# How Does Pricing of Day-ahead Electricity Market Affect Put Option Pricing?

H. Raouf Sheybani* and M. Oloomi Buygi**

**Abstract:** In this paper, impacts of day-ahead market pricing on behavior of producers and consumers in option and day-ahead markets and on option pricing are studied. To this end, two comprehensive equilibrium models for joint put option and day-ahead markets under pay-as-bid and uniform pricing in day-ahead market are presented, respectively. Interaction between put option and day-ahead markets, uncertainty in fuel price, day-ahead market pricing, and elasticity of consumers to strike price, premium price, and day-ahead price are taken into account in these models. By applying the presented models to a test system impact of day-ahead market pricing on equilibrium of joint put option and day-ahead markets are studied.

**Keywords:** Equilibrium of joint put option and day-ahead markets, Option market modeling, Uniform pricing, Pay-as-bid pricing, Put option pricing, Supply function competition.

## 1 Introduction

Option contracts are used by electric power producers in order to hedge themselves against quantity and price risks in physical electricity market [1], [2]. Standard option contract has a specified strike price, a specified delivery period, and a specific Mega Watt size [3]. Option market operators determine strike prices and delivery periods of standard option contracts [4], [5]. An option trader chooses the desired option contract based on the desired delivery period and the desired strike price, and then offers the required MW size and a suitable premium price to buy or sell it. If bids of a seller and a buyer are matched, the deal is done [4].

Option market operators usually compute a price for the option premium based on historical data of option market, which is called option price, and it is announced to market participants in order to help them having a reference for option bidding [6].

Two different energy pricing methods such as uniform and pay-as-bid pricing are used in day-ahead markets [7]. Although uniform pricing is widely used in day-ahead markets, pay-as-bid pricing is used in some electricity markets such as Iranian electricity market. Different methods such as Black-Scholes and Binomial tree models are used for option pricing [8]. Although option and day-ahead markets are linked together, the option pricing models do not consider the impacts of day-ahead market pricing on the option pricing [6], [8].

Impacts of option contracts on the bidding strategies of physical market participants are studied in [9]–[14]. To hedge risk-averse producers and consumers against price risks, an optimal strategy for selecting optional forward contracts is presented in [9]. Optimal bidding strategy of a load serving entity for buying forward and option contracts are determined in [10] in order to hedge the load serving entity against quantity risks in a physical competitive market. An option market beside a physical electricity market is considered in [11] and an approach for calculating optimal strike price from the viewpoint of a market maker is proposed. In [12] and [13], a multi stage stochastic model is presented to determine the optimal strategies of a risk-averse producer in forward, option and pool markets considering price and generation availability risks. Reference [14] develops a stochastic optimization model for determining the bidding strategy of a producer in an energy call option auction. In this auction, bidders can offer both premium and strike prices.

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* The Author is with the Department of Engineering, Ferdowsi University of Mashhad, Vakilabad Highway, Mashhad, Iran.
E-mail: hani.raoof@gmail.com.
** The Authors are with the Department of Engineering, Ferdowsi University of Mashhad, Vakilabad Highway, Mashhad, Iran.
E-mail: m_oloomi@yahoo.com
Corresponding Author: Majid Oloomi Buygi.

The equilibrium of both physical and option markets are studied in [1], [15]–[18]. In [15], a new forward contract with bilateral options is introduced in order to hedge the price volatility risks of buyers and sellers in the physical market. In [16], a two period equilibrium model for financial and physical electricity markets is presented. In [16] strategic producers compete with their rivals by setting their supply functions in a spot market and by setting their generation power in a financial option market. In [17] effects of put and call option contracts on the strategies of producers on a physical market with Cournot competition is studied. Reference [18] evaluates prices of put and call Asian options using interest rate theory and day-ahead market equilibrium. In this approach demand is forecasted and electricity price variability is modeled by calibrating the volatility parameter as an input. The proposed approach is based on day-ahead market equilibrium, while the presented research work is based on the equilibrium of the joint day-ahead and option markets.

The main difference between this work and the available researches is as follows. This research work models option market in more detail and considers the impacts of day-ahead energy pricing on the behavior of day-ahead and option markets by computing the equilibrium of the joint option and day-ahead markets.

In this paper, impacts of day-ahead market pricing on behavior of market participants in option and day-ahead markets and on option pricing are studied from the viewpoint of market regulators. Bids of producers in the option and day-ahead markets are needed for this study. However, bids of producers are unknown and change in different situations in oligopoly markets. In order to overcome this problem and take into account the interaction of market participants, it is assumed that the understudy put option and day-ahead markets have reached to their Nash equilibrium [19-22].

The paper is organized as follows. In Section II two equilibrium model for a joint put option and day-ahead markets under uniform and pay-as-bid pricing in the day-ahead market are presented. By applying the presented models to a four producer power system, impacts of day-ahead market pricing on the equilibrium of joint option and day-ahead markets and on option pricing are studied in Section IV. Concluding remarks are provided in Section V.

## 2 Modeling Joint Option and Day-ahead Electricity Markets

Consider a power system with a physical day-ahead electricity market beside of a financial option market. It is assumed that fuel price changes over the time. Producers and consumers can hedge themselves against risks in quantity and price of trading electric energy by concluding derivative contracts in the option market. Put and call option contracts are two different derivative instruments that are used for hedging and are traded independently.

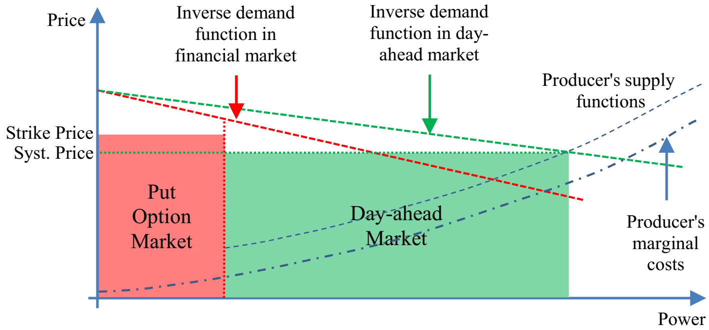


**Fig. 1** Different elasticity in the option market and day-ahead market with uniform pricing.

Option contracts are traded for a specific delivery period several months in advance. Hence, demand has a higher elasticity in put option markets in comparison to day-ahead markets, as shown in Fig. 1. In order to consumers sell put option, the strike price of put option contracts must be equal to or less than the inverse demand function of financial option market, as illustrated in Fig. 1.

In order to study the impacts of day-ahead market pricing on put option pricing, two cases are considered. In the first case uniform pricing and in the second case pay-as-bid pricing is considered for pricing electric energy. Then the equilibrium of the joint option and day-ahead markets are computed for the both cases. Finally, the impacts of day-ahead market pricing on put option pricing are assessed at the market equilibrium.

### 2.1 Assumptions and decision framework

The following assumptions are considered in this paper.

