A Novel DOA Estimation Approach for Unknown Coherent Source Groups with Coherent Signals

S. Shirvani Moghaddam*(C.A.), Z. Ebadi** and V. Tabataba Vakili***

Abstract: In this paper, a new combination of Minimum Description Length (MDL) or Eigenvalue Gradient Method (EGM), Joint Approximate Diagonalization of Eigenmatrices (JADE) and Modified Forward-Backward Linear Prediction (MFBLP) algorithms is proposed which determines the number of non-coherent source groups and estimates the Direction Of Arrivals (DOAs) of coherent signals in each group. First, the MDL/EGM algorithm determines the number of non-coherent signal groups, and then the JADE algorithm estimates the generalized steering vectors considering white/colored Gaussian noise. Finally, the MFBLP algorithm is applied to estimate DOAs of coherent signals in each group. A new Normalized Root Mean Square Error (NRMSE) is also proposed that introduces a more realistic metric to compare the performance of DOA estimation methods. Simulation results show that the proposed algorithm can resolve sources regardless of QAM modulation size. In addition, simulations in white/colored Gaussian noises show that the proposed algorithm outperforms the JADE-MUSIC algorithm in a wide range of Signal to Noise Ratios (SNRs).

Keywords: DOA, EGM, JADE, MDL, MFBLP, RMSE.

1 Introduction

In wireless communications, electromagnetic waves experience fading and multipath phenomena which introduce coherent signals in receiver. Coherent signals lead to rank deficiency in the spatial covariance matrix. Therefore, some second-order-statistics-based subspace methods, such as MUltiple SIgnal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT), fail to correctly resolve the signals, thus the estimation of the Direction Of Arrival (DOA) is not possible [1, 2].

To overcome this problem, many effective methods, such as spatial smoothing [3], Maximum Likelihood (ML) [4], deflation approach [5], method in [6] and method based on Matrix Pencil (MP) [7], are proposed. The spatial smoothing algorithm is based on a preprocessing scheme, which divides an original array into several overlapping subarrays, computes covariance matrix for each subarray, and then applies high-resolution methods to the covariance matrices in order to resolve the signals [3]. In the deflation approach, uncorrelated signals are first estimated and then eliminated and finally coherent signals will be resolved [5]. A different technique presented in [6] utilizes symmetric configuration of Uniform Linear Array (ULA) and constructs non-Toeplitz matrix to resolve signals. The proposed method in [7] utilizes idea of MP on the spatial samples of the data. The MP method can find DOA of coherent signals without spatial smoothing. However, spatial smoothing requires more sensors for preprocessing and the ML approach is computationally intensive. The deflation approach has significant loss of array aperture, because it cannot utilize all the constructed Toeplitz matrices [5] and [8]. Also, the method in [6] suffers from burdensome computation and shortcoming of MP is its requirement for high Signal to Noise Ratio (SNR). Many other efforts based on Fourth Order Cumulants (FOC), such as Steering Vectors DOA (SV-DOA) [9], modified MUSIC [10], methods in [11-14], have been also made for DOA estimation of coherent signals. SV-DOA, first, estimates the steering vectors blindly and then, it utilizes Modified Forward-Backward Linear Prediction (MFBLP) to estimate DOA using the estimated vectors. Modified MUSIC in [10], utilizes switching matrix to construct a new covariance matrix. Method in [11], first,
constructs the data vector which contains all DOA information. Then, it constructs rotation matrix and finally estimates signal DOAs. In [12], Joint Approximate Diagonalization of Eigenmatrices (JADE) and MUSIC were combined, as JADE-MUSIC, to estimate source groups containing coherent signals. Method represented in [13], first, estimates the independent signals, and then eliminates them. Finally, it constructs FOC matrix of coherent signals and also resolves them. Method in [14], combines two algorithms, JADE and MFBLP, to estimate signals. However, SV-DOA requires a large number of snapshots. Computational complexity of the proposed methods in [10] and [12] is relatively high due to spatial spectrum calculation and peak searching. Moreover, the ESPRIT method [11] needs special array geometry [11] and the proposed method in [13] requires a large number of snapshots.

