A New Approach for DOA Estimation of Unknown Non-Coherent Source Groups Containing Coherent Signals

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Abstract

In this paper, a new combination of minimum description length (MDL) or eigenvalue gradient method (EGM), joint approximate diagonalization of eigenmatrices (JADE) and modified forward-backward linear prediction (MFBLP) algorithms is proposed which determines the number of non-coherent source groups and estimates the direction of arrivals (DOAs) of coherent signals in each group. First, MDL/EGM algorithm determines the number of non-coherent signal groups, and then JADE algorithm estimates the generalized steering vectors considering white/colored Gaussian noise. Finally, MFBLP algorithm is applied to estimate DOAs of coherent signals in each group. Also, a new normalized root mean square error (NRMSE) is proposed that introduces a more realistic measure to compare the performance of DOA estimation methods. Simulations in MATLAB show that proposed algorithm can resolve sources regardless of QAM modulation size. In addition, simulations in white/colored Gaussian noises how that in wide range of SNRs, proposed algorithm performs better than JADE-MUSIC algorithm.

Keywords: DOA, JADE, MFBLP, MDL, EGM, RMSE.

1. Introduction

In wireless communications, electromagnetic waves experience fading and multipath phenomena and introduce coherent signals in receiver. Coherent signals cause rank loss of the spatial covariance matrix. Thereby, some second-order-statistics-based subspace methods, such as multiple signal classification (MUSIC) and estimation of signal parameters via rotational invariance techniques(ESPRIT) fail to resolve the signals correctly, and direction of arrival (DOA) estimation is not possible [1-2].

To handle the coherency problem, many effective methods such as spatial smoothing [3], maximum likelihood (ML) [4], deflation approach [5], method in [6] and method based on matrix pencil (MP) [7] are proposed. The spatial smoothing is based on a preprocessing scheme which divides original array into several overlap subarrays, computes covariance matrix for each subarray and applies high-resolution methods to the average of covariance matrices to resolve the

signals [3]. Another technique named as deflation approach, first estimates the uncorrelated signals and eliminates them and finally resolves the coherent signals [5]. Different technique presented in [6] utilizes symmetric configuration of uniform linear array (ULA) and constructs non-Toeplitz matrix to resolve signals. Method in [7] utilizes idea of MP on the spatial samples of the data. The MP method can find DOA of coherent signals without spatial smoothing. However, spatial smoothing requires more sensors for preprocessing and ML approach is computationally intensive. Deflation approach has significant loss of array aperture, because it cannot utilize all the constructed Toeplitz matrices [5] and [8]. Also, method in [6] suffers from burdensome computation and shortcoming of MP is its requirement to high signal to noise ratio (SNR).
Many other efforts based on fourth order cumulants (FOC) such as steering vectors DOA (SV-DOA) [9], modified MUSIC [10], methods in [11], [12], [13] and [14] have also been made for DOA estimation of coherent signals. SV-DOA, first, estimates the steering vectors blindly and then, it utilizes modified forward-backward linear prediction (MFBLP) to estimate DOA using the estimated vectors. Modified MUSIC in [10], utilizes switching matrix to construct a new covariance matrix. Method in [11], first, constructs data vector which contains all DOA information. Then, constructs rotation matrix and finally estimates signal DOAs. In [12], two algorithms joint approximate diagonalization of eigenmatrices (JADE) and MUSIC are combined, as JADE-MUSIC, to estimate source groups containing coherent signals. Method represented in [13], first, estimates the independent signals, and then eliminates them. Finally, it constructs FOC matrix of coherent signals and also resolves them. Method in [14], combines two algorithms, JADE and MFBLP to estimate signals. However, SV-DOA requires large number of snapshots. Computational complexity of methods in [10] and [12] due to spatial spectrum calculation and peak searching is high. Unfortunately, ESPIRIT method [11] needs special array geometry [11] and method in [13] requires large number of snapshots.

All above mentioned DOA estimation algorithms suppose that the number of coherent and/or non-coherent signals is known or it is determined, approximately. In order to evaluate the number of sources, Wax and Kailath suggested the use of the minimum description length (MDL) criterion [15]. Also, many other criteria such as eigenvalue gradient method (EGM) [16] and eigenincrement threshold (EIT) [17] are proposed. EGM criterion, first computes eigenvalues of auto-correlation matrix, then it determines number of sources by checking the difference between neighbor eigenvalues. EIT method utilizes a threshold to evaluate the eigenincrement between neighbor eigenvalues. According to simulation results of [18] for MDL and EGM methods, MDL is used for AWGN channel and EGM for colored noise.

In this paper, the approach of [14] is extended for signal groups. Unlike [14], number of signal groups is considered unknown. Thereby, here, first MDL/EGM is used to detect the number of coherent signal groups with considering white/colored Gaussian noise. Then, generalized steering vectors are estimated with JADE algorithm. Finally, MFBLP algorithm is used to estimate DOAs. Also, to evaluate the performance of the proposed method, its results are compared with

JADE-MUSIC equipped with MDL and EGM techniques as the determiner of the number of signal groups.

The notation$(.)^T$,$(.)^*$, $E(.)$, $(.)^\#$ and $(.)^H$ denote transpose, conjugate, expectation value, pseudo inverse and conjugate transpose, respectively. Also, vector symbols are shown as bolded small letters and matrix symbols as bolded capital letters.

This paper is organized as follows. In Section II, signal model for coherent signals is described. Section III presents MDL and EGM criteria. In Section IV, steering vector estimation and estimating DOAs with MFBLP algorithm are summarized. In section V, simulation results using MATLAB software package are presented. These results include comparison of DOA estimation performance of [12] and proposed algorithm. Finally, conclusions are given in section VI.