- The understudy financial instrument is an European put option with physical delivery. Hereafter, word option is used instead of put option for the sake of simplicity in this paper.
- Market operator determines one or a few strike prices for each standard put option, as it is in real option markets [4]-[5]. Therefore, strike prices are known and deterministic variables.
- Participants of put option market select the desired strike price and bid for premium price to buy or sell the desired option, as it is in real option markets [4]-[5].
- The understudy physical electricity market is an oligopoly day-ahead market with poolco structure, supply function competition, and uniform or pay-as-bid pricing.
- Fuel price changes over the time and is an uncertain variable.
- It is assumed that transmission network has no

constraints to avoid the impact of congestion on the simulation results.

- Load is elastic with constant elasticity both in the physical and financial electricity markets over delivery period. However, the elasticity of the load in option market is higher than its elasticity in day-ahead market.

Delivery period of an option contract usually consists of 24 hours or specified hours of a specified week, month, season, or year. Without loss of generality, it is assumed that delivery period consists of a few specified hours of several consecutive days. These hours are referred to as *study hours*. Hours of the delivery period are numerated as $t_j$ where $j = 1, 2, \ldots, T$.

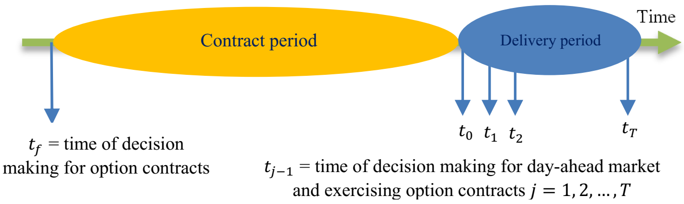


**Fig. 2** Timeline for decision-making by generating firms in option and day-ahead markets.

In order to model uncertainty in fuel price, possible scenarios for fuel prices are identified. Suppose $\Omega$ is the set of possible scenarios for fuel price. In the day-ahead market, load changes during the delivery period. Suppose inverse demand function in the day-ahead market at study hour $t$ of scenario $s$ of the delivery period is as follows.

$$\lambda_{st} = N_t^{Dh} - \gamma^{Dh} Q_{st}^{L} \qquad t = t_1, t_2, \ldots, t_T \;\; \forall s \in \Omega \tag{1}$$

where $\lambda_{st}$ and $Q_{st}^{L}$ are electricity price and total network load at hour $t$ of scenario $s$, respectively. Generation cost of producer $i$ at the study hour $t$ of scenario $s$ is as below.

$$C_i\left(Q_{ist}^{O} + Q_{ist}^{Dh}\right) = \rho_s \left(a_i(Q_{ist}^{O} + Q_{ist}^{Dh}) + \frac{1}{2} b_i (Q_{ist}^{O} + Q_{ist}^{Dh})^2\right) \tag{2}$$

where $\rho_s$ is the fuel price at scenario $s$ in \$/Mbtu, $Q_{ist}^{O}$ is the exercise volume of option contract of producer $i$ at hour $t$ of scenario $s$, $Q_{ist}^{Dh}$ is the day-ahead generation power of producer $i$ at hour $t$ of scenario $s$, and $a_i$ and $b_i$ are coefficients of the cost function of producer $i$ in Mbtu/MWh and Mbtu/MW$^2$h respectively.

It is assumed that each producer offers an affine supply function to independent system operator (ISO) as its bid at day-ahead market. The slope of bid of each producer is assumed to be equal to the slope of its marginal cost function [23]. Each producer determines the intercept of its bid function by maximizing its profit. Producer $i$ bids as follows for hour $t$ of scenario $s$ at day-ahead market.

$$bid_{ist} = \alpha_{ist} Q_{ist}^{Dh} + \frac{1}{2} \rho_s b_i {Q_{ist}^{Dh}}^2 \tag{3}$$

where $bid_{ist}$ and $\alpha_{ist}$ are the bid of producer $i$ and its intercept at hour $t$ of scenario $s$ in the day-ahead market respectively.

Consider a delivery period. Timeline for producers' decision-making in the option and day-ahead markets is shown in Fig. 2. Producers should make the following decisions optimally to maximize their profits over this delivery period:

1) Several months before starting the delivery period each producer should decide about the volume of the option contract that should buy from the option market for this delivery period. Suppose producer $i$ buys $Q_i^O$ MW option contract from the option market at contract time $t_f$ as it is shown in Fig. 2. It means that producer i have the right of selling power up to $Q_i^O$ MW to its option contract counterparty in delivery period.
2) One day before each day of the delivery period fuel price scenario is specified. Suppose scenario $s$ occurs. At this time producer $i$ should decide about its bid, i.e. $\alpha_{ist}$, for each hour $t$ of scenario $s$ at the day-ahead market.
3) One day before each days of the delivery period, producer $i$ should decide what portion of its option contract must be exercised at each study hour of the next day. It is assumed that producer $i$ exercises $Q_{ist}^{O}$ MW of its total option contract, i.e. $Q_i^O$, at hour $t$ of scenario $s$ of the delivery period. Here it is assumed that the exercised volume of option contracts is a continuous variable. In real world it may be a discrete variable with a small step size.

Participating in option market is not mandatory. Hence, producers can be categorized in two sets A and B. Set A consists of the producers that attend in both option and day-ahead markets. Set B consists of the producers that only attend in day-ahead market.

### 2.2 Case 1: Uniform Pricing

In this section, first the optimization problem for each producer in set A and B is modeled assuming uniform pricing for electric energy in the day-ahead market. Then, KKT optimality conditions for each producer in set A and B are extracted. Market Nash equilibrium is computed by solving the KKT conditions of all producers' optimization problems.

The optimization problem for producer $i$ of set A, who participates both in the option and day-ahead markets, is formulated as the following bi-level optimization.

$$\max_{Q^O_{ist},\alpha_{ist},Q^O_i,f_{iK}} E(\pi_i) = \sum_{s\epsilon\Omega}\sum_{t=t_0}^{t_T} p_s\Big(Q^O_{ist}K + Q^{Dh}_{ist}\lambda_{st} - \rho_s\Big(a_i\big(Q^O_{ist}+Q^{Dh}_{ist}\big) + \frac{1}{2}b_i\big(Q^O_{ist}+Q^{Dh}_{ist}\big)^2\Big)\Big) - Q^O_i T f_{iK} e^{rT_C} \quad (4)$$

$s.t.$:

$$(Q^O_{ist} - Q^O_i) \le 0 \qquad \forall s\in\Omega, \forall t\in\mathcal{T} : \omega_{ist} \quad (5)$$

$$K - f_{iK}e^{rT_C} \le N^O - \gamma^O \sum_{i\in A} Q^O_i \quad \forall s\in\Omega, \forall t\in\mathcal{T}: \beta_i \quad (6)$$

$$\max_{Q^L_{st},Q^{Dh}_{jst},Q^{Dh}_{lst}} SW_{st} = \int_{\sum_{j\in A}Q^O_{jst}}^{Q^L_{st}} (N^{Dh}_t - \gamma^{Dh}Q_d)\,dQ_d - \sum_{m\in A\cup B}\Big(\alpha_{mst}Q^{Dh}_{mst} + \frac{1}{2}\rho_s b_m {Q^{Dh}_{mst}}^2\Big) \qquad \forall s\in\Omega, \forall t\in\mathcal{T} \quad (7)$$

