



Adaptive Approximation-Based Control for Uncertain Nonlinear Systems With Unknown Dead-Zone Using Minimal Learning Parameter Algorithm

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Abstract: This paper proposes an adaptive approximation-based controller for uncertain strict-feedback nonlinear systems with unknown dead-zone nonlinearity. Dead-zone constraint is represented as a combination of a linear system with a disturbance-like term. This work invokes neural networks (NNs) as a linear-in-parameter approximator to model uncertain nonlinear functions that appear in virtual and actual control laws. Minimal learning parameter (MLP) algorithm is proposed to decrease the computational load, the number of adjustable parameters, and to avoid the “explosion of learning parameters” problem. An adaptive TSK-type fuzzy system is proposed to estimate the disturbance-like term in the dead-zone description which further will be used to compensate the effect of the dead-zone, and it does not require the availability of the dead-zone input. Then, the proposed method based on the dynamic surface control (DSC) method is designed which avoids the “explosion of complexity” problem. Proposed scheme deals with dead-zone nonlinearity and uncertain dynamics without requiring the availability of any knowledge about them, and it develops a control input without singularity concern. Stability analysis shows that all the signals of the closed-loop system are semi-globally uniformly ultimately bounded and the tracking error converges to the vicinity of the origin. Simulation and comparison results verify the acceptable performance of the presented controller.

Keywords: Dead-Zone Nonlinearity, Minimal Learning Parameter Algorithm, Adaptive Fuzzy Approach, Neural Networks.

1 Introduction

CONTROL of uncertain nonlinear systems with input nonlinearity has widespread applications in industry [1-4]. Dead-zone is one of the non-smooth and nonlinear constraints that exists in the most of the practical systems due to the physical limitations of actuators. In practice, systems with dead-zone input do not respond to the input until the input reaches a particular level. So, it is necessary to compensate the effects of the dead-zone since existence of such nonlinearity decreases the systems performance and

even it may cause the instability of the closed-loop system.

To deal with the dead-zone nonlinearity in uncertain nonlinear systems, some approaches were proposed to design the feedback controller. One approach to compensate the adverse effects of the dead-zone is based on the smooth inverse of the dead-zone nonlinearity. In [5], smooth inverse of dead-zone nonlinearity and the backstepping approach were used to develop an adaptive output feedback control scheme for a class of nonlinear systems with matching condition. An adaptive output feedback backstepping control method was developed for a class of uncertain nonlinear systems without satisfying the matching condition in the presence of the dead-zone [6]. Also, adaptive fuzzy backstepping control method was developed for uncertain pure-feedback nonlinear systems with unmeasurable states and non-matching condition in [7]. However, all of the developed methods

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in [5-7] suffer from the “overparameterization” and “explosion of complexity” problems.

Some works describe the dead-zone nonlinearity as a fuzzy value instead of deterministic value. In [8], adaptive fuzzy dynamic surface control was proposed for uncertain strict-feedback nonlinear systems with fuzzy dead-zone. It assumes that the dead-zone input of the nonlinear system is fuzzy instead of deterministic. After that, some other control schemes were proposed for uncertain nonlinear systems with fuzzy dead-zone, for example see [9-12]. However, all of the proposed works in [8-12] consider nonlinear systems with unit actual and virtual control gains and they propose complex control inputs that make the stability analysis difficult. Furthermore, they require expert's experience for dead-zone description and when there is no knowledge about dead-zone nonlinearity, their application become difficult task.

Some other works model the dead-zone nonlinearity as a combination of a linear system with a disturbance-like term and then apply robust control approaches to design the control scheme [13]. Disturbance observer-based adaptive control was proposed for a class of uncertain nonlinear systems with unknown dead-zone in the canonical controllable form [14]. An adaptive neural control approach was proposed for uncertain nonstrict-feedback nonlinear systems including unknown control gains and input dead-zone nonlinearity [15]. In [16], interval type-2 fuzzy system based dynamic surface control scheme was proposed for uncertain nonlinear systems with unknown asymmetric dead-zone.

However, all of the existing works that focus on control of uncertain nonlinear systems with dead-zone nonlinearity invoke NNs or fuzzy systems (FSs) as a linear-in-parameter approximator to model the uncertain nonlinear dynamics of the system. A main drawback of NNs or FSs-based approaches is that the number of adaptive parameters and adaptation laws depend on the NN nodes or number of fuzzy rules in FS. When NNs or FSs are used for approximation purpose, with increasing of the dimension of the argument vector of the network input, the network nodes grows drastically and consequently the number of adaptive parameters increases significantly and it results a time-consuming process which has high computational burden. This problem is called “explosion of learning parameters” problem.

Also, the proposed approach in some of the existing works is based on availability of the dead-zone input [13, 14]; or some of them assume that the parameters of the dead-zone or their bounds are known [17, 18]. However, in the most of the practical system, dead-zone input is not available and parameters of the dead-zone nonlinearity are poorly known or even they are completely unknown.

To overcome the mentioned difficulties, in this work an adaptive approximation-based control scheme is proposed for uncertain strict-feedback nonlinear

systems with unknown control gains and unknown asymmetric dead-zone nonlinearity. It models the dead-zone constraint as a combination of a linear system with a disturbance-like term. NNs are invoked to approximate the model of the uncertain terms that appear in virtual and actual control inputs. To avoid the “explosion of learning parameters” problem, MLP algorithm is proposed which assumes the norm of the NN weights as an adjustable parameter and tune them based on the adaptive learning laws. No prior knowledge about uncertain dynamics and dead-zone nonlinearity is required. Also, adaptive TSK-type fuzzy system is proposed to approximate the disturbance-like term of the dead-zone model which further will be used to compensate the adverse effects of the dead-zone. Proposed scheme avoids the “explosion of complexity” and “singularity” problems, simultaneously. The proposed scheme compensates the effect of the dead-zone without the requirement for availability of the dead-zone input. The proposed MLP-based control scheme decreases the number of adaptive parameters considerably. It estimate the norm of the weigh vector of the NNs instead of estimating the elements of the weight vector. By using the proposed scheme, number of adaptive parameter for each control input decreases to one parameter. So, the computational load is decreased significantly.

The rest of the paper is presented as follows. Section 2 presents the problem formulation and some preliminary concepts. In Section 3, the proposed adaptive approximation-based control scheme is designed and its stability is investigated. In Section 4, some simulation and comparison results are provided to illustrate the effectiveness of the proposed control approach. Finally, some concluding remarks are given in Section 5.

