Reliability Analysis of FSO Communication Links using Aberrated Divergent Rectangular Partially Coherent Flat-Topped Beam

B. Ghafary*, F. D. Kashani* and E. Kazemian*

Abstract: In the present paper, the effects of aberration on the Bit-Error-Rate (BER) and reliability of Free-Space Optical (FSO) communication links are investigated. Based on aberrated divergent rectangular partially coherent flat-topped beam formula on the receiver plane and considering the atmosphere losses due to absorption, scattering and turbulence, numerical values for Power-In-Bucket (PIB), Signal to Noise Ratio (SNR) and BER are calculated. Using these values, the effects of source parameters on link reliability are described. The results are illustrated by graphs obtained by calculation and simulation.

Keywords: Aberration, Divergence, Free-space Optical Communication (FSO), Rectangular Partially Coherent Flat-Topped Beam.

1 Introduction
In recent years, the demand for a backup and complementary link to the radio frequency technology particularly for the “last mile” in access network based on the FSO system has increased considerably [1, 2]. This is due to a number of key advantages including a large unregulated and license free transmission bandwidth spectrum, a large data transmission, consumption of low power, security as well as immunity to the electromagnetic interference [3-5]. Despite these advantages, the performances of FSO links largely depend on the atmospheric condition [6]. Therefore, various methods have been proposed to overcome or reduce the atmosphere-induced degradation of laser beams [7]. It was shown that partially coherent beams are less affected by turbulent atmosphere than fully coherent beams [8-10]. Thus, a considerable number of investigations have paid attention to the characterizations of partially coherent light propagating through turbulent atmosphere [11-13]. Also because of less effectiveness of Partially Coherent Flat-Topped (PCFT) beams in turbulent atmosphere, these types of beams have been widely studied [14]. However, almost all of the related papers using fully collimated aberration-free case and almost no beam is perfect and fully collimated. As it is well-known, the aberrations may emerge while laser beam passes through an optical system or atmosphere; or it could be inherent like astigmatism in laser diode. On the other hand, almost all beams even when propagated through a good collimator have a small amount of divergence.

As it is mentioned before, atmosphere has a significant influence on link quality as well as source parameters. The quality of FSO links, expressed by availability and BER, is determined by the parameters of link and the statistical properties of the atmosphere [15]. These quality factors are basically affected by some phenomenon, such as atmospheric effects (absorption, scattering and turbulence), Source parameters (order of flatness, initial beam divergence angle, aberration, correlation length) and etc. We analyze the effects of source parameters in the presence of atmospheric phenomena.

The purpose of this paper is to study quality of an FSO link using aberrated divergent Rectangular Partially Coherent Flat-Topped (RPCFT) beam, in the presence of the atmospheric losses. To achieve this goal, the current paper is organized as following. First, we introduce an analytical formula for intensity distribution of partially coherent flat-topped beams which propagated through atmosphere. Then we calculate the atmospheric transmission as well as receiving power, based on numerical methods. Finally, we investigate the amount of SNR and BER for some source conditions while considering the effects of the atmospheric extinction on communication link performance.
2. Propagation Equation of Aberrated Stigmatic Rectangular Partially Coherent Flat-Topped Beam through an Atmospheric Turbulence

We consider a field propagation from the plane \( z = 0 \) into the half space, \( z > 0 \), where the turbulence exists (Fig. 1). RPCFT beam propagation can be investigated by using the cross-spectral density [16]:

\[
\Gamma ( \mathbf{r}_1, \mathbf{r}_2, z) = \left\{ E ( \mathbf{r}_1, z; \omega) E^* ( \mathbf{r}_2, z; \omega) g ( \mathbf{r}_1 - \mathbf{r}_2) \right\}
\]

(1)

where \( E ( \mathbf{r}, z; \omega) \) is the complex electric vector at a point specified by the transverse position vector \( \mathbf{r} \), the asterisk stands for the complex conjugate. Angle brackets represent the average, taken over an ensemble of realizations of the electric field in the sense of the coherence theory in the space-frequency domain. \( g ( \mathbf{r}_1 - \mathbf{r}_2) \) is the degree of the spatial coherence.

