Frequency Estimation of Unbalanced Three-Phase Power System using a New LMS Algorithm

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Abstract: This paper presents a simple and easy implementable Least Mean Square (LMS) type approach for frequency estimation of three phase power system in an unbalanced condition. The proposed LMS type algorithm is based on a second order recursion for the complex voltage derived from Clarke's transformation which is proved in the paper. The proposed algorithm is real adaptive filter with real parameter (not complex) which can be efficiently implemented by DSP. In unbalanced situations, simulation experiments show the advantages and drawbacks of the proposed algorithm in comparison to Complex LMS (CLMS) and Augmented Complex LMS (ACLMS) methods.

Keywords: Adaptive Filter, Frequency Estimation, LMS Algorithm, Power Systems.

1 Introduction

Signal processing has been used in many aspects of power system [1, 2]. Frequency is an important parameter of a power system in monitoring, control and security applications. Besides, accurate and online frequency estimation in a power system is a prerequisite for the future smart grid where the generation, load and topology will be dynamically updated [3]. In addition, unexpected frequency variation from nominal value indicates an emergency situation where quick response should be taken into account. So, fast and accurate frequency estimation is necessary in such conditions.

Various algorithms have been proposed for frequency estimation in power systems. Traditionally, zero crossing information was used for frequency estimation [4]. Because of the degradation of zero crossings by noise and other harmonic distortions, other techniques have been proposed. In [5], a demodulation technique was presented. In addition, Discrete Fourier Transform (DFT) based algorithms were suggested to solve the problem [6-8]. Phase-Locked-Loop (PLL) based algorithms were also used in the literature [9, 10]. Moreover, Kalman filtering approaches were successfully applied [11-15]. In [16], an adaptive notch filter was proposed for frequency estimation of a single phase and then this method was extended to three-phase power systems [17]. A simple Weighted Least Square (WLS) recursive algorithm was suggested in [18] for single phase cases and again was extended to three-

phase systems in [19]. Also, a simple Complex Least Mean Square (CLMS) adaptive filter was introduced in [20]. Recently, the CLMS was extended to an Augmented CLMS based on the concept of widely linear modeling and augmented complex statistics [21]. Moreover, an iterative Minimum Variance Distortion less Response (MVDR) algorithm was proposed in [22] and then an Augmented MVDR approach was suggested recently [23].

Among various algorithms for frequency estimation, some exploit the frequency from single phase (e.g. refer to [4, 7, 8, 11, 16, 18, 24, 25]), while others extract the frequency from all three phases because of more robustness (e.g. refer to [12, 15, 17, 20, 22]). Since using all phases, it provides a more robust algorithm especially when phases undergo some abnormal conditions. Usually, the Clark's transformation is used to convert the three phase signals to a single complex signal. Moreover, recently, unbalanced power systems have gained more attention [15, 21, 23, 26, 27]. This paper proposes a simple and easy-implementable adaptive LMS algorithm for power system frequency estimation based on all three phases in unbalanced condition where the three phases are not exactly the same and especially their amplitudes are different. In this case, the complex signal derived from Clark's transformation has a recursion different to what an LMS technique is based on. According to this recursion, an LMS algorithm is suggested which uses two consecutive samples of the complex signal instead of a single value. Fortunately, the coefficients of this recursion is real and so the proposed overall LMS algorithm is a real adaptive filter and is computationally efficient specially for applying on DSP's.

This paper is organized as follows. In section 2, the preliminaries such as Clark's transformation and the problem are introduced. Section 3 presents an overview of the CLMS and ACLMS algorithms. Then, in section 4, at first a recursion will be proved for the complex signal derived from Clark's transformation. Secondly, the proposed LMS algorithm is presented based on this recursion. In section 5, to explain the suitability of the proposed algorithm, several experiments were conducted in unbalanced cases and with different conditions and the advantage and disadvantages of the proposed algorithm in comparison to its counterparts i.e. CLMS and ACLMS algorithms are illustrated. After that, paper concludes in section 6. Finally, a nomenclature is presented for convenience.