2 Problem Statement and Preliminaries

This work considers uncertain strict-feedback nonlinear systems with unknown dead-zone input and external disturbance in the following form

$$\begin{aligned} \dot{x}_1 &= f_1(\mathbf{x}_1) + g_1(\mathbf{x}_1)x_2 + \omega_1 \\ \dot{x}_2 &= f_2(\mathbf{x}_2) + g_2(\mathbf{x}_2)x_3 + \omega_2 \\ &\vdots \\ \dot{x}_n &= f_n(\mathbf{x}_n) + g_n(\mathbf{x}_n)D(u) + \omega_n \\ y &= x_1 \end{aligned} \quad (1)$$

where $\mathbf{x}_i = [x_1, \dots, x_i]^T \in \mathbb{R}^i$, $i = 1, 2, \dots, n$ represents the state vector, $y \in \mathbb{R}$ denote the output variable and ω_i , $i = 1, 2, \dots, n$ is an external disturbance which is unknown but bounded. $u \in \mathbb{R}$ is the dead-zone input, $f_i(x_i)$, $g_i(x_i)$, $i = 1, 2, \dots, n$ are unknown nonlinear functions and unknown control gains, respectively, and $D(u)$ is the dead-zone output which is modeled as [19, 20]

$$D(u) = \begin{cases} p_l(u - \delta_l) & u \leq \delta_l \\ 0 & \delta_l < u < \delta_r \\ p_r(u - \delta_r) & u \geq \delta_r \end{cases} \quad (2)$$

where $p_l > 0$ and $p_r > 0$ represent the dead zone slopes in the left and right regions; $\delta_l < 0$ and $\delta_r > 0$ are the breakpoints of the left and right regions. Dead-zone model in (2) can be written as a combination of a linear system with a disturbance-like term as follows [19, 20]

$$D(u) = p(u - d(u)) \quad (3)$$

where

$$p = \begin{cases} p_l & u \leq 0 \\ p_r & u > 0 \end{cases} \quad (4)$$

and

$$d(u) = \begin{cases} \delta_l & u \leq \delta_l \\ u & \delta_l < u < \delta_r \\ \delta_r & u \geq \delta_r \end{cases} \quad (5)$$

Substituting (3) in (1), results

$$\begin{aligned} \dot{x}_1 &= f_1(\mathbf{x}_1) + g_1(\mathbf{x}_1)x_2 + \omega_1 \\ \dot{x}_2 &= f_2(\mathbf{x}_2) + g_2(\mathbf{x}_2)x_2 + \omega_2 \\ &\vdots \\ \dot{x}_n &= f_n(\mathbf{x}_n) + g_n(\mathbf{x}_n)(pu - pd(u)) + \omega_n \\ y &= \mathbf{x}_1 \end{aligned} \quad (6)$$

Before designing the proposed scheme, the following assumptions and lemma are considered.

Assumption 1. The desired trajectory y_d , is a sufficiently smooth function of t and $y_d, \dot{y}_d, \ddot{y}_d$ are bounded, i.e., there is a positive constant B_0 such that [21]

$$\Pi_0 := \{(y_d, \dot{y}_d, \ddot{y}_d) : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq B_0\}.$$

Assumption 2. The control gains $g_i(x_1, \dots, x_i), i = 1, \dots, n$ are unknown but there exist known positive constants g_i^m, g_i^M such that $g_i^m \leq |g_i(x_1, \dots, x_i)| \leq g_i^M$ [16].

Without loss of generality, it is assumed that the $g_i^m, i = 1, \dots, n$ is a positive constant.

Assumption 3. There exists positive constant g_i^d such that $|g_i(\cdot)| \leq g_i^d$ for $i = 1, \dots, n$ [16].

Assumption 4. The unknown external disturbance $\omega_i, i = 1, 2, \dots, n$ is unknown but bounded. Also, it is assumed that there exists a positive constant $\bar{\omega}$, such that $\max\{|\omega_1|, |\omega_2|, \dots, |\omega_n|\} \leq \bar{\omega}$.

Lemma 1. For $\forall \varepsilon > 0$ and $l \in R$, the following inequality holds [22]

$$0 \leq |l| - l \tanh\left(\frac{l}{\varepsilon}\right) \leq 0.2785\varepsilon \quad (7)$$

It is worth to note that $\bar{\omega}$ is only used for theoretical analysis and controller design does not require its actual value.

In this work, NN is invoked to approximate the uncertain nonlinear terms that appear in designing the controller. According to the universal approximation property of NN, any continuous function $\rho_i(\mathbf{z}_i)$ can be approximated as [23, 24]

$$\rho_i(\mathbf{z}_i) = \xi_i \Phi_i(\mathbf{z}_i) + \varepsilon_i(\mathbf{z}_i) \quad (8)$$

where $\varepsilon_i(\mathbf{z}_i)$ is an approximation error which is unknown but bounded, i.e., $|\varepsilon_i(\mathbf{z}_i)| \leq \bar{\varepsilon}_i$, q denote the network nodes, $\xi_i \in R^{1 \times q}$ represents ideal weight vector, and $\Phi_i(\mathbf{z}_i) \in R^q$ is the vector of basis functions. In this work, Gaussian type basis function is considered as follows [23, 24]

$$\Phi_i(\mathbf{z}_i) = e^{-\frac{(\mathbf{z}_i - \mathbf{c}_i)^T (\mathbf{z}_i - \mathbf{c}_i)}{\sigma_i^2}} \quad (9)$$

In (8), \mathbf{c}_i and σ_i denote the center and width of the i -th basis function.

3 Proposed Control Scheme and Stability Analysis

In the following, the proposed controller is designed and then its stability is analyzed. To avoid the ‘‘explosion of learning parameters’’ problem, MLP algorithm is invoked.

Let us define the unknown adjustable parameter $\theta_i = \|\xi_i\|^2$ for $i = 1, \dots, n$. In the MLP algorithm, in order to avoid the ‘‘explosion of learning parameters’’ problem, adaptive parameter $\theta_i \in R$ is adjusted instead of adjusting the elements of the weight vector $\xi_i \in R^{1 \times q}$; this results only one adaptive law and one adaptive parameter for approximating one uncertain term. So, the number of adjustable parameters and consequently the computational load is decreased, significantly. In fact, the number of the adjustable parameters in approximating uncertain functions is decreased from q parameters to one parameter. Therefore, the computational time is decreased significantly.