2. Signal Model

To begin, let us assume that N signals; $s_i(k)$; $i = 1,2,\dots,N$, impinge on an M-element antenna array from directions θ_i. Then, the received signal is

$$\boldsymbol{X}(k) = \boldsymbol{A}\boldsymbol{s}(k) + \boldsymbol{n}(k), \quad k = 1, 2, \dots, N_s \tag{1}$$

where N_s is the number of snapshots and $\boldsymbol{A}$ is the array response (manifold) matrix.

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1)\ \boldsymbol{a}(\theta_2) \dots \boldsymbol{a}(\theta_N)] \tag{2}$$

We suppose that a uniformly spaced linear sensor array is used. So, the steering vector, $\boldsymbol{a}(\theta_k)$, can be expressed as follows.

$$\boldsymbol{a}(\theta_k) = [1\ e^{\frac{j2\pi d sin\theta_k}{\lambda}} \dots e^{\frac{j2\pi d(M-1) sin\theta_k}{\lambda}}]^T \tag{3}$$

where d and λ are sensor inter-element spacing and signal wavelength, respectively. In equation (1), $\boldsymbol{s}$ is a vector of signals

$$\boldsymbol{s}(k) = [s_1(k)s_2(k) \dots s_N(k)]^T \tag{4}$$

and $\boldsymbol{n}$ is a vector of additive Gaussian noise.

$$\boldsymbol{n}(k) = [n_1(k)n_2(k) \dots n_M(k)]^T \tag{5}$$

Fourth-order cumulants of Gaussian signals are zero [19]. Therefore, we assume signals are statistically independent and they are non-Gaussian zero-mean complex random processes with symmetric distribution. Also, suppose that there are K statistically independent groups that each group contains G coherent signals. Hence, source vector is given by

$$\boldsymbol{s}(k) = [s_1(k)s_2(k) \dots s_K(k)]^T \tag{6}$$

And

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_{ef_1})\ \boldsymbol{a}(\theta_{ef_2}) \ldots \boldsymbol{a}(\theta_{ef_K})] \tag{7}$$

For instance

$$\boldsymbol{a}(\theta_{ef_1}) = \sum_{m=1}^{G} \alpha_m . a(\theta_{\boldsymbol{m}}) \tag{8}$$

where, α_m is a complex scalar that describes the gain and phase associated to path in coherent signals. Therefore, G signals will be appeared as a single signal that it arrives from new $\theta_{\boldsymbol{ef}}$ direction.

3. Determining the number of sources

An MDL criterion is used to determine the number of independent groups in AWGN scenario. The MDL criterion cannot determine the number of sources in colored noise scenario [18]. So, EGM criterion is used to this purpose in colored noise scenario.

3.1 MDL criterion

Steps of MDL are presented as follows [15].

1) Form the array covariance matrix

$$\boldsymbol{R} = E\{\boldsymbol{X}(t)\boldsymbol{X}^H(t)\} \tag{9}$$

2) The number of groups is determined as the value of $k \in \{0,1, \ldots, M\}$ that MDL is minimized.

$$MDL(k) = -\log \left(\frac{\prod_{i=k+1}^{M} \lambda_i^{1/(M-k)}}{\frac{1}{M-k}\sum_{i=k+1}^{M} \lambda_i}\right)^{(M-k)N_s} + \frac{1}{2}k(2M-k)logN_s \tag{10}$$

where $\lambda_1 > \lambda_2 > \cdots > \lambda_M$ are eigenvalues of the covariance matrix R.

3.2 EGM criterion

Steps of EGM are presented as follows [16].

1) Find the average eigenvalues of the covariance matrix $\boldsymbol{R}$.

$$\Delta\bar{\lambda} = \frac{\lambda_1 - \lambda_M}{M-1} \tag{11}$$

2) Calculate the gradients of all neighbor eigenvalues.

$$\Delta\lambda_i = \lambda_i - \lambda_{i+1}\ ; i = 1, \ldots, M-1 \tag{12}$$

3) Find i_0. i_0 is first i that it satisfy $\Delta\lambda_i \le \Delta\bar{\lambda}$. Number of sources is $i_0 - 1$.

4. Steering vector estimation using JADE algorithm

JADE algorithm is applied to estimate the generalized steering vectors. It is summarized as follows [12].

1) Compute whitening matrix $\boldsymbol{W}$. Whitening process can be expressed as

$$\boldsymbol{Z}(t) = \boldsymbol{W}\boldsymbol{X}(t) \tag{13}$$

2) Form fourth order cumulants of $\boldsymbol{Z}(t)$ as follows.

$$Cum(z_{k_1}, z_{k_2}, z_{l_1}, z_{l_2}) = E\{z_{k_1} z_{k_2} z_{l_1}{}^* z_{l_2}{}^*\} - E\{z_{k_1} z^*{}_{l_1}\}E\{z_{k_2} z^*{}_{l_2}\} - E\{z_{k_1} z^*{}_{l_2}\}E\{z_{k_2} z^*{}_{l_1}\} \tag{14}$$

3) Jointly diagonalize the set $\{\lambda_{zr}, M_{zr}, r = 1,2, \dots, D\}$ by a unitary matrix $\boldsymbol{U}$ that eigenpairs $\{\lambda_{zr}, M_{zr}\}$ corresponds to the D largest eigenvalues.
4) Estimate the array response matrix

$$\boldsymbol{A}' = \boldsymbol{W}^{\dagger}\boldsymbol{U} \tag{15}$$

that $\boldsymbol{W}^{\dagger}$ is Moore-Penrose inverse of whitening matrix. Equation (3) can be expressed as follows.

$$\boldsymbol{a}(\theta_k) = [1\ e^{i\omega_k}\ e^{i2\omega_k} \dots\ e^{i(M-1)\omega_k}]^T \tag{16}$$

with

$$\omega_k = \frac{2\pi f d sin\theta_k}{c} \tag{17}$$

where f is carrier frequency of signals and c is the speed of wave propagation.

