Plane Wave Reflection and Transmission from Uni- and Bi-Axial Chiral Slabs

D. Zarifi*, M. Soleimani* and A. Abdolali*(C.A.)

Abstract: In this paper, the propagation of electromagnetic waves through an infinite slab of uni- or bi- axial chiral medium is analytically formulated for an arbitrary incidence using 4×4 matrix method. In this powerful method, a state vector differential equation is extracted whose solution is given in terms of a transition matrix relating the tangential components of electric and magnetic fields at the input and output planes of the uni- or bi- axial chiral layer. The formulas of the reflection and transmission are then derived. Also, the presented method is verified by some typical examples and the results are compared with the results obtained by the other available methods.

Keywords: Bianisotropic Media, Chiral Media, Reflection and Transmission.

1 Introduction

Unlike the ordinary materials, described by electric permittivity and magnetic permeability, chiral media include a magneto-electric coupling yielding to interesting properties of the electromagnetic fields [1]. Interaction of electromagnetic fields with chiral media has been the subject of many studies over the past decade [2-11] and has led to the introduction of its wide application in different microwave devices such as linear and circular polarization rotators [12-14], microwave absorbers [15], waveguides [16-20], focusing [21], cloaking [22], and circularly polarized antennas [23].

Chiral and Tellegen materials are both subclasses of a more general material, which is called bi-isotropic material. A bi-isotropic material is characterized by four parameters: permittivity (ε), permeability (μ), chirality parameter (κ), and Tellegen parameter (χ); and its constitutive relations are as follows:

$$\mathbf{D} = \varepsilon \mathbf{E} + (\chi - j\kappa)\sqrt{\varepsilon_0 \mu_0}\ \mathbf{H},$$
$$\mathbf{B} = (\chi + j\kappa)\sqrt{\varepsilon_0 \mu_0}\ \mathbf{E} + \mu \mathbf{H} \quad (1)$$

With different values of these parameters, the general bi-isotropic material can be divided into four sub-class materials, which are shown in Table 1.

Table 1 Classification of bi-isotropic media.

	nonchiral ($\kappa = 0$)	**chiral ($\kappa \neq 0$)**
reciprocal ($\chi = 0$)	Simple isotropic medium	Pasteur (or chiral) medium
nonreciprocal ($\chi \neq 0$)	Tellegen medium	General bi-isotropic medium

The uniaxial bianisotropic chiral medium is a special type of bi-anisotropic media where the chirality appears only in one direction [1, 24]. A uniaxial bianisotropic chiral slab can be realized by placing parallel miniature wire spirals mixed in a host dielectric slab [1]. Similarly, the biaxial bianisotropic chiral medium is a special type of bianisotropic media where the chirality appears in two directions. One of the well-known applications of the uniaxial chiral medium is in polarization transformers, whereby any polarization can be transformed to any other polarization [25]. In fact, the ellipticity and handedness of the propagating wave in a uniaxial bianisotropic chiral slab can be changed which makes it possible to design a simple polarization transformer.

The reflection and transmission properties of a plane wave which is incident normally or obliquely from free space to a uniaxial chiral slab have been well studied over the years [26-28]. In previous works, electric and magnetic field equations are written for the uniaxial bianisotropic chiral region and then with determining eigen-polarizations of this layer, boundary conditions are considered at two interfaces. This paper presents the powerful 4×4 matrix method. In this method, Maxwell's equations in the uni- or biaxial chiral region are only cast into a 4×4 matrix formulation, and it is not