The proposed controller is designed in n steps as follows [21]:

Step 1. Define the first error surface as

$$s_1 = y - y_d \quad (10)$$

where y_d is the desired output. Taking the time derivative of (10) and substituting (6) in it, results in

$$\dot{s}_1 = g_1 x_2 + f_1 + \omega_1 - \dot{y}_d \tag{11}$$

In (11), the state variable x_2 is obtained as a virtual control variable. Let us define the uncertain function

$$\rho_1(\mathbf{z}_1) = \left(\frac{1}{g_1} \right) (f_1 - \dot{y}_d) + \frac{\bar{\omega}}{g_1^m} \tanh\left(\frac{s_1 \bar{\omega}}{g_1^m \nu} \right) \quad \text{where}$$

$\mathbf{z}_1 = [x_1, \dot{y}_d]^T$ and use NN to approximate it.

Now, in order to stabilize (11) and eliminate uncertain term $\rho_1(\mathbf{z}_1)$, the following virtual control law is proposed:

$$v_2 = -\frac{1}{2} \hat{\theta}_1 s_1 \Phi^T \Phi_1 - c_1 s_1 \tag{12}$$

where $c_1 > 0$ is a design parameter. To avoid the ‘‘explosion of complexity’’ problem, new state variable \bar{v}_2 is defined which is called filtered virtual control law and it is obtained by passing the virtual control law v_2 from a first-order filter with time constant τ_2 . So, we have

$$\tau_2 \dot{\bar{v}}_2 + \bar{v}_2 = v_2, \quad \bar{v}_2(0) = v_2(0) \tag{13}$$

Step i ($i=2, \dots, n-1$). Define the i -th error surface as

$$s_i = x_i - \bar{v}_i \tag{14}$$

Taking time derivative of (14) and using (6), gives

$$\dot{s}_i = f_i + g_i x_{i+1} + \omega_i - \dot{\bar{v}}_i \tag{15}$$

In (15), the state variable x_{i+1} is obtained as a virtual control input. Let us define uncertain function

$$\rho_i(\mathbf{z}_i) = \frac{1}{g_i} (f_i - \dot{\bar{v}}_i) + \frac{\bar{\omega}}{g_i^m} \tanh\left(\frac{s_i \bar{\omega}}{g_i^m \nu} \right), \quad \text{where}$$

$\mathbf{z}_i = [z_1, \dots, z_i]^T$ and use NN to it.

In order to stabilize (15) and eliminate the uncertain terms $\rho_i(\mathbf{z}_i)$, the following virtual control law is proposed

$$v_{i+1} = -\frac{1}{2} \hat{\theta}_i s_i \Phi^T \Phi_i - c_i s_i \tag{16}$$

where $c_i > 0$ is a design parameter. Similar to the first step, the virtual control law v_{i+1} is passed through a first-order filter with time constant τ_{i+1} to obtain the new variable \bar{v}_{i+1} as

$$\tau_{i+1} \dot{\bar{v}}_{i+1} + \bar{v}_{i+1} = v_{i+1}, \quad \bar{v}_{i+1}(0) = v_{i+1}(0) \tag{17}$$

Step n . Finally, define the n th error surface as

$$s_n = x_n - \bar{v}_n \tag{18}$$

Taking the time derivative of (18) and using (6), we have

$$\dot{s}_n = f_n(\mathbf{x}_n) + g_n(\mathbf{x}_n) p(u - d(u)) + \omega_n - \dot{\bar{v}}_n \tag{19}$$

Let us define $\rho_n(\mathbf{z}_n) = \frac{1}{p g_n} (f_n - \dot{\bar{v}}_n) +$

$$\frac{\bar{\omega}}{p_{\min} g_n^m} \tanh\left(\frac{s_n \bar{\omega}}{p_{\min} g_n^m \nu} \right) \text{ where } \mathbf{z}_n = [x_1, \dots, x_n, \dot{\bar{v}}_n]^T \text{ and}$$

d use NN to approximate uncertain term $\rho_n(\mathbf{z}_n)$. In this step, the following actual control input is designed

$$u = u' + \hat{d}(u') \tag{20}$$

where $c_n > 0$ is a design parameter, and control input u' is proposed as follows

$$u' = -\frac{1}{2} \hat{\theta}_n s_n \Phi^T \Phi_n - c_n s_n \tag{21}$$

Also, $\hat{d}(u')$ in (20) denote the estimation of a disturbance-like term in the dead-zone description.

In this work, an adaptive TSK-type fuzzy system is proposed to approximate it. The proposed TSK-type fuzzy system composed of a set of fuzzy rules in the following form

$$\text{If } u' \text{ is } U_k, \text{ Then } \hat{d}_k = \hat{D}_k, \quad k = 1, \dots, N \tag{22}$$

where U_k is the fuzzy membership function, N is the number of fuzzy rules and \hat{D}_k is the output value of the k -th rule.

Using the singleton fuzzifier, product inference engine and center of gravity defuzzification, the output of the TSK-type fuzzy system is calculated by

$$\hat{d}(u') = \frac{\sum_{k=1}^N \mu_k \hat{d}_k}{\sum_{k=1}^N \mu_k} \tag{23}$$

where μ_k indicates the firing strength of the k -th rule and \hat{d}_k is the output of the k -th rule.

Therefore, output of the proposed TSK-type fuzzy system can be expressed as

$$\hat{d}(u') = \hat{\mathbf{D}}^T \Psi(u') \tag{24}$$

where $\hat{\mathbf{D}} \in R^N$ denote the vector of adjustable parameters and $\Psi(u')$ is defined as

$$\Psi_i^j(u_i') = \frac{\mu_j}{\sum_{j=1}^N \mu_j} \quad (25)$$

According to the universal approximation property of fuzzy systems, there exists ideal weight vector \mathbf{D} such that the uncertain term $d(u)$ can be approximated as [25]

$$d(u) = \mathbf{D}^T \Psi(u) + \delta(u) \quad (26)$$

where $\delta(u)$ denote the unknown but bounded approximation error, i.e., $|\delta(u)| \leq \bar{\delta}$.

So, the control input in (20) can be rewritten as

$$u = -\frac{1}{2} \hat{\theta}_n s_n \Phi_n^T \Phi_n - c_n s_n + \hat{\mathbf{D}}^T \Psi(u') \quad (27)$$

In (11), (15) and (27), adaptive parameters $\hat{\theta}_i, i = 1, \dots, n$ and $\hat{\mathbf{D}}$ are adjusted based on the proposed adaptive laws in (28)

$$\begin{aligned} \dot{\hat{\theta}}_i &= \frac{1}{2} \gamma_i s_i^2 \Phi_i^T \Phi_i - \sigma_i \hat{\theta}_i \\ \dot{\hat{\mathbf{D}}} &= -\beta s_n \Psi - \sigma_d \hat{\mathbf{D}} \end{aligned} \quad (28)$$

where positive constants $\gamma_i, \sigma_i, \beta$, and σ_d are design parameters.

