho}^{(l)}_{\text{min}})^2 \hat{\rho}^{(l)}_{01} + (1 - \hat{\rho}^{(l)}_{\text{min}}) (\hat{\rho}^{(l)}_{02} \hat{\rho}^{(l)}_{12} + \hat{\rho}^{(l)}_{03} \hat{\rho}^{(l)}_{23}) + $$

$$ + \hat{\rho}^{(l)}_{01} (\hat{\rho}^{(l)}_{12})^2 - \hat{\rho}^{(l)}_{02} \hat{\rho}^{(l)}_{13} \hat{\rho}^{(l)}_{23} - \hat{\rho}^{(l)}_{03} \hat{\rho}^{(l)}_{12} \hat{\rho}^{(l)}_{23}, $$

$$ \hat{A}_4^1 = (1 - \hat{\rho}^{(l)}_{\text{min}})^3 - (1 - \hat{\rho}^{(l)}_{\text{min}}) (\hat{\rho}^{(l)}_{02} \hat{\rho}^{(l)}_{12} + \hat{\rho}^{(l)}_{03} \hat{\rho}^{(l)}_{23}) + $$

$$ + 2 \hat{\rho}^{(l)}_{02} \hat{\rho}^{(l)}_{03} \hat{\rho}^{(l)}_{23}, $$

$$ \hat{A}_2^1 = -(1 - \hat{\rho}^{(l)}_{\text{min}})^2 \hat{\rho}^{(l)}_{12} + (1 - \hat{\rho}^{(l)}_{\text{min}}) (\hat{\rho}^{(l)}_{01} \hat{\rho}^{(l)}_{02} + \hat{\rho}^{(l)}_{13} \hat{\rho}^{(l)}_{23}) + $$

$$ + \hat{\rho}^{(l)}_{12} (\hat{\rho}^{(l)}_{03})^2 - \hat{\rho}^{(l)}_{02} \hat{\rho}^{(l)}_{13} \hat{\rho}^{(l)}_{23} - \hat{\rho}^{(l)}_{01} \hat{\rho}^{(l)}_{12} \hat{\rho}^{(l)}_{23}, $$

$$ \hat{A}_3^2 = -(1 - \hat{\rho}^{(l)}_{\text{min}})^2 \hat{\rho}^{(l)}_{13} + (1 - \hat{\rho}^{(l)}_{\text{min}}) (\hat{\rho}^{(l)}_{01} \hat{\rho}^{(l)}_{03} + \hat{\rho}^{(l)}_{12} \hat{\rho}^{(l)}_{23}) + $$

$$ + \hat{\rho}^{(l)}_{13} (\hat{\rho}^{(l)}_{02})^2 - \hat{\rho}^{(l)}_{01} \hat{\rho}^{(l)}_{12} \hat{\rho}^{(l)}_{23} - \hat{\rho}^{(l)}_{02} \hat{\rho}^{(l)}_{12} \hat{\rho}^{(l)}_{23}. $$

(17)
For \( m=3 \), adaptive algorithms depend on estimates of six correlation coefficients and turn out as extremely bulky, therefore, in this case, it is expedient to build ARF in the form of cascade connection of chains of 1st and 2nd orders to simplify the adaptation procedures.

Now we note a connection of above-mentioned algorithms with the already known ones \([1, 5]\). With this goal, at \( m=2 \) for algorithms (16) we find the following limits:

\[
\lim_{\Delta f \to 0} (g_1^{(l)}) = -1 - \left( \frac{T_{l-1}}{T_{l-2}} \right), \quad \lim_{\Delta f \to 0} (g_2^{(l)}) = \frac{T_{l-1}}{T_{l-2}};
\]

\[
\lim_{\text{mod} \to 0} (g_1^{(l)}) = -\frac{2\rho_1^{(l)}}{1 - \rho_2^{(l)}}, \quad \lim_{\text{mod} \to 0} (g_2^{(l)}) = 1;
\]

where \( \Delta f \) is the spectral width of the interference.

In the case of strongly-correlated interference (i.e., at \( \rho_{jk} \to 1 \) or \( \Delta f \to 0 \)), algorithms (16) at arbitrary wobulation deepness, as it follows from (18), are completely coincided with known algorithms basing on the time-varying coefficients according to the wobulation law [1].

At \( \text{mod} \to 0 \), we obtain \( \rho_{jk}^{(l)} \approx \rho_1^{(l)} \approx \rho \), and algorithms (16), as it follows from (19), completely coincide with algorithms of [5]. In addition, at \( \rho \to 1 \) they transform to the classical algorithms with binomial weighting coefficients \( z_k = (-1)^k c_m^k \). The convergence of synthesized algorithms to known ones in the specific cases confirms their authenticity.