5. DOA estimation using MFBLP algorithm

In this paper, MFBLP algorithm is used to estimate DOA of sources. MFBLP algorithm is summarized as follows [9]

1) Choose L for each $M \times 1$ estimated steering vector $\widehat{\boldsymbol{a}}$ that L must satisfy equation (18). In each $\widehat{\boldsymbol{a}}$, there are G coherent signals. Then, Form matrix $\boldsymbol{Q}$ and vector $\boldsymbol{h}$. $\boldsymbol{Q}$ is $2(M-L) \times L$ matrix and h is $2(M-L) \times 1$ vector.

$$G \le L \le M - \frac{G}{2} \tag{18}$$

$$\boldsymbol{Q} = \begin{bmatrix} \hat{a}_L & \hat{a}_{L-1} & \cdots & \hat{a}_1 \\ \hat{a}_{L+1} & \hat{a}_L & \cdots & \hat{a}_2 \\ \hat{a}_{L+2} & \hat{a}_{L+1} & & \hat{a}_3 \\ \vdots & \vdots & & \vdots \\ \hat{a}_{M-1} & \hat{a}_{M-2} & & \hat{a}_{M-L} \\ \hat{a}_2{}^* & \hat{a}_3{}^* & & \hat{a}_{L+1}{}^* \\ \hat{a}_3{}^* & \hat{a}_4{}^* & \cdots & \hat{a}_{L+2}{}^* \\ \hat{a}_4{}^* & \hat{a}_5{}^* & & \hat{a}_{L+3}{}^* \\ \vdots & \vdots & & \vdots \\ \hat{a}_{M-L+1}{}^* & \hat{a}_{M-L+2}{}^* & & \hat{a}_M{}^* \end{bmatrix} \tag{19}$$

$$\boldsymbol{h} = [\hat{a}_{L+1}\hat{a}_{L+2} \ldots \hat{a}_M \hat{a}_1{}^* \hat{a}_2{}^* \ldots \hat{a}_{M-L}{}^*]^T \tag{20}$$

2) Compute a singular value decomposition of Q as
$$\boldsymbol{Q} = \boldsymbol{U}\Lambda\boldsymbol{V}^H \tag{21}$$
Then, set $L - G$ smallest singular values on the diagonal of Λ to 0 and name new matrix $\boldsymbol{\Sigma}$. Dimension of $\boldsymbol{\Sigma}$ is same as dimension of Λ.
3) Compute g as follows
$$\boldsymbol{g} = [g_1 g_2 \cdots g_L]^T = -\boldsymbol{V}\boldsymbol{\Sigma}^{\#}\boldsymbol{U}^H\boldsymbol{h} \tag{22}$$
Then, find roots of the polynomial as
$$H(o) = 1 + g_1 o^{-1} + g_2 o^{-2} + \cdots + g_3 o^{-L} \tag{23}$$

$H(o)$ has L zeros, $o_i = be^{j\omega_k}; i = 1,2, \ldots, L$. G zeros of $H(o)$ lie on unit circle, i.e., $b = 1$.So, G unknown frequencies ω_k are determined. Finally, θ_k can be computed using (17).

6. Proposed Algorithm

A novel approach to joint determining the number and DOA estimation of coherent signals in non-coherent groups is proposed as a combination of MDL/EGM, JADE and MFBLP. In proposed algorithm, MDL and EGM methods are utilized to determine the number of non-coherent groups for white noise and colored noise, respectively. Therefore, a decision stage is utilized to determine the kind of noise. If the power of noise at all frequencies is static then noise is white, otherwise, noise is nonwhite or colored. In other word, the auto-correlation function of a white noise process with a variance of σ^2 is a delta function. The steps of proposed algorithm are illustrated in Table 1.

Table 1. Summary of Proposed DOA Estimation Method

Given	A received signal $\boldsymbol{X}$
Step 1.	Apply MDL/EGM method to $\boldsymbol{X}$ and determine the number of non-coherent source groups.

Step 2. Estimate generalized steering vectors with JADE algorithm. Step 3. Use MFBLP algorithm to estimate DOAs.

7. Simulation Results

In this section, several sets of simulation results are provided to demonstrate the performance of proposed algorithm. JADE-MUSIC Algorithm is selected to be comparative method. In all simulations, a uniform linear array (ULA) with relative inter-element spacing $(\frac{d}{\lambda} = \frac{1}{2})$ is considered and 12 signals impinge on the array. Three source groups of coherent signals are considered which their multipath fading coefficients are $[1, -0.6426 + 0.7266j, 0.8677 + 0.0632j, 0.7319 - 0.1639j]$, $[1, -0.8262 + 0.4690j, 0.1897 - 0.8593j, 0.2049 - 0.7630j]$, $[1, -0.1681 - 0.9045j, -0.7293 - 0.1750j, 0.6102 + 0.1565j$, respectively [12]. Simulations are organized in three sections. In section A, we consider five examples. The results of these examples are summarized in Table 2. In sections B and C, new approach is compared with JADE-MUSIC algorithm in two scenarios of noise, AWGN and colored.

These algorithms are simulated in MATLAB 7.11 software. A PC with an Intel Core i5-2400, 3.1 GHz CPU, and 4 GB RAM is used to run computer codes. Computational time of JADE-MUSIC algorithm for estimation of DOAs of three groups of coherent signals in $SNR = 10dB$ is 968.906828 seconds. However, this time for proposed algorithm is 95.567817 seconds. So, computational time of our algorithm is about 10 times less than computation time of JADE-MUSIC algorithm in similar conditions.

7.1 Five examples

In this section, 5 examples show the effectiveness of proposed method. Two measures, mean value and standard deviation of DOA estimation for each group are defined as

$$Mean = \frac{1}{L}\sum_{k=1}^{L} \hat{\theta}_i(k) \tag{24}$$

$$Standard\ deviation = \sqrt{\frac{1}{LN}\sum_{k=1}^{L}\sum_{i=1}^{N}(\hat{\theta}_i(k) - Mean)^2} \tag{25}$$

where θ_i contains G signal of one group (i.e., N signals of all groups) and $\hat{\theta}_i(k)$ is the estimate of θ_i for the kth Monte Carlo trial and L is the number of Monte Carlo trials.