For the divergent rectangular PCFT source \( \Gamma^{(0)} ( \mathbf{r}_1, \mathbf{r}_2, z = 0; \omega) \) can be written as:

\[
E^{(0)} ( \mathbf{r}_1, z = 0; \omega) = \sum_{m=1}^{M} \sum_{n=1}^{N} \left( - \frac{1}{M+N} \right) \left( \begin{array}{c} M \\ m \\ n \end{array} \right) \left( \begin{array}{c} N \\ m \end{array} \right) \times \exp \left[ - \frac{\mathbf{r}_1^2}{2 \omega_0^2} - \frac{\mathbf{r}_2^2}{2 \omega_0^2} - \frac{i k \left( x_1^2 + y_1^2 \right)}{2 R} \right] \exp \left[ \frac{-r (x_1' - x_2')^2 - I (y_1' - y_2')^2}{2 \sigma_o^2} \right]
\]

(2)

\[
\Gamma^{(0)} ( \mathbf{r}_1, \mathbf{r}_2, z = 0; \omega) = \frac{1}{\sqrt{M+N}} \left\{ \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{p=1}^{M} \sum_{q=1}^{N} \left( - \frac{1}{M+N} \right) \left( \begin{array}{c} M \\ m \\ n \end{array} \right) \left( \begin{array}{c} N \\ m \end{array} \right) \times \exp \left[ - \frac{\mathbf{r}_1^2}{2 \omega_0^2} - \frac{\mathbf{r}_2^2}{2 \omega_0^2} - \frac{i k \left( x_1^2 + y_1^2 \right)}{2 R} \right] \times \exp \left[ - \frac{\mathbf{r}_1^2}{2 \omega_0^2} - \frac{\mathbf{r}_2^2}{2 \omega_0^2} + \frac{i k \left( x_1^2 + y_1^2 \right)}{2 R} \right] \times \exp \left[ - \frac{r (x_1' - x_2')^2 - I (y_1' - y_2')^2}{2 \sigma_o^2} \right] \right\}
\]

(3)

where \( w_{0x} \) and \( w_{0y} \) are the waist sizes of an elliptical Gaussian beam in \( x \)- and \( y \)-directions, respectively. \( M \) and \( N \) are orders of flatness in \( x \)- and \( y \)- directions. \( \sigma_o \) is the correlation length of the Gaussian Schell Model (GSM).

Assuming that the degree of the spatial coherence only depends on the distance between points \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \), and one can represent the degree of spatial coherence as a finite sum of Gaussian functions with different parameters, as Eq. (3).

The cross-spectral density function at these two points: \( (\mathbf{r}_1', z) \) and \( (\mathbf{r}_2', z) \) in transverse plane \( z = z_0 \) for constant \( z_0 > 0 \) are given by [9]:

\[
\Gamma ( \mathbf{r}_1', \mathbf{r}_2', z; \omega) = \left( \frac{k}{2 \pi z} \right)^{3} \int d^3\mathbf{r}_1' \int d^3\mathbf{r}_2' \times \exp \left[ - \frac{i k}{2 z} \left( \frac{1}{2} \left( \mathbf{r}_1' - \mathbf{r}_1 \right)^2 - \frac{1}{2} \left( \mathbf{r}_2 - \mathbf{r}_2 \right)^2 \right) \right] \exp \left[ \frac{\left( \mathbf{r}_1' - \mathbf{r}_1 \right)^2 + \left( \mathbf{r}_2 - \mathbf{r}_2 \right)^2}{2 \sigma_o^2} \right]
\]

