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Research Paper

Synthesis of Antenna Arrays of Maximum Directivity for a Specified Sidelobe Level

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Abstract: Linear and planar antenna arrays are synthesized to have maximum directivity for a specified sidelobe level. The directivity is maximized subject to a given SLL. The beamwidth and the zeros of array factor are studied as well as the directivity. Maximum directivity-arrays are compared through some examples with super-directive, uniform, Dolph-Chebyshev and Riblet-Chebychev arrays to find a complete definition of optimum arrays. Also, the optimum value of *n*-bar is intuitively found for Taylor arrays.

Keywords: Antenna Array Synthesis, Dolph-Chebyshev Arrays, Riblet-Chebyshev Arrays, Maximum-Directivity Arrays, Taylor Arrays.

1 Introduction

SYNTHESIS of linear and planar antenna arrays with specified sidelobe levels, directivity and beamwidth is important for many applications such as communication and radar systems. The directivity, beamwidth and sidelobe level (SLL) are three important characteristics of antenna arrays which are dependent on the inter-distances between the antennas and the excitation currents of the antennas. In this regard, some facts are known:

1. Uniformly excited linear arrays have the maximum possible directivity when the inter-distances are equal or more than a half wavelength. The sidelobe level of uniform arrays is about -13.2 dB [1, 2].
2. Super-directive linear arrays have the maximum possible directivity when the inter-distances are less than a half wavelength. The sidelobe level of super-directive arrays is about -13.2 dB [1-3].
3. Dolph-Chebyshev arrays having inter-distances equal or more than half a wavelength, render equi-ripple sidelobes of arbitrary SLL. They have a minimum beamwidth for a given sidelobe level or have a minimum sidelobe level for a given beamwidth [1-2, 4].
4. Riblet-Chebyshev arrays having inter-distances less than half a wavelength, render equi-ripple sidelobes of arbitrary SLL. They have a minimum beamwidth for a given sidelobe level or have a minimum sidelobe level for a given beamwidth [1, 2, 5].

Here, we synthesize arrays, called Maximum-Directivity arrays, to have maximum possible directivity for a given or prefixed sidelobe level. Also, a comprehensive comparison between three types of arrays, namely maximum-directivity, Chebyshev and uniform arrays, is done. Then a complete definition of optimum arrays versus three parameters, sidelobe levels, directivity and beamwidth, is concluded. Finally, it is shown that Taylor arrays are near to maximum-directivity arrays. The optimum value of *n*-bar for Taylor arrays is found intuitively.

The paper is organized as follows: In Section II, linear arrays are synthesized optimally. In Section III, some examples are presented to synthesize maximum-directivity arrays. In Section IV, synthesis of planar array to have maximum directivity is addressed.

2 Synthesis of Maximum-Directivity Linear Arrays

A linear antenna array consists of N identical antennas of uniform inter-distances d on the z axis. The excitation current of the n-th antenna is $I_n = A_n \exp(jn\alpha_0)$ where $\alpha_0 = -kd\cos\theta_0$, in which θ_0 is the angle of maximum radiation. Also $k = 2\pi/\lambda$ where λ is the wavelength in the free space.

The array factor of a linear antenna array is given by

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$$F(\psi)=\sum_{n=0}^{N-1} A_n \exp(jn\psi) \tag{1}$$

where ψ is a real variable defined as $\psi = kd\cos\theta + \alpha_0$. A period of $F(\psi)$ is from $-\pi$ to $+\pi$ whereas its visible region is from $\psi = -kd + \alpha_0$ to $\psi = +kd + \alpha_0$.

The directivity of linear antenna arrays resulted from the visible region of $F(\psi)$, can be obtained from (A2) in the appendix as follows [1-3].

$$D=\frac{\left(\sum_{n=0}^{N-1} A_n\right)^2}{\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} A_n A_m \exp\left(j(n-m)\alpha_0\right)\mathrm{sinc}\left(\frac{kd}{\pi}(n-m)\right)} \tag{2}$$

The directivity in (2) can be made maximum by suitable adopting the amplitudes A_n so that the following error function gets minimum.

$$error = D^{-1} \tag{3}$$

Solely minimization of the error function in (3) gives us super-directive or uniform array, depending on the inter-distances. Super-directive arrays, like uniformly excited arrays, have a sidelobe level about -13.2dB irrespective of d, which is out of our control.

