



Optimum Distributed Energy Management of Residential Consumers in Presence of Rooftop Photovoltaic Panels

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Abstract: In this paper, a new energy management method is proposed for residential consumers based on a distributed algorithm. Consumers could participate in demand response programs by managing their schedulable and deferrable loads as well as using of photovoltaic (PV) systems. In the proposed method, the Alternating Direction Method of Multiplier (ADMM) is used to model the distributed management and scheduling of buildings electricity consumption. By implementing the distributed algorithm, a large number of residential consumers can update their consumption parameters by online communication with the central controller in parallel. The results confirm that residential customers are able to reduce their electricity bill by modifying their electricity consumption patterns without reducing their welfare.

Keywords: Demand Response, Residential Consumers, Distributed Algorithm, Electricity Bill.

1 Introduction

IN recent years, the unprecedented increase in power demand along with the delay in the construction and commissioning of new power plants have highly reduced stability and reliability of power system. However, the rise in electricity demand is not a challenge during all periods of a year except for peak demand periods. In most countries such as the UK and US, heating, ventilating and air conditioning (HVAC) systems are the largest consumers of electricity energy and a major contributor to peak demand [1]. Electricity demand reaches a peaks in winter and summer due to increase in heating and cooling demands and results in high wholesale electricity prices and reduced reliability due to limited generation reserve margins. Insufficient

forward planning on renewable generation and demand side management may lead to system contingencies, or in extreme cases, system blackouts [2]. Therefore, demand response (DR) programs are defined as an approach for managing electrical consumptions in terms of production in the electricity market [3, 4]. However, the real-time pricing by activating demand sides leads to increase in social welfare [5], but small-scale consumers do not have enough technical and economic information to participate in the open electricity market [6]. There have been a number of researches on residential DR in recent years, which have used a centralized approach to manage energy consumptions of buildings and homes. In [7], a control strategy for all controllable loads in a single house based on time of use tariffs has been presented. The implementation of demand response programs resulted in a reduction in generation cost, electricity end-use expenditure and carbon emission. In [8], a controller that curtails peak load by managing the set-point temperature of HVAC has been proposed. It resulted in saving electricity cost while maintaining reasonable thermal comfort. The main purpose of the DR program is to schedule resources and equipment efficiently so that both consumers and distribution companies take advantage [9]. Therefore, the main purpose of DR is to shift the load from peak time to off-peak time and somehow extend the consumption graph to a flat pattern. After different intervals are defined for

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using electricity, most companies increase price over peak time and offer low prices for off-peak intervals [10]. In [11], an intelligent home energy management system aiming at minimizing the cost of electricity has been presented. The presence of electric vehicles along with distributed generation units has also been taken into account which leads to complicating the management of energy resource. In [4], end-users' willingness to shift their appliances usage has been modeled probabilistically using parameters that were estimated from real data. Then, an algorithm for computing day-ahead prices has been developed. Also, another algorithm has been developed to estimate and refine user reaction to the prices. Based on user behavior, the algorithms let the provider to dynamically regulate the offered prices. The pricing model has reduced the required communications. However, the aggregate demand over each day has been considered constant and photovoltaic generation has not taken into account. The smart grid features related to load side are more relevant than other sectors. By introducing Advanced Metering Infrastructure (AMI) [10, 12] and energy controllers [12, 13], more distributed algorithms in the field of the power system energy scheduling could be proposed. Using the AMI, the power consumption information of each consumer will be available at any time. In general, these controllers will communicate with each other and will be controlled by a hierarchical system [13, 14]. In the latter case, retailers may themselves control these controllers in order to reduce their risk. It may be possible for the system operator to use these controllers to enhance the security of the system. It is worth noting that this state of affairs may conflict with the privacy of consumers. In most of the existing electricity markets, the share of accountability is low. In these markets, only large consumers have direct access to the luck of the wholesale market and small consumers are not eligible to participate in these markets for two main reasons. The first reason is that the necessity of the participation of small consumers in the electricity market is to measure their actual consumption at all times [15, 16]. Consumers should always be aware of the price of electricity in different periods. The use of such measurement and communication systems in the electric network requires a high cost as explained in [16, 18]. The second reason is that the number of these consumers is high, so the information that should be collected or sent by the central controller is very high, and it is not possible in a real condition. In [19], the load response model is presented using the concept of demand elasticity pricing in which variations of demand are considered as a linear function of price. For this reason, the authors in [20] presented other mathematical models by considering non-linear changes in demand by proposing the load response program using the concept of stretching. In the extended model, energy consumption data and electricity prices are exchanged

between different players. In [21], a similar game has been formulated for traditional consumers and energy consumers who have energy resources and energy storage. In [22], an algorithm has been presented to find the optimal energy consumption levels for each customer aiming at maximizing the aggregate utility of all customers. However, the algorithm was not able to significantly shave the peak load. Also, with increasing the number of end-users, the algorithm may not converge or find the optimum solution.