The ARF system (transfer) function in plain is defined as a superposition of \( p \) specific system functions by the following equation:

\[
H(z_l) = \frac{1}{p} \sum_{l=m+1}^{m+p} \sum_{k=0}^{m} g_k^{(l)} z_l^{-k} \exp \left( i \sum_{i=1}^{k} \phi_{i-l} \right),
\]

where \( z_l = \exp(i \alpha_{lf}) \).

In accordance with the given system function, we can synthesize the ARF structural circuit, which differs from described in [5] by a presence of an additional weighting unit corresponding to unit at the coefficient \( g_m^{(l)} \), by means of estimate delay of inter-period correlation coefficients and then calculating weighting coefficients according to adaptive algorithms (16), and by peculiarities of ARF units synchronization, taking into account the repetition period wobulation.

4. The Adaptive Algorithm Analysis

Let us analyze benefits achieved by offered algorithms and defining by expression

\[
\Delta \mu = \frac{\mu}{\mu_{kn}},
\]

where \( \mu \) is the improvement coefficient of ARF on the base of synthesized algorithms, \( \mu_{kn} \) is the improvement coefficient of ARF on the base of known algorithms.

\[
Fig. 1. \text{The improvement coefficient benefit vs. the clutter parameter}
\]
At eight-multiple \( (p=8) \) wobbulation of the repetition period and the wobbulation deepness \( \text{mod}=25\% \), Figure 1 shows functions of benefit \( \Delta \mu \) in ARF improvement coefficient \( \mu \) vs. normalized spectral width of interference \( \beta \) basing on non-adaptive time-varying algorithms compared with RF basing on non-adaptive time-varying algorithms [1]. From curves we can see that offered adaptation to clutter correlations characteristics at wobbulation of the repetition period allows obtaining essential benefits in clutter rejection effectiveness, which achieve 1.8 and 4.2 dB, relatively, at ARF order \( m=2 \) and 3.

Figure 2 shows the curves at \( m=3 \) and \( \beta=0.05 \) illustrating the effect of wobbulation parameters on the benefit in ARF effectiveness basing on offered algorithms compared to ARF with stationary algorithms [5]. We see that at variation of wobbulation deepness within the limits of \( \text{mod}=50\% \) (depending on the wobbulation parameters), significant (up to 15 dB) are achieved, which correspond to the wobbulation cross-law. At the linear wobbulation law the similar benefit value does not exceed 5 dB.

These benefits can be explained as follows. In ARF with stationary algorithms the variation on an inter-pulse interval leads to deformation of ARF velocity characteristics in the rejection zone, which essentially decreases the clutter rejection effectiveness. The processing algorithm offered here combines adaptation to clutter correlation characteristics and variation in time of the weighting vector due to which additional benefits may be achieved in the clutter rejection effectiveness.

The analysis of ARF velocity characteristics on the base of offered algorithms shows that at \( m < p \), together with maximization of the clutter rejection effectiveness, application of adaptive algorithms (16) and (17) for calculation of weighting coefficients gives the smoothed enough ARF characteristic in the pass-band at any placing of repetition periods in the group. In addition, ARF of \( m = p \)-order is invariant according to chosen effectiveness criterion (8) with respect to various placing of repetition periods within the limits of the wobbulation pe-riod. This allows elimination of contradiction between requirements to increase the effectiveness of clutter rejection and to improve uniformity of the velocity characteristic at choice of repetition periods alteration in the group.

5. Conclusions

Algorithms synthesized in suggested paper, under conditions of a priori ambiguity of the spectral-correlation clutter characteristics, allow adaptation to an argument and a modulus of the relevant correlation function without approximation of its shape, taking into account wobbulation laws and parameters for repetition periods.

Application of synthesized algorithms allows obtaining essential benefits (compared to known algorithms of radar signal processing) in effectiveness of signal extraction of moving targets on the clutter background under conditions of a priori uncertainty of interference characteristics at repetition period wobbulation allowing elimination of blend target velocities. Achievement of this benefit is caused by combination of processing algorithm adaptation to unknown interference parameters with the time adjustment in accordance with the wobbulation law.

Application of algorithms obtained at ARF design allows effectiveness increase of radar signal extraction from moving targets in much wider range of their radial velocities on the clutter background with known spectral-correlation properties. At that, the complication of the hardware-software implementation of ARF and growth of processing time of arrived data are required in accordance with more complicated offered algorithms compared to known ones.

References


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