Iranian Journal of Electrical & Electronic Engineering, 2014.
Paper first received 15 Oct. 2013 and in revised form 19 Nov. 2013.
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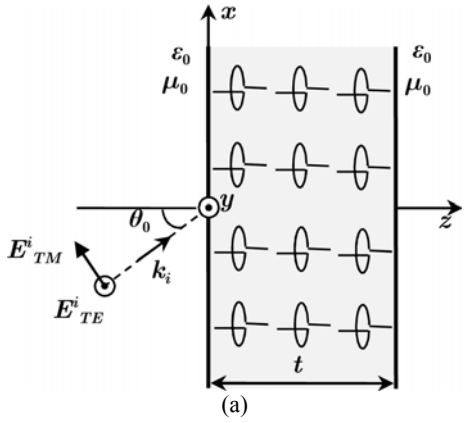
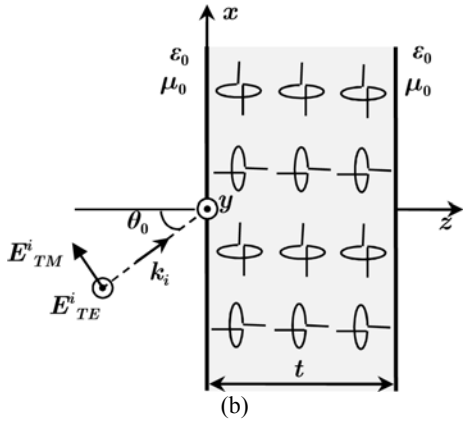


(a)



(b)

Fig. 1. (a) Uniaxial chiral slab. The axis of uniaxial slab is parallel to the *z*-axis and the interfaces. (b) Biaxial chiral slab. The chirality appears in both *x* and *z* directions.

necessary to specify the unintelligible eigen-polarizations of this layer. Then, the complete solution is derived by combining boundary conditions at the interfaces with the transition matrix.

The paper is organized as follows: In Section 2, the differential equations describing the uni- and biaxial bianisotropic chiral slab are extracted, and then the formulas of the reflection and transmission are derived. In Section 3, the validation of the presented method is discussed along with example calculations.

2 General Formulations

The problem geometry is shown in Fig. 1. Consider a uniaxial or biaxial chiral slab which is confined between two infinitely extended planes, $z = 0$ and $z = t$, and placed in free space. The constitutive relations in such a reciprocal biaxial chiral medium can be written as:

$$\begin{aligned}\mathbf{D} &= \left[\varepsilon_x \hat{x}\hat{x} + \varepsilon_y \hat{y}\hat{y} + \varepsilon_z \hat{z}\hat{z}\right].\mathbf{E} - j\kappa\sqrt{\varepsilon_0\mu_0}\,(\hat{x}\hat{x} + \hat{z}\hat{z}).\mathbf{H} \\ \mathbf{B} &= \left[\mu_x \hat{z}\hat{z} + \mu_y \hat{y}\hat{y} + \mu_z \hat{z}\hat{z}\right].\mathbf{H} + j\kappa\sqrt{\varepsilon_0\mu_0}\,(\hat{x}\hat{x} + \hat{z}\hat{z}).\mathbf{E}\end{aligned} \tag{2}$$

where **E**, **D**, **H**, and **B** are the electric field, electric flux density, magnetic field, and magnetic flux density, respectively. In Eq. (2), κ is the chirality parameter, and ε_0 and μ_0 are permittivity and permeability of free space. Clearly, in uniaxial chiral slab shown in Fig. 1(a), the chirality in the *x* direction is disappeared.

It is assumed that an arbitrarily polarized plane wave is incident from free space to the biaxial chiral slab at an oblique angle θ_0. Substituting the constitutive equations into Maxwell's equations, considering $\partial/\partial y = 0$ and $\partial/\partial x = -jk_0 \sin(\theta_0) = -jk_x$ where k_0 is the wave number in vacuum, the differential equations describing biaxial chiral layer are given by

$$\frac{\partial E_x}{\partial z} = -jk_x E_z - j\omega\mu_y H_y \tag{3}$$

$$-\frac{\partial E_y}{\partial z} = \frac{\omega\kappa}{c} E_x - j\omega\mu_x H_x \tag{4}$$

$$-jk_x E_y = \frac{\omega\kappa}{c} E_z - j\omega\mu_z H_z \tag{5}$$

$$\frac{\partial H_x}{\partial z} = j\omega\varepsilon_y E_y - jk_x H_z \tag{6}$$