In examples 1-4, three groups of coherent signals from $[10^\circ, 20^\circ, 28^\circ, 45^\circ]$, $[5^\circ, 25^\circ, 35^\circ, 55^\circ]$ and $[40^\circ, 60^\circ, 15^\circ, 30^\circ]$, AWGN noise, $N_s = 3000$, $SNR = 10dB$ and $M = 10$ are considered. Also, 1000 Monte-Carlo runs are made and modulation type is changed for each group. Examples 1-4 show that the standard deviation of each group depends on its QAM modulation size.

Table 2. Mean value and standard deviation of estimated DOAs for different QAM modulation sizes

$$(N_s = 3000, SNR = 10dB, L = 1000, M = 10)$$

Example	Original DOAs Group 1	2	3	Mean value of estimated DOAs Group 1	2	3	Standard deviation of estimated DOAs Group 1	2	3	QAM sizes Group 1	2	3
1	10	5	40	9.9602	4.9959	40.0356	0.6451	0.0640	0.3040	4	4	4
	20	25	60	19.5954	24.9967	60.0072						
	28	35	15	27.7200	35.0150	14.9507						
	45	55	30	45.1492	55.0040	29.8760						
2	10	5	40	9.9989	4.9984	40.0084	0.2341	0.1439	0.3724	16	4	4
	20	25	60	20.0110	24.9964	59.9959						
	28	35	15	28.0414	34.9918	14.9911						
	45	55	30	44.9913	54.9985	29.9084						
3	10	5	40	10.0023	4.9998	39.9589	0.3142	0.0583	1.5370	16	16	4
	20	25	60	20.0617	25.0060	59.9792						
	28	35	15	28.1217	34.9959	14.9041						
	45	55	30	44.9772	55.0033	29.7219						
4	10	5	40	9.9486	4.9960	40.0351	0.8234	0.0678	0.4101	16	16	16
	20	25	60	19.6705	24.9958	60.0196						
	28	35	15	27.9217	35.0098	14.9537						
	45	55	30	45.1371	54.9964	29.9010						
5	10	5	30	9.9724	4.9922	30.0244	0.3604	0.0922	0.4341	4	4	4
	20	20	10	19.6927	19.9723	10.0076						
	28	35	20	27.6218	34.9877	19.8944						
	45	55	40	45.1076	55.0101	40.0127						

In example 5, it is assumed that three groups of coherent signals coming from $[10^\circ, 20^\circ, 28^\circ, 45^\circ]$, $[5^\circ, 20^\circ, 35^\circ, 55^\circ]$ and $[30^\circ, 10^\circ, 20^\circ, 40^\circ]$. In other word, three independent signals approach the array from exactly the same direction (i.e., 20°) and two independent signals approach from direction 10°. This algorithm can estimate DOAs relatively well because coherent signals in each group are combined to form a steering vector. It means, three steering vectors are distinct from each other.

7.2 Comparison of proposed method and JADE-MUISC considering AWGN

In this section, it is assumed that three groups of coherent signals approach the array of sensors at $[10^{\circ},20^{\circ},28^{\circ},45^{\circ}]$, $[5^{\circ},20^{\circ},35^{\circ},55^{\circ}]$ and $[30^{\circ},10^{\circ},20^{\circ},40^{\circ}]$, respectively. Modulation size of each coherent signal group is 4-QAM and $SNR = 10dB$. The MUSIC spectrums for each group by JADE-MUSIC algorithm are shown, in Figures 1-3. As depicted in Figure 1, JADE-MUSIC can resolve first coherent signal groups. But, Figures 2, 3, show that JADE-MUSIC algorithm cannot resolve one and two signals of second and third groups, respectively. However, estimated DOAs by proposed algorithm is more accurate with respect to JADE-MUSIC algorithm.

