IUST Iran University of Science and Technology

Iranian Journal of Electrical and Electronic Engineering

Journal Homepage: ijeee.iust.ac.ir

Research Paper

New Robust Stability Criteria for Uncertain Neutral Time-Delay Systems With Discrete and Distributed Delays

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Abstract: In this study, delay-dependent robust stability problem is investigated for uncertain neutral systems with discrete and distributed delays. By constructing an augmented Lyapunov-Krasovskii functional involving triple integral terms and taking into account the relationships between the different delays, new less conservative stability and robust stability criteria are established first using the delay bi-decomposition approach then generalized with the delay N-decomposition technique. Some integral inequalities are employed to deal with the cross terms and few free weighing matrices are introduced to reduce the conservatism. The proposed criteria are expressed in terms of linear matrix inequalities. The effectiveness of the proposed stability conditions is illustrated by a numerical example.

Keywords: Asymptotic Stability, Linear Matrix Inequality, Lyapunov-Krasovskii Functional, Neutral Systems, Robust Stability, Time-Delay Systems.

1 Introduction

TIME-DELAY has been widely studied during the past two decades because many practical systems include after-effects phenomena in their inner dynamics such as networked control systems, transportation of energy or information, process control systems, biological and mechanical systems [1].

The problem of the stability of time-delay systems has attracted a great deal of interest because in many cases time-delay leads to performance degradation and is considered as a major source of instability [2].

Since delay-dependent stability conditions are known to be less conservative than delay-independent stability criteria, especially when the size of the delay is small, reducing conservatism by establishing delay-dependent stability conditions with the maximum allowable delay bound has been a challenging task during the last several years [3-5].

The first step to derive a less conservative stability condition is the choice of the Lyapunov-Krasovskii functional (LKF). It has been demonstrated that constructing an appropriate LKF is an efficient tool to obtain stability criteria in terms of linear matrix inequalities (LMIs) [6-9]. A literature review shows that other important techniques are introduced to reduce conservatism in delay-dependent stability criteria such as model transformation [10, 11], augmented LKF [12-14], convex combination based conditions [15-17], free weighting matrices [18-20], delay fractioning approaches [21, 22], and integral inequalities exploiting [23-25].

In the respect of using integral inequalities, Jensen's inequality [26] has been widely employed to bound the cross terms resulting from the differentiation of the LKF as well as Park's inequality [27], Moon's inequality [28] and the Wirtinger-based inequality [29].

Though neutral systems with discrete and distributed delays are important practically and theoretically, only a limited number of results has been dedicated to studying the stability of this class of systems [30-34]. In order to obtain robust stability conditions for neutral systems with discrete and distributed delays, Li and Zhu [30]

Iranian Journal of Electrical and Electronic Engineering, 2020.
Paper first received 23 September 2018, revised 31 July 2019, and accepted 05 August 2019.
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constructed a modified LKF and introduced some free weighting matrices. Sun *et al*. [31] constructed a new form of LKF with triple integral terms then derived new stability and robust stability conditions without using any free weighting matrices. A delay decomposition approach is employed in [32] to obtain some stability criteria with larger maximum allowable delay bound. By using fewer decision variables, the authors in [33] derived improved stability criteria. Taking into account the relationships between the discrete, neutral, and distributed delays, Chen *et al*. [34] established some less conservative stability conditions for neutral systems with discrete and distributed delays.

Motivated by the above discussions, in this paper, we consider the interconnected information between neutral delay, discrete delay, and distributed delay to construct an augmented LKF with triple integral terms. Some integral inequalities are used to deal with the cross terms in the derivative of the LKF and some free weighting matrices are introduced to reduce conservatism. New less conservative stability and robust stability criteria in terms of LMIs are derived based on the delay decomposition approach. The introduction of the derivative of the state with respect to the distributed delay plays a key role in establishing the stability conditions.

The rest of this paper is organized as follows: In Section 2, the robust stability problem for uncertain neutral systems with discrete and distributed delays is formulated and some important lemmas are given. Section 3 presents the stability and robust stability criteria for the nominal and the uncertain system based on the delay bi-decomposition approach then generalized using the delay *N*-decomposition idea. A numerical example is given in Section 4 to demonstrate the effectiveness of the proposed stability conditions. Section 5 concludes the paper.

Notations. Throughout this paper, the superscript T stands for matrix transposition. $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and set of all $n\times n$ real matrices, respectively. $P > 0$ (≥ 0) means that P is a real symmetric and positive definite (positive semi-definite) matrix. I and 0 are used to denote identity and zero matrices, respectively. The symmetric terms in a symmetric matrix are denoted by *. $\|.\|$ refers to the induced matrix 2-norm. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible with algebraic operations.

2 Problem Formulation

Consider the following uncertain neutral system with discrete and distributed delays:

$$\dot{x}(t)-(C+\Delta C)\dot{x}(t-\tau)=(A+\Delta A)x(t)+(B+\Delta B)x(t-h)+(D+\Delta D)\int_{t-r}^{t}x(s)ds \quad (1)$$

$$x(t)=\phi(t), \quad \forall t\in[-\max\{\tau,h,r\},0] \quad (2)$$

where $x(t)\in\mathbb{R}^n$ is the state vector, τ, h, and r denote the neutral delay, discrete delay, and distributed delay, respectively, $\phi\in\mathbb{R}^n$ is a continuous vector valued initial function. A, B, C, and $D\in\mathbb{R}^{n\times n}$ are known real constant matrices, $\Delta A(t)$, $\Delta B(t)$, $\Delta C(t)$, and $\Delta D(t)$ denote the time-varying parameter uncertainties and are assumed to be of the following form:

$$\left[\Delta A(t)\ \Delta B(t)\ \Delta C(t)\ \Delta D(t)\right]=L\,F(t)\left[E_a\ E_b\ E_c\ E_d\right] \quad (3)$$

where L, E_a, E_b, E_b, E_c, and E_d are known real constant matrices with appropriate dimensions and $F(t)$ is an unknown continuous time-varying matrix function satisfying

$$F^T(t)F(t)\leq I \quad (4)$$

For system (1), it is also assumed that the condition $\|C+\Delta C(t)\|\leq I$ holds, which is necessary for guaranteeing the asymptotic stability.

The objective of this paper is to determine the maximum allowable delay bounds of the considered system that ensure the robust stability. To this end, the following lemmas are exploited.