$s.t.$:

$$Q^L_{st} = \sum_{j\in A}\big(Q^O_{jst}+Q^{Dh}_{jst}\big) + \sum_{l\in B} Q^{Dh}_{lst} : \lambda_{st} \quad (8)$$

$$0 \le Q^{Dh}_{mst} \qquad \forall m\in A\cup B : \underline{\mu}_{mst} \quad (9)$$

$$Q^{Dh}_{jst} \le \overline{Q_j} - Q^O_{jst} \qquad \forall j\in A : \overline{\mu}_{jst} \quad (10)$$

$$Q^{Dh}_{lst} \le \overline{Q_l} \qquad \forall l\in B : \overline{\mu}_{lst} \quad (11)$$

$$Q^O_{ist} \ge 0, Q^O_i \ge 0, f_{iK} \ge 0 \quad \forall s\in\Omega, \forall t\in\mathcal{T} \quad (12)$$

where $E(\pi_i)$ is the expected profit of producer $i$ over delivery period, $SW_{st}$ is social welfare of day-ahead market at hour $t$ of scenario $s$, $\overline{Q_j}$ is the maximum generation capacity of producer $j$ in MW, $N^O$ is the intercept of inverse demand function in option market, $\gamma^O$ is the slope of inverse demand function in the option market, $\mathcal{T}$ is the set of study hours in delivery period, $K$ is strike price of the option contract in \$/MWh, $f_{iK}$ is the premium bid of producer $i$ at strike price $K$ in the option market in \$/MWh, $p_s$ is the probability of scenario $s$, $r$ is interest rate, $T_C$ is trading period in year or duration time between contract time and start of delivery period, $\omega_{ist}$ is the dual variable of upper capacity limit for exercising option contract of producer $i$ at study hour $t$ of scenario $s$, $\beta_i$ is the dual variable of consumer elasticity constraint in the option market, and $\underline{\mu}_{ist}$ and $\overline{\mu}_{ist}$ are the dual variables of lower and upper capacity limits of producer $i$ at study hour $t$ of scenario $s$ in day-ahead market respectively.

The first term of objective function (4) denotes the income of producer $i$ from the exercising option contracts at different hours of the delivery period. The second term of (4) denotes income of producer $i$ from the physical day-ahead market over the delivery period assuming uniform pricing. The sum of third to sixth terms of (4) which are located inside parenthesis indicates the total generation cost of producer $i$ over the delivery period. The last term of (4) denotes the cost of buying put option contract.

Decision-making about exercising option by producer $i$ at hour $t$ of scenario $s$ is modeled by maximizing $\big(Q^O_{ist}K + Q^{Dh}_{ist}\lambda_{st}\big)$ in the objective function, considering the fact that demand function is constant at hour $t$ of scenario $s$. If strike price $K$ is greater than day-ahead market price $\lambda_{st}$, the profit of producer $i$ at scenario $s$ is maximized if $Q^O_{ist}K$ is maximized, i.e., if $Q^O_{ist}$ is equal to $Q^O_i$, or if producer $i$ exercises its option contract. If strike price $K$ is smaller than $\lambda_{st}$, the profit of producer $i$ is maximized if $Q^{Dh}_{ist}\lambda_{st}$ is maximized, i.e., if $Q^O_{ist}$ is equal to zero or if the producer $i$ does not exercise its option contract.

Inequalities (5) impose the upper limit of producer $i$ for exercising of option contract at every hour of the delivery period. Constraints (6) model the elasticity of load in the option market. Constraints (7) to (13) model the ISO's optimization problem in the day-ahead market. In objective function (7), social welfare of the day-ahead market at hour $t$ of scenario $s$ is maximized. Equation (8) demonstrated the balance of generation and demand in option and day-ahead markets. Inequalities (9) to (11) enforce generation limits of producers at all hours of all scenarios over the delivery period.

In order to summarize the formulas, vectors $\boldsymbol{V}_{is}$ and $\boldsymbol{Y}_s$, and scalars $D_s$, $u_{st}$ and $w_s$ are defined as follows.

$$H_{is} = 1 + \gamma^{Dh}\sum_{(m\in A\cup B|m\neq i)}\Big(\frac{1}{\rho_s b_m}\Big) \qquad \forall i\in A\cup B, \forall s\in\Omega \quad (13)$$

$$D_s = \rho_s + \gamma^{Dh}\sum_{m\in A\cup B}\Big(\frac{1}{b_m}\Big) \qquad \forall s\in\Omega \quad (14)$$

$$V_{is}(m) = \begin{cases} -H_{is}/D_s & m = i \\ \gamma^{Dh}/(\rho_s b_m D_s) & m \neq i \end{cases} \qquad \forall m\&i\in A\cup B, \forall s\in\Omega \quad (15)$$

$$Y_s(m) = \gamma^{Dh}/(\rho_s b_m D_s) \qquad \forall m\in A\cup B, \forall s\in\Omega \quad (16)$$

$$u_{st} = N^{Dh}_t/D_s \qquad \forall s\in\Omega, \forall t\in\mathcal{T} \quad (17)$$

$$w_s = \gamma^{Dh}/D_s \qquad \forall s\in\Omega \quad (18)$$

where $\boldsymbol{\alpha_{st}}$ is a $n(A\cup B)\times 1$ vector which consists of intercepts of bids of all producers and $\boldsymbol{Q^O_{st}}$ is a $n(A)\times 1$ vector which consists of volume of option contracts of producers that attend in the option market.

The KKT conditions of each producer $i$ of set A are as below.

$$\frac{\partial\mathcal{L}_i}{\partial\alpha_{ist}} = -p_s\Bigg(\Big(\frac{\gamma^{Dh}+\rho_s b_i H_{is}}{{b_i}^2 D_s}\Big)\big(u_{st} + \boldsymbol{V}^T_{is}\boldsymbol{\alpha}_{st} - w_s\boldsymbol{1}^T\boldsymbol{Q}^O_{st}\big) - \Big(\frac{\rho_s H_{is}}{b_i D_s}\Big)\big(u_{st} + \boldsymbol{Y}^T_s\boldsymbol{\alpha}_{st} - w_s\boldsymbol{1}^T\boldsymbol{Q}^O_{st}\big) + \Big(\frac{\rho_s H_{is}}{b_i D_s}\Big)\big(a_i + b_i Q^O_{ist}\big)\Bigg) + \underline{\mu}_{ist} - \overline{\mu}_{ist} = 0 \qquad \forall t\in\tau, \forall s\in\Omega \quad (19)$$

$$\Bigg\{\frac{\partial\mathcal{L}_i}{\partial Q^O_{ist}} = -p_s\Bigg(K - \Big(\frac{\rho_s\gamma^{Dh}}{b_i D_s}\Big)\big(u_{st} + Y^T_s\alpha_{st} - w_s 1^T Q^O_{st}\big) -$$