All above mentioned DOA estimation algorithms suppose that the number of coherent and/or non-coherent signals is known or it is determined, approximately. In order to evaluate the number of sources, Wax and Kailath suggested the use of the Minimum Description Length (MDL) criterion [15]. Also, many other criteria such as Eigenvalue Gradient Method (EGM) [16] and Eigen Increment Threshold (EIT) [17] were proposed. The EGM criterion, first computes eigenvalues of auto-correlation matrix, then it determines the number of sources by checking the difference between neighbor eigenvalues. The EIT method utilizes a threshold to evaluate the eigenincrement between neighbor eigenvalues. According to the simulation results in [18] for MDL and EGM methods, the MDL is more appropriate for AWGN channel but EGM is suitable for colored noise.

In this paper, we extend the work in [14] for estimating signal groups. Unlike [14], the number of signal groups is considered to be unknown. In the proposed approach, first MDL/EGM is used to detect the number of coherent signal groups with considering white/colored Gaussian noise. Then, generalized steering vectors are estimated with the JADE algorithm. Finally, MFBLP algorithm is used to estimate DOAs. We also compare the performance of the proposed scheme with JADE-MUSIC equipped with MDL and EGM techniques to determine the number of signal groups.

Here we use notations (.)^T, (.)^*, E(,), (.)^g and (.)^H which denote transpose, conjugate, expectation value, pseudo inverse and conjugate transpose, respectively. Also, vectors are denoted by bold small letters and matrices are denoted by bold capital letters.

This paper is organized as follows. In Section 2, the signal model for coherent signals is described. Section 3 presents MDL and EGM criteria. In Section 4, steering vector estimation and estimating DOAs with MFBLP algorithm are summarized. In Section 5, simulation results are presented. These results include comparison of DOA estimation performance of [12] and the proposed algorithm. Finally, conclusions are drawn in Section 6.

2 Signal Model
Let us assume that N signals; s_i(k); i=1, 2, ..., N, impinge on an M-element antenna array from directions θ, Then, the received signal is

\[ X(k) = As(k) + n(k), \quad k = 1, 2, ..., N_s \]  (1)

where \( N_s \) is the number of snapshots and \( A \) is the array response (manifold) matrix given by:

\[ A = [a(\theta_1) \ a(\theta_2) ... a(\theta_N)] \]  (2)

We suppose that a uniformly spaced linear sensor array is used. So, the steering vector, \( a(\theta_k) \), can be expressed as follows.

\[ a(\theta_k) = [1 \ e^{j2\pi\sin\theta_k} \ ... \ e^{j2\pi(M-1)\sin\theta_k}]^T \]  (3)

where \( d \) and \( \lambda \) are sensor inter-element spacing and signal wavelength, respectively. In Eq. (1), \( s \) is a vector of signals:

\[ s(k) = [s_1(k) \ s_2(k) \ ... \ s_N(k)]^T \]  (4)

and \( n \) is a vector of additive Gaussian noise.

\[ n(k) = [n_1(k) \ n_2(k) \ ... \ n_M(k)]^T \]  (5)

Since the fourth-order cumulants of Gaussian signals are zero [19], we assume that signals are statistically independent and they are non-Gaussian zero-mean complex random processes with symmetric distribution. Also, we suppose that there are \( K \) statistically independent groups that each group contains \( G \) coherent signals. Hence, source vector is given by:

\[ s(k) = [s_1(k) \ s_2(k) \ ... \ s_K(k)]^T \]  (6)

and

\[ A = [a(\theta_{ef_1}) \ a(\theta_{ef_2}) ... a(\theta_{ef_K})] \]  (7)

For instance

\[ a(\theta_{ef_i}) = \sum_{m=1}^{G} a_m a(\theta_m) \]  (8)

where, \( a_m \) is a complex scalar that describes the gain and phase associated to path in coherent signals. Therefore, \( G \) signals will be appeared as a single signal that it arrives from new \( \theta_{ef} \) direction.

3 Determining the Number of Sources
An MDL criterion is used to determine the number of independent groups in AWGN scenario. The MDL criterion cannot determine the number of sources in colored noise scenario [18]. So, EGM criterion is used for this purpose in colored noise scenario.