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon_x E_x + \frac{\omega\kappa}{c} H_x \tag{7}$$

$$-jk_x H_y = j\omega\varepsilon_z E_z + \frac{\omega\kappa}{c} H_z \tag{8}$$

By eliminating E_z and H_z from these equations, one can write:

$$\frac{d}{dz}\begin{pmatrix}\mathbf{E}_T \\ \mathbf{H}_T\end{pmatrix} = \Gamma\begin{pmatrix}\mathbf{E}_T \\ \mathbf{H}_T\end{pmatrix} \tag{9}$$

where $\mathbf{E}_T = (E_x\ ,\ E_y)$ and $\mathbf{H}_T = (H_x\ ,\ H_y)$ are the transverse components of electric and magnetic fields, respectively; and the elements of the $\mathbf{\Gamma}$ matrix are given by:

$$\mathbf{\Gamma} = \begin{pmatrix} 0 & \dfrac{\omega\kappa}{c\left(\varepsilon_z^r \mu_z^r - \kappa^2\right)}\sin^2\theta_0 \\ -\dfrac{\omega}{c}\kappa & 0 \\ 0 & j\omega\varepsilon_0\left(\varepsilon_y^r - \dfrac{\varepsilon_z^r}{\varepsilon_z^r \mu_z^r - \kappa^2}\sin^2\theta_0\right) \\ -j\omega\varepsilon_0\varepsilon_x^r & 0 \\ 0 & -j\omega\mu_0\left(\mu_y^r - \dfrac{\mu_z^r}{\varepsilon_z^r \mu_z^r - \kappa^2}\sin^2\theta_0\right) \\ j\omega\mu_0\mu_x^r & 0 \\ 0 & \dfrac{\omega\kappa}{c\left(\varepsilon_z^r \mu_z^r - \kappa^2\right)}\sin^2\theta_0 \\ -\dfrac{\omega}{c}\kappa & 0 \end{pmatrix} \tag{10}$$

where r subscript for ε and μ indicates relative permittivity and permeability, respectively; and c and ω are the speed of light in vacuum and the angular frequency, respectively. We may define a 4×4 transition matrix $\mathbf{\Phi}$ that relates the transverse components of electric and magnetic fields at the two boundaries of the slab

$$\begin{pmatrix} \mathbf{E}_T(z=0) \\ \mathbf{H}_T(z=0) \end{pmatrix} = \mathbf{\Phi} \begin{pmatrix} \mathbf{E}_T(z=t) \\ \mathbf{H}_T(z=t) \end{pmatrix} \tag{11}$$

Similar to state-space equations in linear control systems [29, 30], it can be easily seen that the transition matrix $\mathbf{\Phi}$ is given by

$$\mathbf{\Phi} = e^{-\mathbf{\Gamma} t} \tag{12}$$

For the computation of the matrix, many methods have been proposed [29] such as expansion of $\mathbf{\Phi}$ in a power series, Cayley-Hamilton theorem, *expm* command in MATLAB, etc.

By introducing the reflection and transmission matrices, T and R, we can write:

$$\mathbf{E}_T^r(0) = \mathbf{R}\,\mathbf{E}_T^i(0) = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{pmatrix} \mathbf{E}_T^i(0) \tag{13}$$

$$\mathbf{E}_T^t(t) = \mathbf{T}\,\mathbf{E}_T^i(0) = \begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix} \mathbf{E}_T^i(0) \tag{14}$$

where the superscripts i, r, and t denote the incident, reflected, and transmitted field, respectively. If the transition matrix $\mathbf{\Phi}$ is partitioned into four 2×2 submatrices, such that

$$\mathbf{\Phi} = \begin{pmatrix} (\mathbf{\Phi}_1)_{2\times2} & (\mathbf{\Phi}_2)_{2\times2} \\ (\mathbf{\Phi}_3)_{2\times2} & (\mathbf{\Phi}_4)_{2\times2} \end{pmatrix}, \tag{15}$$