The error between \bar{v}_{i+1} and v_{i+1} is defined as a boundary layer error as $e_{i+1} = \bar{v}_{i+1} - v_{i+1}$ for $i = 1, \dots, n - 1$.

Taking the time derivative of e_{i+1} and using (13) and (16), results in

$$\begin{aligned} \dot{e}_{i+1} &= -\frac{e_{i+1}}{\tau_{i+1}} \\ &+ B_{i+1}(s_1, \dots, s_{i+1}, e_2, \dots, e_i, \hat{\theta}_1, \dots, \hat{\theta}_i, y_0, \dot{y}_0, \ddot{y}_0) \end{aligned} \quad (29)$$

where B_{i+1} is a continuous and bounded function which satisfies $|B_{i+1}| \leq \bar{B}_{i+1}$ [26].

Now, stability of the proposed approach is analyzed. For this, the following Lyapunov function is considered

$$V = \frac{1}{2} \left(\sum_{i=1}^{n-1} \frac{1}{g_i} s_i^2 + \frac{1}{pg_n} s_n^2 + \sum_{i=1}^{n-1} e_{i+1}^2 + \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\theta}_i^2 + \frac{1}{\beta} \tilde{\mathbf{D}}^T \tilde{\mathbf{D}} \right) \quad (30)$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\tilde{\mathbf{D}} = \mathbf{D} - \hat{\mathbf{D}}$.

Taking the time derivative of (30), results in

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n-1} \left(\frac{1}{g_i} s_i \dot{s}_i - \frac{1}{2g_i} s_i^2 \dot{g}_i \right) + \frac{1}{pg_n} s_n \dot{s}_n \\ &- \frac{1}{2pg_n} s_n^2 \dot{g}_n + \sum_{i=1}^{n-1} e_{i+1} \dot{e}_{i+1} - \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\theta}_i \dot{\tilde{\theta}}_i - \frac{1}{\beta} \tilde{\mathbf{D}}^T \dot{\tilde{\mathbf{D}}} \end{aligned} \quad (31)$$

Substituting (11), (15) and (19) in (31), results

$$\begin{aligned} \dot{V} &= \frac{1}{g_1} s_1 (f_1 + g_1 x_2 + \omega_1 - \dot{y}_d) - \frac{1}{2g_1^2} s_1^2 \dot{g}_1 \\ &+ \sum_{i=2}^{n-1} \left(\frac{1}{g_i} s_i (f_i + g_i x_{i+1} + \omega_i - \dot{v}_i) - \frac{1}{2g_i^2} s_i^2 \dot{g}_i \right) \\ &+ \frac{1}{pg_n} s_n (f_n - \dot{v}_i + pg_n u + \omega_n - pg_n d(u)) \\ &- \frac{1}{2pg_n} s_n^2 \dot{g}_n + \sum_{i=1}^{n-1} e_{i+1} \dot{e}_{i+1} - \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\theta}_i \dot{\tilde{\theta}}_i - \frac{1}{\beta} \tilde{\mathbf{D}}^T \dot{\tilde{\mathbf{D}}} \end{aligned} \quad (32)$$

According to Assumption 3 and Lemma 1, we have:

$$\begin{cases} \frac{1}{g_i} s_i \omega_i(t) \leq \frac{1}{g_i^m} |s_i| \bar{\omega} \leq \frac{1}{g_i^m} s_i \bar{\omega} \tanh\left(\frac{s_i \bar{\omega}}{g_i^m v}\right) + 0.2785 v_i, \\ i = 1, \dots, n-1 \\ \frac{1}{pg_n} s_n \omega_n(t) \leq \frac{1}{p_{\min} g_n^m} |s_n| \bar{\omega} \leq \frac{1}{p_{\min} g_n^m} s_n \bar{\omega} \tanh\left(\frac{s_n \bar{\omega}}{p_{\min} g_n^m v}\right) \\ + 0.2785 v_n \end{cases} \quad (33)$$

Also, substituting $e_{i+1} = \bar{v}_{i+1} - v_{i+1}$ in (14), gives

$$x_{i+1} = s_{i+1} + e_{i+1} + v_{i+1}, i = 1, \dots, n - 1 \quad (34)$$

Substituting (33) and (34) into (32), and also considering definitions of $\rho_1(z_1), \rho_i(\mathbf{z}_i), \dots, \rho_n(\mathbf{z}_n)$ and using NN approximation in (8), one can write (32) as follows

$$\begin{aligned} \dot{V} &\leq s_1 (\xi_1^T \Phi_1 + \varepsilon_1 + s_2 + e_2 + v_2) \\ &+ \sum_{i=2}^{n-1} s_i (\xi_i^T \Phi_i + \varepsilon_i + s_{i+1} + e_{i+1} + v_{i+1}) \\ &+ s_n (\xi_n^T \Phi_n + \varepsilon_n + u - d(u)) \\ &- \frac{1}{2g_1^2} s_1^2 \dot{g}_1 - \frac{1}{2g_i^2} s_i^2 \dot{g}_i - \frac{1}{2pg_n^2} s_n^2 \dot{g}_n \\ &+ \sum_{i=1}^{n-1} e_{i+1} \dot{e}_{i+1} - \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\theta}_i \dot{\tilde{\theta}}_i - \frac{1}{\beta} \tilde{\mathbf{D}}^T \dot{\tilde{\mathbf{D}}} \\ &+ 0.2785 \left(\sum_{i=1}^n v_i \right) \end{aligned} \quad (35)$$

Now, consider the Young inequalities [27]

$$\begin{cases} s_i \xi_i^T \Phi_i + s_i \varepsilon_i \leq \frac{1}{2} \theta_i s_i^2 \Phi_i^T \Phi_i + \frac{1}{2} + \frac{1}{2} s_i^2 + \frac{1}{2} \bar{\varepsilon}_i^2 \\ s_i \xi_i^T \Phi_i + s_i \varepsilon_i \leq \frac{1}{2} \theta_i s_i^2 \Phi_i^T \Phi_i + \frac{1}{2} + \frac{1}{2} s_i^2 + \frac{1}{2} \bar{\varepsilon}_i^2, i = 2, \dots, n-1 \\ s_n \xi_n^T \Phi_n + s_n \varepsilon_n \leq \frac{1}{2} \theta_n s_n^2 \Phi_n^T \Phi_n + \frac{1}{2} + \frac{1}{2} s_n^2 + \frac{1}{2} \bar{\varepsilon}_n^2 \end{cases} \quad (36)$$