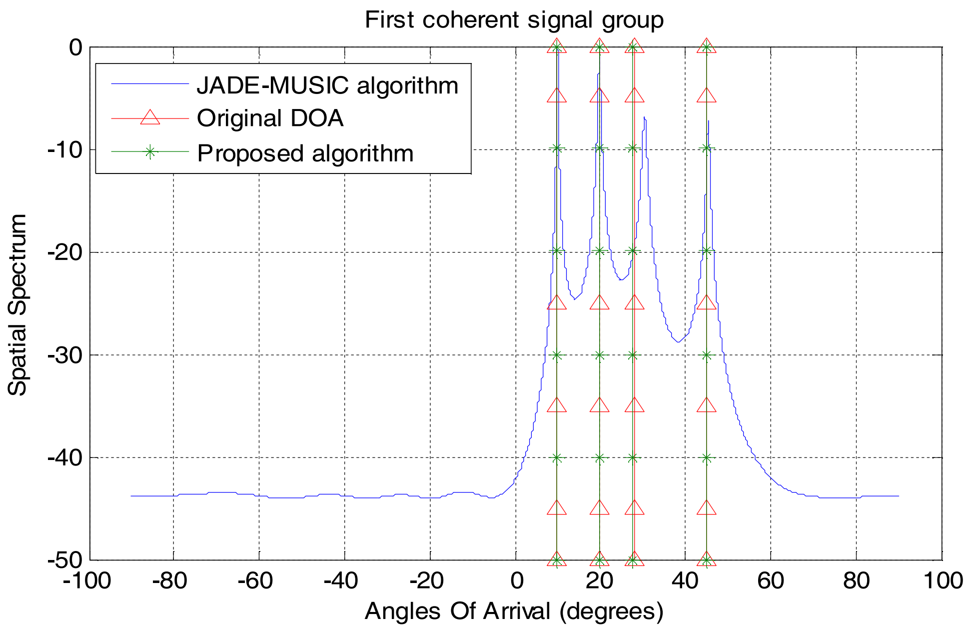

Figure 1. MUSIC spectrum for first coherent signal group

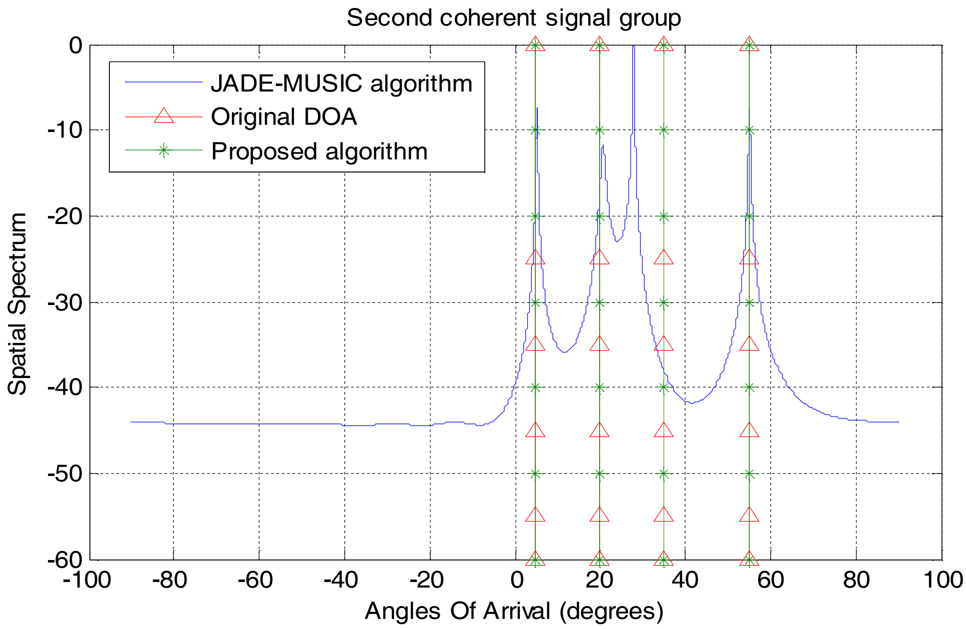

Figure 2. MUSIC spectrum for second coherent signal group

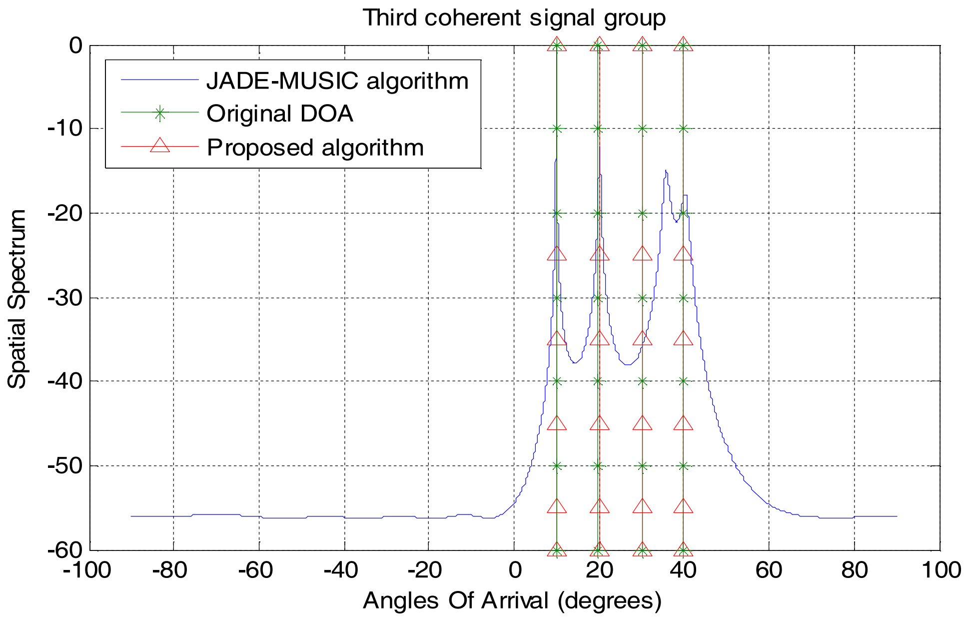

Figure 3. MUSIC spectrum for third coherent signal group

To better comparison of two algorithms, root mean square error (RMSE) is utilized as performance metric. RMSE is defined as

$$RMSE = \sqrt{\frac{1}{LN}\sum_{k=1}^{L}\sum_{i=1}^{N}(\hat{\theta}_i(k) - \theta_i)^2} \tag{26}$$

where $\hat{\theta}_i(k)$ is the estimate of θ_i for the kth Monte Carlo trial and N is the total number of signals.L is the number of Monte Carlo trials. To calculate RMSE, according to previous works [1], [3], [6], [9], [12], it is considered that one original DOA is estimated two times when the number of estimated DOAs is less than original DOAs. However, the algorithm cannot resolve those DOAs, so RMSE is not accurate performance measure; especially, when two original DOAs are close to each other. For example, when two sources coming from 20° and 25° and the algorithm could not resolve 25°, then it is considered that algorithm estimates 20° two times. It means, a new measure is needed that can show the performance of DOA estimation algorithms, accurately. In this research, this performance metric is suggested, namely Shirvani-Ebadi Normalized RMSE (SE-NRMSE).