To make sidelobe level of a maximum-directive array equal to an arbitrary value, the error function (3) must be minimized subject to it. Hence, optimum arrays should be defined those that have maximum directivity subject to a specified sidelobe level, SLL. To design such optimum arrays, any appropriate constrained minimization method can be used. Here, we have used the "trust-region-reflective" constrained optimization algorithm which is available in the MATLAB environment as *fmincon* function.

3 Examples and Discussion

To verify the proposed method to synthesize maximum-directivity arrays, some examples are presented for broadside radiation, $\theta_0 = 90°$. It is seen from (2) that the angle of maximum radiation θ_0 has no effect on directivity when d is exactly equal to 0.5λ. Also, one can see that this angle has a little effect on the maximum-directivity when it is slightly changed about the broadside angle, i.e. $\alpha_0 = 0°$.

First, a linear array with $N = 21$ antennas of inter-distance $d = 0.5\lambda$ is designed to have maximum-directivity subject to SLL = -10, -15, -20, and -25dB. Figs. 1-4 compare the resultant patterns and their directivities with those of uniformly excited array which is super-directive, in fact. Directivity of uniformly excited arrays is about $2Nd/\lambda$ [1, 2]. Fig. 5 shows the required amplitude of the antennas. The maximum amplitude does not occur at the center antenna for SLL larger than -13.2dB. It is seen from Figs. 1-5 that the more SLL reduces the more the resulted patterns and amplitudes approache those of Dolph-Chebyshev's array.

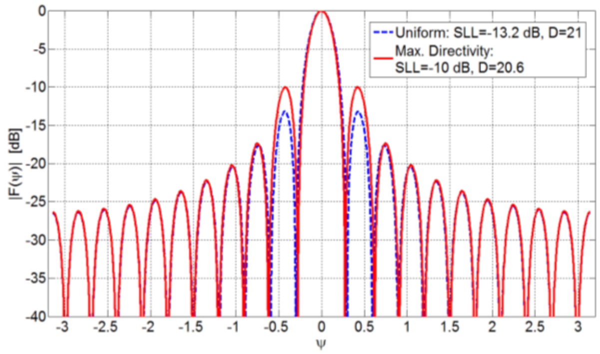

Fig. 1 Designed pattern of maximum-directivity for $d = 0.5\lambda$ and SLL = -10dB.

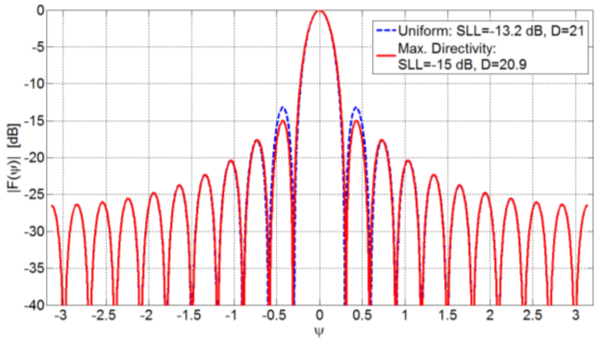

Fig. 2 Designed pattern of maximum-directivity for $d = 0.5\lambda$ and SLL = -15dB.

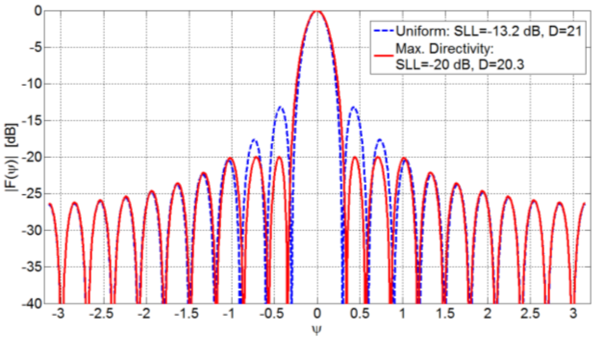

Fig. 3 Designed pattern of maximum-directivity for $d = 0.5\lambda$ and SLL = -20dB.