2 Preliminaries

The voltages in a three-phase power system can be represented in a discrete time form as

$$v_a(k) = V_a(k)\cos(\omega k\Delta T + \varphi) + n_a$$
$$v_b(k) = V_b(k)\cos\left(\omega k\Delta T + \varphi - \frac{2\pi}{3}\right) + n_b \qquad (1)$$
$$v_c(k) = V_c(k)\cos\left(\omega k\Delta T + \varphi + \frac{2\pi}{3}\right) + n_c$$

where $V_i(k)$ is the peak value of i'th voltage phase at time instant k, n_i is the noise of i'th voltage phase, $\Delta T = \frac{1}{f_s}$ is the sampling interval with f_s being the sampling frequency, φ is the initial phase of fundamental component, and $\omega = 2\pi f$ is the angular frequency with f being the system frequency. The noises are assumed to be independent white Gaussian with variances σ_n^2. The Clark's transform convert the three phases (v_a, v_b and v_c) to a zero-sequence component v_0 and direct and quadrature components v_α and v_β as the form of [28]:

$$\begin{pmatrix} v_0(k) \\ v_\alpha(k) \\ v_\beta(k) \end{pmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{pmatrix} v_a(k) \\ v_b(k) \\ v_c(k) \end{pmatrix} \qquad (2)$$

where the factor $\sqrt{\frac{2}{3}}$ is used to ensure that the system is invariant under this transformation. In balanced condition where $V_a(k) = V_b(k) = V_c(k)$, we have $v_0(k) = 0$, $v_\alpha(k) = A\cos(\omega k\Delta T + \varphi)$ and $v_\beta(k) = A\cos\left(\omega k\Delta T + \varphi + \frac{\pi}{2}\right)$ where v_α and v_β are orthogonal components. Thus, a complex voltage signal is defined as:

$$v(k) = v_\alpha(k) + jv_\beta(k) \qquad (3)$$

where in practice, this complex signal was extensively used for frequency estimation in three-phase power systems [5, 12, 17, 19, 20]. Naturally, the harmonics that are mainly zero-sequence $v_0(k)$ are blocked by this transformation [5]. Thus, the zero-sequence was discarded in all these references and we did the same.

In general, it has been shown that the complex voltage can be written in the form [21]:

$$v(k) = A(k)e^{j(\omega k\Delta T + \varphi)} + B(k)e^{-j(\omega k\Delta T + \varphi)} \qquad (4)$$

where

$$A(k) = \frac{\sqrt{6}(V_a(k) + V_b(k) + V_c(k))}{6} \qquad (5)$$
$$B(k) = \frac{\sqrt{6}(2V_a(k) - V_b(k) - V_c(k))}{12} - \frac{\sqrt{2}(V_b(k) - V_c(k))}{4} \qquad (6)$$

In other words, when $V_a(k)$, $V_b(k)$, and $V_c(k)$ are not identical, we have a clockwise and a counterclockwise rotating complex vector resulting a certain degree of non-circularity [21]. When $V_a(k)$, $V_b(k)$, and $V_c(k)$ are identical, we have $B(k) = 0$. Thus, we have the following recursion:

$$v(k+1) = A(k+1)e^{j(\omega(k+1)\Delta T + \varphi)}$$
$$= v(k)e^{j(\omega\Delta T)} = v(k)w(k) \qquad (7)$$

where $w(k) = e^{j(\omega\Delta T)}$ and the assumption $A(k+1) = A(k)$ is used.

3 CLMS and ACLMS Algorithms

Based on Eq. (7), the CLMS algorithm was proposed for frequency estimation and summarized as [20, 21]:

$$\hat{v}(k+1) = v(k)w(k) \qquad (8)$$
$$e(k) = v(k+1) - \hat{v}(k+1) \qquad (9)$$
$$w(k+1) = w(k) + \mu e(k)v^*(k) \qquad (10)$$

where $w(k) = e^{j(\omega_k\Delta T)}$ is the weight coefficient at time instant k, μ is the step size, $\hat{v}(k+1)$ is the estimate of the desired signal $v(k+1)$ and $e(k)$ is the estimation error. Also, the system frequency can be estimated as

$$\hat{f}(k) = \frac{1}{2\pi\Delta T}\sin^{-1}(Im(w(k))) \qquad (11)$$

Unfortunately, in unbalanced condition, the recursion Eq. (7) does not hold and the CLMS algorithm start to break down. In this case, the complex signal $v(k)$ is exactly expressed as in Eq. (4). In ACLMS algorithm [21], it is suggested to estimate the complex signal from both $v(k)$ and its complex conjugate $v^*(k)$. Therefore, we have:

$$\hat{v}(k+1) = v(k)h(k) + v^*(k)g(k) \qquad (12)$$

where $h(k)$ and $g(k)$ are the filter weight coefficients corresponding to the standard and conjugate updates at time instant k, respectively [21]. The estimation error $e(k)$ and cost function $J(k)$ is defined as:

$$e(k) = v(k+1) - \hat{v}(k+1) \qquad (13)$$
$$J(k) = |e(k)|^2 = e^*(k)e(k) \qquad (14)$$

where the update of weight coefficients $h(k)$ and $g(k)$ is based on this cost function and is obtained by an steepest descent recursion in the following form:

$$h(k+1) = h(k) - \mu\nabla_h J(k) \qquad (15)$$
$$g(k+1) = g(k) - \mu\nabla_g J(k) \qquad (16)$$

where $\nabla_h J(k)$ and $\nabla_g J(k)$ are the gradients of the cost function with respect to weight coefficients. The final recursion for updating the coefficients is [21]:

$$h(k+1) = h(k) + \mu e(k)v^*(k) \qquad (17)$$

$$g(k+1) = g(k) + \mu e(k)v(k) \qquad (18)$$

where the above coefficient updates are the basic relations of ACLMS algorithm. In order to estimate the frequency based on the weight coefficients, it is shown that [21]:

$$\hat{f}(k) = \frac{1}{2\pi\Delta T}\sin^{-1}(Im(h(k) + a_1(k)g(k))) \qquad (19)$$

where the coefficient $a_1(k)$ is defined as:

$$a_1(k) = \frac{-jIm(h(k)) + j\sqrt{Im^2(h(k)) - |g(k)|^2}}{g(k)} \qquad (20)$$

when balanced condition holds, the weight coefficient $g(k) = 0$ and ACLMS frequency estimation Eq. (19) simplifies to the standard CLMS frequency estimation Eq. (11).

4 The Proposed LMS Algorithm

Inspiring from the recursive relation for single phase voltage [18], a recursion can be derived for the complex voltage in Eq. (4). Eq. (4) is rewritten as the sum of two clockwise and counter clockwise voltages as the from:

$$v(k) = v_+(k) + v_-(k) \qquad (21)$$

where $v_+(k)$ and $v_-(k)$ are:

$$v_+(k) = A(k)e^{j(\omega k\Delta T + \varphi)} \qquad (22)$$

$$v_-(k) = B(k)e^{-j(\omega k\Delta T + \varphi)} \qquad (23)$$

To find a recursion for complex voltage $v(k)$, it is straightforward to consider three consecutive time instants $k-1$, k and $k+1$. Assuming the amplitudes $V_a(k)$, $V_b(k)$, and $V_c(k)$ are constant over these three samples, then based on Eq. (5) and Eq. (6), we have $A(k-1) = A(k) = A(k+1)$ and $B(k-1) = B(k) = B(k+1)$. Thus, it can be expressed:

$$v(k-1) = e^{-j\omega\Delta T}v_+(k) + e^{j\omega\Delta T}v_-(k) \qquad (24)$$

$$v(k) = v_+(k) + v_-(k) \qquad (25)$$

$$v(k+1) = e^{j\omega\Delta T}v_+(k) + e^{-j\omega\Delta T}v_-(k) \qquad (26)$$

where the elimination of $v_+(k)$ and $v_-(k)$ in Eqs. (24)-(26), leads to the following recursion:

$$v(k+1) = 2\cos(\omega\Delta T)v(k) - v(k-1)$$
$$= [2\cos(\omega\Delta T) \quad -1]\begin{bmatrix} v(k) \\ v(k-1) \end{bmatrix} \qquad (27)$$