Lemma 1. [3, 26] For any constant symmetric matrix $M\in\mathbb{R}^n$, scalars a and b satisfying $a < b$, and vector function ω: $[a, b]\rightarrow\mathbb{R}^n$ such that the integrations concerned are well defined, then

$$\left(\int_a^b\omega(s)ds\right)^T M\left(\int_a^b\omega(s)ds\right)\leq(b-a)\int_a^b\omega^T(s)M\,\omega(s)ds \quad \text{(i)}$$

$$\left(\int_a^b\int_{a\,t+\theta}^{t}\omega(s)ds\,d\theta\right)^T M\left(\int_a^b\int_{a\,t+\theta}^{t}\omega(s)ds\,d\theta\right)\leq\frac{\left(b^2-a^2\right)}{2}\int_a^b\int_{a\,t+\theta}^{t}\omega^T(s)M\,\omega(s)dsd\theta \quad \text{(ii)}$$

Lemma 2. [3] Let U, V, W, and M be real matrices of appropriate dimensions with M satisfying $M = M^{\mathrm{T}}$, then $M + UVW + W^{\mathrm{T}}V^{\mathrm{T}}U^{\mathrm{T}} < 0$ for all V satisfying $V^{\mathrm{T}}V\leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that $M + \varepsilon^{-1}UU^{\mathrm{T}} + \varepsilon W^{\mathrm{T}}W < 0$

3 Main Results

In this section, the robust stability of uncertain neutral systems with discrete and distributed delays is studied. Based on the delay bi-decomposition approach, a new less conservative robust stability criterion is presented then generalized using the delay *N*-decomposition technique. First, we propose the asymptotic stability condition for the following nominal system:

$$\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + Bx(t-h) + D\int_{t-r}^{t} x(s)ds \quad (5)$$

Theorem 1. For given scalars τ, h, and r, the nominal system (5) is asymptotically stable if there exist matrices
$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} \\ * & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} \\ * & * & P_{33} & P_{34} & P_{35} & P_{36} \\ * & * & * & P_{44} & P_{45} & P_{46} \\ * & * & * & * & P_{55} & P_{56} \\ * & * & * & * & * & P_{66} \end{bmatrix} > 0,$$
$Q > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $S_i = \begin{bmatrix} S_{i1} & S_{i2} \\ * & S_{i3} \end{bmatrix} > 0$, $U_j = \begin{bmatrix} U_{j1} & U_{j2} \\ * & U_{j3} \end{bmatrix} > 0$, $W_j = \begin{bmatrix} W_{j1} & W_{j2} \\ * & W_{j3} \end{bmatrix} > 0$, $Z_1 > 0$, $Z_2 > 0$, and M_k ($i = 1, 2, 3, 4$, $j = 1, 2$, $k = 1, 2, 3, 4, 5$) such that the LMI (6) holds.

$$\Xi = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & P_{13} & 0 & \Xi_{19} & \Xi_{1,10} & \Xi_{1,11} & \Xi_{1,12} & \Xi_{1,13} \\
* & \Xi_{22} & \Xi_{23} & -P_{26} & P_{12}^T & P_{22} & P_{23} & 0 & \Xi_{29} & \Xi_{2,10} & -P_{46} & -W_{12}^T & 0 \\
* & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & 0 & \Xi_{39} & \Xi_{3,10} & \Xi_{3,11} & 0 & -W_{22}^T \\
* & * & * & \Xi_{44} & 0 & 0 & 0 & -S_{22} & -P_{46}^T & \Xi_{4,10} & \Xi_{4,11} & 0 & 0 \\
* & * & * & * & \Xi_{55} & \Xi_{56} & 0 & 0 & P_{14} & P_{15} & \Xi_{5,11} & 0 & 0 \\
* & * & * & * & * & \Xi_{66} & 0 & 0 & P_{24} & P_{25} & \Xi_{6,11} & 0 & 0 \\
* & * & * & * & * & * & -S_{13} & 0 & P_{34} & P_{35} & P_{36} & 0 & 0 \\
* & * & * & * & * & * & * & -S_{23} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Xi_{99} & U_{11} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Xi_{10,10} & U_{21} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \Xi_{11,11} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \Xi_{12,12} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & \Xi_{13,13}
\end{bmatrix} < 0 \quad (6)$$