$$\rho_s(u_{st} + V_{is}^T\alpha_{st} - w_s 1^T Q_{st}^O) + \left(\frac{\rho_s\gamma^{Dh} - \rho_s b_i D_s}{b_i D_s}\right)(a_i + b_i Q_{ist}^O)\Big) + \omega_{ist} + \underline{\mu}_{ist}\left(\frac{\gamma^{Dh}}{H_{is}}\right) + \overline{\mu}_{ist} b_i \rho_s\Big\} \geq 0 \perp Q_{ist}^O \geq 0 \quad \forall t \in \tau, \forall s \in \Omega \quad (20)$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial Q_i^O} = f_{iK} T e^{rT_C} - \sum_{s=1}^{S}\sum_{t=t_0}^{t_T}\omega_{ist} + \beta_i\left(K - f_{iK}e^{rT_C} - N^O + \gamma^O\sum_{j\in A}Q_j^O\right) + \beta_i\gamma^O\sum_{j\in A}Q_j^O\right\} \geq 0 \perp Q_i^O \geq 0 \quad (21)$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial f_{iK}} = Q_i^O T e^{rT_C} - \beta_i e^{rT_C}\sum_{j\in A}Q_j^O + \overline{\varphi}_{iK} - \underline{\varphi}_{iK}\right\} \geq 0 \perp f_{iK} \geq 0 \quad (22)$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial \omega_{ist}} = (Q_i^O - Q_{ist}^O)\right\} \geq 0 \perp \omega_{ist} \geq 0, \quad \forall t \in \tau, \forall s \in \Omega \quad (23)$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial \beta_i} = N^O - \gamma^O\sum_{j\in A}Q_j^O - K + f_{iK}e^{rT_C}\right\} \geq 0 \perp \beta_i \geq 0 \quad (24)$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial \underline{\mu}_{ist}} = \frac{1}{b_i}(u_{st} + \boldsymbol{V}_{is}^T\boldsymbol{\alpha}_{st} - w_s\boldsymbol{1}^T\boldsymbol{Q}_{st}^O)\right\} \geq 0 \perp \underline{\mu}_{ist} \geq 0 \quad \forall t \in \tau, \forall s \in \Omega \quad (25)$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial \overline{\mu}_{ist}} = \left(\overline{Q_i} - Q_{ist}^O - \frac{1}{b_i}(u_{st} + \boldsymbol{V}_{is}^T\boldsymbol{\alpha}_{st} - w_s\boldsymbol{1}^T\boldsymbol{Q}_{st}^O)\right)\right\} \geq 0 \perp \overline{\mu}_{ist} \geq 0 \quad \forall t \in \tau, \forall s \in \Omega \quad (26)$$

where $\mathcal{L}_i$ is the Lagrangian of the optimization problem of producer $i$ of set A.

Every producer $k$ of set B, participates only in the day-ahead market. Therefore, by ignoring (5), (6) and (12), and setting $Q_{ist}^O$ and $Q_i^O$ equal to zero in (4) and (10), the optimization problem of producer $k$ of set B is obtained. In the same way, the KKT conditions of each producer $k$ of set B can be extracted by omitting (20), (21), (22), (23), and (24) and by setting $Q_{ist}^O$ and $Q_i^O$ equal to zero in (19) and (25).

The maximization problem (4) to (12) is a non-concave optimization. The dual optimization problem of the optimization problem (4) to (12) can be written as follows:

$$Min\ g_i\left(\omega_{ist}, \beta_{it}, \underline{\mu}_{ist}, \overline{\mu}_{ist}\right) = \text{Min}\left(\sup_{Q_{ist}^O, \alpha_{ist}, Q_i^O, f_{iK}} \mathcal{L}_i\right)$$

$$s.t.: \quad \omega_{ist} \geq 0, \beta_{it} \geq 0, \underline{\mu}_{ist} \geq 0, \overline{\mu}_{ist} \geq 0 \ \ \forall s \in \Omega, \forall t \in \mathcal{T}$$

Since $\mathcal{L}_i$ is a second order function of variables $Q_i^O$, $Q_{ist}^O$, $f_{iK}$, and $\alpha_{ist}$, supremum of $\mathcal{L}_i$ can be computed by equating derivatives of $\mathcal{L}_i$ versus primal variables to zero, computing primal variables versus dual variables, and substituting them in $\mathcal{L}_i$. In the process of computing $g_i(.)$ no new constraint is added to the dual problem. Since $L_i$ is an affine function versus dual variables and $g_i(.)$ is equal to the supremum of some affine functions, the dual problem is a convex optimization problem [24]. The dual problem is convex minimization and all its constraints are affine and hence the dual problem is strong dual [24]. Therefore, the primal problem is strong dual and the KKT conditions of the primal problem guarantee the global maximum.

### 2.2 Case 2: Pay-as-Bid Pricing

In this section, it is supposed that the day-ahead market has a pay-as-bid pricing. By replacing term $\left(Q_{ist}^{Dh}\lambda_{st}\right)$ in (4) with $\left(\alpha_{ist}Q_{ist}^{Dh} + \frac{1}{2}\rho_s b_i {Q_{ist}^{Dh}}^2\right)$, optimization problem (4) to (12) for producer $i$ in set A is obtained assuming pay-as-bid pricing in day-ahead market. The KKT conditions of this producer can be rewritten by replacing equations (19) and (20) with equations (27) and (28), respectively.

$$\frac{\partial \mathcal{L}_i}{\partial \alpha_{ist}} = -p_s\left(\left(\frac{1}{b_i}\right)(u_{st} + \boldsymbol{V}_{is}^T\boldsymbol{\alpha}_{st} - w_s\boldsymbol{1}^T\boldsymbol{Q}_{st}^O) - \alpha_{ist}\left(\frac{H_{is}}{b_i D_s}\right) + \left(\frac{\rho_s H_{is}}{b_i D_s}\right)(a_i + b_i Q_{ist}^O)\right) + \underline{\mu}_{ist} - \overline{\mu}_{ist} = 0 \quad \forall t \in \tau, \forall s \in \Omega \quad (27)$$

$$\left\{\frac{\partial \mathcal{L}_i}{\partial Q_{ist}^O} = -p_s\left(K - \alpha_{ist}\left(\frac{\gamma^{Dh}}{b_i D_s}\right) + \rho_s\left(\frac{\gamma^{Dh}}{b_i D_s}\right)(a_i + b_i Q_{ist}^O)\right) + \omega_{ist} + \underline{\mu}_{ist}\left(\frac{\gamma^{Dh}}{H_{is}}\right) + \overline{\mu}_{ist}\rho_s b_i\right\} \geq 0 \perp Q_{ist}^O \geq \quad \forall t \in \tau, \forall s \in \Omega \quad (28)$$

The optimization problem and KKT conditions of producer $k$ of set B can be obtained as the same way that it was explained in Section II. B. It can be shown that the maximization problem of each producer in pay-as-bid case is a strong dual optimization as the same way that is described in Section II.B.