So, it is more straightforward to estimate the desired signal $v(k+1)$ based on two preceding samples $v(k)$ and $v(k-1)$ instead of $v(k)$ and $v^*(k)$. Thus, in this new proposed LMS algorithm, the estimated complex voltage is:

$$\hat{v}(k+1) = w(k)v(k) - v(k-1)$$
$$= [w(k) \quad -1]\begin{bmatrix} v(k) \\ v(k-1) \end{bmatrix} \qquad (28)$$

where weight coefficient is defined as $w(k) = 2\cos(\omega\Delta T)$. To update weight coefficient $w(k)$, we use the cost function $J(k) = |e(k)|^2 = e^*(k)e(k)$ where estimated error is $e(k) = v(k+1) - \hat{v}(k+1)$. The steepest descent update will be:

$$w(k+1) = w(k) - \mu\nabla_{w(k)}J(k) \qquad (29)$$

where some mathematical simplifications leads to the final recursion of the new proposed LMS algorithm:

$$w(k+1) = w(k) + 2\mu Re(v(k)e^*(k)) \qquad (30)$$

where $Re(.)$ is real part operator. This real part is appeared in the update because the weight coefficient $w(k) = 2\cos(\omega\Delta T)$ is real. The frequency estimator based on this new algorithm is:

$$\hat{f}(k) = \frac{1}{2\pi\Delta T}\cos^{-1}(\frac{w(k)}{2}) \qquad (31)$$

5 Simulations

The new proposed LMS algorithm and its frequency estimator in Eqs. (30) and (31) was applied to estimate the power system frequency from discrete time samples of three-phase voltage signals. Different experiments were conducted to evaluate the performance of the proposed algorithm (we nominated it as Modified LMS or MLMS) in comparison to the counterpart algorithms i.e. CLMS and ACLMS. Simulations were performed in the MATLAB programming environment with a sampling frequency of 5 kHz. The simulation results are averaged over 100 independent runs.

5.1 Experiment 1

In first experiment, the power system was in normal condition at 50 Hz and in noiseless case. At first, three phases are balanced ($V_a(k) = V_b(k) = V_c(k) = 1$). At time $t = 0.05$, an extra 0.1-per-unit (p.u) magnitude was imposed at phases b and c, together with 0.05-p.u. on phase a ($V_a(k) = 1.05, V_b(k) = 1.1, V_c(k) = 1.1$). Also, at $t = 0.05$ a 50 percent voltage sag was happened at phase c ($V_a(k) = 1.05, V_b(k) = 1.1, V_c(k) = 0.5$). All step sizes were set to μ=0.01 and all algorithms were initialized at 50.5 Hz. The estimated frequency of all three algorithms (CLMS, ACLMS and MLMS) are shown in Fig. 1. In balanced case $t < 0.05$ s, all three algorithms were converged to 50 Hz without oscillation and MLMS had faster convergence. In unbalanced case $0.05 < t < 0.15$ s, CLMS had an inevitable oscillation with frequency 100 Hz and with 0.2 Hz amplitude. Also, ACLMS and MLMS converged to 50 Hz with and without oscillation. After voltage drop of phase c at $t = 0.15$ s, CLMS algorithm failed to converge. MLMS and ACLMS converged after voltage sag, at $t = 0.22$ s and $t = 0.35$ s, respectively. So, MLMS had faster convergence after voltage drop.

5.2 Experiment 2

To compare proposed MLMS algorithm with CLMS and ACLMS, in noisy conditions, a bias and variance analysis was performed. At first, we set the variance of noise of $\sigma_n = 0.001$. In noisy cases, step sizes of all algorithms were selected as 0.0001. The three phases were at all times in unbalanced case with peak amplitudes $V_a(k) = 1.1, V_b(k) = 1, V_c(k) = 1$. The results of estimated frequency are shown in Fig. 2. This figure shows the faster convergence of MLMS in comparison to CLMS and ACLMS. It also shows a large bias of CLMS.
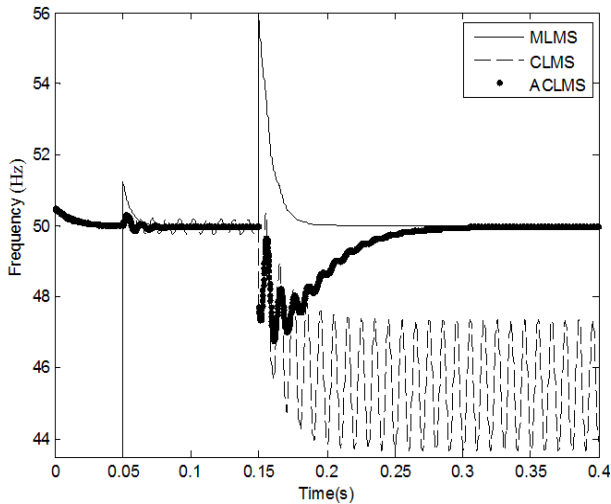
Fig. 1 Frequency estimation under unbalanced conditions. To generate an unbalanced condition, an extra 0.1 p.u. was imposed on phase b and phase c, plus a 0.05 p.u. magnitude on phase a at $t = 0.05\ s$. A 50% voltage sag in the third channel occurred at $t = 0.15\ s$. The frequency was initialized at 50.5 Hz with the true system frequency set to 50 Hz.
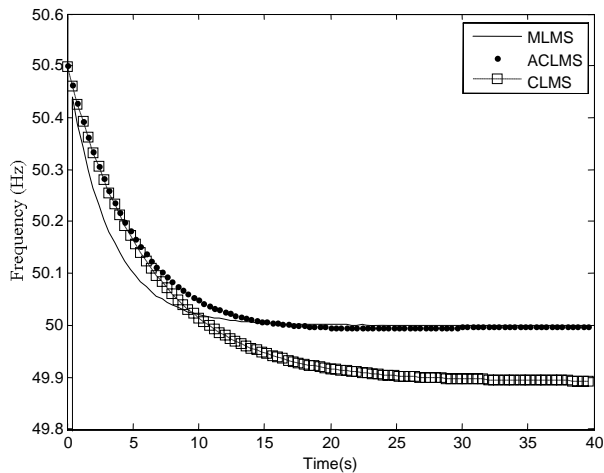