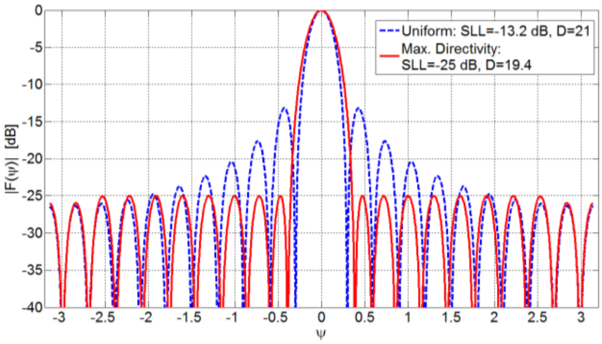

Fig. 4 Designed pattern of maximum-directivity for $d = 0.5\lambda$ and SLL = -25dB.

Second, the inter-distances are changed. Figs. 6 and 7 show the resultant patterns having maximum directivity and SLL = -20 dB for $d = 0.6\lambda$ and $d = 0.45\lambda$, respectively. The zeros of patterns tend to compress together and gather in the visible region, in the case of $d < 0.5\lambda$. This fact is also seeable in Fig. 8 which shows the locus of the zeros on the Schelkunoff's unit circle [6] for $d = 0.45\lambda$, $d = 0.5\lambda$, and $d = 0.6\lambda$. Also, Fig. 9, shows the super-directive, Riblet-Chebyshev (SLL = -20 dB) and Dolph-Chebyshev (SLL = -20 dB) patterns.

The required amplitudes of the antennas are shown in Figs. 5 and 10 for $d = 0.6\lambda$ and $d = 0.45\lambda$, respectively. It is seen from Fig. 10 that maximum-directivity arrays, like super-directive and Riblet-Chebyshev arrays, have fluctuating amplitudes when the inter-distances between the antennas is less than half a wavelength.

Third, Figs. 11 and 12 compare the directivity and null to null beamwidth of the synthesized arrays with those of Chebyshev arrays versus side lobe ratio SLR, respectively. It is seen that the resultant maximum directivity decreases as SLR increases beyond 13.2dB which corresponds to super-directive or uniform arrays. Also, for larger SLRs, the maximum directivity tends to directivity of Dolph-Chebyshev or Riblet-Chebyshev for $d \geq 0.5\lambda$ and $d < 0.5\lambda$, respectively. Analogous results hold for the beamwidth versus SLR. Also, Fig. 13 compares the directivity of the three maximum-directivity arrays having $N = 11$, 21, and 31 antennas with those of Chebyshev arrays for $d = 0.5\lambda$.

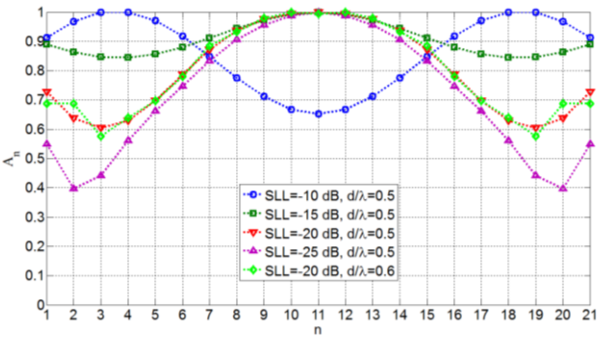

Fig. 5 Required amplitudes of antennas for maximum-directivity arrays for $d = 0.5\lambda$ and $d = 0.6\lambda$.