Using the inequalities in (34), and substituting (11), (15) and (26) in (33), gives

$$\begin{aligned} \dot{V} \leq & s_1 s_2 + s_1 e_2 + \frac{1}{2} \tilde{\theta}_1 s_1^2 \boldsymbol{\varphi}_1^T \boldsymbol{\varphi}_1 - c_1 s_1^2 \\ & + \frac{1}{2} + \frac{1}{2} \bar{\varepsilon}_1^2 + \frac{1}{2} s_1^2 - \frac{1}{2g_1^2} s_1^2 \dot{g}_1 \\ & + \sum_{i=2}^{n-1} \left(s_i s_{i+1} + s_i e_{i+1} + \frac{1}{2} \tilde{\theta}_i s_i^2 \boldsymbol{\varphi}_i^T \boldsymbol{\varphi}_i - c_i s_i^2 \right) \\ & + \sum_{i=1}^{n-1} \left(\frac{1}{2} + \frac{1}{2} \bar{\varepsilon}_i^2 + \frac{1}{2} s_i^2 - \frac{1}{2g_i^2} s_i^2 \dot{g}_i \right) \\ & + \frac{1}{2} \tilde{\theta}_n s_n^2 \boldsymbol{\varphi}_n^T \boldsymbol{\varphi}_n - c_n s_n^2 + s_n (\hat{d} - d) + \frac{1}{2} + \frac{1}{2} \bar{\varepsilon}_n^2 \\ & + \frac{1}{2} s_n^2 - \frac{1}{2p_{\min} g_n^2} s_n^2 \dot{g}_n + \sum_{i=1}^{n-1} e_{i+1} \dot{e}_{i+1} - \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\theta}_i \dot{\theta}_i \\ & - \frac{1}{\beta} \tilde{\mathbf{D}}^T \dot{\tilde{\mathbf{D}}} + 0.2785 \left(\sum_{i=1}^n \nu_i \right) \end{aligned} \quad (37)$$

Consider the following facts

$$\begin{cases} s_1 s_2 \leq \frac{1}{2} s_1^2 + \frac{1}{2} s_2^2 \\ s_1 e_2 \leq \frac{1}{2} s_1^2 + \frac{1}{2} e_2^2 \\ s_i s_{i+1} \leq \frac{1}{2} s_i^2 + \frac{1}{2} s_{i+1}^2 \\ s_i e_{i+1} \leq \frac{1}{2} s_i^2 + \frac{1}{2} e_{i+1}^2 \\ e_{i+1} B_{i+1} \leq \frac{1}{2} e_{i+1}^2 + \frac{1}{2} \bar{B}_{i+1}^2 \end{cases} \quad (38)$$

Substituting (38) in (37), results in

$$\begin{aligned} \dot{V} \leq & s_1^2 \left(\frac{3}{2} - c_1 - \frac{1}{2g_1^2} \dot{g}_1 \right) + \sum_{i=2}^{n-1} s_i^2 \left(2 - c_i - \frac{1}{2g_i^2} \dot{g}_i \right) \\ & + s_n^2 \left(\frac{3}{2} - c_n - \frac{1}{2p_{\min} g_n^2} \dot{g}_n \right) + \sum_{i=1}^{n-1} e_{i+1}^2 \left(1 - \frac{1}{\tau_{i+1}} \right) \\ & - \sum_{i=1}^n \frac{1}{\gamma_i} \tilde{\theta}_i \left(\dot{\theta}_i - \frac{1}{2} \gamma_i s_i^2 \boldsymbol{\varphi}_i^T \boldsymbol{\varphi}_i \right) - \frac{1}{\beta} \tilde{\mathbf{D}}^T \left(\dot{\tilde{\mathbf{D}}} + \beta s_n \boldsymbol{\Psi} \right) \\ & + 0.2785 \left(\sum_{i=1}^n \nu_i \right) + \frac{n}{4} + \frac{1}{2} \sum_{i=1}^n \bar{\varepsilon}_i^2 + \frac{1}{2} \bar{\delta}^2 + \frac{1}{2} \sum_{i=1}^{n-1} \bar{B}_{i+1}^2 \end{aligned} \quad (39)$$

Also, by substituting the adaptive laws (28) in (39), results

$$\begin{aligned} \dot{V} \leq & s_1^2 \left(\frac{3}{2} - c_1 - \frac{1}{2g_1^2} \dot{g}_1 \right) + \sum_{i=2}^{n-1} s_i^2 \left(2 - c_i - \frac{1}{2g_i^2} \dot{g}_i \right) \\ & + s_n^2 \left(\frac{3}{2} - c_n - \frac{1}{2p_{\min} g_n^2} \dot{g}_n \right) + \sum_{i=1}^{n-1} e_{i+1}^2 \left(1 - \frac{1}{\tau_{i+1}} \right) \\ & - \sum_{i=1}^n \frac{\sigma_i}{2\gamma_i} \tilde{\theta}_i^2 - \frac{\sigma_d}{2\beta} \tilde{\mathbf{D}}^T \tilde{\mathbf{D}} + \sum_{i=1}^n \frac{\sigma_i}{2\gamma_i} \theta_i^2 + \frac{\sigma_d}{2\beta} D^T D \end{aligned}$$

$$+ \frac{n}{4} + \frac{1}{2} \sum_{i=1}^n \bar{\varepsilon}_i^2 + \frac{1}{2} \bar{\delta}^2 + \frac{1}{2} \sum_{i=1}^{n-1} \bar{B}_{i+1}^2 + 0.2785 \left(\sum_{i=1}^n \nu_i \right) \quad (40)$$