The equilibriums of the joint option and day-ahead markets for the both cases are calculated by solving the set of KKT conditions of optimization problems of all producers.

## 3 Case Study

In this section the proposed models are applied to a four-producer power system. Generators of producers 1 to 4 are the same as generators of areas 1 to 4 of IEEE 300-bus test system. The marginal cost function of each producer is computed by aggregating the marginal cost functions of his or her generators and fitting an affine function to it. Capacities of the producers and coefficients of their marginal cost functions are given in Table I.

Suppose producers 1 and 2 hedge themselves against price volatility of the day-ahead market by buying European put option one year before starting of delivery period, i.e. trading period is one year or $T_C = 1$.

**Table 1** Characteristics of producers.

| | Number of producer | Coefficients of marginal cost functions | | Generation capacity (GW) |
|---|---|---|---|---|
| | | $a_i(Mbtu/MWh)$ | $b_i(Mbtu/MW^2h)$ | |
| Set A | 1 | 0.4989 | 0.0002505 | 11.40 |
| | 2 | 1.2352 | 0.0001012 | 12.00 |
| Set B | 3 | 1.3005 | 0.0001211 | 8.721 |
| | 4 | 0.8829 | 0.0105000 | 0.558 |

Producers 3 and 4 only participate in day-head market, i.e. producers 1 and 2 are in set A and producers 3 and 4 are in set B as it is shown in Table I. Suppose at contract time $t_f$, twenty scenarios for fuel price over delivery period are identified. Fuel prices and their probabilities for different scenarios calculated using distribution of fuel price. It is assumed that distribution of fuel price is $\mathcal{N}(15,1.5)$ \$/Mbtu.

Suppose delivery period of the understudy option contracts consists of a single hour of ten consecutive days. It is assumed that although demand changes during the delivery period, the slope of demand function in the day-ahead market remains constant and equal to ($\gamma^{Dh} = \$-0.0002/MW^2h$). Demand in different hours of delivery period is specified with $N_t^{Dh}$ as it is given in Table II.

**Table 2** Intercept of day-ahead demand function at different hours of the delivery period.

| Day $t$ of delivery period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $N_t^{Dh}$ (\$/MWh) | 44 | 39 | 49 | 48 | 48.5 | 47 | 48 | 43 | 40 | 46 |

The intercept and the slope of inverse demand function in the option market is equal to $\$45/MWh$ and $\$-0.0004/MW^2h$, respectively. Simulation results show that by setting $K$ equal to zero, no option contracts are concluded in the equilibrium of joint option and day-ahead markets. In this situation, the minimum and maximum prices of day-ahead market over all scenarios and all hours of delivery period for the uniform and pay-as-bid cases are $\lambda_{unif}^{min} = \$32.5/\text{MWh}$, $\lambda_{unif}^{max} = \$46.8/\text{MWh}$, $\lambda_{PaB}^{min} = \$32.5/\text{MWh}$, and $\lambda_{PaB}^{max} = \$47.8/\text{MWh}$, respectively. $\lambda_{PaB}^{min}$ is energy price at the intersection of supply and demand functions in pay-as-bid pricing case. In order to consider strike prices greater than, equal to, and less than day-ahead market price in the study, which in the finance parlance are named *in the money*, *at the money*, and *out the money* strike prices respectively, it is assumed that the strike price varies from \$15/MWh to \$60/MWh with step size \$1/MWh. At each strike price, equilibrium of the joint option and day-ahead markets is calculated for the uniform and pay-as-bid cases considering fuel price uncertainty and load change in the delivery period. At each equilibrium point, premium price of option contracts, expected value of day-ahead market price, expected profit of each producer, and the expected value of total social welfare of the joint option and day-ahead markets are computed for the uniform and pay-as-bid cases and they are compared.

Based on simulation results, the optimal premium bids of the first and second producers are equal at the equilibrium of the joint option and day-ahead markets in both cases. In each case, if bid of a producer is a little smaller than the bid of another one, maximum possible contracts are concluded with this producer and the profit of the other producer decreases noticeably. Premium price at the equilibrium of the joint option and day-ahead markets are shown in Fig. 3 for different strike prices and for the both uniform and pay-as-bid cases.

In some option markets, a settlement premium price is computed for each day of trading period by the market regulator in order to use in mark-to-marketing process [4]. In financial markets, usually, settlement premium price of a day is equal to weighted average of premium prices of option contracts that are traded in that day or in a part of that day [4]. For each strike price, the related settlement premium price is used as an option price and is announced to option market participants as a quotation for the next day [4]. Hence, computing the settlement premium prices at equilibrium of the joint option and day-ahead markets can be considered as a method for option pricing. Since premium bids of the producers are equal at the equilibrium of the joint option and day-ahead markets in both cases, the settlement premium price is equal to optimal premium bids of producers in both cases. As Fig. 3 shows, at each specific strike price, optimal/settlement premium price of the pay-as-bid case is greater than or equal to optimal/settlement premium price of the uniform case.

Total volume of concluded option contracts of all producers for the uniform and pay-as-bid cases are shown in Fig. 4. In the uniform case, if strike price is less than $\lambda_{unif}^{min} + f_K e^{rT_C}$, concluding option contract is not profitable for producers and no option contract is concluded, where $f_K$ is the settlement premium price of option market at the equilibrium of joint option and day-ahead markets. As strike price exceeds $\lambda_{unif}^{min} + f_K e^{rT_C}$, concluding option contract gets profitable for producers and the volume of concluded option contracts increases rapidly as it is shown in Fig.4. But, since in the pay-as-bid case payment to each producer is equal to its bid that is smaller than or equal to system price, producers trade option contracts for strike prices even lower than $\lambda_{PaB}^{min} + f_K e^{rT_C}$, as it is shown in Fig. 4. In both uniform and pay-as-bid cases, as strike price increases the volume of concluded option contracts increases until it is restricted by the demand function.