To define SE-NRMSE, consider the algorithm resolves H sources, where $H < N$. It is assumed that if $\left|\hat{\theta}_{ij} - \theta_{ij}\right| < Th_j; i = 1, \dots, G, j = 1, \dots, K$. So the algorithm could resolve the signals. It is also assumed that the total number of angles that satisfy $\left|\hat{\theta}_{ij} - \theta_{ij}\right| < Th_j$ is $; \leq H$. Where θ_{ij} is ith θ in jth group and Th_j is threshold for jth group of coherent signals. In other word, Th_j is defined as half of the difference between two nearest angles in jth group. Thereby, $NMSE_{ij}$ is calculated by equation (27).

$$NMSE_{ij} = \left(\frac{\hat{\theta}_{ij}(k) - \theta_{ij}}{Th_j}\right)^2; i = 1, \dots, G, j = 1, \dots, K \tag{27}$$

Note that $0 \leq NMSE_{ij} \leq 1$. In equation (27), if $\hat{\theta}_{ij} = \theta_{ij}$ then $NMSE_{ij} = 0$ and if $\hat{\theta}_{ij} = \theta_{ij} \pm Th_j$ then $NMSE_{ij} = 1$.

$$\boldsymbol{nmse} = [NMSE_{ij}]_{1\times F} \tag{28}$$

If $\left|\hat{\theta}_{ij} - \theta_{ij}\right| \geq Th_j; i = 1, \dots, G$ and $j = 1, \dots, K$, so the algorithm could not resolve the signals. Total number of angles that satisfy $\left|\hat{\theta}_{ij} - \theta_{ij}\right| \geq Th_j$ is $H - F$. Then $NRMSE_{ij}$ is given by equation (29).

$$NRMSE_{ij} = \frac{1}{N} \; ; \; i = 1, \dots, G \tag{29}$$

DOA estimation algorithm also could not resolve $N - H$ signals, especially in low SNRs and/or small number of snapshots. So, $NRMSE_{ij}$ of these signals is calculated by equation (29) again. Thereby, NRMSE for each Monte Carlo trial is calculated by equation (30)

$$NRMSE(l) = \frac{N-H}{N} + \frac{H-F}{N} + \frac{\sqrt{F}}{N}\sqrt{\sum_{f=1}^{F} NMSE(f)}$$

$$= 1 - \frac{F}{N}\left(1 - \sqrt{\frac{\sum_{f=1}^{F} NMSE(f)}{F}}\right); l = 1,2, \dots, L \tag{30}$$

where $NMSE(f)$ is fth element of $\boldsymbol{nmse}$ in equation (28). Finally, total SE-NRMSE is defined as

$$SE - NRMSE = \frac{1}{L}\sum_{l=1}^{L} NRMSE(l) \tag{31}$$

RMSE and SE-NRMSE versus SNR are shown in Figures 4 and 5, respectively. In these simulations, 200 Monte-Carlo runs with $N_s = 2000$ are made. As shown in Figure 4, when SNR increases from $-2dB$, proposed algorithm begins to demonstrate smaller RMSE than JADE-MUSIC algorithm. When SNR is below $-2dB$, RMSE of proposed algorithm is a bit more than RMSE of JADE-MUSIC algorithm.

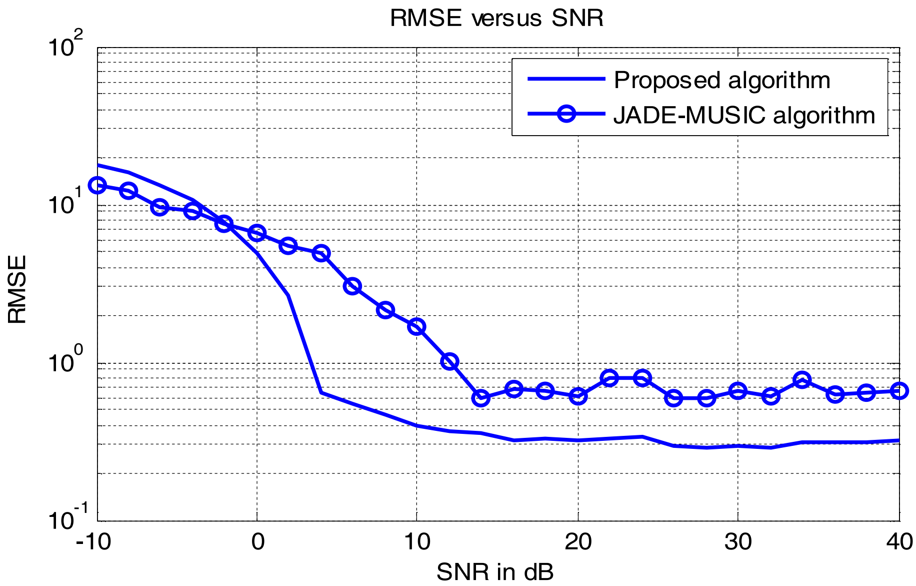

Figure 4. RMSE versus SNR for AWGN scenario

To more accurate look at the performance of two algorithms, SE-NRMSE is plotted in Figure 5. Figure 5 illustrates that SE-NRMSE of proposed algorithm is less than SE-NRMSE of JADE-MUSIC algorithm in all SNR values.