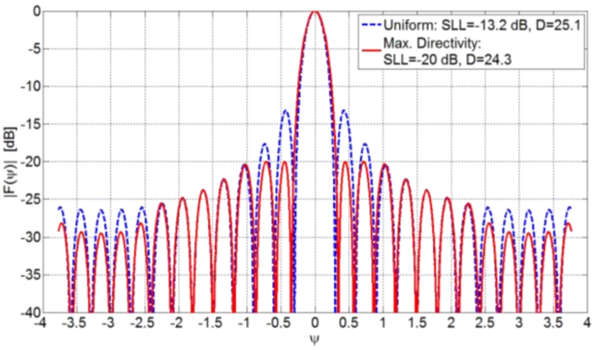

Fig. 6 Designed pattern of maximum-directivity for $d = 0.6\lambda$ and SLL = -20dB.

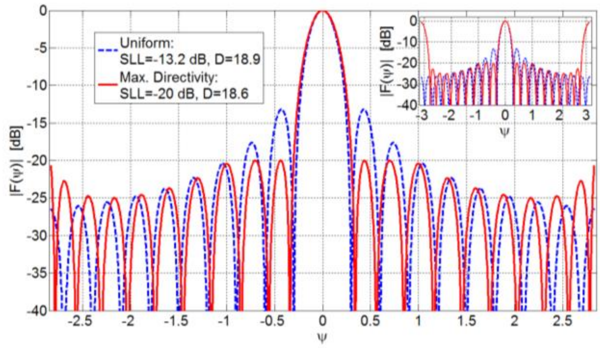

Fig. 7 Designed pattern of maximum-directivity for $d = 0.45\lambda$ and SLL = -20dB.

*Main figure: visible region, Sub-figure: region of periodicity.

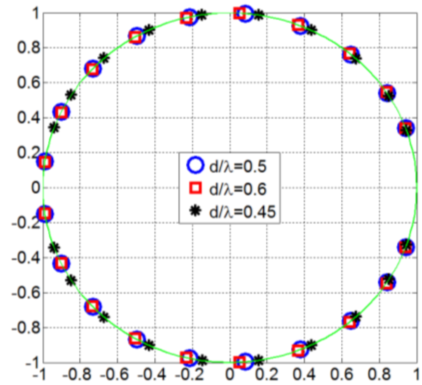

Fig. 8 Locus of zeros of synthesized patterns on the Schelkunoff's unit circle.

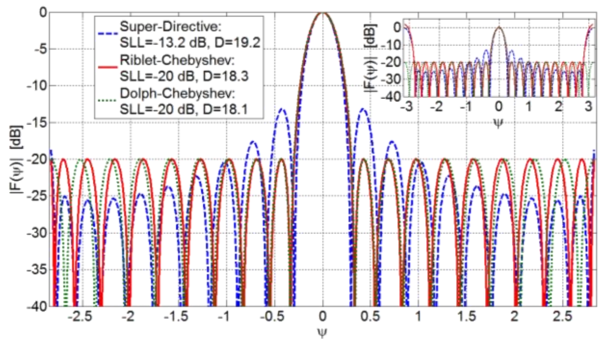

Fig. 9 Super-directive, Riblet-Chebyshev, and Dolph-Chebyshev patterns for $d = 0.45\lambda$ and SLL = -20dB.

*Main figure: visible region, Sub-figure: region of periodicity.

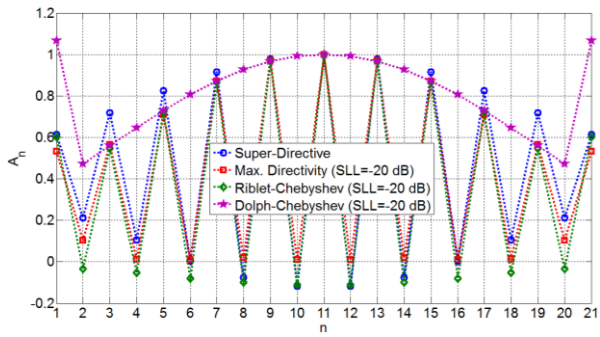

Fig. 10 Required amplitudes of patterns for $d = 0.45\lambda$.