3.1 MDL Criterion
Steps of MDL are presented as follows [15]:

1) Form the array covariance matrix as follows

\[ R = E\{X(t)X^H(t)\} \]  (9)
2) The number of groups is determined as the value of 
ke {0, 1, 2, ..., M} that minimizes MDL

\[
MDL(k) = -\log \left( \frac{1}{M-k} \sum_{i=k+1}^{M} \lambda_i \right)^{(M-k)/N_s} + \frac{1}{2} \log \left( 2M - k \right)
\]

(10)

where \( \lambda_i > \lambda_{i+1} \); \( i = 0, 1, 2, ..., M-1 \)

3.2 EGM Criterion

Steps of EGM are presented as follows [16].

1) Find the average eigenvalues of the covariance matrix \( R \):
\[
\Delta \lambda = \frac{\lambda_i - \lambda_M}{M-1}
\]

(11)

2) Calculate the gradients of all neighbor eigenvalues.
\[
\Delta \lambda_i = \lambda_i - \lambda_{i+1} ; \ i = 1, 2, ..., M - 1
\]

(12)

3) Find \( i_0 \) which is the smallest \( i \) that satisfies \( \Delta \lambda_i \leq \Delta \lambda \).

The number of sources is \( i_0-1 \).

4 Steering Vector Estimation using JADE Algorithm

JADE algorithm is applied to estimate the generalized steering vectors. It is summarized as follows [12].

1) Compute whitening matrix \( W \). Whitening process can be expressed as follows
\[ Z(t) = WX(t) \]

(13)

2) Form fourth-order cumulants of \( Z(t) \) as follows.
\[
\text{Cum}(z_{k_1}z_{k_2}z_{l_1}z_{l_2}) = \\
E[z_{k_1}z_{k_2}z_{l_1}z_{l_2}] - E[z_{k_1}z_{l_1}]E[z_{k_2}z_{l_2}] - E[z_{k_1}z_{l_2}]E[z_{k_2}z_{l_1}]
\]

(14)

3) Jointly diagonalize the set \( \{ \lambda_{M-r}, M-r, r = 1, 2, ..., D \} \) by a unitary matrix \( U \) that eigenpairs \( \{ \lambda_{M-r}, M-r \} \) corresponds to the D largest eigenvalues.

4) Estimate the array response matrix
\[ A' = W^t U \]

(15)

in which \( U \) is Moore-Penrose inverse of whitening matrix.

\[ a(\theta_k) = [1 e^{j \omega k} e^{j 2 \omega k} ... e^{j (M-1) \omega k}]^T \]

(16)

with
\[ \omega_k = \frac{2 \pi f \sin \theta_k}{c} \]

(17)

where \( f \) is the carrier frequency of signals and \( c \) is the speed of wave propagation.

JADE algorithm proposed by Cardoso is an ICA method which has been successfully applied to the beam-forming and DOA estimation. By using JADE algorithm, the array response vectors can be estimated without any prior knowledge about the array manifold.

5 DOA Estimation Using MFBLP Algorithm

In this paper, MFBLP algorithm is used to estimate DOA of sources. MFBLP algorithm is summarized as follows [9].

1) Choose \( L \) for each \( M \times 1 \) estimated steering vector \( \hat{a} \) that \( L \) must satisfy Eq. (18).

\[
G \leq L \leq M - \frac{G}{2}
\]

(18)

2) Compute a singular value decomposition of \( Q \) as:
\[ Q = U \Sigma V^H \]

(19)

3) Compute \( g \) as follows
\[ g = [g_1 g_2 ... g_L] = -V \Sigma^H U^H h \]

(20)

3) Compute \( g \) as follows
\[ g = [g_1 g_2 ... g_L] = -V \Sigma^H U^H h \]

(20)

Then, set \( L \)-\( G \) smallest singular values on the diagonal of \( \Sigma \) to 0 and name new matrix \( \Sigma \). Dimension of \( \Sigma \) is the same as the dimension of \( D \).