In [23], a two-step energy management method for multicarrier energy systems has been proposed. In the first step, an optimization problem has been solved for each home to achieve the desired energy cost where the system operator running another optimization problem to minimize the total power losses in the second step. In [24], a data analytical demand response management scheme for residential customers aiming at reducing the peak load demand has been presented. The scheme is based on the analysis of consumers' consumption data collected from smart homes. The results showed that the method increases the savings of the residential customers by reducing their total electricity bills. In [25], a day ahead demand response scheduling using distributed algorithm is proposed in which the aggregate cost, utility, and retailer's profit have simultaneously been optimized. Also, the interaction between the agents has modeled by Stackelberg game and solved using a fully distributed adaptive diffusion. An electricity load scheduling method by using a Benders algorithm has been presented in [26]. The method has been formulated in a distributed manner while the private information of the residences is protected. Also, the near-optimal load scheduling for each residence has been obtained.

Due to the advantages of renewable energies, the implementation of rooftop Photovoltaic (PV) system for residential customers have grown rapidly worldwide. Electric companies try to develop electricity buying-back schemes to encourage customers to generate more renewable energy. In [9], a day-ahead pricing scheme considering renewable energy and demand side management has been proposed. The problem has been formulated as a convex optimization dual problem in a distributed manner which aimed to achieve maximum benefits for both electric companies and customers. In [27], an energy sharing model for a microgrid including customers PVs has been presented. Also, the price-based DR has been considered to design an equivalent cost model in terms of economic cost and users' willingness.

The reviewed researches have taken valuable steps in DR implementation in homes and buildings, but in most of them, minimizing the costs of each equipment is considered as the main objective of the Home Energy Management (HEM) module, which may be in conflict with total cost reduction of the utility.

In this paper, the Alternating Direction Method of Multipliers (ADMM) is used to model electricity

scheduling and management of residential consumers. In this method, many residential consumers can update their requested load in parallel and simultaneously by communicating with the central controller where the subsystems are not interconnected. Consumers will also be able to reduce their electricity bills by modifying their consumption patterns without losing their welfare.

The remainder of the paper is organized as follows: The next section reviews the relationship and architecture between home energy management and consumers. Section 3 describes the distributed energy management algorithm. Simulation results and the discussions are provided in Section 4 and conclusions are given in Section 5.

2 System Model

In this paper, a group of residential buildings and houses within a radial low voltage distribution network is considered. The relationship between consumers and the Energy Management System (EMS) is defined by using AMI. Each household includes a series of interruptible loads such as lighting, schedulable loads such as Air-Conditioners (ACs), and deferrable loads such as washing machines and dishwashing machines.

2.1 System Overview

In general, the goal of creating smart homes is to increase the comfort of its inhabitants and save energy. In a given house, electrical load can be divided into three main categories. The larger one includes base loads such as lighting systems that cannot be transferred to other hours. The second category includes schedulable loads such as ACs. The third category entails deferrable loads such as washing machines and dishwashers which can be transferred to other hours. In this paper, the total load of each residential customer is given by $d_{h,t}$ which includes base loads $d_{h,t}^{base}$, schedulable loads ($d_{h,t}^S$), deferrable loads ($d_{h,t}^{def}$) and the power generation of PV system ($d_{h,t}^{PV}$).

$$d_{h,t} = d_{h,t}^{base} + d_{h,t}^S + d_{h,t}^{def} - d_{h,t}^{PV} \quad (1)$$

In the proposed method, a bidirectional smart meter is implemented to use the net metering program [28]. Distribution companies purchase the net amount of electricity which end-users produce beyond their own consumption. For example, whenever the PV generation of a residential customer exceeds its electricity consumption, the smart meter runs backwards.

Programmable loads have the following limitations to ensure consumer welfare:

$$\sum_{t=1}^T d_{h,t}^S = s_h^S \quad (2)$$

$$0 \leq d_{h,t}^S \leq d_{h,t}^{max} \quad (3)$$

where $d_{h,t}^{max}$ is the maximum schedulable load per hour and s_h^S is the total schedulable loads for each customer. The PV systems are also considered as negative load. Equations (2) and (3) are the constraints for maintaining the welfare of each customer in case of schedulable loads. It means that the total requested demand during 24 hours must be provided but the dispatching of loads is carried out by the proposed algorithm. In practice, amount of $d_{h,t}^{max}$ is determined based on utility policies as well as the allowable demand or ampere of a single residential customer [29, 30].