To guarantee the stability of the closed-loop system, design parameters are chosen as follows:

$$\begin{cases} c_1 \geq \frac{3}{2} + \frac{\kappa}{2g_1^m} + \frac{g_1^d}{2(g_1^m)^2} \\ c_i \geq 2 + \frac{\kappa}{2g_i^m} + \frac{g_i^d}{2(g_i^m)^2}, \quad i = 2, 3, \dots, n-1 \\ c_n \geq \frac{3}{2} + \frac{\kappa}{2g_n^m p_{\min}} + \frac{g_n^d}{2p_{\min} (g_n^m)^2} \\ \tau_{i+1} \leq \frac{2}{\kappa + 2}, \quad i = 1, 2, \dots, n-1 \\ \sigma_i \geq \kappa, \quad i = 1, 2, \dots, n \\ \sigma_d \geq \kappa \end{cases} \quad (41)$$

where, p_{\min} denote the minimum bound of the dead-zone slopes and κ is a positive constant. Considering (41), and comparing (40) with (30) reveals that (40) can be rewritten as

$$\dot{V} \leq -\kappa V + \bar{m} \quad (42)$$

where

$$\begin{aligned} \bar{m} = & \frac{1}{2} \left(\sum_{i=1}^n \frac{\sigma_i}{\gamma_i} \theta_i^2 + \frac{\sigma_d}{\beta} \mathbf{D}^T \mathbf{D} + \frac{n}{2} + \sum_{i=1}^n \bar{\varepsilon}_i^2 + \bar{\delta}^2 \right. \\ & \left. + \sum_{i=1}^{n-1} \bar{B}_{i+1}^2 + 0.2785 \left(\sum_{i=1}^n \nu_i \right) \right) \end{aligned}$$

Solving (40) yields

$$V \leq \frac{\bar{m}}{\kappa} + \left(V(0) - \frac{\bar{m}}{\kappa} \right) e^{-\kappa t} \quad (43)$$

From (41), it is obtained that all signals of the closed-loop system are semi-globally uniformly ultimately bounded and the tracking error can be made arbitrarily small by proper selection of the design parameters. ■

Remark 1. It is worth to note that in comparison with [16], the number of adaptive parameters in each step of the controller design is decreased from two vectors to one parameter. So, the proposed method in this work reduces the number of adaptive parameters and online computational load, considerably. Furthermore, the proposed scheme in this work is more general than the proposed one in [16]. The proposed strategy based on the MLP algorithm in this work not only can be applied to the proposed scheme in [16] but also it can be applied for all of the linear-in-parameter approximator-based control scheme. Since it eliminates the ‘‘explosion of

learning parameters” problem that exists in the most of the linear-in-parameter approximator-based control schemes.

4 Simulation Results

To investigate performance of the proposed approach, inverted pendulum system is simulated. This system is described by the following differential equations [28]

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)D(u) + \omega_2 \\ y = x_1 \end{cases} \quad (44)$$

where

$$f_2(x_1, x_2) = \frac{g_a \sin x_1 - \frac{mlx_2^2 \cos x_1 \sin x_1}{m_c + m}}{l \left(\frac{4}{3} - \frac{m \cos^2 x_1}{m_c + m} \right)},$$

$$g_2(x_1, x_2) = \frac{\frac{\cos x_1}{m_c + m}}{l \left(\frac{4}{3} - \frac{m \cos^2 x_1}{m_c + m} \right)}$$

and $D(u)$ denote the dead-zone input, and ω_2 represents the external disturbance. The parameters of the system were set to $m_c = 1kg$, $m = 0.1kg$, $l = 0.5m$ and $g_a = 9.8m/s^2$.

In order to construct the proposed control scheme, at first, two NNs composed of 25 neurons were constructed to approximate uncertain terms $\rho_1(\mathbf{z}_1)$ and $\rho_2(\mathbf{z}_2)$. The center of the Gaussian activation functions were chosen arbitrarily and the width of the membership functions was set to 0.05.

Also, an adaptive TSK-type fuzzy system was designed to approximate disturbance-like term in the dead-zone description. The proposed adaptive fuzzy systems composed of 7 membership functions and 7 fuzzy rules.

Fig. 1 shows the membership functions of the proposed adaptive TSK-type fuzzy system.

Remark 2. Number and type of the membership functions are arbitrary chosen by the designer.

In the following, dead-zone parameters were set to $p_r = p_l = 0.8$ and $\delta_r = -\delta_l = 0.2$, the learning rates and design parameters of the proposed scheme were set to $c_1 = 5$, $c_2 = 12$, $\gamma_2 = 4$, $\beta = 1$, $\sigma_d = 0.01$, $\sigma_2 = 0.01$ and initial conditions were chosen as $x_1(0) = -x_2(0) = 0.2$.

To design the proposed controller, the following virtual and actual control laws are chosen

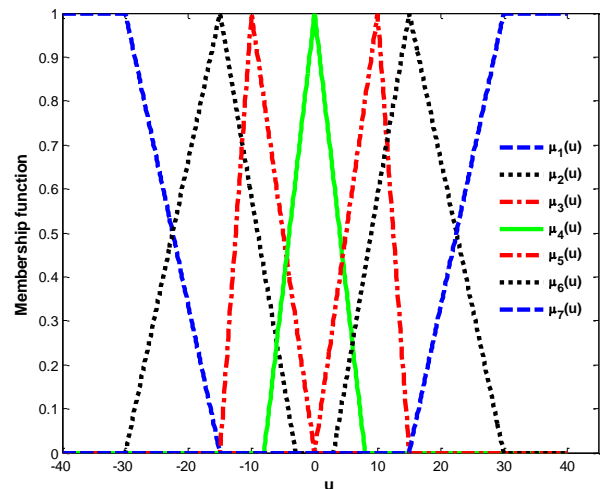


Fig. 1 Membership functions of an adaptive TSK-type fuzzy system.

$$\begin{cases} v_2 = -\frac{1}{2} \hat{\theta}_1 s_1 \Phi^T \Phi_1 - c_1 s_1 \\ u = -\frac{1}{2} \hat{\theta}_2 s_2 \Phi^T \Phi_2 - c_2 s_2 + \hat{D}^T \Psi(u') \end{cases} \quad (45)$$

To investigate the performance of the presented controller, tracking of a step command with magnitude $\pi/6$ is performed. The control objective is to track the desired trajectory $y_d = \pi/6$. The reference signal y_d , \dot{y}_d and \ddot{y}_d are obtained from the following filter [14]

$$\frac{y_d}{y_c} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (46)$$

where $\xi = 0.9$ and $\omega_n = 0.9$. Figs. 2-5 show the simulation results. Also, external disturbance $\omega_2(t)$ in (42) is applied as the same as [14]:

$$\omega_2(t) = \begin{cases} 0.1 \sin t & t \leq 20 \\ -5 & t > 20 \end{cases} \quad (47)$$

To verify the superior performance of the proposed scheme, the results are compared with the composite learning-adaptive fuzzy control (CL-AFC) scheme in [14] and they are shown in Figs. 2-6. Angular position and the desired output y_d are shown in Fig. 2(a). Angular velocity is depicted in Fig. 2(b). The error surfaces s_1 and s_2 are depicted in Fig. 3.