3) Compute \( g \) as follows
\[ g = [g_1 g_2 ... g_L] = -V \Sigma^H U^H h \]

(20)

Then, find roots of the polynomial as
\[ H(o) = 1 + g_1 o^{-1} + g_2 o^{-2} + ... + g_L o^{-L} \]

(23)

H(o) has L zeros, \( o_i = be^{j \theta_k} \); \( i = 1, 2, ..., L \). G zeros of H(o) lie on unit circle, i.e., \( b=1 \). So, G unknown frequencies \( \theta_k \) are determined. Finally, \( \theta_k \) can be computed by using Eq. (17).

6 Proposed Algorithm

A novel approach to jointly determine the number and DOA estimation of coherent signals in non-coherent groups is proposed as a combination of MDL/EGM, JADE and MFBLP. In the proposed algorithm, MDL and EGM methods are utilized to determine the number of non-coherent groups for white noise and colored noise, respectively. Therefore, a decision stage is utilized to determine the kind of noise. If the power of noise at all frequencies is constant then noise is white, otherwise, noise is nonwhite or colored. In other word, the auto-correlation function of a white noise process with a variance of \( \sigma^2 \) is a delta function. The steps of the proposed algorithm are illustrated in Table 1.
7 Simulation Results

In this section, several sets of simulation results are provided to demonstrate the performance of the proposed algorithm. JADE-MUSIC algorithm is selected to be a comparative method. In all simulations, a uniform linear array with relative inter-element spacing \((d/\lambda=0.5)\) is considered and 12 signals impinge on the array is considered. Three source groups of coherent signals are considered which their multipath fading coefficients are \([1, -0.6426+0.7266j, 0.8677+0.0632j, 0.7319-0.1639j], [1, -0.8262+0.4690j, 0.1897-0.8593j], [0.2049-0.7630j, 0.6102+0.1565j], [1, -0.1681-0.8677+0.0632j, 0.7319-0.1639j], [1, -0.6426+0.7266j, 0.8677+0.0632j, 0.7319-0.1639j]) respectively. In other word, three independent signals approach the array from exactly the same direction (i.e., 20°) and two independent signals approach from direction 10°. This algorithm can estimate DOAs relatively well because coherent signals in each group are combined to form a steering vector. It means, three steering vectors are distinct from each other.

7.2 Comparison of Proposed Method and JADE-MUSIC Considering AWGN

In this section, it is assumed that three groups of coherent signals approach the array of sensors at \([10°, 20°, 28°, 45°], [5°, 20°, 35°, 55°] and [30°, 10°, 20°, 40°]\), respectively. Modulation size of each coherent signal group is 4-QAM and SNR=10 dB.

The MUSIC spectrums for each group by JADE-MUSIC algorithm are shown in Figs. 1-3. As depicted in Fig. 1, JADE-MUSIC can resolve the first coherent signal groups. But, Figs. 2 and 3 show that JADE-MUSIC algorithm cannot resolve one and two signals of the second and third groups, respectively. However, estimated DOAs by the proposed algorithm is more accurate with respect to JADE-MUSIC algorithm.

To better compare the two algorithms, Root Mean Square Error (RMSE) is utilized as the performance metric. RMSE is defined as follows:

\[
RMSE = \sqrt{\frac{1}{LN} \sum_{k=1}^{L} \sum_{i=1}^{N} (\theta_{i\text{est}}(k) - \theta_i)^2}
\]  

(26)

where \(\theta_{i\text{est}}(k)\) is the estimate of \(\theta_i\) for the k-th Monte Carlo trial, \(N\) is the total number of signals, and \(L\) is the number of Monte Carlo trials.

Table 1 Summary of the proposed DOA estimation method.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Apply MDL/EGM method to (X) and determine the number of non-coherent source groups.</td>
</tr>
<tr>
<td>2.</td>
<td>Estimate generalized steering vectors with JADE algorithm.</td>
</tr>
<tr>
<td>3.</td>
<td>Use MFBLP algorithm to estimate DOAs.</td>
</tr>
</tbody>
</table>

In examples 1-4, we consider three groups of coherent signals from \([10°, 20°, 28°, 45°], [5°, 25°, 35°, 55°] , [40°, 60°, 15°, 30°]\) and AWGN noise, \(N_s=3000\), SNR=10 dB and \(M=10\). Also, 1000 Monte-Carlo runs are made and modulation type is changed for each group. Examples 1-4 show that the standard deviation of each group depends on its QAM modulation size.