where $\Xi_{11} = P_{14} + P_{14}^T + P_{15} + P_{15}^T + P_{16} + P_{16}^T - 2R_1 - 2R_2 - 2R_3 + S_{11} + S_{21} + hS_{31} + rS_{41} - \frac{1}{h}S_{33} - \frac{1}{r}S_{43} + (\tau-h)^2U_{11} + (h-r)^2U_{21} + W_{11} + W_{21} - \frac{2}{\tau}Z_1 - \frac{2}{h}Z_2 + M_1A + A^TM_1^T$,
$\Xi_{12} = -P_{14} + P_{24}^T + P_{25}^T + P_{26}^T$, $\Xi_{13} = -P_{15} + P_{24}^T + P_{35}^T + P_{36}^T + \frac{1}{h}S_{33} + M_1B + A^TM_2^T$, $\Xi_{14} = -P_{16} + \frac{1}{r}S_{43}$, $\Xi_{15} = P_{11} + (\tau-h)^2U_{12} + (h-r)^2U_{22} + S_{12} + S_{22} + hS_{32} + rS_{42} - M_1 + A^TM_3^T$, $\Xi_{16} = P_{12} + M_1C + A^TM_4^T$, $\Xi_{19} = P_{44} + P_{45}^T + P_{46}^T + \frac{2}{\tau}R_1$, $\Xi_{1,10} = P_{45} + P_{55} + P_{56}^T + \frac{2}{h}R_2 - \frac{1}{h}S_{32}^T$,
$\Xi_{1,11} = P_{46} + P_{56} + P_{66} + \frac{2}{r}R_3 - \frac{1}{r}S_{42}^T + M_1D + A^TM_5^T$,
$\Xi_{1,12} = W_{12} + \frac{2}{\tau}Z_1$, $\Xi_{1,13} = W_{22} + \frac{2}{h}Z_2$, $\Xi_{22} = -P_{24} - P_{24}^T - U_{13} - W_{13}$, $\Xi_{23} = -P_{25} - P_{34}^T + U_{13}$, $\Xi_{29} = -P_{44} + U_{12}^T$, $\Xi_{2,10} = -P_{45} - U_{12}^T$, $\Xi_{33} = -P_{35} - P_{35}^T - S_{11} - \frac{1}{h}S_{33} - U_{13} - U_{23} - W_{23} + M_2B + B^TM_2^T$, $\Xi_{34} = -P_{36} + U_{23}$, $\Xi_{35} = P_{13}^T - M_2 + B^TM_3^T$, $\Xi_{36} = P_{23}^T + M_2C + B^TM_4^T$, $\Xi_{37} = P_{33} - S_{12}$,
$\Xi_{39} = -P_{45}^T - U_{12}^T$, $\Xi_{3,10} = -P_{55} + \frac{1}{h}S_{32}^T + U_{12}^T + U_{22}^T$,
$\Xi_{3,11} = -P_{56} - U_{22}^T + M_2D + B^TM_5^T$, $\Xi_{44} = -S_{21} - \frac{1}{r}S_{43} - U_{23}$, $\Xi_{4,10} = -P_{56}^T - U_{22}^T$, $\Xi_{4,11} = -P_{66} + \frac{1}{r}S_{42}^T + U_{22}^T$,
$\Xi_{55} = Q + \frac{\tau^2}{2}R_1 + \frac{h^2}{2}R_2 + \frac{r^2}{2}R_3 + S_{13} + S_{23} + hS_{33} + rS_{43} + (\tau-h)^2U_{13} + (h-r)^2U_{23} + \frac{\tau}{2}Z_1 + \frac{h}{2}Z_2 - M_3 - M_3^T$, $\Xi_{56} = M_3C - M_4^T$, $\Xi_{5,11} = P_{16} + M_3D - M_5^T$, $\Xi_{66} = -Q + M_4C + C^TM_4^T$, $\Xi_{6,11} = P_{26} + M_4D + C^TM_5^T$, $\Xi_{99} = -\frac{2}{\tau^2}R_1 - U_{11}$, $\Xi_{10,10} = -\frac{2}{h^2}R_2 - \frac{1}{h}S_{31} - U_{11} - U_{21}$,
$\Xi_{11,11} = -\frac{2}{\tau^2}R_3 - \frac{1}{r}S_{41} - U_{21} + M_5D + D^TM_5^T$, $\Xi_{12,12} = W_{13} - W_{11} - \frac{2}{\tau}Z_1$, $\Xi_{13,13} = W_{23} - W_{21} - \frac{2}{h}Z_2$.

Proof. Construct the following Lyapunov-Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) \quad (7)$$

where $V_1(t) = \eta^T(t)P\eta(t) + \int_{t-\tau}^{t} \dot{x}^T(s)Q\dot{x}(s)ds$,

$$V_2(t) = \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds\,d\theta\,d\lambda + \int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds\,d\theta\,d\lambda + \int_{-r}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)R_3\dot{x}(s)ds\,d\theta\,d\lambda,$$

$$V_3(t) = \int_{t-h}^{t} \omega^T(s)S_1\omega(s)ds + \int_{t-r}^{t} \omega^T(s)S_2\omega(s)ds + \int_{-h}^{0}\int_{t+\theta}^{t} \omega^T(s)S_3\omega(s)ds\,d\theta + \int_{-r}^{0}\int_{t+\theta}^{t} \omega^T(s)S_4\omega(s)ds\,d\theta,$$

$$V_4(t) = (\tau-h)\int_{-\tau}^{-h}\int_{t+\theta}^{t} \omega^T(s)U_1\omega(s)ds\,d\theta + (h-r)\int_{-h}^{-r}\int_{t+\theta}^{t} \omega^T(s)U_2\omega(s)ds\,d\theta,$$

$$V_5(t) = \int_{t-\frac{\tau}{2}}^{t} \alpha^T(s)W_1\alpha(s)ds + \int_{-\frac{\tau}{2}}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds\,d\theta + \int_{t-\frac{h}{2}}^{t} \beta^T(s)W_2\beta(s)ds + \int_{-\frac{h}{2}}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)ds\,d\theta,$$

where

$$\eta^T(t) = \left[x^T(t) \quad x^T(t-\tau) \quad x^T(t-h) \quad \int_{t-\tau}^{t} x^T(s)ds \quad \int_{t-h}^{t} x^T(s)ds \quad \int_{t-r}^{t} x^T(s)ds \right],$$

$$\omega^T(s) = \left[x^T(s) \quad \dot{x}^T(s) \right], \alpha^T(s) = \left[x^T(s) \quad x^T\left(s - \frac{\tau}{2}\right) \right],$$

$$\beta^T(s) = \left[x^T(s) \quad x^T\left(s - \frac{h}{2}\right) \right].$$

The time derivative of $V(t)$ along the trajectory of the nominal system (5) is given by

$$\dot{V}_1(t) = 2\eta^{\mathrm{T}}(t) P \dot{\eta}(t) + \dot{x}^{\mathrm{T}}(t) Q \dot{x}(t) - \dot{x}^{\mathrm{T}}(t-\tau) Q \dot{x}(t-\tau) \tag{8}$$

$$\dot{V}_2(t) = \dot{x}^{\mathrm{T}}(t)\left[\frac{\tau^2}{2}R_1 + \frac{h^2}{2}R_2 + \frac{r^2}{2}R_3\right]\dot{x}(t) - \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R_1 \dot{x}(s) ds\, d\theta - \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R_2 \dot{x}(s) ds\, d\theta - \int_{-r}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R_3 \dot{x}(s) ds\, d\theta \tag{9}$$

$$\begin{aligned}\dot{V}_3(t) = &\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_{11}+S_{21} & S_{12}+S_{22} \\ * & S_{13}+S_{23} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\ &- \begin{bmatrix} x(t-h) \\ \dot{x}(t-h) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_{11} & S_{12} \\ * & S_{13} \end{bmatrix} \begin{bmatrix} x(t-h) \\ \dot{x}(t-h) \end{bmatrix} \\ &- \begin{bmatrix} x(t-r) \\ \dot{x}(t-r) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_{21} & S_{22} \\ * & S_{23} \end{bmatrix} \begin{bmatrix} x(t-r) \\ \dot{x}(t-r) \end{bmatrix} \\ &+ \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} hS_{31}+rS_{41} & hS_{32}+rS_{42} \\ * & hS_{33}+rS_{43} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\ &- \int_{t-h}^{t} \omega^{\mathrm{T}}(s) S_3 \omega(s) ds - \int_{t-r}^{t} \omega^{\mathrm{T}}(s) S_4 \omega(s) ds \end{aligned} \tag{10}$$