Fourth, Figs. 14 and 15 show the ratio of the zeros of the array factor, ψ_{zn}, of maximum-directivity and Dolph-Chebyshev arrays, respectively, to those of uniform array, in the range of $0<\psi<\pi$ for N=21. Also, Fig. 16 shows the ratio of the zeros of the array factor, ψ_{zn}, of maximum-directivity array to those of Dolph-Chebyshev array. It is seen that the some first zeros of the maximum-directivity array are almost (not exactly) a constant proportion of those of Dolph-Chebyshev and the other following zeros are almost equal to those of uniform array. This property of maximum-directivity array is the same basis which is used to design Taylor n-bar arrays [1, 2]. In view of Figs. 2-4, 14, and 16, the optimum value of n-bar could be the number of that zero at which the sidelobe level of Dolph-Chebyshev starts to be smaller than that of uniform array. Hence, as desired SLL decreases the value of n-bar increases.

According to the above facts and figures, one can define optimum arrays as follows:

1. Optimum arrays in this sense that they have minimum beamwidth for a given sidelobe level are Chebysheve arrays. Dolph-Chebyshev for $d \geq 0.5\lambda$ and Riblet-Chebyshev for $d < 0.5\lambda$.
2. Maximum-directivity arrays are optimum arrays in this sense that they have maximum directivity for a given sidelobe level. Conversely, maximum-directivity arrays give minimum sidelobe level for a specified directivity.
3. Maximum-directive arrays tend to Chebyshev arrays for sidelobe levels less than a specified value, e.g., -17, -22 and -25 dB for N=11, 21 and 31 antennas, respectively, while d is 0.5λ.
4. The zeros of array factor of maximum-directivity arrays are almost the zeros of Taylor n-bar arrays.

4 Synthesis of Maximum-Directivity Planar Arrays

A rectangular planar antenna array can be comprised of $M\times N$ antennas on the xy plane. The excitation current of the mn-th element is $I_{mn} = A_{mn}\exp(jm\alpha_x + jn\alpha_y)$, where $\alpha_x = -kd_x \sin\theta_0 \cos\varphi_0$ and $\alpha_y = -kd_y \sin\theta_0 \sin\varphi_0$. The array factor of planar arrays is given by

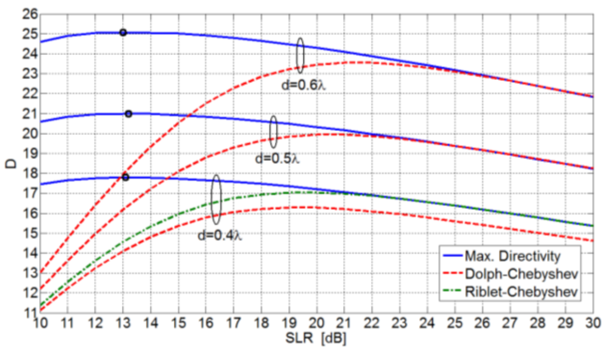

Fig. 11 Directivity of maximum-directivity arrays and Chebyshev arrays versus SLR for N = 21.
*O: uniform or super-directive arrays.

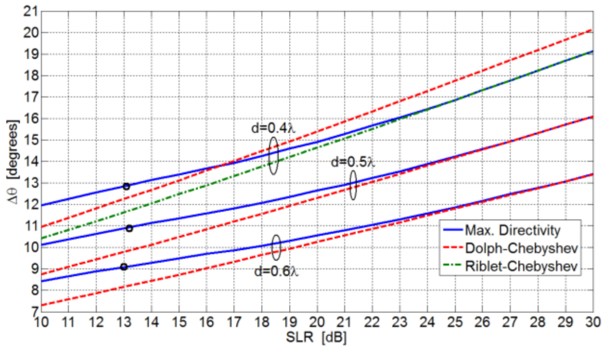

Fig. 12 Null to null beamwidth of maximum-directivity arrays and Chebyshev arrays versus SLR for N = 21.
*O: uniform or super-directive arrays.