In example 5, it is assumed that three groups of coherent signals coming from \([10°, 20°, 28°, 45°], [5°, 20°, 35°, 55°] \) and \([30°, 10°, 20°, 40°]\). In other word, three independent signals approach the array from exactly the same direction (i.e., 20°) and two independent signals approach from direction 10°. This algorithm can estimate DOAs relatively well because coherent signals in each group are combined to form a steering vector. It means, three steering vectors are distinct from each other.
Table 2  Mean value and standard deviation of estimated DOAs for different QAM modulation sizes (N_s=3000, SNR=10 dB, L=1000, M=10).

<table>
<thead>
<tr>
<th>Example</th>
<th>Original DOAs</th>
<th>Mean value of estimated DOAs</th>
<th>Standard deviation of estimated DOAs</th>
<th>QAM sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Group</td>
<td></td>
<td>Group</td>
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<tr>
<td></td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
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<tr>
<td>1</td>
<td>10 5 40</td>
<td>9.9602 4.9959 40.0356</td>
<td>0.6451 0.0640 0.3040</td>
<td>4 4 4</td>
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<tr>
<td></td>
<td>20 25 60</td>
<td>19.5954 24.9967 60.0072</td>
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<tr>
<td></td>
<td>28 35 15</td>
<td>27.7200 35.0150 14.9507</td>
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<tr>
<td></td>
<td>45 55 30</td>
<td>45.1492 55.0040 29.8760</td>
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<tr>
<td>2</td>
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<td>16 4 4</td>
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<tr>
<td></td>
<td>20 25 60</td>
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<tr>
<td></td>
<td>28 35 15</td>
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<td>45 55 30</td>
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<td>28.1217 34.9959 14.9041</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>45 55 40</td>
<td>45.1076 55.0101 40.0127</td>
<td></td>
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</tr>
</tbody>
</table>

Fig. 1 MUSIC spectrum for the first coherent signal group.

Fig. 2 MUSIC spectrum for the second coherent signal group.
To define SE-NRMSE, consider that the algorithm resolves $H$ sources, where $H < N$. It is assumed that if $|\theta_{\text{est}} - \theta_{ij}| < T_{hj}$; $i=1, 2, \ldots, G$, and $j=1, 2, \ldots, K$, the algorithm can resolve the signals. It is also assumed that the total number of angles that satisfy $|\theta_{\text{est}} - \theta_{ij}| < T_{hj}$ is not greater than $H$, $\theta_{ij}$ is the $i$-th $\theta$ in the $j$-th group and $T_{hj}$ is threshold for the $j$-th group of coherent signals. In other word, $T_{hj}$ is defined as half of the difference between two nearest angles in the $j$-th group. Thereby, $\text{NMSE}_{ij}$ is calculated by Eq. (27). Note that $0 \leq \text{NMSE}_{ij} \leq 1$. In Eq. (27), if $\theta_{\text{est}} = \theta_{ij}$ then $\text{NMSE}_{ij} = 0$ and if $\theta_{\text{est}} = \theta_{ij} \pm T_{hj}$ then $\text{NMSE}_{ij} = 1$.

$$\text{NMSE}_{ij} = \left( \frac{\theta_{\text{est}(k)} - \theta_{ij}}{T_{hj}} \right)^2; i = 1, \ldots, G, j = 1, \ldots, K \quad (27)$$

$$\text{nmse} = [\text{NMSE}_{ij}]_{1\times F} \quad (28)$$

If $|\theta_{\text{est}} - \theta_{ij}| \geq T_{hj}$; $i=1, 2, \ldots, G$ and $j=1, 2, \ldots, K$, so the algorithm could not resolve the signals. The total number of angles that satisfy $|\theta_{\text{est}} - \theta_{ij}| \geq T_{hj}$ is $H-F$. Then $\text{NRMSE}_{ij}$ is given by Eq. (29).