$$\begin{aligned}\dot{V}_4(t) = &\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} (\tau-h)^2 U_{11} & (\tau-h)^2 U_{12} \\ * & (\tau-h)^2 U_{13} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\ &- (\tau-h)\int_{t-\tau}^{t-h} \omega^T(s) U_1 \omega(s) ds \\ &+ \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} (h-r)^2 U_{21} & (h-r)^2 U_{22} \\ * & (h-r)^2 U_{23} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\ &- (h-r)\int_{t-h}^{t-r} \omega^T(s) U_2 \omega(s) ds \end{aligned} \tag{11}$$

$$\begin{aligned}\dot{V}_5(t) = &\begin{bmatrix} x(t) \\ x\left(t-\frac{\tau}{2}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} W_{11} & W_{12} \\ * & W_{13} \end{bmatrix} \begin{bmatrix} x(t) \\ x\left(t-\frac{\tau}{2}\right) \end{bmatrix} \\ &- \begin{bmatrix} x\left(t-\frac{\tau}{2}\right) \\ x(t-\tau) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} W_{11} & W_{12} \\ * & W_{13} \end{bmatrix} \begin{bmatrix} x\left(t-\frac{\tau}{2}\right) \\ x(t-\tau) \end{bmatrix} \\ &+ \begin{bmatrix} x(t) \\ x\left(t-\frac{h}{2}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} W_{21} & W_{22} \\ * & W_{23} \end{bmatrix} \begin{bmatrix} x(t) \\ x\left(t-\frac{h}{2}\right) \end{bmatrix} \\ &- \begin{bmatrix} x\left(t-\frac{h}{2}\right) \\ x(t-h) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} W_{21} & W_{22} \\ * & W_{23} \end{bmatrix} \begin{bmatrix} x\left(t-\frac{h}{2}\right) \\ x(t-h) \end{bmatrix} \\ &+ \dot{x}^T(t)\left[\frac{\tau}{2}Z_1 + \frac{h}{2}Z_2\right]\dot{x}(t) \\ &- \int_{t-\frac{\tau}{2}}^{t} \dot{x}^{\mathrm{T}}(s) Z_1 \dot{x}(s) ds - \int_{t-\frac{h}{2}}^{t} \dot{x}^{\mathrm{T}}(s) Z_2 \dot{x}(s) ds \end{aligned} \tag{12}$$

By the integral inequalities in Lemma 1, one can obtain that

$$-\int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R_1 \dot{x}(s) ds\, d\theta \le -\frac{2}{\tau^2}\left[\tau x(t) - \int_{t-\tau}^{t} x(s) ds\right]^{\mathrm{T}} R_1 \left[\tau x(t) - \int_{t-\tau}^{t} x(s) ds\right] \tag{13}$$

$$-\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R_2 \dot{x}(s) ds\, d\theta \le -\frac{2}{h^2}\left[h x(t) - \int_{t-h}^{t} x(s) ds\right]^{\mathrm{T}} R_2 \left[h x(t) - \int_{t-h}^{t} x(s) ds\right] \tag{14}$$

$$-\int_{-r}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R_3 \dot{x}(s) ds\, d\theta \le -\frac{2}{r^2}\left[r x(t) - \int_{t-r}^{t} x(s) ds\right]^{\mathrm{T}} R_3 \left[r x(t) - \int_{t-r}^{t} x(s) ds\right] \tag{15}$$

$$-\int_{t-h}^{t} \omega^{\mathrm{T}}(s) S_3 \omega(s) ds \le -\begin{bmatrix} \int_{t-h}^{t} x(s) ds \\ x(t)-x(t-h) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \frac{1}{h}S_{31} & \frac{1}{h}S_{32} \\ * & \frac{1}{h}S_{33} \end{bmatrix} \begin{bmatrix} \int_{t-h}^{t} x(s) ds \\ x(t)-x(t-h) \end{bmatrix} \tag{16}$$

$$-\int_{t-r}^{t} \omega^{\mathrm{T}}(s) S_4 \omega(s) ds \le -\begin{bmatrix} \int_{t-r}^{t} x(s) ds \\ x(t)-x(t-r) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \frac{1}{r}S_{41} & \frac{1}{r}S_{42} \\ * & \frac{1}{r}S_{43} \end{bmatrix} \begin{bmatrix} \int_{t-r}^{t} x(s) ds \\ x(t)-x(t-r) \end{bmatrix} \tag{17}$$

$$-(\tau-h)\int_{t-\tau}^{t-h} \omega^{\mathrm{T}}(s) U_1 \omega(s) ds \le -\begin{bmatrix} \int_{t-\tau}^{t} x(s) ds - \int_{t-h}^{t} x(s) ds \\ x(t-h)-x(t-\tau) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} U_{11} & U_{12} \\ * & U_{13} \end{bmatrix} \begin{bmatrix} \int_{t-\tau}^{t} x(s) ds - \int_{t-h}^{t} x(s) ds \\ x(t-h)-x(t-\tau) \end{bmatrix} \tag{18}$$

$$-(h-r)\int_{t-h}^{t-r} \omega^T(s) U_2 \omega(s) ds \le -\begin{bmatrix} \int_{t-h}^{t} x(s) ds - \int_{t-r}^{t} x(s) ds \\ x(t-r)-x(t-h) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} U_{21} & U_{22} \\ * & U_{23} \end{bmatrix} \begin{bmatrix} \int_{t-h}^{t} x(s) ds - \int_{t-r}^{t} x(s) ds \\ x(t-r)-x(t-h) \end{bmatrix} \tag{19}$$

$$-\int_{t-\frac{\tau}{2}}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)ds \le -\frac{2}{\tau}\left[x(t)-x\left(t-\frac{\tau}{2}\right)\right]^{\mathrm{T}}Z_1\left[x(t)-x\left(t-\frac{\tau}{2}\right)\right] \quad (20)$$

$$-\int_{t-\frac{h}{2}}^{t}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)ds \le -\frac{2}{h}\left[x(t)-x\left(t-\frac{h}{2}\right)\right]^{\mathrm{T}}Z_2\left[x(t)-x\left(t-\frac{h}{2}\right)\right] \quad (21)$$

$$\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)+\dot{V}_4(t)+\dot{V}_5(t)+\gamma \quad (22)$$

where

$$\gamma=2\rho^{\mathrm{T}}(t)M\times\left[Ax(t)+Bx(t-h)-\dot{x}(t)+C\dot{x}(t-\tau)+D\int_{t-r}^{t}x(s)ds\right]=0 \quad (23)$$

with $\rho^{\mathrm{T}}(t)=\left[x^{\mathrm{T}}(t)\quad x^{\mathrm{T}}(t-h)\quad \dot{x}^{\mathrm{T}}(t)\quad \dot{x}^{\mathrm{T}}(t-\tau)\quad \int_{t-r}^{t}x^{\mathrm{T}}(s)ds\right]$, $M^{\mathrm{T}}=\left[M_1^{\mathrm{T}}\quad M_2^{\mathrm{T}}\quad M_3^{\mathrm{T}}\quad M_4^{\mathrm{T}}\quad M_5^{\mathrm{T}}\right]$.