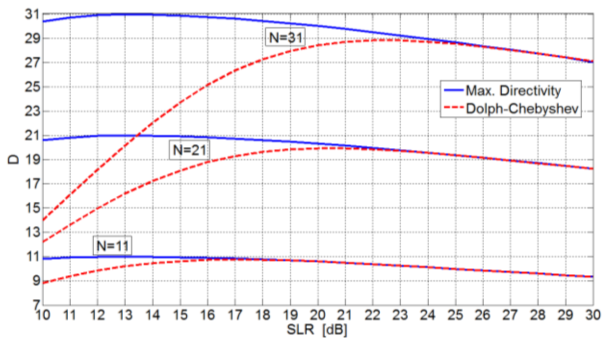

Fig. 13 Directivity of maximum-directivity arrays and Chebyshev arrays versus SLR for $d = 0.5\lambda$.

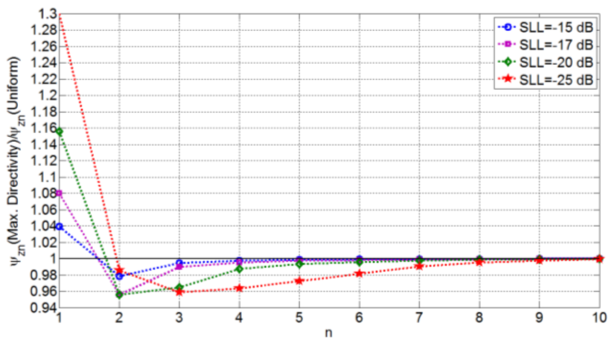

Fig. 14 The ratio of the zeros of the array factor of maximum-directivity array to those of uniform array.

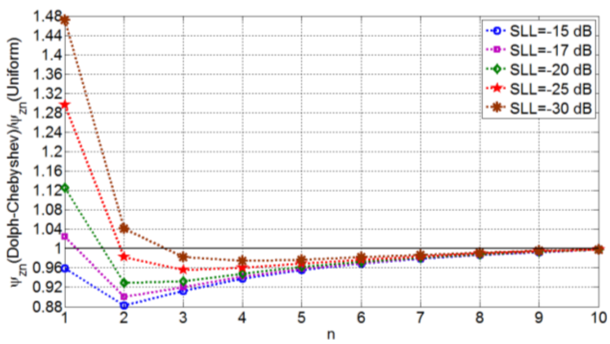

Fig. 15 The ratio of the zeros of the array factor of maximum-directivity array to those of uniform array.

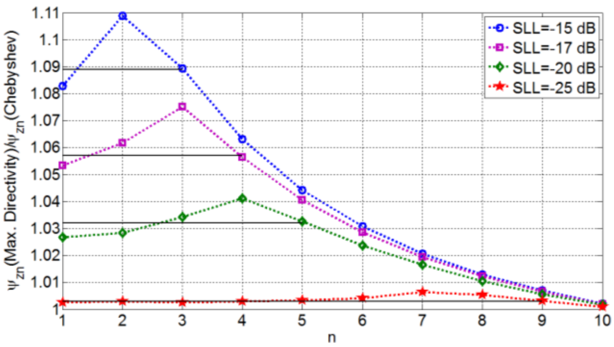

Fig. 16 The ratio of the zeros of the array factor of maximum-directivity array to those of Dolph-Chebyshev array.

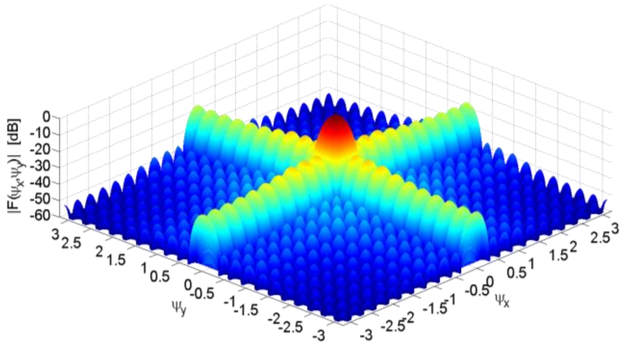

Fig. 17 Designed pattern of planar array of maximum directivity for $d = 0.5\lambda$ and SLL = -20dB.