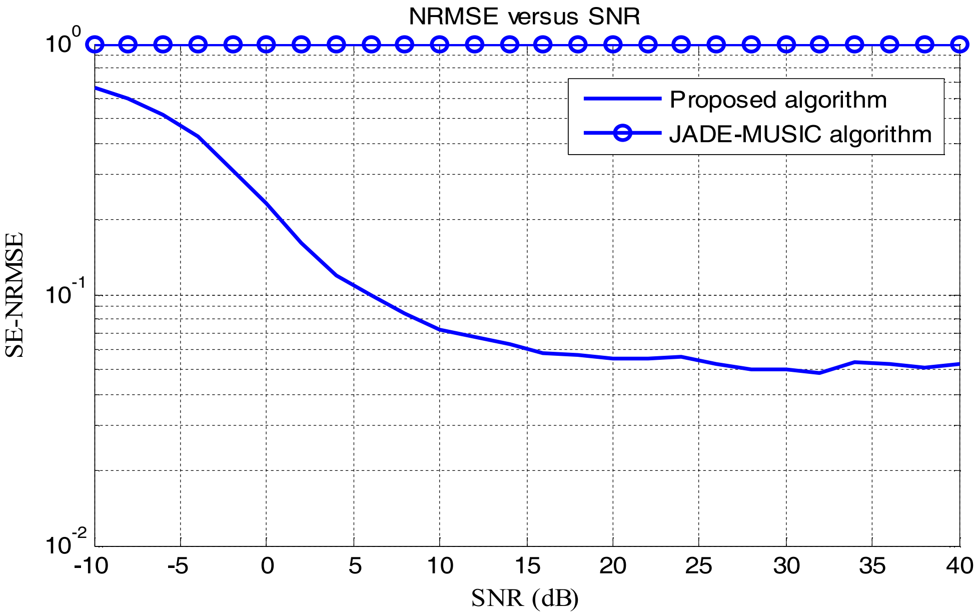

Figure 5. SE-NRMSE versus SNR for AWGN scenario

7.3 Comparison of proposed method and JADE-MUSIC considering colored noise

In this section, it is assumed that three groups of coherent signals approach the array of sensors at $[0^\circ, 10^\circ, 20^\circ, 28^\circ]$, $[25^\circ, 33^\circ, 10^\circ, 19^\circ]$ and $[30^\circ, 8^\circ, 19^\circ, 26^\circ]$, respectively. Modulation size of each coherent signal group is 4-QAM, $SNR = 10dB$ and noise is colored Gaussian. As we know, the power spectral density of the colored noise is not uniform across the entire frequency spectrum. Hence, the colored Gaussian additive noise is generated by passing the AWGN through a first-order auto regressive (AR1) filter with coefficienta_0. Correlation coefficient a_0, is 0.7 and 200 Monte-Carlo runs with $N_s = 2000$ is made.

Figure 6 shows that MDL criterion cannot detect the number of coherent signal groups in colored Gaussian noise scenario. It also shows that detection probability of EGM criterion is 100%, in $SNR \geq -2dB$. So, in $SNR \leq -2dB$, it is assumed that the number of coherent signal groups is known and in $SNR \geq -2dB$, the number of independent groups is detected by EGM, exactly. Hence, in colored noise scenario, EGM is selected as the determiner of the number of source groups.

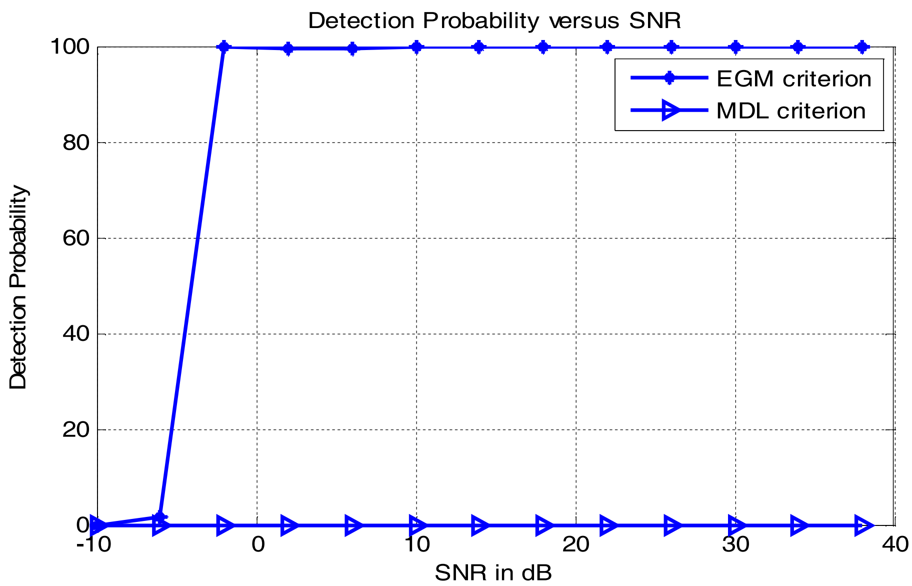

Figure 6. Detection Probability versus SNR for AWGN scenario

As shown in Figure 7, RMSE of proposed algorithm in $SNR > -6dB$ is lower than JADE-MUSIC algorithm and vice versa. Figure 8 shows that, in wide range of SNRs proposed algorithm demonstrate smaller SE-NRMSE than JADE-MUSIC algorithm.