Substituting the inequalities (13)-(21) into (22), it follows that

$$\dot{V}(t)\le\zeta^{\mathrm{T}}(t)\Xi\zeta(t) \quad (24)$$

where $\zeta^{\mathrm{T}}(t)$=[$x^{\mathrm{T}}(t)$ $x^{\mathrm{T}}(t-\tau)$ $x^{\mathrm{T}}(t-h)$ $x^{\mathrm{T}}(t-r)$ $\dot{x}^{\mathrm{T}}(t)$ $\dot{x}^{\mathrm{T}}(t-\tau)$ $\dot{x}^{\mathrm{T}}(t-h)$ $\dot{x}^{\mathrm{T}}(t-r)$ $\int_{t-\tau}^{t}x^{\mathrm{T}}(s)ds$ $\int_{t-h}^{t}x^{\mathrm{T}}(s)ds$ $\int_{t-r}^{t}x^{\mathrm{T}}(s)ds$ $x^{\mathrm{T}}(t-\frac{\tau}{2})$ $x^{\mathrm{T}}(t-\frac{h}{2})$]. If $\Xi<0$ then $\dot{V}(t)<0$ and the nominal system (5) is asymptotically stable. This completes the proof.

Remark 1. The augmented LKF chosen in this paper contains the term $\int_{t-r}^{t}\omega^{\mathrm{T}}(s)S_2\,\omega(s)ds$ which plays a key role to obtain less conservative stability conditions because it takes into account the relationship between $x(t-r)$ and $\dot{x}(t-r)$ in the derivative of the LKF

Remark 2. The number of free weighting matrices introduced in [30, 32, 33] is 21, 10, and 8, respectively. In Theorem 1, only 5 free weighting matrices are involved.

Remark 3. The augmented vector $\eta(t)$ is used in $V_1(t)$ in which every term is important to give less conservative stability conditions. When removing any term from $\eta(t)$, the results may become more conservative.

Now, we consider the robust stability of the uncertain system (1). Based on Theorem 1, we can easily obtain the following condition:

Theorem 2. For given scalars τ, h, and r the uncertain system (1) is robustly asymptotically stable if there exist matrices $P=\begin{bmatrix}P_{11}&P_{12}&P_{13}&P_{14}&P_{15}&P_{16}\\ *&P_{22}&P_{23}&P_{24}&P_{25}&P_{26}\\ *&*&P_{33}&P_{34}&P_{35}&P_{36}\\ *&*&*&P_{44}&P_{45}&P_{46}\\ *&*&*&*&P_{55}&P_{56}\\ *&*&*&*&*&P_{66}\end{bmatrix}>0$, $Q>0$, $R_1>0$, $R_2>0$, $R_3>0$, $S_i=\begin{bmatrix}S_{i1}&S_{i2}\\ *&S_{i3}\end{bmatrix}>0$, $\mathrm{U}_j=\begin{bmatrix}U_{j1}&U_{j2}\\ *&U_{j3}\end{bmatrix}>0$, $\mathrm{W}_j=\begin{bmatrix}W_{j1}&W_{j2}\\ *&W_{j3}\end{bmatrix}>0$, $Z_1>0$, $Z_2>0$, $M_k(i=1,2,3,4,j=1,2,k=1,2,3,4,5)$, and scalar ε such that the following LMI holds

$$\begin{bmatrix}\Psi&\Gamma\\ *&-\varepsilon I\end{bmatrix}<0 \quad (25)$$

where $\Psi=\Xi+\varepsilon YY^{\mathrm{T}}$, $Y^{\mathrm{T}}=[E_a\ 0\ E_b\ 0\ 0\ E_c\ 0\ 0\ 0\ 0\ E_d\ 0\ 0]$, $\Gamma^{\mathrm{T}}=[L^{\mathrm{T}}M_1^{\mathrm{T}}\ 0\ L^{\mathrm{T}}M_2^{\mathrm{T}}\ 0\ L^{\mathrm{T}}M_3^{\mathrm{T}}\ L^{\mathrm{T}}M_4^{\mathrm{T}}\ 0\ 0\ 0\ 0\ L^{\mathrm{T}}M_5^{\mathrm{T}}\ 0\ 0]$ and Ξ is defined in Theorem 1.

Proof. Replacing matrices A, B, C, and D in Theorem 1 with $A+LF(t)E_a$, $B+LF(t)E_b$, $C+LF(t)E_c$, and $D+LF(t)E_d$ respectively yields

$$\Xi+\Gamma F(t)\mathrm{Y}^{\mathrm{T}}+\mathrm{Y}F^{\mathrm{T}}(t)\Gamma^{\mathrm{T}}<0 \quad (26)$$

Using Lemma 2 one can obtain:

$$\Xi+\varepsilon\mathrm{Y}\mathrm{Y}^{\mathrm{T}}+\varepsilon^{-1}\Gamma\Gamma^{\mathrm{T}}<0 \quad (27)$$

By Schur complement, it is easy to see that (27) is equivalent to (25). This completes the proof.