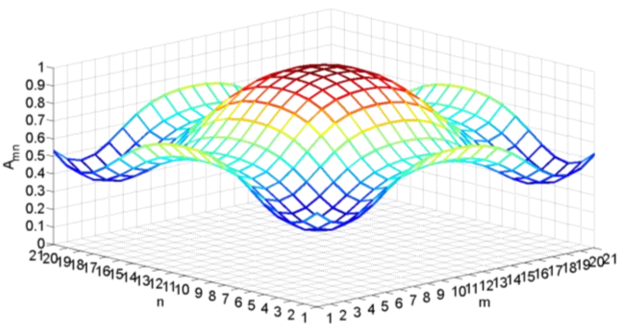

Fig. 18 Designed pattern of planar array of maximum directivity for $d = 0.5\lambda$ and SLL = -20dB.

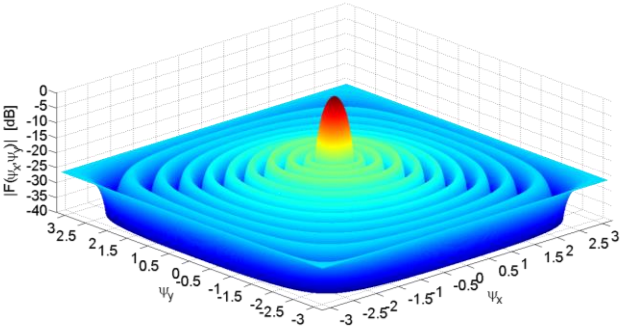

Fig. 19 Designed pattern of planar array of ring-type sidelobes for $d = 0.5\lambda$ and SLL = -20dB.

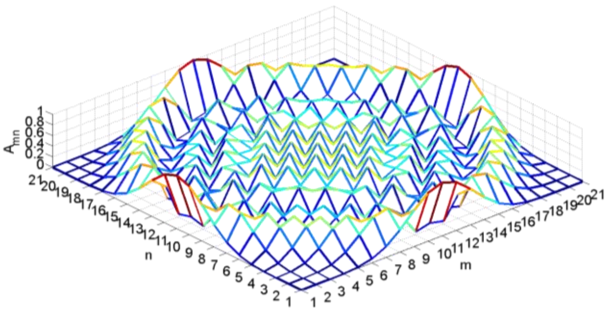

Fig. 20 Required amplitudes of antennas for planar array of ring-type sidelobes for $d = 0.5\lambda$ and SLL = -20dB.

$$F(\psi_x,\psi_y) = \sum_{m=0}^{M}\sum_{n=0}^{N} A_{mn} \exp\left(j(m\psi_x + n\psi_y)\right) \tag{4}$$

where ψ_x and ψ_y are real variables defined below.

$$\psi_x = kd_x \sin\theta\cos\varphi + \alpha_x \tag{5}$$

$$\psi_y = kd_y \sin\theta\sin\varphi + \alpha_y \tag{6}$$

Usually, the pattern of planar arrays are equated as the product of patterns of two similar linear arrays by supposing $A_{mn} = A_m A_n$. In this prevalent case, the array factor and directivity of the planar arrays are given as follows, respectively [1, 2, 7].

$$\begin{aligned} F(\psi_x,\psi_y) &= F_x(\psi_x)F_y(\psi_y) \\ &= \sum_{m=0}^{M} A_m \exp(jm\psi_x)\sum_{n=0}^{N} A_n \exp(jn\psi_y) \end{aligned} \tag{7}$$

$$D \cong \pi D_x D_y \cos\theta_0 \tag{8}$$

where $F_x(\psi_x)$ and $F_y(\psi_y)$ are associated linear arrays and D_x and D_y are their directivities, respectively. Equation (8) has been proved in detail in the Appendix, as (A5).

It is deduced from (8) that to maximize directivity of a planar array it is needed to maximize directivity of its associated linear arrays. So, optimum design of planar arrays reduces to optimum design of two linear arrays.