Fig. 2 Frequency estimation in unbalanced case. Phase a has an extra 0.1 p.u. magnitude at all times.

To compare the algorithms, the bias and variance in various noisy conditions were calculated (Figs. 3 and 4). For this, SNR is defined as $SNR \triangleq 10\ log_{10}(\frac{0.5}{\sigma_n^2})$ and bias term is computed as $Bias \triangleq \frac{1}{Card(C_s)}\sum_{k \in C_s}|\hat{f}(k) - f_0|$ where C_s is the convergence set. While after $t = 6\ s$, all algorithms were converged. So, we used $C_s = \{k|6 < k\Delta T < 8\}$ with the $Card(C_s) = 2f_s = 10000$. From Fig. 3 and Fig. 4, it is obvious that in noisy cases (i.e. $SNR < 40\ dB$), the ACLMS has the best result among three algorithms. But, in very low noise cases (i.e. $SNR > 40dB$), the MLMS algorithm outperforms ACLMS.
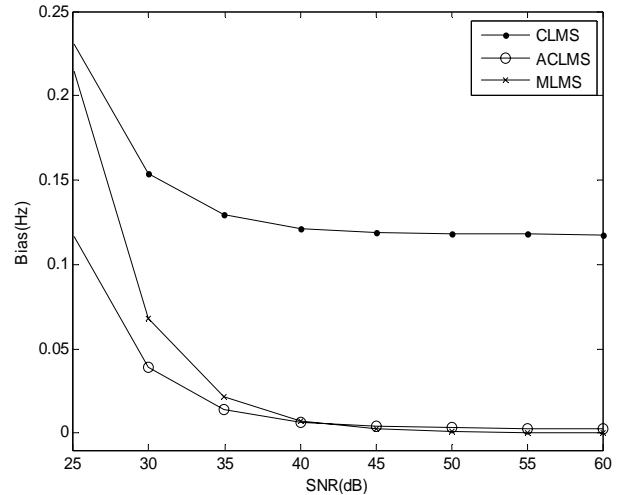

Fig. 3 Bias error of frequency estimation of three algorithms in various noisy conditions.
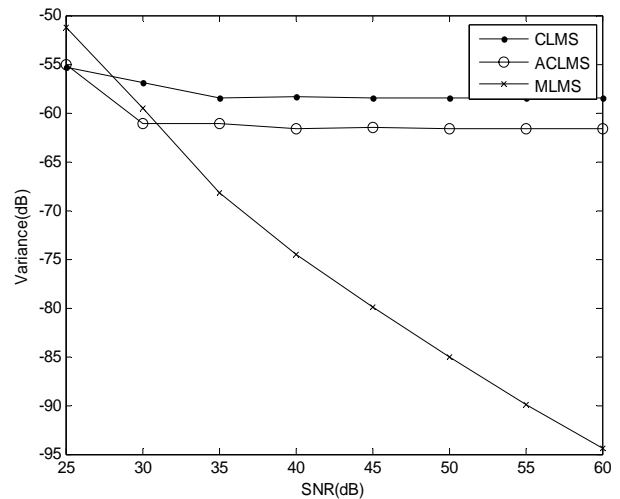

Fig. 4 Variance of frequency estimation of three algorithms in various noisy conditions.