Remark 4. Theorem 2 deals with the robust stability of uncertain neutral systems with discrete and distributed delays. It is based on the delay bi-decomposition idea which can be generalized using the delay N-decomposition approach and less conservative results may be obtained. First, we apply the delay N-decomposition technique to the nominal system (5)

Theorem 3. For given scalars τ, h, and r the system (5) is asymptotically stable if there exist matrices $P=\begin{bmatrix}P_{11}&P_{12}&P_{13}&P_{14}&P_{15}&P_{16}\\ *&P_{22}&P_{23}&P_{24}&P_{25}&P_{26}\\ *&*&P_{33}&P_{34}&P_{35}&P_{36}\\ *&*&*&P_{44}&P_{45}&P_{46}\\ *&*&*&*&P_{55}&P_{56}\\ *&*&*&*&*&P_{66}\end{bmatrix}>0$, $Q>0$, $R_1>0$, $R_2>0$, $R_3>0$, $S_i=\begin{bmatrix}S_{i1}&S_{i2}\\ *&S_{i3}\end{bmatrix}>0$, $\mathrm{U}_j=\begin{bmatrix}U_{j1}&U_{j2}\\ *&U_{j3}\end{bmatrix}>0$, $Z_1>0$, $Z_2>0$, $M_k(i=1,2,3,4,j=1,2,k=1,2,3,4,5)$, and $G=\begin{bmatrix}G_{11}&G_{12}&\dots&G_{1N}\\ *&G_{22}&\dots&G_{2N}\\ *&\vdots&\ddots&\vdots\\ *&*&\dots&G_{NN}\end{bmatrix}>0$, $H=\begin{bmatrix}H_{11}&H_{12}&\dots&H_{1N}\\ *&H_{22}&\dots&H_{2N}\\ *&\vdots&\ddots&\vdots\\ *&*&\dots&H_{NN}\end{bmatrix}>0$, such that the following LMI holds

$$\Xi'=\begin{bmatrix}\Xi_1'&\Xi_2'\\ *&\Xi_3'\end{bmatrix}<0 \quad (28)$$

where Ξ'_1, Ξ'_2, and Ξ'_3 are defined in (29)-(31). The terms Ξ_{ij} are defined in Theorem 1 and

$$\Xi'_1=\begin{bmatrix}(1,1) & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & P_{13} & 0 & \Xi_{19} & \Xi_{1,10} & \Xi_{1,11} & (1,12) & (1,13)\\ * & (2,2) & \Xi_{23} & -P_{26} & P_{12}^T & P_{22} & P_{23} & 0 & \Xi_{29} & \Xi_{2,10} & -P_{46} & -G_{1N}^T & 0\\ * & * & (3,3) & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & 0 & \Xi_{39} & \Xi_{3,10} & \Xi_{3,11} & 0 & -H_{1N}^T\\ * & * & * & \Xi_{44} & 0 & 0 & 0 & -S_{22} & -P_{46}^T & \Xi_{4,10} & \Xi_{4,11} & 0 & 0\\ * & * & * & * & (5,5) & \Xi_{56} & 0 & 0 & P_{14} & P_{15} & \Xi_{5,11} & 0 & 0\\ * & * & * & * & * & \Xi_{66} & 0 & 0 & P_{24} & P_{25} & \Xi_{6,11} & 0 & 0\\ * & * & * & * & * & * & -S_{13} & 0 & P_{34} & P_{35} & P_{36} & 00 & 0\\ * & * & * & * & * & * & * & -S_{23} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & * & \Xi_{99} & U_{11} & 0 & 0 & 0\\ * & * & * & * & * & * & * & * & * & \Xi_{10,10} & U_{21} & 0 & 0\\ * & * & * & * & * & * & * & * & * & * & \Xi_{11,11} & 0 & 0\\ * & * & * & * & * & * & * & * & * & * & * & (12,12) & 0\\ * & * & * & * & * & * & * & * & * & * & * & * & (13,13)\end{bmatrix}<0 \tag{29}$$

$$\Xi'_2=\begin{bmatrix}G_{13} & H_{13} & G_{14} & H_{14} & \cdots & G_{1N} & H_{1N}\\ -G_{2N}^T & 0 & -G_{3N}^T & 0 & \cdots & -G_{N-1,N}^T & 0\\ 0 & -H_{2N}^T & 0 & -H_{3N}^T & \cdots & 0 & -H_{N-1,N}^T\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0\\ (12,14) & 0 & (12,16) & 0 & \cdots & (12,2N+8) & 0\\ 0 & (13,15) & 0 & (13,17) & \cdots & 0 & (13,2N+9)\end{bmatrix} \tag{30}$$

$$\Xi'_3=\begin{bmatrix}(14,14) & 0 & (14,16) & 0 & \dots & (14,2N+8) & 0\\ * & (15,15) & 0 & (15,17) & \dots & 0 & (15,2N+9)\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ * & * & * & * & \dots & (2N+8,2N+8) & 0\\ * & * & * & * & \dots & * & (2N+9,2N+9)\end{bmatrix} \tag{31}$$

$$(1,1)=P_{14}+P_{14}^T+P_{15}+P_{15}^T+P_{16}+P_{16}^T-2R_1-2R_2-2R_3+S_{11}+S_{21}+hS_{31}+rS_{41}-\frac{1}{h}S_{33}-\frac{1}{r}S_{43}+(\tau-h)^2U_{11}+(h-r)^2U_{21}$$
$$+G_{11}+H_{11}-\frac{N}{\tau}Z_1-\frac{N}{h}Z_2+M_1A+A^TM_1^T,$$
$$(1,12)=G_{12}+\frac{N}{\tau}Z1,$$
$$(1,13)=H_{12}+\frac{N}{h}Z_2,$$
$$(2,2)=-P_{24}-P_{24}^T-U_{13}-G_{NN},$$
$$(3,3)=-P_{35}-P_{35}^T-S_{11}-\frac{1}{h}S_{33}-U_{13}-U_{23}-H_{NN}+\mathrm{M}_2\mathrm{B}+\mathrm{B}^\mathrm{T}\mathrm{M}_2^\mathrm{T},$$
$$(5,5)=Q+\frac{\tau^2}{2}R_1+\frac{h^2}{2}R_2+\frac{r^2}{2}R_3+S_{13}+S_{23}+hS_{33}+rS_{43}+(\tau-h)^2U_{13}+(h-r)^2U_{23}+\frac{\tau}{N}Z_1+\frac{h}{N}Z_2-M_3-M_3^T,$$
$$(12,12)=G_{22}-G_{11}-\frac{N}{\tau}Z_1,$$
$$(13,13)=H_{22}-H_{11}-\frac{N}{h}Z_2,$$
$$(12,14)=G_{23}-G_{12},$$
$$(12,16)=G_{24}-G_{13},$$
$$(12,2N+8)=G_{2N}-G_{1,N-1},$$

$$(13,15) = H_{23} - H_{12},$$
$$(13,17) = H_{24} - H_{13},$$
$$(13,2N+9) = H_{2N} - H_{1,N-1},$$
$$(14,14) = G_{33} - G_{22},$$
$$(14,16) = G_{34} - G_{23},$$
$$(14,2N+8) = G_{3N} - G_{2,N-1},$$
$$(15,15) = H_{33} - H_{22},$$
$$(15,17) = H_{34} - H_{23},$$
$$(15,2N+9) = H_{3N} - H_{2,N-1},$$
$$(2N+8,2N+8) = G_{NN} - G_{N-1,N-1},$$
$$(2N+9,2N+9) = H_{NN} - H_{N-1,N-1}$$