2.2 Electricity Pricing Strategy

In this section, the electricity generation cost model is initially described. Then, a pricing scheme is proposed for this cost model. The final electricity generation cost is increased by ξ_t for one kWh in a thermal power unit. In an electric power system, demand fluctuation could add the second cost to the total cost (including the cost of electricity companies) due to the construction of new power plants to meet demands at peak periods [31]. The cost function is given by:

$$cost = \sum_{t=1}^T \xi_t \sum_h d_{h,t} + \mu \sum_{t=1}^T \left(\sum_{t=1}^T d_{h,t} - A_{daily} \right)^2 \quad (4)$$

In which, μ is the cost of the load fluctuations, and A_{daily} is the average power consumption per day given as:

$$A_{daily} = \left(\frac{1}{T} \right) \sum_{t=1}^T \sum_h d_{h,t} \quad (5)$$

Electricity pricing is a sophisticated method and varies widely in the electricity market of different countries. In this paper, the electricity price consists of two parts. The first part is the basic price, and the second part is the demand fluctuations. The base price (P_{base}) is determined according to the total sum of base loads for all consumers. The base price is proportional to the sum of base loads at specific time t that is equal to:

$$P_{base,t} = \xi_1 \left(\sum_h d_{h,t}^{base} \right) \quad (6)$$

$$A_{base} = \left(\frac{1}{T} \right) \sum_{t=1}^T \sum_h d_{h,t}^{base} \quad (7)$$

ξ_1 is obtained based on the average daily value of the base load (A_{base}). If only the base price is considered, there is a low incentive for consumers to change their electricity consumption patterns. If a large number of loads are scheduled in periods of lower energy costs, another peak in the demand curve will be produced. In order to motivate consumers to reduce the cost of new ones, in Eq. (4), the latter cost should be introduced based on how the demand curve fluctuates.

$$\pi = \sum_h d_{h,t} - A_{base} \quad t = [1, 2, \dots, 24] \quad (8)$$

The additional price term $P_{\pi,t}$ related to this difference is added to the price model. So, the fluctuation price can be written as follows:

$$P_{\pi,t} = \begin{cases} \xi_2, & \pi > A_{base} \\ 0, & otherwise \end{cases} \quad (9)$$

$$\xi_h = \sum_{t=1}^T (P_{base,t} + P_{\pi,t}) d_{h,t} \quad (10)$$

By considering the fluctuation of prices, the consumers will manage their consumptions aiming at flattening the demand curve of the distribution network.

2.3 Uncertainty

The demand forecasting uncertainty is considered with the help of Gaussian random variables within the proposed model as follows:

$$d_{h,t}^{base} = d_{h,t}^{base0} + \varepsilon_{h,t}^{base} \quad (11)$$

where $d_{h,t}^{base0}$ is the actual load of each consumer h , and $\varepsilon_{h,t}^{base}$ is Gaussian random error with distribution $\mathcal{N}(0, \sigma_{base}^2)$, which shows the prediction error in the mathematical model. In this model, it is assumed that $\varepsilon_{h,t}^{base}$ is not similar in all consumers. To investigate the uncertainty of the PV output power, the short-term prediction method from [32] has been used. Assuming the forecast error as Gaussian error, the linear model is obtained as follows:

$$d_{h,t}^{PV} = d_{h,t}^{PV0} + \varepsilon_{h,t}^{PV} \quad (12)$$

where $d_{h,t}^{PV}$ represents the actual power generation of solar panels for each consumer (h) and $\varepsilon_{h,t}^{PV}$ is the prediction error that is considered as a random error in the model. As all consumers are in the same area with common weather patterns, the $\varepsilon_{h,t}^{PV}$ becomes similar. Therefore, the noise cycle is equal to:

$$\varepsilon_{h,t}^{PV} = e_i^{PV} + e_{h,t}^{PV} \quad (13)$$

where e_h^{PV} is Gaussian random error with distribution $\mathcal{N}(0, \sigma_1^2)$, which indicates that the prediction error of PV systems is the same for all consumers. In addition, $e_{h,t}^{PV}$ represents a Gaussian random error with the distribution $\mathcal{N}(0, \sigma_0^2)$.

3 Distributed Algorithm

The main purpose of the consumers is to minimize

electricity bills. In addition, utilities also plan to cut the total bill in order to reduce the high cost of peak load supply. Since minimizing the total billing of all users will flatten the load curve, the cost of the fluctuation will be lower for utilities. In other words, an electricity utility could minimize the total bills if its income can cover the cost of power generation, and it can be guaranteed by choosing appropriate ξ_1, ξ_2 . Therefore, the constants ξ_1, ξ_2 of the utility are tuned to minimize the total bills. The objective function is defined as:

$$\min_{\{d_h\}} \sum_h P_{base}^T d_h + \sum_h P_{\pi}^T d_h \quad (14)$$

The results of optimization by solving Equation (14) with the centralized method do not fulfill the required efficiency due to its large dimensions and high constraints and the computation time increases with the number of consumers. In addition, consumers must report their energy consumption to their utility, which violates privacy issues. Hence, the objective function in Eq. (14) can be solved by the decentralized ADMM method. So, the optimization problem is introduced with the introduction of auxiliary variables z_h as follows:

$$\min_{\{d_h\}, \{z_h\}} \sum_h P_{base}^T d_h + (\rho/2) \|d_h - z_h\|_2^2 + \sum_h P_{\pi}^T z_h \quad (15)$$

s.t: (1)–(3).