As an example, consider a planar array with 21×21 antennas of inter-distance $d = 0.5\lambda$ to have maximum directivity supposing SLL = -20dB. Considering Figs. 3 and 5, the array factor and required amplitudes of the antennas resulted in as shown in Figs. 17 and 18, respectively. Directivity of array factor shown in Fig. 17 is calculated as 1294 which accords with (8) considering $D_x = D_y = 20.3$.

A way to have square planar arrays having ring-type sidelobes is applying a suitable transformation to a linear array [8-10]. Fig. 19 shows synthesized planar array of ring-type sidelobes. Also, Fig. 20 shows the required amplitudes of antennas. Directivity of this pattern is calculated as 806 which is less than that resulted from product of two linear arrays. So, planar arrays synthesized by the product of two linear arrays have higher directivity than the ring-type planar arrays.

5 Conclusion

Linear antenna arrays were synthesized to have maximum directivity for a prefixed sidelobe level. It was concluded that Chebyshev arrays are better than other arrays when the beamwidth is more important than directivity for a given sidelobe level. However, the maximum-directivity arrays are better than other arrays when directivity is more important than beamwidth for a given sidelobe level. Also, it was shown that Taylor arrays are near to maximum-directivity arrays. The optimum value of *n*-bar for Taylor arrays is found intuitively. It was shown that maximum-directivity planar arrays can be obtained by two maximum-directivity linear arrays.

Appendix

Here, the directivity of planar arrays shown in (8) is proved.

The following relation is obtained from this definition $\psi = kd\cos\theta + \alpha_0$ for linear arrays.

$$d\psi = -kd\sin\theta\, d\theta \tag{A1}$$

Utilizing (A1), yields the following relation for directivity of linear arrays.

$$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} |\bar{F}(\theta)|^2 \sin\theta\, d\theta\, d\varphi} = \frac{2}{\int_0^{\pi} |\bar{F}(\theta)|^2 \sin\theta\, d\theta} = \frac{2kd}{\int_{-kd+\alpha_0}^{kd+\alpha_0} |\bar{F}(\psi)|^2\, d\psi} \tag{A2}$$

It is known from mathematics that there is a Jacobian determinant between two groups of surface differentials. Hence, the following relation is there.

$$d\psi_x d\psi_y = \begin{vmatrix} \dfrac{\partial\psi_x}{\partial\theta} & \dfrac{\partial\psi_x}{\partial\varphi} \\ \dfrac{\partial\psi_y}{\partial\theta} & \dfrac{\partial\psi_y}{\partial\varphi} \end{vmatrix} d\theta\, d\varphi = (kd_x)(kd_y)\cos\theta\sin\theta\, d\theta\, d\varphi \tag{A3}$$

Utilizing (A3), yields the following relation for directivity of planar arrays.

$$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi/2} |\bar{F}(\theta,\varphi)|^2 \sin\theta\, d\theta\, d\varphi} \cong \frac{4\pi(kd_x)(kd_y)\cos\theta_0}{\int_{-kd_y+\alpha_y}^{kd_y+\alpha_y}\int_{-kd_x+\alpha_x}^{kd_x+\alpha_x} |\bar{F}(\psi_x,\psi_y)|^2\, d\psi_x d\psi_y} \tag{A4}$$

The approximation in (A4) is related to large planar arrays which most of their radiation power is around the angle θ_0. Substituting (7) in (A4) and considering (A2), gives us the directivity of a planar array with respect to directivities of two linear arrays, as follows

$$D \cong \pi\cos\theta_0 \frac{2kd_x}{\int_{-kd_x+\alpha_x}^{kd_x+\alpha_x} |\bar{F}_x(\psi_x)|^2\, d\psi_x} \tag{A1}$$

$$\times \frac{2kd_y}{\int_{-kd_y+\alpha_y}^{kd_y+\alpha_y} |\bar{F}_y(\psi_y)|^2\, d\psi_y} = \pi D_x D_y \cos\theta_0 \tag{A5}$$

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