Proof. We replace $V(t)$ in the proof of Theorem 1 by:

$$\tilde{V}(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + \tilde{V}_5(t) \tag{32}$$

where

$$\begin{aligned}\tilde{V}_5(t) = &\int_{t-\frac{\tau}{N}}^{t} \tilde{\alpha}^{\mathrm{T}}(s) G \tilde{\alpha}(s) ds + \int_{-\frac{\tau}{N}}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) Z_1 \dot{x}(s) ds\, d\theta \\ &+ \int_{t-\frac{h}{N}}^{t} \tilde{\beta}^{\mathrm{T}}(s) W_2 \tilde{\beta}(s) ds + \int_{-\frac{h}{N}}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) Z_2 \dot{x}(s) ds\, d\theta\end{aligned} \tag{33}$$

with

$$\tilde{\alpha}^{\mathrm{T}}(s) = \left[x^{\mathrm{T}}(s) \quad x^{\mathrm{T}}\left(s - \frac{\tau}{N}\right) \quad x^{\mathrm{T}}\left(s - \frac{2\tau}{N}\right) \quad \ldots \quad x^{\mathrm{T}}\left(s - \frac{(N-1)\tau}{N}\right) \right]$$

$$\tilde{\beta}^{\mathrm{T}}(s) = \left[x^{\mathrm{T}}(s) \quad x^{\mathrm{T}}\left(s - \frac{h}{N}\right) \quad x^{\mathrm{T}}\left(s - \frac{2h}{N}\right) \quad \ldots \quad x^{\mathrm{T}}\left(s - \frac{(N-1)h}{N}\right) \right]$$

Differentiating $\tilde{V}(t)$ along the trajectory of the nominal system (5), one can obtain (8)-(11) and

$$\begin{aligned}\dot{\tilde{V}}_5(t) = &\begin{bmatrix} x(t) \\ x\left(t-\frac{\tau}{N}\right) \\ \vdots \\ x\left(t-\frac{(N-1)\tau}{N}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} G_{11} & G_{12} & \ldots & G_{1N} \\ * & G_{22} & \ldots & G_{2N} \\ * & \vdots & \ddots & \vdots \\ * & * & \ldots & G_{NN} \end{bmatrix} \begin{bmatrix} x(t) \\ x\left(t-\frac{\tau}{N}\right) \\ \vdots \\ x\left(t-\frac{(N-1)\tau}{N}\right) \end{bmatrix} \\ &- \begin{bmatrix} x\left(t-\frac{\tau}{N}\right) \\ x\left(t-\frac{2\tau}{N}\right) \\ \vdots \\ x(t-\tau) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} G_{11} & G_{12} & \ldots & G_{1N} \\ * & G_{22} & \ldots & G_{2N} \\ * & \vdots & \ddots & \vdots \\ * & * & \ldots & G_{NN} \end{bmatrix} \begin{bmatrix} x\left(t-\frac{\tau}{N}\right) \\ x\left(t-\frac{2\tau}{N}\right) \\ \vdots \\ x(t-\tau) \end{bmatrix} \\ &+ \begin{bmatrix} x(t) \\ x\left(t-\frac{h}{N}\right) \\ \vdots \\ x\left(t-\frac{(N-1)h}{N}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} H_{11} & H_{12} & \ldots & H_{1N} \\ * & H_{22} & \ldots & H_{2N} \\ * & \vdots & \ddots & \vdots \\ * & * & \ldots & H_{NN} \end{bmatrix} \begin{bmatrix} x(t) \\ x\left(t-\frac{h}{N}\right) \\ \vdots \\ x\left(t-\frac{(N-1)h}{N}\right) \end{bmatrix} \\ &- \begin{bmatrix} x\left(t-\frac{h}{N}\right) \\ x\left(t-\frac{2h}{N}\right) \\ \vdots \\ x(t-h) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} H_{11} & H_{12} & \ldots & H_{1N} \\ * & H_{22} & \ldots & H_{2N} \\ * & \vdots & \ddots & \vdots \\ * & * & \ldots & H_{NN} \end{bmatrix} \begin{bmatrix} x\left(t-\frac{h}{N}\right) \\ x\left(t-\frac{2h}{N}\right) \\ \vdots \\ x(t-h) \end{bmatrix} \\ &+ \dot{x}^{\mathrm{T}}(t)\left[\frac{\tau}{N} Z_1 + \frac{h}{N} Z_2\right]\dot{x}(t) - \int_{t-\frac{\tau}{N}}^{t} \dot{x}^{\mathrm{T}}(s) Z_1 \dot{x}(s) ds \\ &- \int_{t-\frac{h}{N}}^{t} \dot{x}^{\mathrm{T}}(s) Z_2 \dot{x}(s) ds\end{aligned} \tag{34}$$

Using the integral inequalities in Lemma 1, one can obtain

$$-\int_{t-\frac{\tau}{N}}^{t} \dot{x}^{\mathrm{T}}(s) Z_1 \dot{x}(s) ds \le -\frac{N}{\tau}\left[x(t) - x\left(t-\frac{\tau}{N}\right)\right]^{\mathrm{T}} Z_1 \left[x(t) - x\left(t-\frac{\tau}{N}\right)\right] \tag{35}$$

$$-\int_{t-\frac{h}{N}}^{t} \dot{x}^{\mathrm{T}}(s) Z_2 \dot{x}(s) ds \le -\frac{N}{h}\left[x(t) - x\left(t-\frac{h}{N}\right)\right]^{\mathrm{T}} Z_2 \left[x(t) - x\left(t-\frac{h}{N}\right)\right] \tag{36}$$

$$\dot{\tilde{V}}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \dot{\tilde{V}}_5(t) + \gamma \tag{37}$$