\[
\Gamma^{(0)} ( \mathbf{r}_1', \mathbf{r}_2', z_0; \omega) \exp \left[ - \frac{i k}{2 z} \left( \frac{1}{2} \left( \mathbf{r}_1' - \mathbf{r}_1 \right)^2 - \frac{1}{2} \left( \mathbf{r}_2 - \mathbf{r}_2 \right)^2 \right) \right] \times \exp \left[ \frac{\left( \mathbf{r}_1' - \mathbf{r}_1 \right)^2 + \left( \mathbf{r}_2 - \mathbf{r}_2 \right)^2}{2 \sigma_o^2} \right]
\]

\[
\exp \left[ \frac{\left( \mathbf{r}_1' - \mathbf{r}_1 \right)^2 + \left( \mathbf{r}_2 - \mathbf{r}_2 \right)^2}{2 \sigma_o^2} \right]
\]

where \( k (\mathbf{r}_1, \mathbf{r}_2) = \exp \left[ i k \left[ \varphi (\mathbf{r}_1') - \varphi (\mathbf{r}_2') \right] \right] \), \( \varphi (\mathbf{r}') = C_s \left( r^{(x'^2 - y'^2)} \right) \)

\[
(6)
\]

where \( \varphi \) and \( \varphi \) are wave aberration function for astigmatism and astigmatism coefficient, respectively. By substituting the Eqs. (4, 6) in Eq. (5), choosing \( \mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r} \), and calculating the related integral, the average intensity distribution at output plane, \( \left\{ I (\mathbf{r}, z; \omega) \right\} \), is obtained as:

Fig. 1 System diagram of aberrated beam propagation
\begin{align*}
\langle I (\vec{r}, z; \omega) \rangle &= \frac{k^2 (-1)^{m+r+p}}{z^2 (MN)^2} \times \\
& \sum_{m,n,p,q} \left( \frac{M}{n} \right) \left( \frac{N}{p} \right) \left( \frac{N}{q} \right) \times \\
& \exp\left( -\frac{k^2}{r} \left( \frac{\beta_y}{\beta_0} y^2 + \frac{\beta_x}{\beta_0} x^2 \right) \right) \\
\beta_1 &= \frac{m+q}{w_0}, \quad \beta_2 = \frac{n+p}{w_0}, \\
\beta_3 &= 4\alpha_2 \alpha_4 - \alpha_2^2, \quad \beta_4 = 4\alpha_1 \alpha_3 - \alpha_2^2, \\
\alpha_1 &= 1 + \frac{r}{2\sigma^2} - \frac{ik}{2 \sigma} \left( \frac{1}{z} + \frac{1}{R} \right) + M - ikC_a, \\
\alpha_2 &= \frac{m+q}{w_0}, \quad \frac{1}{2\sigma^2} - \frac{ik}{2 \sigma} \left( \frac{1}{z} + \frac{1}{R} \right) + M + ikC_s, \\
\alpha_3 &= \frac{m+q}{w_0}, \quad \frac{1}{2\sigma^2} - \frac{ik}{2 \sigma} \left( \frac{1}{z} + \frac{1}{R} \right) + M + ikC_s, \\
\alpha_4 &= \frac{m+q}{w_0}, \quad \frac{1}{2\sigma^2} - \frac{ik}{2 \sigma} \left( \frac{1}{z} + \frac{1}{R} \right) + M - ikC_s, \\
\alpha_5 &= \frac{m+q}{w_0}, \quad 2M, \quad \frac{1}{\sigma^2} - 2M \\
M &= \begin{cases} 
0.5465C_s^{\frac{1}{5}} \lambda^{\frac{2}{5}} k^2 & \text{Tatarskii spectrum} \\
0.49(C_s)^{\frac{1}{5}} \lambda^{\frac{2}{5}} z^\frac{3}{5} & \text{Kolmogorov spectrum}
\end{cases}
\end{align*}

where $C_s$ being the refractive index structure parameter and $L_i$ is the inner scale of turbulence. On the other hand, as the detector just can receive power in the special bucket, the received power must be calculated at a circular area with radius “a” as follow \([19]\):

\[
P_a = \iint_{0 \leq r \leq a} I (\vec{r}, z) \, r \, d\varphi \, dr
\]