The algorithm is updated for solving in the following steps as follows:

$$d_h^{i+1} = \arg \min_{d_h \in F} P_{base}^T d_h + v_h^{iT} (d_h - z_h^i) + (\rho/2) \|d_h - z_h^i\|_2^2 \quad (16)$$

$$\{z_h^{i+1}\} = \arg \min_{z_h} \sum_h (v_h^{iT} (d_h^{i+1} - z_h) + (\rho/2) \|d_h^{i+1} - z_h\|_2^2) + \sum_h P_{\pi}^T z_h \quad (17)$$

$$v_h^{i+1} = v_h^i + \rho (d_h^{i+1} - z_h^{i+1}), \quad \forall h \quad (18)$$

Compared to other methods, in this algorithm, if any value greater than zero is chosen for the coefficient ρ , it converges to the optimal solution.

$$r^i := \|A d_h^i + B z_h^i - c\| \quad (19)$$

$$s^i := \|\rho A^T B (z_h^i - z_h^{(i-1)})\| \quad (20)$$

The stopping criterion would be

$$\|r^i\|_2 \leq \epsilon^{pri} \quad (21)$$

$$\|s^i\|_2 \leq \epsilon^{dual} \quad (22)$$

More explanations about the detail of the procedure are available in [14].

4 Case Study

The proposed method has been examined on a real distribution low voltage feeder with 210 residential consumers. The test feeder base on GIS data is shown in Fig.1. Since ACs are used in these areas in the summer, the annual peak load occurs in this season. The Billing data of the utility indicates that electricity consumption of residential consumers during the summer is equal to

the total energy consumed by them in the other three seasons of the year. The consumption profile of the distribution substation at the peak time (15:30) is recorded by a data logger and is shown in Table 1. According to the proposed method, the consumption rate is classified with intervals of 3.3 kWh/Day as presented in Table 2. It should be mentioned that the total amount of costumer bills on July 5, 2017 was 284\$. According to the algorithm shown in Fig. 2, the problem has been investigated using the ADMM method in two different scenarios.

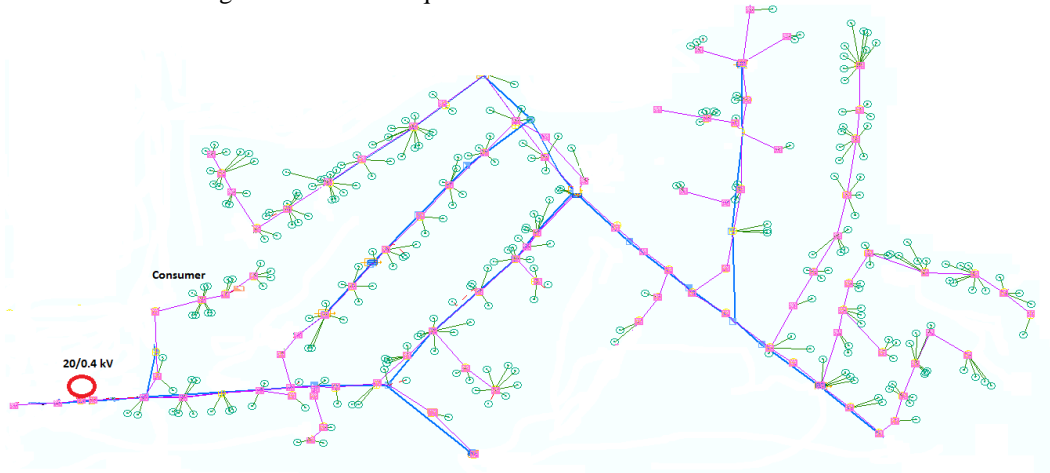


Fig. 1 Test feeder based on GIS data.

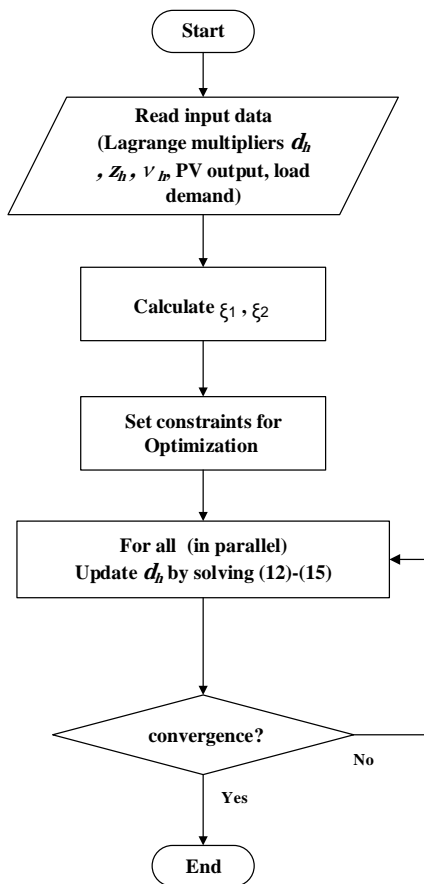


Fig. 2 Flowchart of the proposed ADMM algorithm.

Table 1 Output of Data logger on the studied day at 15:30.