Equation (11) may be rewritten as

$$\mathbf{E}_T^i(0) + \mathbf{E}_T^r(0) = \mathbf{\Phi}_1\,\mathbf{E}_T^t(t) + \mathbf{\Phi}_2\,\mathbf{H}_T^t(t) \tag{16}$$

$$\mathbf{H}_T^i(0) + \mathbf{H}_T^r(0) = \mathbf{\Phi}_3\,\mathbf{E}_T^t(t) + \mathbf{\Phi}_4\,\mathbf{H}_T^t(t) \tag{17}$$

By introducing wave impedance matrix Z_0 to relate the electric and magnetic fields in the free space regions, we can write $\mathbf{E}_T^i(0) = Z_0\,\mathbf{H}_T^i(0)$, $\mathbf{E}_T^r(0) = -Z_0\,\mathbf{H}_T^r(0)$ and $\mathbf{E}_T^t(t) = Z_0\,\mathbf{H}_T^t(t)$, where

$$\mathbf{Z}_0 = \begin{pmatrix} 0 & \sqrt{\dfrac{\mu_0}{\varepsilon_0}}\cos\theta_0 \\ \dfrac{-\sqrt{\mu_0/\varepsilon_0}}{\cos\theta_0} & 0 \end{pmatrix} \tag{18}$$

Substituting aforesaid equations in Eq. (17), we have:

$$Z_0^{-1}\mathbf{E}_T^i(0) - Z_0^{-1}\mathbf{E}_T^r(0) = \mathbf{\Phi}_3\,\mathbf{E}_T^t(t) + \mathbf{\Phi}_4 Z_0^{-1}\mathbf{E}_T^t(t) \tag{19}$$

Using Eqs. (16) and (19) and by considering Eqs. (13) and (14), one can write

$$\mathbf{R} = \left[\mathbf{\Phi}_1 Z_0 + \mathbf{\Phi}_2 - Z_0\left(\mathbf{\Phi}_3 Z_0 + \mathbf{\Phi}_4\right)\right] \left[\mathbf{\Phi}_1 Z_0 + \mathbf{\Phi}_2 + Z_0\left(\mathbf{\Phi}_3 Z_0 + \mathbf{\Phi}_4\right)\right]^{-1} \tag{20}$$

$$\mathbf{T} = 2\,Z_0\left[\mathbf{\Phi}_1 Z_0 + \mathbf{\Phi}_2 + Z_0\left(\mathbf{\Phi}_3 Z_0 + \mathbf{\Phi}_4\right)\right]^{-1} \tag{21}$$

Once **R** and **T** matrices were determined, co- and cross-reflection and transmission coefficients could be identified. For instance, according to Fig. 1, the co- and cross-reflection coefficients of the biaxial chiral slab at $z = 0$ can be expressed as the following

$$R_{TE-TE} = \left[\frac{E_{TE}^r}{E_{TE}^i}\right]_{E_{TM}^i=0} = R_{yy} \tag{22}$$

$$R_{TM-TM} = \left[\frac{E_{TM}^r}{E_{TM}^i}\right]_{E_{TE}^i=0} = R_{xx} \tag{23}$$

$$R_{TE-TM} = \left[\frac{E_{TE}^r}{E_{TM}^i}\right]_{E_{TE}^i=0} = R_{yx}\cos\theta_0 \tag{24}$$

$$R_{TM-TE} = \left[\frac{E_{TM}^r}{E_{TE}^i}\right]_{E_{TM}^i=0} = \frac{R_{xy}}{\cos\theta_0} \tag{25}$$

The reader should note that the main difference between this method and conventional methods that use full wave analysis is that here state transition matrix is used. In fact, in this method it is not necessary to obtain eigen-polarizations of the uni- or biaxial chiral medium through solving complex wave equation and then using boundary conditions which results in deriving complex nonlinear equations. To conclude, although these different approaches are equivalent generally, the proposed method allows avoiding nonlinearity and complexity of the problem, and may result convenient from computational and practical points of view. Briefly, this procedure can be summarized as follows:

- Consider Γ matrix using Eq. (10).
- Compute Φ matrix using Eq. (12).
- Obtain R and T matrices using Eqs. (20) and (21).