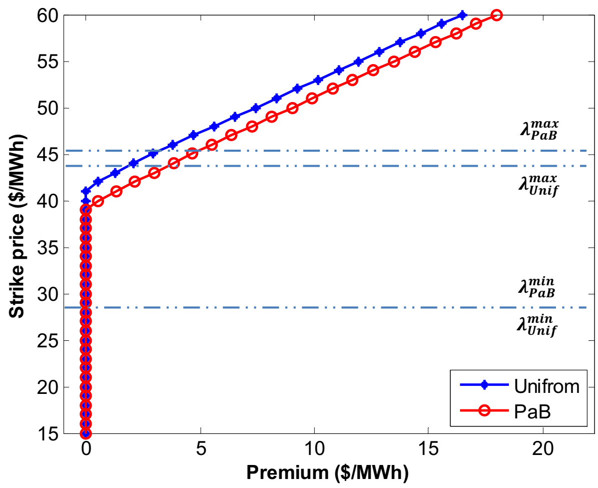


**Fig. 3** Settlement premium price in uniform and pay-as-bid cases.

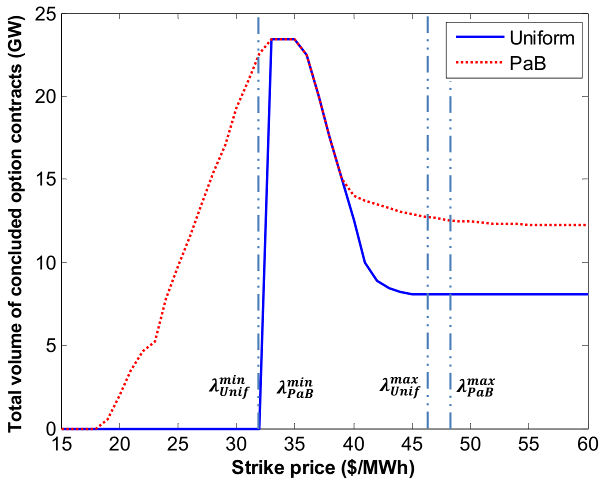


**Fig. 4** Total volume of concluded option contracts of all producers in uniform and pay-as-bid cases.

When strike price exceeds $35/MWh, the price of buying electricity from option market, i.e. $K - f_K e^{rT_C}$, reaches to maximum acceptable price of consumers, which is determined by demand function (6), and total volume of concluded option contracts decrease in both cases, as shown in Fig. 4.

Since option premium price of the pay-as-bid case is greater than option premium price of the uniform case, concluding option contract is more profitable for consumers in the pay-as-bid case in comparison to the uniform case. This is why at each premium price greater than $39/MWh total volume of concluded option contract in the pay-as-bid case is greater than the uniform case, as illustrated in Fig. 4. Note that low strike prices are profitable for consumers and hence the behavior of producers determines the concluded volume of option contracts, and high strike prices are profitable for producers and hence the behavior of consumers determines the concluded volume of option contracts.

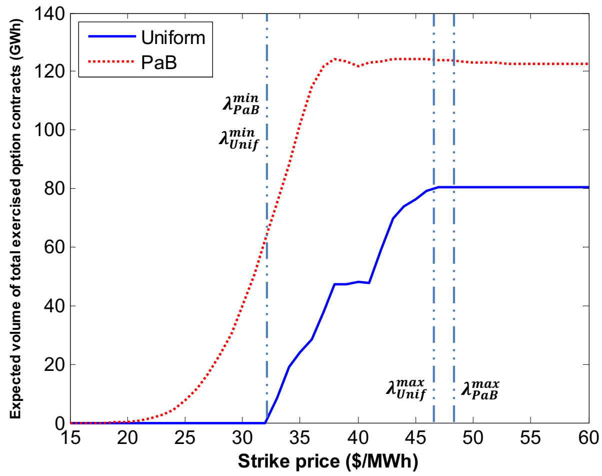


**Fig. 5** Expected volume of total exercised option contracts over delivery period in uniform and pay-as-bid cases.

Expected volumes of total exercised option contracts of all producers for the uniform and pay-as-bid cases over the delivery period are shown in Fig. 5. The expected is computed over all scenarios of the delivery period and total indicates summation over all hours of the delivery period and all producers.

As it is shown in Fig. 5, in the uniform case total volume of concluded option contract increases as strike price exceeds $\lambda_{Unif}^{min} + f_{iK} e^{rT_C}$ and it also get profitable for producers to exercise their option, and consequently the expected volume of total exercised option contracts increases. In the pay-as-bid case total volume of concluded option contract increases as strike price exceeds $bid_{PaB}^{min} + f_K e^{rT_C}$ and it also get profitable for some producers to exercise their option, and consequently the expected volume of total exercised option contracts increases, as it is shown in Fig. 5, where $bid_{PaB}^{min}$ is the minimum of bids of all producers that participate in option market over all scenarios in pay-as-bid case.

For strike prices greater than $\lambda_{Unif}^{min} + f_K e^{rT_C}$ in uniform case or $bid_{PaB}^{min} + f_K e^{rT_C}$ in pay-as-bid case, the total exercised volume of option contracts increase, however the concluded volume of option contracts decrease in both case due to load elasticity. The expected volume of total exercised option contracts in pay-as-bid case is greater than it in uniform case as shown in Fig. 5.

In both cases for strike prices greater than $\lambda_{Unif}^{max}$ or $\lambda_{PaB}^{max}$, constraint (6) gets actives and term $K - f_K e^{rT_C}$ in (6) remain constant. Thus, concluded and exercised volumes of option contracts remain constants, as shown in Fig. 4 and Fig. 5, respectively.

The expected value of day-ahead market price over all hours and scenarios for uniform and pay-as-bid cases are shown in Fig. 6.

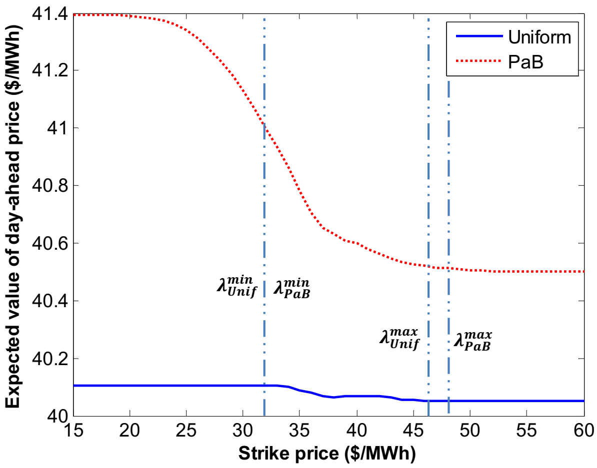


**Fig. 6** Expected value of day-ahead market price in uniform and pay-as-bid cases.

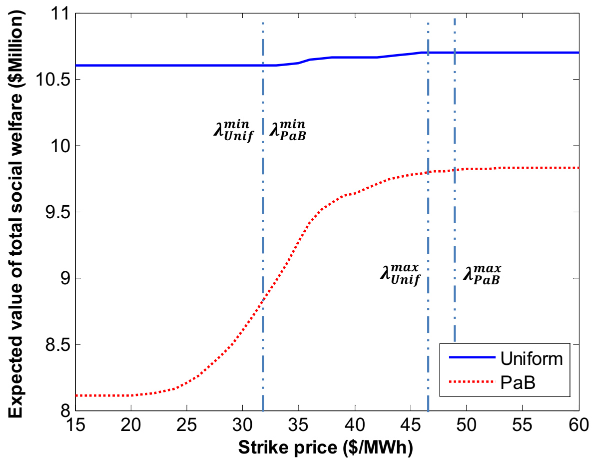


**Fig. 7** Expected value of total social welfare of the joint option and day-ahead markets in uniform and pay-as-bid cases.