104MT911240					
1396/04/14		15:30		1	
V1	231.9 v	V2	235.6 v	V3	231.2 v
V12	404.8 v	V23	404.2 v	V31	401.1 v
I1	635 A	I2	621.6 A	I3	522.9 A
Iu	0 A	Iave	593.1 A		
Vu	3.9 v	Vave	232.9 v		
PF1	95.3 %	PF2	92.6 %	PF3	92.9 %
P1	139.3 kW	P2	128.7 kW	P3	111.5 kW
Q1	43.6 KVAR	Q2	69.5 KVAR	Q3	43.8 KVAR
S1	146 KVA	S2	146.4 KVA	S3	119.8 KVA
Ptotal	379.5 kW	Qtotal	157 KVAR	Stotal	412.3 KVA
THDV1	1 %	THDV2	1 %	THDV3	1 %
THDI1	6 %	THDI2	6 %	THDI3	6 %
F	49.97 Hz	T	31 °C		

Table 2 Energy price steps.

Demand Load (Kwh/Day)	ζ_1 (€/kWh)	ζ_2 (€/kWh)
0 – 3.3	0.9	1.35
3.3 – 6.6	0.976	1.5
6.6 – 10	1.4	2.2
10 – 13.3	2.06	3.4
13.3 – 16.6	2.58	4.02
16.6 – 20	3.13	4.9
> 20	4.11	6.07

4.1 Results of Scenario 1

In this scenario, it is assumed that 210 households have participated in the demand response programs. In this group, the deferrable loads such as washing machine and dishwashing machine can be operated between 12:00 and 23:00, and the power consumption is randomized between 1.2 and 2 and the maximum power consumption was 6 kWh per day [29]. The maximum schedulable load $d_{h,t}^{max}$ is also 3 kWh [29]. In order to model this group, the parameter ρ is considered to be 0.8. Fig. 3 shows the daily consumption of this specific day.

As shown, the daily consumption peak occurred between 13 to 16. To calculate the base load, the 5-day load curve is selected since this curve has the most

similarity in terms of temperature. Then, the average value of the three curves is calculated. This average curve is considered as the base load on the target day [33]. The simulation is performed by the proposed method and the basic load profile is shown in Fig. 4.

As illustrated in Fig. 5, the consumption curve in the studied day has become flat. The total load in peak hours ranged from 13 to 16 at 1250 kWh, which was 21 percent lower than the case without optimization. In a large network, this value results in a significant economic saving. Total consumption has decreased about 3%, which reflects the maintenance of the welfare of the costumers. The total electricity bills of customers also amounted to 262\$, which had a significant decrease compared to the state without optimization.

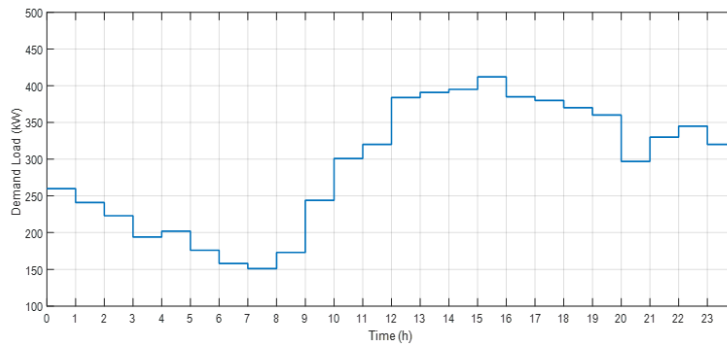


Fig. 3 Total hourly load of the studied day.

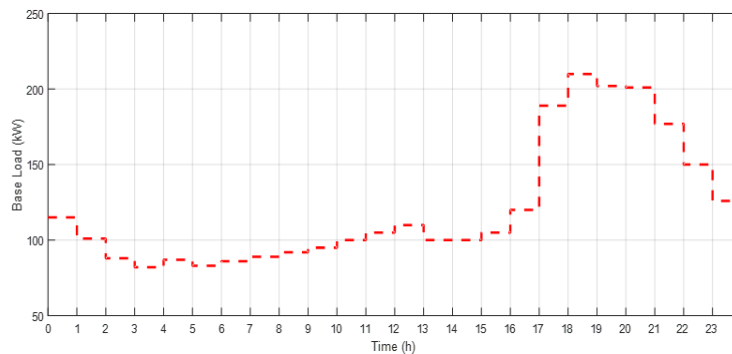


Fig. 4 Total hourly base load.

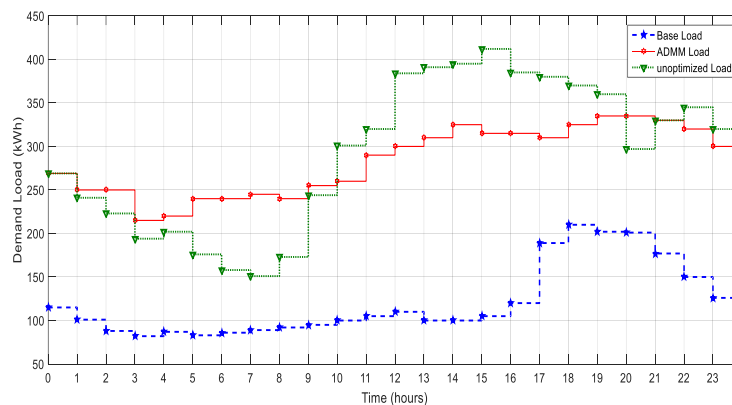


Fig. 5 Hourly load of tree scheduling methods in scenario 1.