Also, Fig. 4 illustrates the control input. Norm of the adjustable parameters are shown in Fig. 5. As it is obtained from the results the proposed scheme compensates the effects of the dead-zone and also compensates the effect of the external disturbance without using any special control term. However, in [14], H_∞ control law and disturbance observer were used to compensate the effect of the dead-zone. Also, number

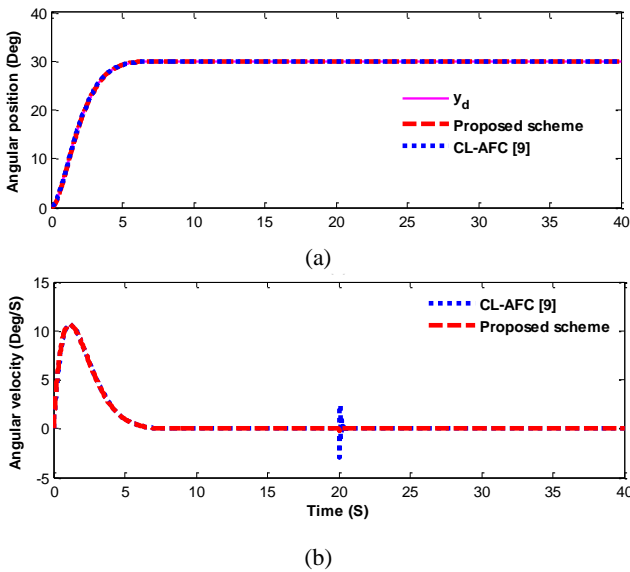


Fig. 2 a) Angular position, and b) Angular velocity in the presence of deterministic disturbance.

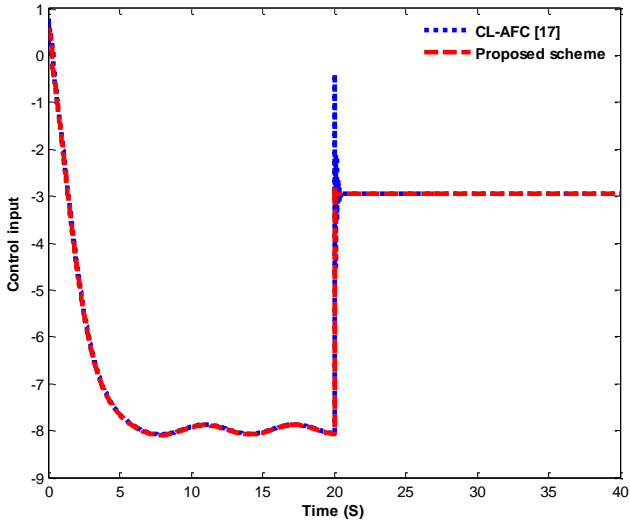


Fig. 4 Control input in the presence of deterministic disturbance.

of nodes and adaptive parameters are reported in Table 1.

As is obtained from the reported results in Table 1, however the node number of networks are same for two approaches but the number of adjustable parameters by the proposed scheme is less than that of [14]. Furthermore, the proposed control scheme in [14] requires the dead-zone input while the proposed scheme does not require it.

Also, simulation results are provided to verify the robustness of the proposed approach against exogenous disturbance as a random signal. For this, the following random signal was generated and applied to the system

$$\omega_2(t) = \begin{cases} 1+0.01\text{randn}() & t \leq 20 \\ 2+0.05\text{randn}() & t > 20 \end{cases} \quad (48)$$

Simulation results are shown in Figs. 6-9. Fig. 6

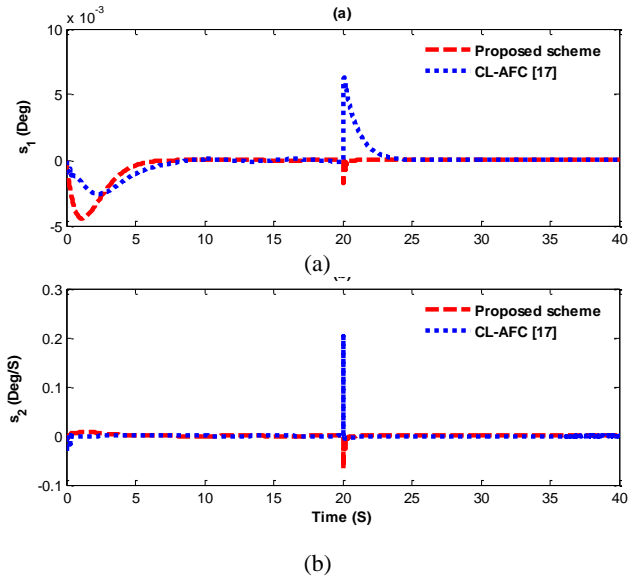


Fig. 3 a) First error surface, and b) Second error surface in the presence of deterministic disturbance.

Table 1 Comparison of the proposed scheme and proposed controllers in [14].

Scheme	Criteria	Number of NN Nodes	Number of Adjustable Parameters	Availability of Dead-Zone Input
Proposed Scheme		50	9	No
CL-AFC [14]		50	51	Yes

shows the angular position and angular velocity. Sliding surfaces s_1 and s_2 are depicted in Fig. 7. Also, Figs. 8 and 9 illustrate the control input and norm of the adjustable parameters, respectively. Obtained results demonstrate that the proposed controller not only can compensate the adverse effects of the dead-zone nonlinearity but also it can preserve the performance of the proposed scheme in the presence of stochastic disturbance.

5 Conclusions

An adaptive approximation-based controller was proposed for uncertain strict-feedback nonlinear systems with unknown control gains and unknown dead-zone nonlinearity. Dead-zone is modeled as a combination of a linear system with a disturbance-like term. NN is invoked to approximate the uncertain term in controller design. MLP algorithm is used to avoid the

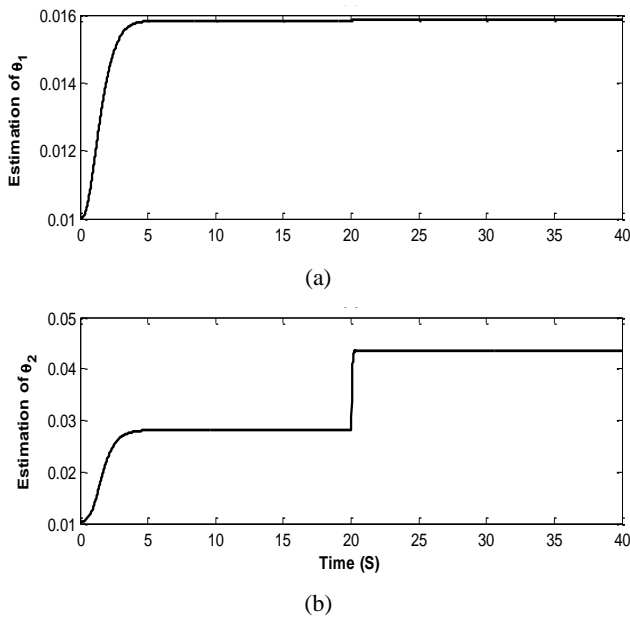


Fig. 5 a) Estimation of θ_1 , and b) Estimation of θ_2 in the presence of deterministic disturbance.