5.3 Experiment 3

Third experiment was done to illustrate the advantage of the proposed MLMS algorithm over CLMS and ACLMS in sag conditions. The step size set to 0.0001, 0.001 and 0.01 for MLMS, CLMS and ACLMS, respectively. At first, the power system is balanced with $V_a(k) = V_b(k) = V_c(k) = 1$. At $t = 6$ s, a complete voltage sag is happened at phase a ($V_a(k) = 0, V_b(k) = V_c(k) = 1$).

The simulation results show that the MLMS algorithm converge faster than ACLMS after sag, while step sizes are selected in such that the ACLMS has faster convergence at balanced case. So, MLMS has higher tracking ability than ACLMS. Also, CLMS diverge after sag condition. The results are shown in Fig. 5.
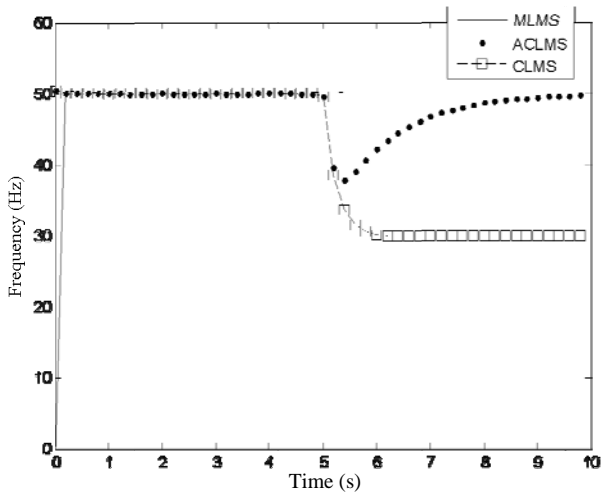
Fig. 5 The frequency estimation of three algorithms after a 100% voltage sag.

5.4 Experiment 4

Forth experiment was conducted to illustrate the sensitivity of algorithms to variations of amplitudes. Step sizes are set as μ=0.0001 for all three algorithms. At first, the power system is balanced. After converging the algorithms, at $t = 8$ s, a 1 Hz variation is imposed on three phases with different amplitudes. They are:

$$V_a(k) = 1 + 0.05\sin(2\pi t)$$
$$V_b(k) = 1 + 0.1\sin(2\pi t) \tag{32}$$
$$V_c(k) = 1 + 0.15\sin(2\pi t)$$

The estimated frequency is depicted in Fig. 6. After amplitude variation, ACLMS and CLMS started to drift from 50 Hz, but MLMS was not influenced by amplitude variation. Thus, MLMS is more robust to amplitude variation than ACLMS and CLMS.
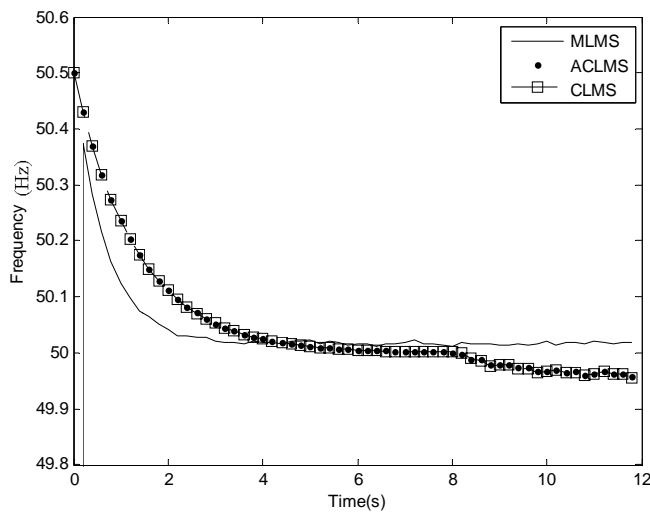