4.2 Results of Scenario 2

Considering the well sunlight conditions in the area and the policies of the utility to encourage consumers to install local solar systems, this approach has been greeted by consumers. In this scenario, it is assumed that 20 houses have installed PV system with a capacity of 5 kW on their roofs. The meter is bidirectional and the data transmission system is wireless. The output power of the panels is given in Fig. 6 which is for July 5, 2017. In Fig. 7, the consumption graph is shown in the proposed method. The consumption curve is more flattened than the one in the previous scenario. The result shows that the total bill of the costumers becomes 250 \$ which is lower than the previous ones. The total consumed load at peak hours (hours 13-16) dropped by 11.2% compared to previous scenario, resulting from the benefit of the PV system. Table 3 compares the results for the discussed cases.

In Fig. 8, the convergence of the algorithm is represented by the ADMM method. As known, the algorithm has reached up to 40 repetitions with an accuracy of 10^{-4} . Fig. 9 shows the normal probability distribution chart to examine the uncertainty of the base load and the output power of a panel at 12:00. In order to investigate the uncertainty of the initial load, first, the average load was obtained for 100 days with three

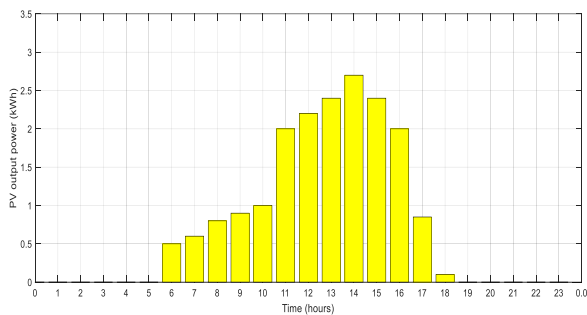


Fig. 6 The output power of the panel.

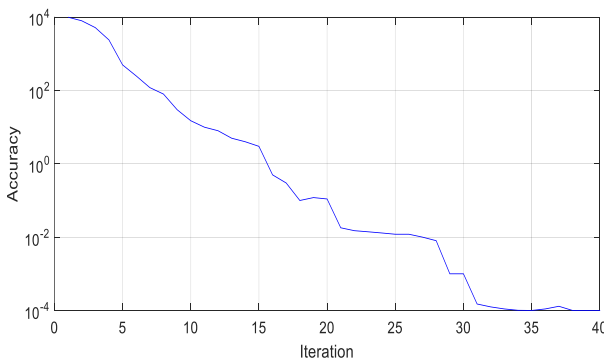


Fig. 8 Convergence of primal residuals.

standard deviations on each side of the average and one hundred random samples at 12:00. According to these analyses, σ_{base} were obtained in the range of 0.7 kW to 2.3 kW. In addition, the average output power of a panel for the selected 100 days is done at 12:00 and three standard deviations on each side of the average and one hundred random samples for the PV system are also considered in this study. According to these evaluations, σ_0 is within a range of 0.2 kW to 2 kW and σ_1 is considered as 0.01 kW.

Figs. 10 and 11 depict electricity bills for the proposed method and the case without optimization. As shown, the total amount of electricity bills increases with rising uncertainty impact. Also, with regard to equal conditions of the PV system for all consumers, a higher sensitivity is observed in Fig. 11.

In order to analyze the advantage of the distributed method to the centralized one [34-35], a case study is carried out via both of two methods. Both methods are modeled in MATLAB R2017a on a core i5, 2.6 GHz processor with 8 GB of RAM. Results of total cost and computation time are given in Table 4. The results showed that total costs from the two methods are very close. On the other hand, the computational time of ADMM method is much shorter than the one in the centralized method.

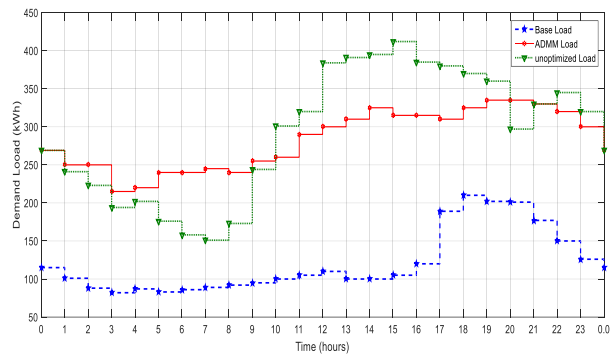


Fig. 7 Total load of tree scheduling methods in scenario 2.

Table 3 Comparison of the total cost under three scheduling methods.

Methods	Costs (\$)
Un-optimization	284
Scenario1	262
Scenario2	250

Table 4 Comparison of the total cost under three scheduling methods.