If total exercised volume of option contracts for hour $t$ of scenario $s$ increases, residual demand for day-ahead market decreases and consequently, day-ahead market price decreases. Thus, the day-ahead market price decreases when option contracts are exercised in uniform and pay-as-bid cases, as shown in Fig. 6.

Based on Fig. 6, the day-ahead market price in pay-as-bid case is greater than the day-ahead market price in uniform case. Also, it is found from Fig. 6 that the day-ahead market price is more sensitive to strike prices in pay-as-bid case than uniform case.

Expected value of total social welfare of the joint option and day-ahead markets over delivery period is calculated as follow.

$$E(TSW_s) = \sum_s \sum_{t=t_0}^{t_T} p_s \left(\left[N_t Q_{st}^L - \frac{1}{2}\gamma {Q_{st}^L}^2\right] - \left(\sum_{j\in A} \rho_s \left[a_j\left(Q_{jst}^{Dh} + Q_{jst}^O\right) + \frac{1}{2} b_j \left(Q_{jst}^{Dh} + Q_{jst}^O\right)^2\right] + \sum_{l\in B} \rho_s \left[a_l Q_{lst}^{Dh} + \frac{1}{2} b_l {Q_{lst}^{Dh}}^2\right]\right)\right) \quad (29)$$

where $TSW_s$ is the total social welfare of both option and day-ahead markets in scenario $s$. Expected value of total social welfare at the equilibrium of the joint option and day-ahead markets is illustrated in Fig. 7 for uniform and pay-as-bid cases. If expected volume of total exercised option contracts for hour $t$ of scenario $s$ increases, day-ahead market price decreases, total consumption increases due to price elasticity of load, and consequently total social welfare at hour $t$ of scenario $s$ increases. Therefore, variations in expected value of total social welfare are in the same direction of variations in expected volume of total exercised option contracts, as it is seen in Fig. 5 and Fig. 7. Total social welfare in pay-as-bid case is highly dependent on strike prices in comparison to uniform case, as illustrated in Fig. 7.

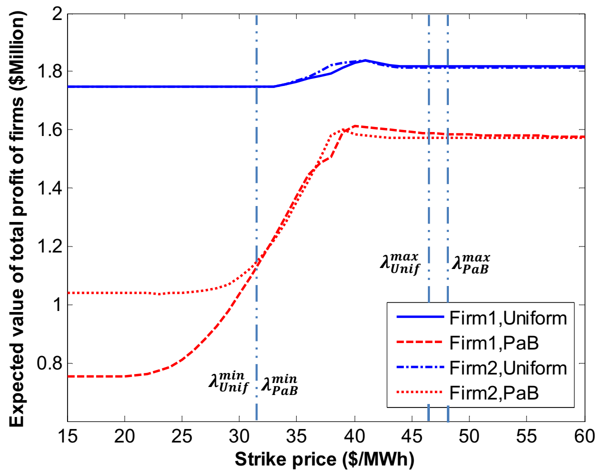


**Fig. 8** Expected value of total profit of the first and second producers in uniform and pay-as-bid cases.

Expected value of total profit of the first and second producers from both option and day-ahead markets are illustrated in Fig. 8 for uniform and pay-as-bid cases.

As it is shown in Fig. 8, expected value of total profit of the first and second producers in the uniform case increase as strike price exceeds $\lambda_{Unif}^{min} + f_K e^{rT_C}$, where strike price is enough high to encourage producers to buy put option and it is also less than maximum acceptable price of consumers and encourage them to sell put option. In uniform case, maximum expected value of total profit of each producer occurs between $\lambda_{Unif}^{min}$ and $\lambda_{Unif}^{max}$ at the highest strike price at which its optimal premium bid is still zero. In the pay-as-bid case, profit of the first and second producers start to increase from strike prices lower than $\lambda_{PAB}^{min} + f_K e^{rT_C}$, as illustrated in Fig. 8. Also, maximum expected value of total profit of each producer occurs between $\lambda_{PAB}^{min}$ and $\lambda_{PAB}^{max}$ in this case. Based on (6), as optimal premium bids of producers increase from zero, total cost of each producer increases and consequently expected value of total profit of each producer decreases in both cases, as shown in Fig. 8. For high strike prices, variation of premium price are so that $K - f_K e^{rT_C}$ remains constant at the equilibrium in both cases. Therefore, profits of producers in set A remain constant at high strike prices. Since generation of producers in day-ahead market and day-ahead market price do not changed for these strike prices, profits of producers in set B remain constant, too.

Financial markets operators usually exercise restrictions to premium prices [4]. Before applying any restrictions, Market regulators are willing to know how market participants react to restrictions. During a specified day of trading period, premium prices change in small range. To study the behavior of market at different premium prices, it is assumed that premium prices of all producers are equal at each day of trading period and it is an exogenous variable as strike price.

In this paper the set of premium price-strike price pairs at which option contracts are concluded is referred to as *option contract area*. It is desire for market regulator to determine the option contract area. To this end, market equilibrium is computed for different pairs of strike-premium prices and the pairs of strike-premium prices at which option contracts are concluded are determined. To compute market equilibrium, $f_K$ is considered an exogenous variable in presented model in sections II.B and II.C. The option contract areas in pay-as-bid and uniform cases are shown in Fig. 9 and Fig. 10, respectively. The option contract areas are specified with OCA in these figures. By comparing these figures, it is clear that the option contract area in pay-as-bid case is much wider than contract area in uniform case. It means that the put option market is more activated and liquid beside the day-ahead market with pay-as-bid pricing than the day-ahead market with uniform pricing.

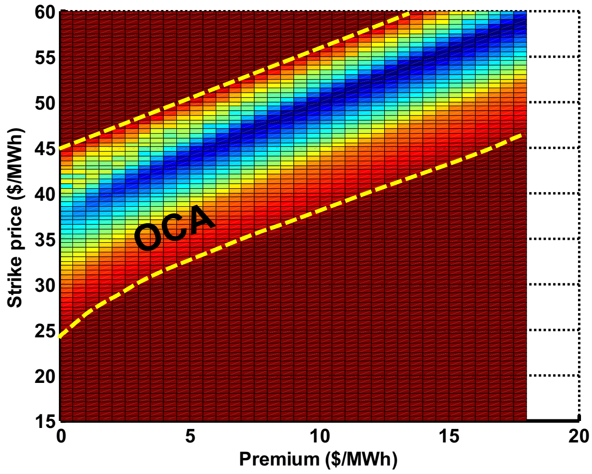


**Fig. 9** Option contract area in pay-as-bid case.

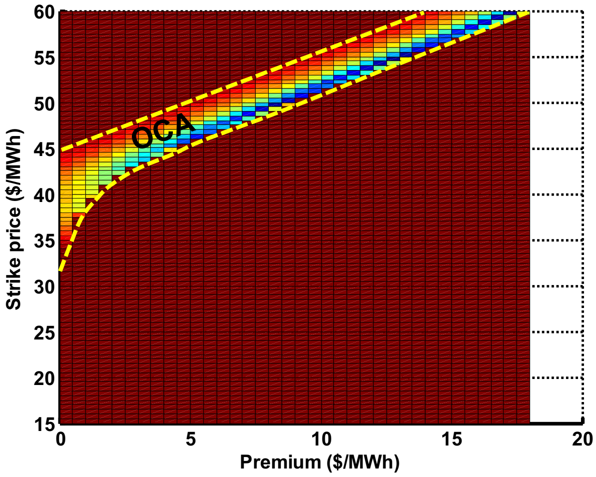


**Fig. 10** Option contract area in uniform case.

## 4 Conclusion

In this paper, the impacts of day-ahead market pricing on the behavior of market participants in the option and day-ahead markets and on the option pricing are studied. Simulation results show that the concluded and exercised volumes of the option contracts are increased when day-ahead market has pay-as-bid pricing rather than uniform pricing. It is also found from simulation that the day-ahead market price, total social welfare and profit of producers are more sensitive to strike prices in the pay-as-bid case rather than the uniform case.