Substituting Eq. (7) in Eq. (8), the power on the surface of optical receiver can be calculated numerically. By using the amount of power on the receiver aperture, another useful parameter for characterizing beam quality which is a measure of laser power within a given bucket, PIB, can be calculated as \([20]\):

\[
\text{PIB} = \iint_{0 \leq r \leq a} I (\vec{r}, z) \, r \, d\varphi \, dr
\]

In received power calculation, the atmospheric effects (absorption and scattering losses) and optical losses have not been included. However, the effect of atmospheric turbulence is taken into account as $M$ parameter. To have an appropriate link budgeting, it is necessary to apply transmission function of atmospheric and optical systems. In the next section, the atmospheric transmission function due to absorption and scattering is calculated.

3. Atmospheric Optical Transmission

Atmospheric transmission is affected by absorption, scattering and turbulence. In the previous section the effects of turbulence take into account with averaging on intensity distribution. The transmission of radiation, $T_{\text{at}}$, traveling through the atmosphere (due to absorption and scattering) can be described by Beer’s law as follow \([21]\):

\[
T_{\text{at}} = \frac{P_a(z)}{P(0)} = e^{-\sigma z}
\]

where $P_a(z)/P(0)$ is the ratio between detected power, $P_a(z)$, at the location $z$ and initially launched power, $P(0)$, and $\sigma$ is the attenuation or total extinction coefficient (per unit length). The attenuation is given as \([22]\):

\[
\sigma = \frac{3.91}{V} \left( \frac{\lambda}{550 \text{nm}} \right)^q
\]

where

\[
\begin{align*}
V &= \text{Visibility (in km)} \\
\lambda &= \text{Wavelength (in nm)} \\
q &= \text{The size distribution of the scattering particles} \\
&= 1.6 \text{ for high visibility ($V > 50$ km)} \\
&= 1.3 \text{ for averaging visibility ($6 \text{ km} < V < 50$ km)} \\
&= 0.161 V^{0.34} \text{ for haze visibility ($1 \text{ km} < V < 6$ km)} \\
&= V^{0.5} \text{ for mist visibility ($0.5 \text{ km} < V < 1$ km)} \\
&= 0 \text{ for fog visibility ($V < 0.5$ km)}
\end{align*}
\]

Atmospheric transmission ($T$) for some weather conditions are calculated based on Bear’s law and are collected in Table 1 along with their visibility ($V$). Therefore the total optical transmission of the FSO communication link path can be calculated as follow:

\[
T_{\text{total}} = T_{\text{at}} \cdot T_{\text{opt}}
\]

where $T_{\text{opt}}$ is transmission of optical elements (lenses of transmitter and receiver). As mentioned before, the effects of turbulence on intensity and consequently received power is considered in previous section.
Table 1 A number of weather conditions along with their visibility and transmission.

<table>
<thead>
<tr>
<th>Weather condition</th>
<th>T (dB/km)</th>
<th>V (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light mist</td>
<td>-2.4203</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>-1.9095</td>
<td>3.5</td>
</tr>
<tr>
<td>Very light mist</td>
<td>-1.5379</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>-1.2583</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>-1.0424</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>-0.8722</td>
<td>5.5</td>
</tr>
</tbody>
</table>

4 Some Useful Communication Link Parameters

The quality of FSO link expressed by availability and BER, is determined by the parameters of link and statistical properties of the atmosphere. The BER parameter for OOK modulated signal is a function of signal to noise ratio (SNR). SNR can be given as follow:

$$\text{SNR} = \frac{P_r}{P_n}$$  \hspace{1cm} (13)

where $P_r$ and $P_n$ are received power at the detector and detectors NEP (Noise equivalent power), respectively. It is worth mentioning that the other noise such as thermal noise have a small amount and can be neglected. $P_r$ is calculated as bellow:

$$P_r = P_T T_{\text{total}}$$  \hspace{1cm} (14)

in which $P_T$ and $T_{\text{total}}$ are received power at receiver’s aperture plane without considering atmosphere effects based on Eq. (8) and total optical transmission, respectively. The relation between BER and SNR for OOK modulated signal is as follow [7]:

$$\text{BER} = \frac{1}{2} \text{erfc} \left( \frac{1}{2 \sqrt{2}} \sqrt{\text{SNR}} \right)$$  \hspace{1cm} (15)