Method	Comp. Time (sec)	Costs (\$)
ADMM	814	262.01
Centralized	2432	259.15

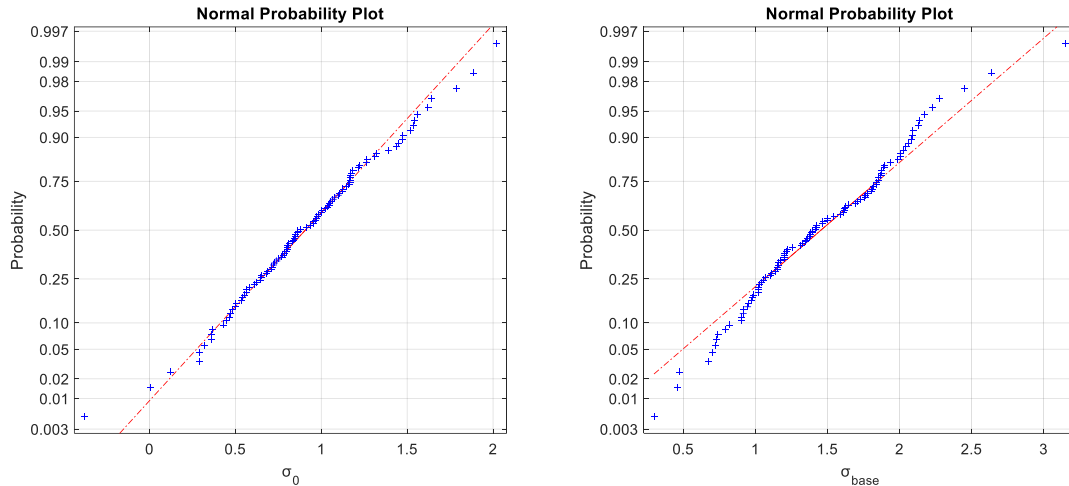


Fig. 9 Normal probability distribution for base load and PV output power at 12:00.

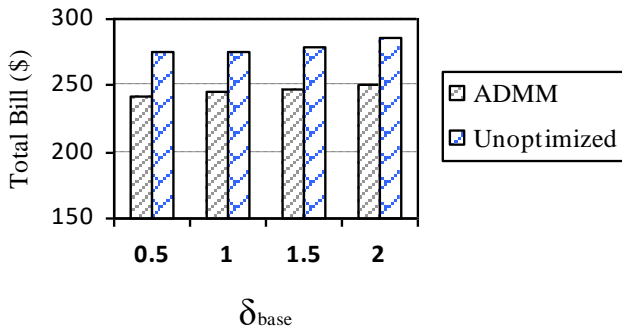


Fig. 10 Total bills with two methods with the effect of uncertainty on base load.

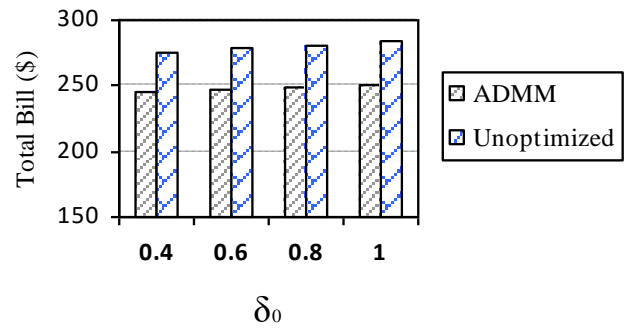


Fig. 11 Total bills with two methods with the effect of uncertainty on panel output power.

5 CONCLUSION

The expansion of smart grid technologies could significantly influence DR scheduling. In a distribution system with a large number of residential consumers, the energy management problem would be complex, and difficult to solve. In this paper, a distributed algorithm has been presented to manage the electricity consumption of a large number of residential customers. Due to the large number of buildings, the decentralized optimization approach leads to speed up convergence of the energy managing and scheduling. The results of simulations reveal that the demand curve was more flattened by using the proposed method. Using the real data in the model showed that the implementation of demand response programs with distributed algorithm could reduce the cost of electricity bills and encourage consumers to manage their consumption.

Appendix

A1 Alternating Direction Method of Multiplier (ADMM) Algorithm

ADMM algorithm is capable of solving convex problems in a decentralized manner. As an alternative to solve the main complex problem, ADMM converts the main problem to several smaller ones and gives an

acceptable solution by solving them separately. The procedure of the algorithm has been summarized below.