3 Numerical Examples and Results

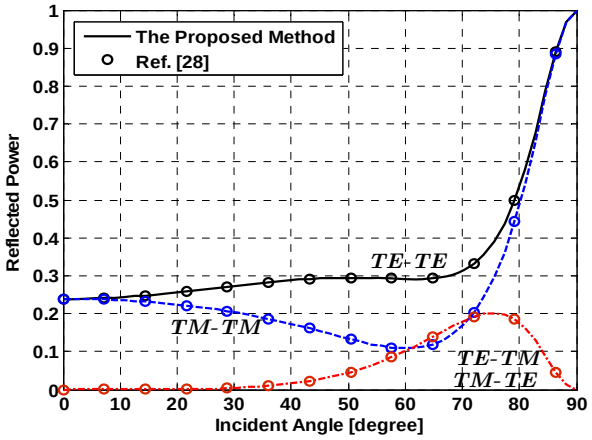
In this section, three examples are provided to illustrate the applicability of the proposed method for

analyzing scattering from uni- or biaxial chiral slabs. In these examples, special cases are considered in order to compare the obtained results with those of previously published methods and achieve the validity of the presented method.

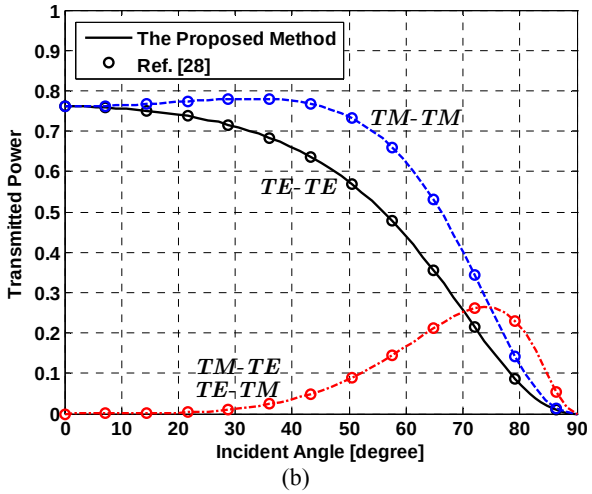
In these numerical examples, we use *expm* command based on Pad´e approximation with scaling and squaring to save computational time in handling the exponential of the matrices in MATLAB.

3.1 Example 1 (Uniaxial Chiral Slab)

As the first example, consider a uniaxial chiral slab with thickness of t = 5 mm, and the electromagnetic parameters $\varepsilon_x = \varepsilon_y = 3\varepsilon_0$, $\varepsilon_z = 4\varepsilon_0$, $\mu_x = \mu_y = \mu_z = \mu_0$, and κ_z=1.5. Assume a plane wave with unity amplitude and excitation frequency 10 GHz obliquely illuminates the slab. The reflected and transmitted power ($|R|^2$ and $|T|^2$) versus the angle of incidence obtained by the proposed method and the exact results presented in [28] are shown in Fig. 2. Comparison between the results, illustrates the good behavior of the proposed method.



(a)



(b)

Fig. 2. (a) The reflected power and (b) the transmitted power, as a function of incident angle for uniaxial chiral slab.

3.2 Example 2 (Biaxial Chiral Slab)

In the second example, to show robustness of the presented method, the problem of reflecting of a plane wave from an infinite perfect electric conductor (PEC) or perfect magnetic conductor (PMC) backed biaxial chiral slab is considered.