$$\text{NRMSE}_{ij} = \frac{1}{N}; i = 1, \ldots, G \quad (29)$$

DOA estimation algorithm also could not resolve $N-H$ signals, especially in low SNRs and/or small number of snapshots. So, $\text{NRMSE}_{ij}$ of these signals is calculated by Eq. (29) again. Thereby, NRMSE for each Monte Carlo trial is calculated by Eq. (30).

$$\text{NRMSE}(i) = \frac{N - H}{N} + \frac{H - F}{N} \sqrt{\frac{F}{N}} \sum_{f=1}^{F} \text{NMSE}(f) \quad (30)$$

$$= 1 - \frac{F}{N} \left( 1 - \frac{\sum_{f=1}^{F} \text{NMSE}(f)}{F} \right), l = 1, 2, \ldots, L$$

where $\text{NMSE}(f)$ is the $f$-th element of $\text{nmse}$ in Eq. (28). Finally, the total SE-NRMSE is defined as

$$SE - NRMSE = \frac{1}{L} \sum_{l=1}^{L} \text{NRMSE}(l) \quad (31)$$

RMSE and SE-NRMSE versus SNR are shown in Figs. 4 and 5, respectively. In these simulations, 200 Monte-Carlo runs with $N_s=2000$ are made. As shown in Fig. 4, when SNR increases from -2 dB, the proposed algorithm begins to demonstrate smaller RMSE than the JADE-MUSIC algorithm. When SNR is below -2 dB, RMSE of the proposed algorithm is a bit more than RMSE of the JADE-MUSIC algorithm. To clearly show the performance, SE-NRMSE is plotted for both algorithms in Fig. 5. It illustrates that SE-NRMSE of the proposed algorithm is less than SE-NRMSE of JADE-MUSIC algorithm in all SNR values.
7.3 Comparison of the Proposed Method and JADE-MUSIC Considering Colored Noise

In this section, it is assumed that three groups of coherent signals approach the array of sensors at \([0°,10°,20°,28°]\), \([25°,33°,10°,19°]\) and \([30°,8°,19°,26°]\), respectively. Modulation size of each coherent signal group is 4-QAM, \(\text{SNR} = 10\) dB and noise is colored Gaussian. As we know, the power spectral density of the colored noise is not uniform across the entire frequency spectrum. Hence, the colored Gaussian additive noise is generated by passing the AWGN through a first-order auto regressive (AR1) filter with coefficient \(a_0\). Correlation coefficient \(\alpha\), is 0.7 and 200 Monte-Carlo runs with \(N_s = 2000\) is made.

Fig. 6 shows that MDL criterion cannot detect the number of coherent signal groups in colored Gaussian noise scenario. It also shows that the detection probability of EGM criterion is 100 %, in \(\text{SNR} \geq -2\) dB. So, in \(\text{SNR} \leq -2\) dB, it is assumed that the number of coherent signal groups is known and in \(\text{SNR} \geq -2\) dB, the number of independent groups is detected by EGM, exactly. Hence, in colored noise scenario, EGM is selected as the determiner of the number of source groups.

As shown in Fig. 7, RMSE of the proposed algorithm in \(\text{SNR} > -6\) dB is lower than JADE-MUSIC algorithm and vice versa. Fig. 8 shows that, in wide range of SNRs the proposed algorithm demonstrates a smaller SE-NRMSE than the JADE-MUSIC algorithm.

In Table 3, the effect of correlation coefficients on RMSE and SE-NRMSE of two algorithms is investigated. Three levels of SNR, low, moderate and high are equal to -10, 10 and 30 dB, respectively. For each SNR, \(\alpha\) is 0.1, 0.3, 0.5 and 0.8. As reported in this table, in low and moderate SNRs, RMSE of the proposed algorithm is higher than RMSE of the JADE-MUSIC algorithm.