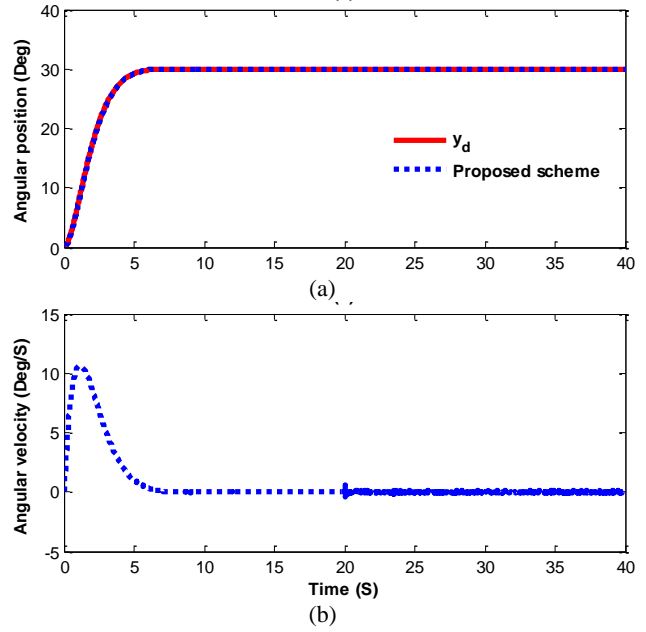


Fig. 6 a) Angular position, and b) Angular velocity in the presence of stochastic disturbance.

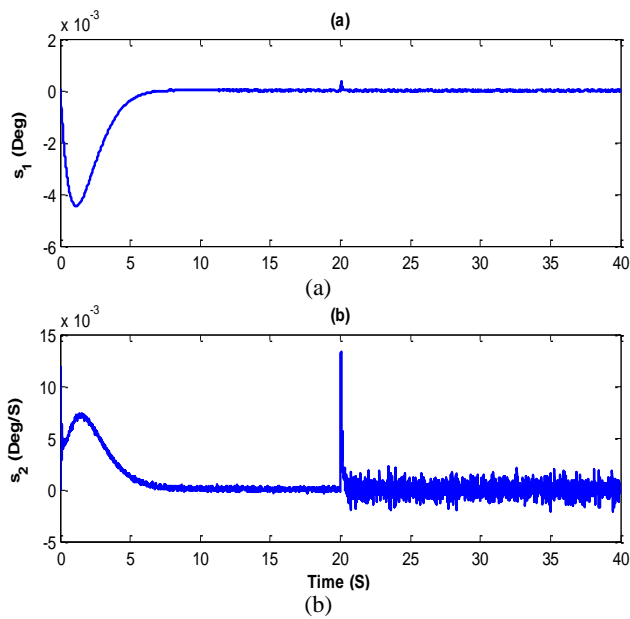


Fig. 7 a) First sliding surface, and b) Second sliding surface in the presence of stochastic disturbance.

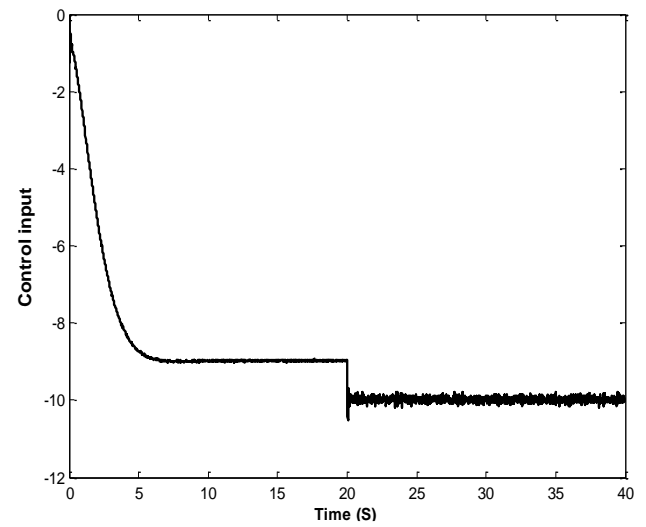


Fig. 8 Control input in the presence of stochastic disturbance.

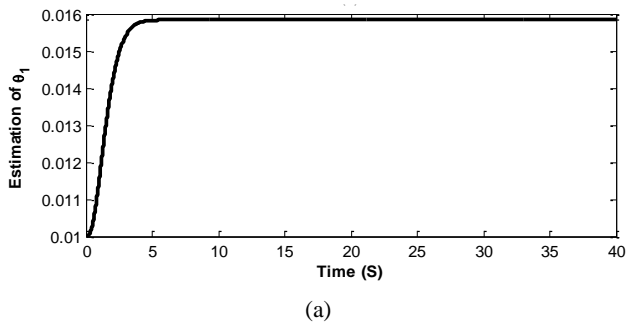


Fig. 9 a) Estimation of θ_1 , and b) Estimation of θ_2 in the presence of stochastic disturbance.

“explosion of learning parameters” problem and to decrease the number of adaptive parameters and online computational time. Also, an adaptive TSK-type fuzzy system was proposed to approximate the uncertain disturbance-like term in the dead-zone description. Stability analysis shows that all the signals of the closed-loop system are semi-globally uniformly ultimately bounded. Unlike most of the existing approaches, the presented approach does not require the availability of the dead-zone input and it does not merge the disturbance-like term with uncertain dynamics. Furthermore, the proposed controller avoids the “explosion of learning parameters”, “explosion of complexity” and “singularity” problems, simultaneously. Simulation and comparison results on an inverted pendulum system and a single link robot manipulator illustrate the effectiveness and superior performance of the proposed approach. Considering the presented controller in this work, control of uncertain nonlinear systems with input nonlinearity and actuator fault is proposed as a future work.

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control of nonlinear

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