A1.1 Dual Ascent

This model is introduced in the following procedure:

$$\begin{aligned} \min f(x) \\ Ax = b \end{aligned} \tag{A.1}$$

Model (A.1) has no difference with the defined model in the following:

$$\begin{aligned} \min_x \max_{\lambda} f(x) + \lambda^T (Ax - b) \\ Ax = b \end{aligned} \tag{A.2}$$

Because it is equivalent to:

$$\begin{cases} f(x) & Ax = b \\ \infty & Ax \neq b \end{cases} \\ f(x) + \lambda^T (Ax - b) = L(x, \lambda) \tag{A.3}$$

So, model (A.1) can be written as follows:

$$\begin{aligned} \min_x \max_{\lambda} L(x, \lambda) \\ Ax = b \end{aligned} \tag{A.4}$$

which its duality will be as follows:

$$\begin{aligned} \max_{\lambda} \min_x L(x, \lambda) \\ Ax = b \end{aligned} \tag{A.5}$$

Thus, the answer of the main problem and dual problem is:

$$x^* = \arg \min_x \max_{\lambda} L(x, \lambda) \tag{A.6}$$

$$\lambda^* = \arg \max_{\lambda} \min_x L(x, \lambda) \tag{A.7}$$

Therefore, in the case of a strong duality property we will have:

$$x^* = \arg \min_x L(x, \lambda^*) \tag{A.8}$$

By repetitively solving the following expression, the received solution tends to the optimum solution.

$$x^{(k+1)} := \arg \min_x L(x, \lambda^k) \tag{A.9}$$

$$\lambda^{(k+1)} := \lambda^k + a^k (Ax^{(k+1)} - b) \tag{A.10}$$

The explained algorithm has an important characteristic titled decomposability, which is explained in the followings.

A1.2 Dual Decomposition

Let function $f(x)$ be a function that has the ability to be decomposed in to $f_i(x_i)$ functions. In this case, function $f(x)$ is written as $\sum f_i(x_i)$. Therefore, Lagrangian model of Eq. (A.1) will be as follows:

$$L(x, \lambda) = \sum_{i=1}^N \left[f_i(x_i) + \lambda^T \left(A_i x_i - \frac{b}{N} \right) \right] \tag{A.11}$$

For solving this, the first term in right hand side of Eq. (A.11) is replaced by the following equation:

$$x_i^{(k+1)} := \arg \min_{x_i} L(x_i, \lambda^k) \tag{A.12}$$

Therefore, we only need a particular part of variables for solving each equation. In fact, the initial large model is converted to several small models. This algorithm is called dual decomposition. The difference between augmented Lagrangian and Lagrangian is in adding a specific term.

$$\begin{aligned} \min f(x) \\ Ax = b \end{aligned} \tag{A.13}$$

$$\begin{aligned} \mathcal{L}_{\rho}(x, \lambda) = f(x) + \lambda^T (Ax - b) \\ \mathcal{L}_{\rho}(x, \lambda) = f(x) + \lambda^T (Ax - b) \\ + (\rho/2) \|Ax - b\|_2^2 \end{aligned} \tag{A.14}$$

The ρ parameter in the above expression is a penalty factor that can have any value larger than zero in the initial condition. If augmented Lagrangian is implemented instead of Lagrangian in the dual algorithm, solving the following equations repetitively leads to a convergence toward the optimum results.

$$x^{(k+1)} := \arg \min_x L_{\rho}(x, \lambda^k) \tag{A.15}$$

$$\lambda^{(k+1)} := \lambda^k + \rho (Ax^{(k+1)} - b) \tag{A.16}$$

$$\begin{aligned} \mathcal{L}_{\rho}(x, \lambda) = f(x) + \lambda^T (Ax - b) \\ + (\rho/2) \|Ax - b\|_2^2 \end{aligned} \tag{A.17}$$

By joining decomposition ability of dual method and the strength of augmented Lagrangian method tendency, ADMM algorithm is identified as a strong tool.

$$\begin{aligned} \min f(x) + g(z) \\ Ax + Bz = c \\ \mathcal{L}_{\rho}(x, z, \lambda) = f(x) + g(z) + \lambda^T (Ax + Bz - c) \\ + (\rho/2) \|Ax + Bz - c\|_2^2 \end{aligned}$$

$$x^{(k+1)} := \arg \min_x L_{\rho}(x, z^k, \lambda^k) \tag{A.18}$$

$$z^{(k+1)} := \arg \min_z L_{\rho}(x^{(k+1)}, z, \lambda^k) \tag{A.19}$$

$$\lambda^{(k+1)} := \lambda^k + \rho (Ax^{(k+1)} + Bz^{(k+1)} - c) \tag{A.20}$$

Comparing to other decomposition methods, in ADMM algorithm, if any value larger than zero is chosen for penalty factor (ρ), it will be converged toward the optimum solution. Compared to dual decomposition, This algorithm is guaranteed to meet an optimal solution under less restrictive conditions. Let

$$r^k := \|Ax^k + Bz^k - c\| \tag{A.21}$$

$$s^k := \rho \|A^T B (z^k - z^{(k-1)})\| \tag{A.22}$$

They are viewed as the residuals for primal and dual feasibility. Under mild conditions, it can be shown that

$$\lim_{k \rightarrow \infty} (r)^k = 0,$$

$$\lim_{k \rightarrow \infty} (s)^k = 0$$

The criterion to stop the algorithm would be

$$\|r^k\|_2 \leq \epsilon^{pri} \tag{A.23}$$

$$\|s^k\|_2 \leq \epsilon^{dual} \tag{A.24}$$

One can refer to [14] for details.

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