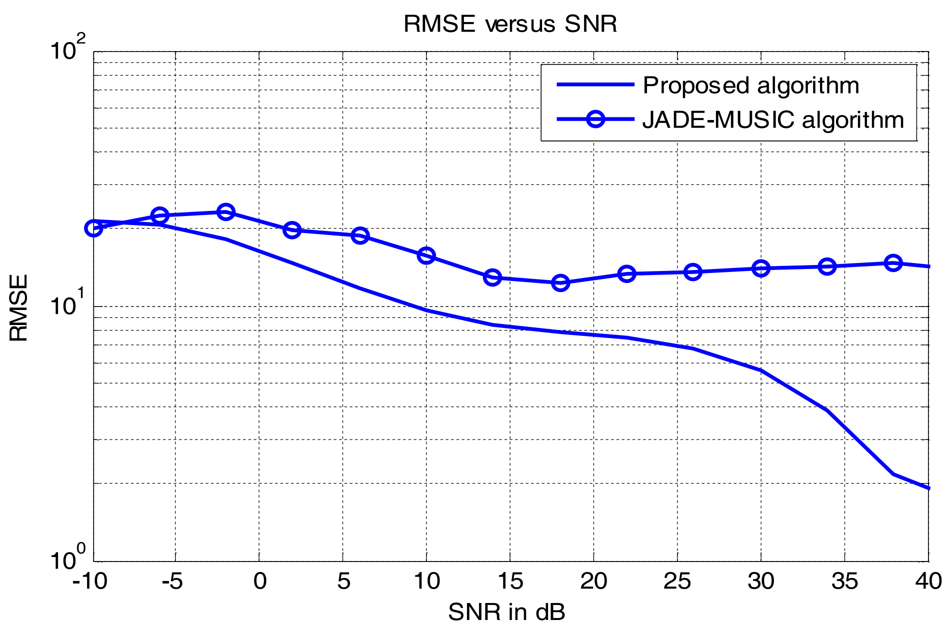

Figure 7. RMSE versus SNR for colored noise scenario

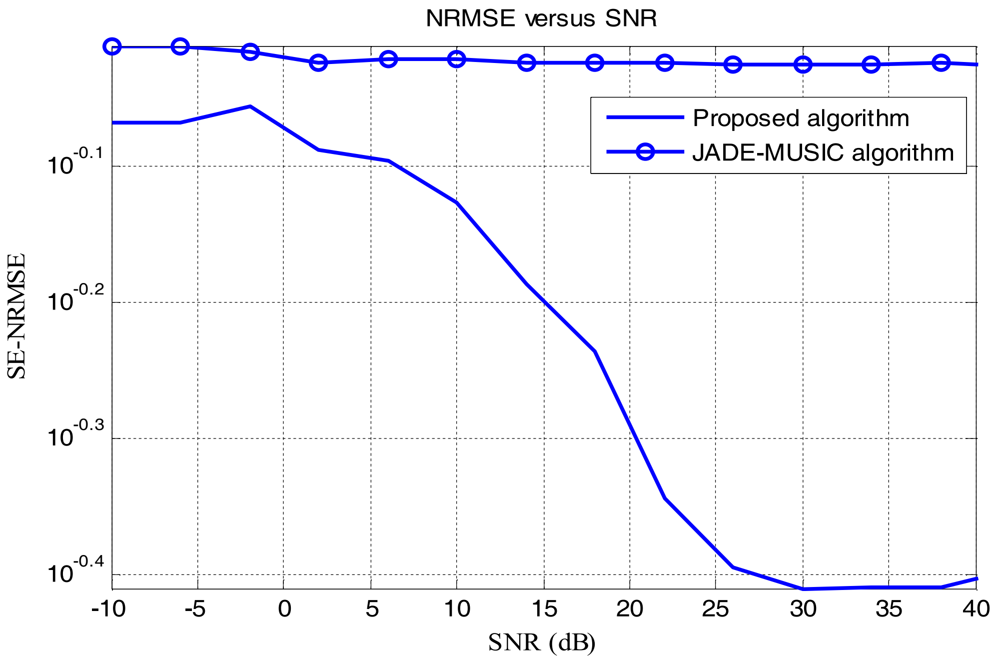

Figure 8. SE-NRMSE versus SNR for colored noise scenario

In Table 3, the effect of correlation coefficient on RMSE and SE-NRMSE of two algorithms is investigated. Three levels of SNR, low, moderate and high are equal to -10, 10 and 30 dB, respectively. For each SNR, α is $0.1, 0.3, 0.5$ and 0.8. As reported in this table, in low and moderate SNRs, RMSE of proposed algorithm is higher than RMSE of JADE-MUSIC algorithm. In high SNRs, RMSE of proposed algorithm is lower than RMSE of JADE-MUSIC algorithm. Also, Table 3 shows that SE-NRMSE of proposed algorithm is lower than SE-NRMSE of JADE-MUSIC algorithm, in low, moderate and high SNRs regardless of α.

Table 3. RMSE and SE-NRMSE for JADE-MUSIC and proposed algorithms in different correlation coefficients ($N_s = 2000$, $M = 10$)

α	SNR (dB)	JADE-MUSIC RMSE	JADE-MUSIC SE-NRMSE	Proposed Algorithm RMSE	Proposed Algorithm SE-NRMSE
0.1	-10	5.5764	0.9324	7.4890	0.7810
	10	5.2157	0.9230	9.0826	0.5004
	30	5.5731	0.9228	3.8195	0.3972
0.3	-10	8.6095	0.9533	13.8726	0.8808
	10	4.963	0.9234	11.1608	0.5561
	30	5.3064	0.9223	3.8171	0.3911
0.5	-10	13.161	0.9654	22.3197	0.8491
	10	5.2537	0.9272	14.4367	0.6323
	30	5.3998	0.9221	3.4659	0.3997

0.8	-10 10 30	13.749 8.2846 5.3258	0.9746 0.9517 0.9241	23.8089 18.0275 5.6303	0.8824 0.8846 0.4214

8. Conclusion

In this paper, a new algorithm as a combination of MDL/EGM, JADE and MFBLP is proposed which is appropriate for DOA estimation of source groups containing coherent signals. MDL criterion is used to determine the number of independent groups in AWGN scenario, JADE algorithm is utilized to estimate the steering vectors and finally, MFBLP algorithm is applied to each steering vectors to estimate DOAs. It should be noted that instead of MDL, EGM criterion is used to detect the number of groups in colored noise scenario.

Also, in this research, a new normalized RMSE performance measure is suggested that show the performance of each algorithm more accurately. To evaluate the performance of proposed algorithm and also compare the numerical results with conventional JADE-MUISC algorithm, several Monte-Carlo simulations are run in both AWGN and colored noise channels. Simulation results show that, for different QAM modulation sizes of coherent signals, proposed method can resolve sources. When AWGN is considered, in $SNR > -2dB,$ RMSE of proposed algorithm is less than RMSE of JADE-MUSIC algorithm and in wide range of $SNRs$, SE-NRMSE of proposed algorithm is less than SE-NRMSE of JADE-MUSIC algorithm. Also, simulation results show the effect of colored Gaussian noise on RMSE and SE-NRMSE. In $SNR > -6dB$, RMSE, and in wide range of SNRs, SE-NRMSE of new algorithm is less than JADE-MUSIC algorithm. It also shows that, in low, moderate and high SNRs, in all coefficient correlation ranges, proposed algorithm shows lower SE-NRMSE. Besides above mentioned superiorities of new proposed method, it needs 10 times lower computational complexity compared to JADE-MUSIC algorithm.

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