Equations (9, 13, 15) are the basis of the next section calculations.

5 Results and Discussions

In this section we calculate the amount of PIB, SNR and BER based on the presented analysis while considering the effects of source parameters (such as order of flatness, aberration coefficient, initial beam divergence angle, ...) in the presence of atmosphere attenuation (absorption, scattering and turbulence effects). Turbulence condition assumed in our simulation is strong turbulent regime ($C_n^2 = 10^{-14} \text{ m}^{-2/3}$). We consider that $N=M$ and $w_{0x} = w_{0y}$, which is equivalent to square symmetry.

In our simulation we used rectangular partially coherent beam which its initial divergence angle is 1.5 mrad. Fig. 2 shows the effects of astigmatic aberration on intensity distribution. As it is shown, any increase in astigmatism coefficient causes an increase in beam spread.

In order to described and analyze the effect of astigmatism coefficient on link quality, Power In special Bucket (PIB) is shown for some astigmatic coefficient (see Fig. 3).

As it is shown, increasing astigmatism coefficient and consequently an increase in beam spread, causes a decrease in PIB value. Fig. 4 shows SNR and BER versus propagation path length for different astigmatism coefficients. As it is expected any decrease in PIB value leads to decrease in received power, SNR and therefore BER is grown. It is clear that astigmatism coefficient has a strong effect on availability of link. The other parameter that can have an influence on beam propagation is the order of flatness.
The effects of order of flatness are shown in Fig. 5. In this figure, SNR and BER values show versus propagation path length for different order of flatness. At it is known [13], increasing order of flatness causes increased beam spread. Therefore, the increasing order of flatness causes a decrease in SNR value and leads to BER arise.

The normalized intensity distribution of astigmatic rectangular PCFT beams are plotted versus (a) “x” and (b) “y” for different correlation lengths, A) \( \sigma_0 = \infty \), B) \( \sigma_0 = 0.025 \text{ m} \), C) \( \sigma_0 = 0.005 \text{ m} \), D) \( \sigma_0 = 0.0025 \text{ m} \). The other parameters are chosen as: \( \lambda = 1550 \text{ nm} \), \( \theta = 1.5 \text{ mrad} \), \( C_a = 0.05 \text{ m}^{-1} \), \( C_s = 10^{-14} \text{ m}^{-2/3} \), \( w_{0x} = w_{0y} = 0.025 \text{ m} \), \( M = N = 4 \), \( V = 4 \text{ km} \).
For analyzing correlation length on SNR and BER, at first, normalized intensity distribution is shown for four different correlation length values (see Fig. 6).

It is shown that the correlation length does not affect intensity distribution. Therefore, it is expected that it has no effect on SNR and BER. Fig. 7 shows the effects of correlation length on link quality factors. Our analysis shows that source parameters have significant effects on link design.

6 Conclusions

In this article, the effects of some source parameters such as order of flatness and astigmatism coefficient on PIB, SNR and BER of FSO link using aberrated divergent rectangular partially coherent flat-topped beam are analyzed. The effects of atmospheric phenomena such as absorption, scattering and turbulence are taken into account. It is shown any increase in order of flatness and astigmatism coefficient causes a decrease in PIB and SNR values which it is leads to an increase in BER. It is shown that the correlation length has no effect on link quality. Finally, we hope that the presented derived analytical results find its application to problems involving free space optical communication systems as well as optical imaging.

References


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