Substituting the inequalities (13)-(19), (35) and (36) into (37), it follows that

$$\dot{\tilde{V}}(t) \le \zeta'^{\mathrm{T}}(t)\, \Xi' \zeta'(t) \tag{38}$$

where

$\zeta^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-\tau)\ \ x^{\mathrm{T}}(t-h)\ \ x^{\mathrm{T}}(t-r)\ \ \dot{x}^{\mathrm{T}}(t)\ \ \dot{x}^{\mathrm{T}}(t-\tau)\ \ \dot{x}^{\mathrm{T}}(t-h)$ $\dot{x}^{\mathrm{T}}(t-\tau)\ \ \int_{t-\tau}^{t} x^{\mathrm{T}}(s) ds\ \ \int_{t-h}^{t} x^{\mathrm{T}}(s) ds\ \ \int_{t-r}^{t} x^{\mathrm{T}}(s) ds\ \ x^{\mathrm{T}}(t-\tau/N)$ $x^{\mathrm{T}}(t-h/N)\ \ x^{\mathrm{T}}(t-2\tau/N)\ \ x^{\mathrm{T}}(t-2h/N)\ \ \ldots\ \ x^{\mathrm{T}}(t-(N-1)\tau/N)$ $x^{\mathrm{T}}(t-(N-1)h/N)]$.

If $\Xi' < 0$ then $\dot{\tilde{V}}(t) < 0$ and the nominal system (5) is asymptotically stable. This completes the proof.

Remark 5. In [32], the number of free weighting matrices increases with respect to N when incorporating the idea of the delay N-decomposition approach, however in Theorem 3, the number of the free weighing matrices remains the same as in Theorem 1.

Remark 6. Based on Theorem 3 and using similar techniques as in the proof of Theorem 2 to deal with the uncertainties, one can obtain the following robust stability criterion for uncertain neutral systems with discrete and distributed delays.

Theorem 4. For given scalars τ, h, and r the uncertain system (1) is robustly asymptotically stable if there exist matrices $P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} \\ * & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} \\ * & * & P_{33} & P_{34} & P_{35} & P_{36} \\ * & * & * & P_{44} & P_{45} & P_{46} \\ * & * & * & * & P_{55} & P_{56} \\ * & * & * & * & * & P_{66} \end{bmatrix} > 0$, $Q > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $S_i = \begin{bmatrix} S_{i1} & S_{i2} \\ * & S_{i3} \end{bmatrix} > 0$, $U_j = \begin{bmatrix} U_{j1} & U_{j2} \\ * & U_{j3} \end{bmatrix} > 0$, $Z_1 > 0$, $Z_2 > 0$, M_k($i = 1, 2, 3, 4$, $j = 1, 2$, $k = 1, 2, 3, 4, 5$), scalar ε and matrices $G = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ * & G_{22} & \cdots & G_{2N} \\ * & \vdots & \ddots & \vdots \\ * & * & \cdots & G_{NN} \end{bmatrix} > 0$, $H = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1N} \\ * & H_{22} & \cdots & H_{2N} \\ * & \vdots & \ddots & \vdots \\ * & * & \cdots & H_{NN} \end{bmatrix} > 0$, such that the following LMI holds

$$\begin{bmatrix} \Psi' & \Gamma' \\ * & -\varepsilon I \end{bmatrix} < 0 \qquad (39)$$

where $\Psi' = \Xi' + \varepsilon Y' Y'^T$ with $Y'^T = [Y^T \ 0 \ \dots \ 0]$ and $\Gamma'^T = [\Gamma^T \ 0 \ \dots \ 0]$, and Y and Γ are defined in Theorem 2. Ξ' is defined in Theorem 3.

Remark 7. It is clear that Theorem 4 for $N \geq 3$ is less conservative than Theorem 2 because of the incorporation of the delay N-decomposition approach. It will be shown in the numerical example.

4 Numerical Example

In this section, a numerical example is given to demonstrate the effectiveness of the proposed method. To this end, we consider the uncertain system (1) with the following parameters [30-34]

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, B = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix}, C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix},$$

$$D = \begin{bmatrix} -0.12 & -0.12 \\ -0.12 & -0.12 \end{bmatrix}, \ L = I, \ E_a = E_b = E_c = E_d = 0.1I,$$

and $\tau = 0.1$.

Table 1 shows the maximum upper bound on the distributed delay r that guarantees the stability of the uncertain system for different values of the discrete delay h. It can be seen that our method is less conservative than previous methods. On the other hand, Table 2 gives the maximum upper bound on the discrete delay h for different values of r. Clearly, our results are more effective and less conservative than other results.

5 Conclusion

In this paper, new less conservative criteria for robust stability of uncertain neutral systems with discrete and distributed delays are presented. An augmented LKF is constructed taking into account the interconnected information between neutral delay, discrete delay, and distributed delay. A delay decomposition approach is proposed and some integral inequalities are employed. A numerical example is given to show that the proposed criteria are less conservative than some existing results.

Table 1 Maximum allowable upper bounds of r for different h.

Approach	***h***						Number of variables
	0.1	0.5	1	1.5	1.6	1.7	
Li and Zhu [30]	6.64	5.55	1.62	-	-	-	119
Sun *et al*. [31]	6.67	6.12	2.75	1.31	0.93	0.42	103
Chen *et al.* [33]	6.67	6.02	3.19	1.50	1.25	1.02	100
Liu **and Huang** [32]	6.65	6.21	4.95	1.36	1.07	0.81	130
Liu and Huang [32] (N = 3)	6.65	6.21	5.01	1.54	1.18	0.92	160
Chen *et al.* [34]	6.67	6.67	5.52	1.95	1.55	1.46	166
Th.2	6.67	6.67	6.21	2.69	2.06	1.65	197
Th.4 **(N = 3)**	6.67	6.67	6.23	2.98	2.26	1.85	219

Table 2 Maximum allowable upper bounds of h for different r.

Approach	***r***					
	1	2	3	4	5	6
Li and Zhu [30]	1.12	0.93	0.77	0.65	0.55	0.43
Sun *et al.* [31]	1.58	1.20	0.95	0.77	0.64	0.51
Chen *et al.* [33]	1.71	1.71	1.09	0.83	0.68	0.51
Liu and Huang [32]	1.62	1.40	1.27	1.14	0.99	0.63
Liu and Huang [32] (N = 3)	1.67	1.43	1.30	1.16	1.00	0.63
Chen *et al.* [34]	1.90	1.49	1.34	1.20	1.07	0.91
Th.2	1.90	1.61	1.46	1.35	1.22	1.05
Th.4 (N = 3)	1.93	1.66	1.49	1.38	1.25	1.06

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