Fig. 6 Impact of oscillatory variation of amplitude on frequency estimation.
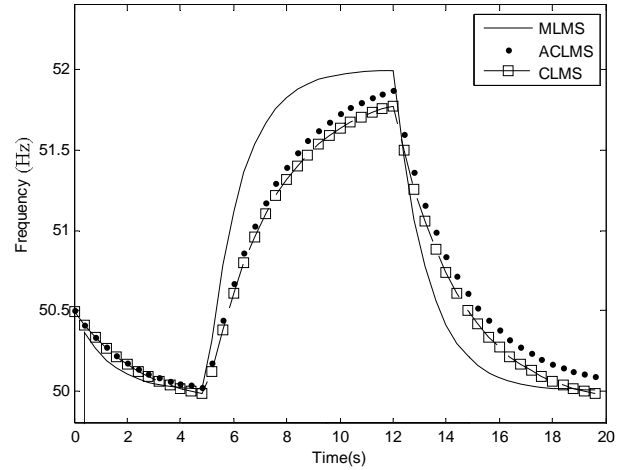

Fig. 7 Frequency estimation under sudden frequency changes.

Table 1 Run time of algorithms.

MLMS	CLMS	ACLMS
23.4ms	25.4ms	56.9ms

5.5 Experiment 5

In this experiment, frequency tracking ability of the algorithms was investigated using sudden frequency changes. The power system was in unbalanced condition with amplitudes $V_a(k) = 1.1, V_b(k) = V_c(k) = 1$. The frequency of power system was changed from 50 Hz to 52 Hz at $t = 5$ s. Then, the frequency was changed back to 50Hz at $t = 13$ s. All step sizes were set to 0.0001. The results are shown in Fig. 7. Because of unbalancedness, CLMS has a considerable bias for frequency estimation. But, ACLMS and MLMS have negligible bias. Also, MLMS has higher tracking ability than ACLMS.

5.6 Experiment 6

To illustrate the simplicity of the proposed algorithm in comparison to CLMS and ACLMS, the sixth experiment was performed. The power system was in unbalanced condition with amplitudes $V_a(k) = 1.1, V_b(k) = V_c(k) = 1$. The frequency of power system was set to 50 Hz. The initial guess of the frequency was assumed to be 50.2 Hz. A duration of 8 s was simulated and the average run time of all three algorithms was calculated and reported in Table 1. It confirms that the proposed algorithm is simpler than CLMS and ACLMS.

5.7 Experiment 7

To investigate the effect of the sampling frequency, the following experiment was performed. The system frequency is selected as 50 Hz and the initial value of all three algorithms was assumed to be 50.5 Hz. The variance of noise is assumed to be $\sigma_n = 0.001$. Step sizes of all algorithms were selected as 0.0001. Different values were selected for sampling frequency. For each sampling frequency, the final bias error was

measured. Also, the convergence time of the three algorithms, when the mean square error reaches the final value, were calculated. The final bias errors and convergence times of three algorithms are shown in Figs. 8 and 9, respectively. The former figure shows that at low sampling frequencies (less than 2 kHz), the suggested algorithm has less bias error than ACLMS algorithm. But, at higher sampling frequencies, the ACLMS algorithm outperforms the proposed algorithm. The latter figure shows that the fastest algorithm is the proposed MLMS algorithm. Moreover, the figure shows that by increasing the sampling frequency, the convergence time of all algorithms is decreased.

5.8 Experiment 8

To investigate the effect of the harmonics on three algorithms, this experiment was performed. Similar to [21], a balanced 10% third harmonic and a 5% fifth harmonic of the fundamental frequency $f = 50$ Hz, were added into unbalanced three-phase power system at $t = 0.05$ s. Fig. 10 shows the estimated frequency of
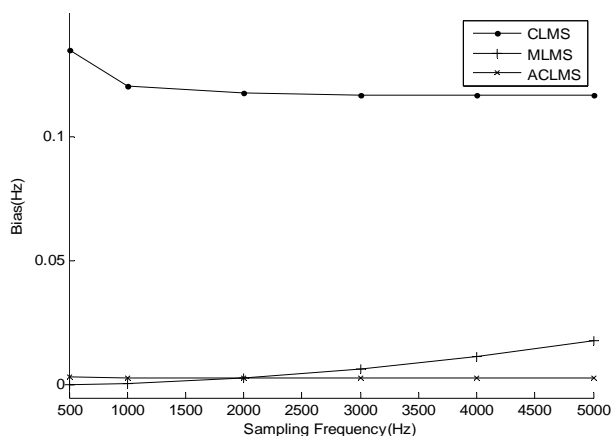