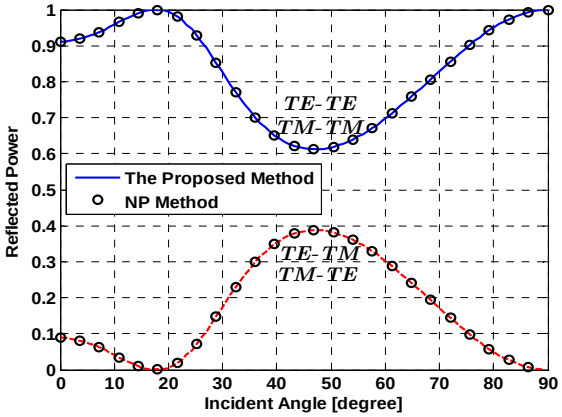
Assuming a PEC boundary condition at $z = t$, we consider $\mathbf{E}_T^t(t)=0$ and then rewrite Eqs. (16) and (19). After simple matrix manipulations, we obtain:

$$\mathbf{R}=\left[\mathbf{\Phi}_2-Z_0\mathbf{\Phi}_4\right]\left[\mathbf{\Phi}_2+Z_0\mathbf{\Phi}_4\right]^{-1} \tag{26}$$

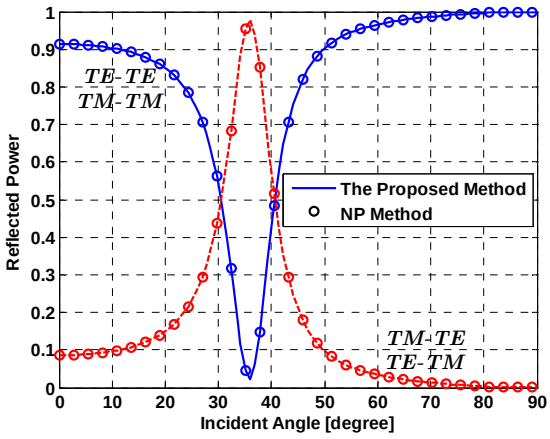
Furthermore, assuming a PMC boundary condition at $z=t$, $\mathbf{H}_T^t(t)=0$ should be considered and Eqs. (16) and (19) should be rewritten. In this case, the matrix $\mathbf{R}$ may be found through simple matrix manipulations.

$$\mathbf{R}=\left[\mathbf{\Phi}_1-Z_0\mathbf{\Phi}_3\right]\left[\mathbf{\Phi}_1+Z_0\mathbf{\Phi}_3\right]^{-1} \tag{27}$$

Assume a plane wave with unity amplitude and frequency 1 GHz obliquely illuminates a PEC or PMC backed biaxial chiral layer with thickness t=5 cm whose constitutive parameters are $\varepsilon_x=\varepsilon_y=2\varepsilon_0$, $\varepsilon_z=5\varepsilon_0$, $\mu_x=\mu_y=3\mu_0$, $\mu_z=\mu_0$, and $\kappa_x=\kappa_z=2$.



(a)



(b)

Fig. 3. The reflected power as a function of incident angle for PEC backed (a), and PMC backed (b) biaxial chiral slab.

Fig. 3 shows the reflected and transmitted power obtained from the proposed method versus the angle of incidence. In order to verify the accuracy, the obtained results by the Notation of Propagators (NP) method [31], which is based on cascading thin linear layers, are also illustrated in Fig. 3. Apparently, there is an excellent agreement between the results of the two different methods.

4 Conclusions

This paper presents an analytic formulation for reflection and transmission problems involving uni- or bi- axial chiral layers. In the presented method, a 4×4 transition matrix that relates the transverse components of electric and magnetic fields at the two boundaries of the slab is employed and is combined with the boundary conditions. The presented approach is very systematic and it can be simply implemented in programming languages supporting matrix manipulations such as MATLAB. The validity of the presented method is achieved by providing some numerical examples and comparing the obtained results with those of other available methods for two special cases. As an interesting, significant, and applicable property of the presented method, it is not necessary to specify the eigen-polarizations of the uni- or bi- axial chiral layer. In addition, the presented method can be used to analysis of the reflection and transmission problems involving more complex bianisotropic layers. In fact, the Γ-matrix should be only calculated for more complex layers and the next steps to obtain reflection and transmission matrices are the same procedure describing in this paper.

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