![Fig. 6 Detection probability versus SNR for AWGN scenario.](image)

![Fig. 7 RMSE versus SNR for colored noise scenario.](image)

![Fig. 8 SE-NRMSE versus SNR for colored noise scenario.](image)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\text{SNR} (\text{dB}))</th>
<th>JADE-MUSIC</th>
<th>Proposed Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>SE-NRMSE</td>
<td>RMSE</td>
</tr>
<tr>
<td>0.1</td>
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In high SNRs, RMSE of the proposed algorithm is lower than that of the JADE-MUSIC algorithm. Also, Table 3 shows that SE-NRMSE of the proposed algorithm is lower than that of the JADE-MUSIC algorithm, in low, moderate and high SNRs regardless of $\alpha$.

8 Conclusion

In this paper, we proposed a new algorithm for DOA estimation of source groups with coherent signals by combining MDL/EGM, JADE and MFBLP algorithms. MDL criterion was used to determine the number of independent groups in AWGN scenario, JADE algorithm was utilized to estimate the steering vectors, and finally MFBLP algorithm was applied to each steering vectors to estimate DOAs. It should be noted that instead of MDL, the EGM criterion was used to detect the number of groups in colored noise scenario.

Also, a new normalized RMSE performance metric was suggested which showed the performance of each algorithm more accurately. To evaluate the performance of the proposed algorithm and also compare the numerical results with conventional JADE-MUSIC algorithm, several Monte-Carlo simulations were run in both AWGN and colored noise channels. Simulation results show that, for different QAM modulation sizes of coherent signals, the proposed method can completely resolve sources. When AWGN is considered, in SNR larger than -2 dB RMSE of the proposed algorithm is less than RMSE of JADE-MUSIC algorithm and in wide range of SNRs, SE-NRMSE of the proposed algorithm is less than SE-NRMSE of JADE-MUSIC algorithm. Also, simulation results show the effect of colored Gaussian noise on RMSE and SE-NRMSE. In SNR larger than -6 dB, RMSE, and in wide range of SNRs, SE-NRMSE of new algorithm is less than JADE-MUSIC algorithm. It also shows that, in low, moderate and high SNRs, in all coefficient correlation ranges, the proposed algorithm shows lower SE-NRMSE. Besides above mentioned superiorities of the new proposed method, it needs 10 times lower computational complexity compared to the JADE-MUSIC algorithm.

References


Shahriar Shirvani Moghaddam received the B.Sc. degree from Iran University of Science and Technology (IUST), Tehran, Iran, both in Electrical Engineering, in 1992 and 1995, respectively. Also, he received the Ph.D. degree in Electrical Engineering from Iran University of Science and Technology (IUST), Tehran, Iran, in 2001. He has published more than 70 refereed international scientific journal and conference papers, 2 text books on digital communications and one book chapter on MIMO systems. Since 2003, he has worked at the Faculty of Electrical Engineering, Shahid Rajae Teacher Training University (SRTTU), Tehran, Iran. He was nominated as the best researcher and the best teacher in the SRTTU University in 2010 (2012, 2013) and 2011 (2013), respectively. Currently, he is an associate professor in Digital Communications Signal Processing (DCSP) research laboratory of SRTTU. His research interests include power control and beam-forming in MIMO and cellular relay networks, cognitive radio communications, adaptive antenna array beam-forming and direction of arrival estimation.

Z. Ebadi received the B.Sc. degree from Islamic Azad University Arak Branch, Iran and M.Sc. degree from Islamic Azad University Tehran South Branch, Iran both in Electrical Engineering, in 2008 and 2012, respectively. She is lecturer in several Institutes of Higher Education. Her research interests include Direction of Arrival (DOA) estimation of wideband and narrowband signals, determining the number of radio sources, antenna beam-forming, and channel estimation of wireless communication systems.

Vahid Tabataba Vakili received the B.Sc. degree from Sharif University of Technology, Tehran, Iran, in 1970, the M.Sc. degree from the University of Manchester, Manchester, U.K., in 1973, and the Ph.D. degree from the University of Bradford, Bradford, U.K., in 1977, all in Electrical Engineering. In 1985, he joined the Department of Electrical Engineering, Iran University of Science and Technology, Tehran. In 1997, he was promoted as Professor. He has served as the Head of the Communications Engineering Department and as the Head of postgraduate studies. His research interests are in the areas of mobile cellular systems, interference cancellation for CDMA systems, ultra-wideband communication systems, satellite communication systems, and space-time processing and coding.