In option pricing, the premium price in pay-as-bid case is greater than the premium price in uniform case at each strike prices. Thus, pay-as-bid pricing in the day-ahead market leads to higher option prices than uniform pricing. On the other side, pay-as-bid pricing in the day-ahead market increases the liquidity of the option contracts in comparison to uniform pricing.

## References

[1] W. Ruiqing, L. Yuzeng, and Z. Shaohua, “Analysis of supply function equilibrium in electricity markets with financial options contracts,” *in DRPT2008*, Nanjing, China, 2008.

[2] S. J. Deng and S. S. Oren, “Electricity derivatives and risk management,” *Energy*, Vol. 31, No. 6, pp. 940–953, 2006.

[3] J. C. Hull, *Options, Futures, and Other Derivatives*, Business and Economics Power system oscillations. Boston: Prentice Hall, 2012.

[4] “Australian electricity futures and options,” Available at: https://asxenergy.com.au/products /electricity futures, accessed: 2015-09-30.

[5] “European electricity exchange,” Available at: https://www.eex.com/en/products/power/power-derivatives-market, accessed: 2016-05-30.

[6] E. Hjalmarsson, “Does the black- scholes formula work for electricity markets? a nonparametric

approach," *Working Papers in Economics*, No. 101, pp. 1–65, 2003.

[7] S. Soleymani, "Strategic bidding of Gencos under two pricing mechanisms: Pay-as-bid and uniform pricing," *GCC Conference and Exhibition (GCC), 2011 IEEE*, 2011.

[8] Q. Bian and Z. Lu, "The valuation of optional financial contract in electricity market," *in International Conference on Sustainable Power Generation and Supply (SUPERGEN 2012)*, pp. 1–6, 2012.

[9] T. W. Gedra, "Optional forward contracts for electric power markets," *Power Systems, IEEE Transactions on*, Vol. 9, No. 4, pp. 1766–1773, 1994.

[10] Y. Oum, S. S. Oren, and S. J. Deng, "Hedging quantity risks with standard power options in a competitive wholesale electricity market," *Naval Research Logistics*, Vol. 53, No. 7, pp. 697–712, 2006.

[11] G. A. V. S´anchez, J. M. Alzate, A. I. Cadena, and J. M. Benavides, "Setting up standard power options to hedge price-quantity risk in a competitive electricity market: the colombian case," *Power Systems, IEEE Transactions on*, Vol. 26, No. 3, pp. 1493–1500, 2011.

[12] S. Pineda and A. J. Conejo, "Managing the financial risks of electricity producers using options," *Energy Economics*, Vol. 34, No. 6, pp. 2216–2227, 2012.

[13] S. Pineda and A. J. Conejo, "Using electricity options to hedge against financial risks of power producers," *Journal of Modern Power Systemsand Clean Energy*, Vol. 1, No. 2, pp. 101–109, 2013.

[14] B. Bezerra, L. A. Barroso, and M. V. Pereira, "Bidding strategies with fuel supply uncertainty in auctions of long-term energy call options," *Power Systems, IEEE Transactions on*, Vol. 26, No. 2, pp. 653–660, 2011.

[15] T. S. Chung, S. H. Zhang, C. W. Yu, and K. P. Wong, "Electricity market risk management using forward contracts with bilateral options," *in Generation, Transmission and Distribution, IEE Proceedings*, Vol. 150, No. 5. IET, pp. 588–594, 2003.

[16] W. Ruiqing, L. Yuzeng, and Z. Shaohua, "Equilibrium for spot and options markets with capacity constraints," *in Chinese Control Conference*, pp. 758–762, 2008.

[17] Z. Shaohua, F. Xinhua, and W. Xian, "Effects of option contracts on electricity markets: A cournot equilibrium analysis," *in Asia-Pacific Power and Energy Engineering Conference (APPEEC)*, 2012.

[18] V. Fanelli, L. Maddalena, and S. Musti, "Asian options pricing in the day-ahead electricity market," Sustainable Cities and Society, Elsevier, Vol. 27, Nov. 2016.

[19] W. Ruiqing, L. Yuzeng, and Z. Shaohua, "Joint equilibrium analysis for options and spot gaming in electricity markets with transmission constraints," *in Intelligent Control and Automation, 7th World Congress on*, pp. 6497–6502, 2008.

[20] M. O. Buygi, H. Zareipour, and W. D. Rosehart, "Impacts of large-scale integration of intermittent resources on electricity markets: A supply function equilibrium approach," *Systems Journal, IEEE*, Vol. 6, No. 2, pp. 220–232, 2012.

[21] P. D. Klemperer and M. A. Meyer, "Supply function equilibria in oligopoly under uncertainty," *Econometrica*, Vol. 57, No. 6, pp. 1243–1277, 1989.

[22] R. Baldick, "Electricity market equilibrium models: the effect of parametrization," *Power Systems, IEEE Transactions on*, Vol. 17, No. 4, pp. 1170–1176, 2002.

[23] B.F. Hobbs, C.B. Metzler, and J.S. Pang, "Strategic gaming analysis for electric power systems: an MPEC approach," *Power Systems, IEEE Transactions on*, Vol. 15, No. 2, pp. 638–645, 2000.

[24] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2009.

**Hani Raouf Sheybani** received the B.S. and M.S. degrees in electrical engineering from the Ferdowsi University of Mashhad, Mashhad, Iran, in 2006 and 2009, respectively. He is currently pursuing the Ph.D. degree in the Electrical Engineering Department at the Ferdowsi University of Mashhad. His research interests include financial electricity markets, restructuring in power system, and restoration of power system.

**Majid Oloomi Buygi** (S'03-M'09) received the Ph.D. degree in electrical engineering in 2004 from Darmstadt University of Technology, Darmstadt, Germany. He was the Director of the Electric Power Engineering Department, Shahrood University of Technology, Shahrood, Iran, from 2005 to 2008. He was with the University of Calgary, Calgary, AB, Canada, as a post-doctoral fellow in 2009. He joined Ferdowsi University of Mashhad, Mashhad, Iran, in 2010. His main research interests include power system operation and planning.