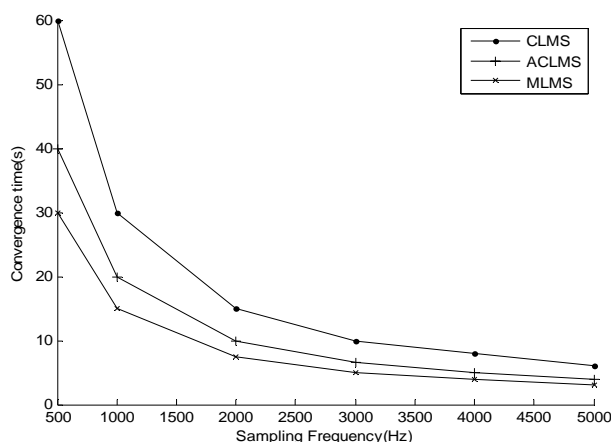**Fig. 8** Bias errors of three algorithms versus sampling frequency.

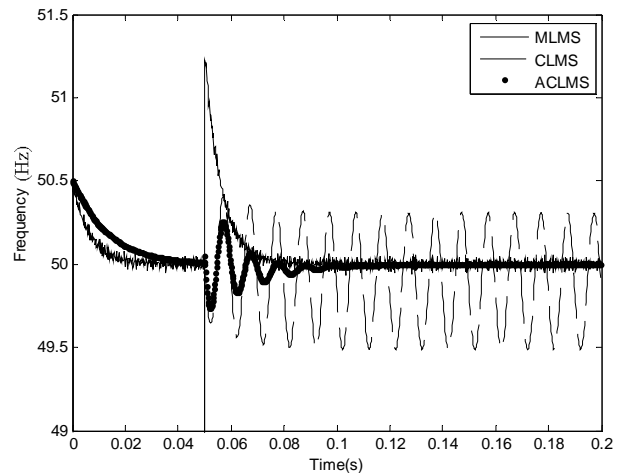**Fig. 9** Convergence times of three algorithms versus sampling frequency.

Fig. 10 Frequency estimation for phase voltages contaminated by a balanced 10% third harmonic and a 5% fifth harmonic.

the three algorithms. It shows the oscillatory response of ACLMS and CLMS algorithms and not oscillatory response of the proposed algorithm. It also shows the similar convergence time for ACLMS and MLMS, while ACLMS has a lower variation error around the final value.

6 Conclusions

In this article, a new LMS algorithm for frequency estimation was proposed. The proposed algorithm relies on a recursion which was proved in this paper. This recursion suggests to infer the next sample of the complex voltage based on two preceding samples instead of just previous sample in CLMS or based on previous sample and its conjugate in ACLMS. Extensive simulations show the advantages and drawbacks of the proposed algorithm in comparison to CLMS and ACLMS. Simulation experiments showed that the proposed algorithm has faster convergence, higher frequency tracking ability, less sensitivity to amplitude variation, more accuracy in very low noise cases and low sampling frequencies and less run time in comparison to CLMS and ACLMS. On the other hand, ACLMS outperforms our proposed algorithm in noisy cases and in high sampling frequencies. Besides, ACLMS has lower error variation in cases where harmonics are contaminated with three phases.

7 Nomenclature

LMS	Least Mean Square
CLMS	Complex Least Mean Square
ACLMS	Augmented Complex Least Mean Square
DSP	Digital Signal Processor
DFT	Discrete Fourier Transform
PLL	Phased Lock Loop
MVDR	Minimum Variance Distortionless Response

$v_a(k), v_b(k), v_c(k)$	Voltage of three phases at time instant k
$V_a(k), V_b(k), V_c(k)$	Voltage amplitudes of three phases at time instant k
f	System frequency
$\omega = 2\pi f$	Angular Frequency
f_s	Sampling frequency
$\Delta T = \frac{1}{f_s}$	Sampling interval
n_a, n_b, n_c	Noise voltages of three phases
v_0	Zero sequence voltage
v_α, v_β	Quadrature component voltages
v	Complex voltage
$\hat{f}(k)$	Estimated frequency at time instant k

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