A Modified Proportional Navigation Guidance for Accurate Target Hitting

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Abstract: When a detector sensitive to the target plume IR seeker is used for tracking airborne targets, the seeker tends to follow the target hot point which is a point farther away from the target exhaust and its fuselage. In order to increase the missile effectiveness, it is necessary to modify the guidance law by adding a lead bias command. The resulting guidance is known as target adaptive guidance (TAG).

First, the pure proportional navigation guidance (PPNG) in 3-dimensional state is explained in a new point of view. The main idea is based on the distinction between angular rate vector and rotation vector conceptions. The current innovation is based on selection of line of sight (LOS) coordinates. A comparison between two available choices for LOS coordinates system is proposed. An improvement is made by adding two additional terms. First term includes a cross range compensator which is used to provide and enhance path observability, and obtain convergent estimates of state variables. The second term is new concept lead bias term, which has been calculated by assuming an equivalent acceleration along the target longitudinal axis. Simulation results indicate that the lead bias term properly provides terminal conditions for accurate target interception.

Keywords: IR Homing Missile, Pure Proportional Navigation Guidance, Target Adaptive Guidance.

1 Introduction

Most IR missiles with a reticle seeker for target tracking use detectors sensitive to the center of Infra-red (IR) radiation waves emitted from different parts of the target. Missiles with a detector sensitive to the target exhaust or nozzle can not attack a target in a head-on mode, because in this case the aircraft nozzle is situated in their blind area and hence can not be observed. However missiles with detectors sensitive to the target plume have no blind points and are omni-directionally capable of being fired, which results in increased capability of these missiles. Therefore, when an IR seeker with a detector sensitive to the target plume is used for tracking airborne targets, the seeker tends to follow the target hot point that is a point farther away from the target exhaust and outside of its fuselage. Hence, most available IR missiles having homing guidance based on LOS measurements by seeker show a

week performance against threats, and this point affects the behavior of small missiles with small interval of fuse performance. In order to increase the missile effectiveness, it is necessary to modify the guidance law by adding a lead bias command. This modification can be accomplished such that the chosen point for guidance is transformed into a valuable point on the target fuselage (see Fig. 1).

In practice, this modification can be implemented by adding a lead bias to the seeker head output, without a change in the tracking loop. However, lead bias computation can not be easily performed. Due to lack of pieces of information such as the relative missile-to-target range, closing velocity and the data relative to target velocity vector, we can not easily compute the lead bias term. To compute the lead bias, we can use an estimator for estimation of the required parameters such as: range, closing velocity, angle of target velocity vector with LOS and etc. Only few numbers of papers can be found about this subject in the literature. In [1], the TAG has been introduced as a solution to this problem and a lead bias term has been proposed in which a measurable IR tracking gimbal angle has been presented as a implementation. However, TAG algorithm requires estimation of the target heading
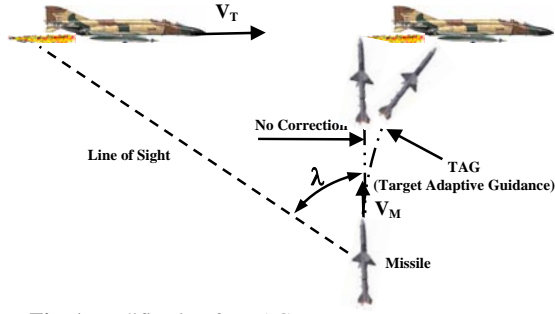
Fig. 1 Modification for TAG

Fig. 2 LOS coordinates definition

angle, which is particularly not observable for non-maneuvering targets [2]. In [3] the MPNG law for IR homing missiles has been introduced in which a lead bias term as [1] has been considered. The proposed guidance law is the same conventional PNG law in which an additional term, including cross range compensator, is considered to provide observability enhancing trajectory and also to obtain convergent estimate of state variable required for TAG modification [4]. In [5], by assuming an accurate estimate of the inverse time-to-go to be available, the TAG approximation by an IR seeker has been proposed using IR intensity measurements of the target.

In this paper, TAG for passive homing missiles in 3-dimensional state is explained. Modeling has been performed by using LOS coordinates with a particular definition. Using this LOS coordinates the acceleration command is simply explained in missile body coordinates. To obtain convergent estimates of those state variables that unavailable from IR trackers, a sequential U-D EKF in modified spherical coordinate (MSC) combined with modified proportional navigation guidance (MPNG) law are proposed. The proposed guidance law is the traditional PPNG law with two additional terms, which has been calculated in 3-dimensional case. First term includes a cross range compensator [6] that is used to provide and enhance trajectory observability and obtain convergent estimates of state variables. The second is lead bias term that is generalized form [3] that with a new concept has been calculated out of modeling method by assuming an equivalent acceleration along the target longitudinal axis.

Simulation results indicate that the lead bias term considered for MPNG properly provides terminal conditions for missile-target homing interception.

2 The LOS Coordinate System Selection

The missile and target relative positional vector has been illustrated in Fig. 2. Assume that $\vec{r}_m$ and $\vec{r}_t$ are location vectors of missile and target in the inertial coordinate system *{I}* respectively.
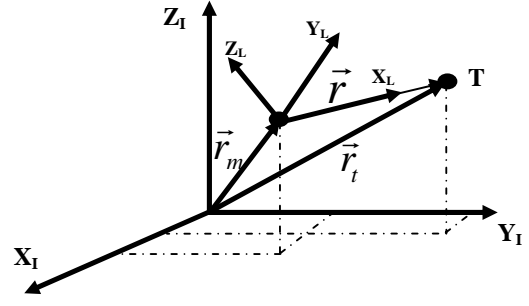
Theorem 1: Assuming that $\vec{\Omega}$ is the rotation vector of $\vec{r}$ relative to the inertial coordinate system, and $\vec{\omega}_{IL}$ is the angular rate vector of LOS coordinate system relative to the inertial coordinate system, and the LOS coordinate system is constructed such that its X-axis lies along $\vec{r}$, then $\vec{\Omega}$ shall be the image of $\vec{\omega}_{IL}$ on a plane perpendicular to $\vec{r}$.

Proof: The LOS vector will be defined as follows,

$$\vec{r} = \vec{r}_t - \vec{r}_m \tag{1}$$

By taking the derivative Eq. (1), we will obtain,

$$\dot{\vec{r}} = \vec{v}_t - \vec{v}_m \tag{2}$$

A vector such as $\vec{\Omega}$ perpendicular to $\vec{r}$ can be found, where $\dot{\vec{r}}$ can be decomposed into two components as follows,

$$\dot{\vec{r}} = r\,\vec{1}_r + \vec{\Omega} \times \vec{r} \tag{3}$$

The following equation can easily be obtained from Eq. (3),

$$\vec{\Omega} = \frac{\vec{r} \times (\vec{v}_t - \vec{v}_m)}{r^2} \tag{4}$$

Therefore, $\vec{\Omega}$ is only dependant on $\vec{r}$, $\vec{v}_t$ and $\vec{v}_m$. Now, we choose the LOS coordinate system (*{L}*) such that its X- axis lies along $\vec{r}$. According to Coriolis Law, we know that,

$$P_I\,\vec{r} = P_L\,\vec{r} + \vec{\omega}_{IL} \times \vec{r} \tag{5}$$

(P_I (P_L) = differential operator with respect to *{I}* (*{L}*)) since X-axis of LOS coordinate is along $\vec{r}$, Eq. (6) can be replaced by,

$$\dot{\vec{r}} = r\,\vec{1}_r + \vec{\omega}_{IL} \times \vec{r} \tag{6}$$

Therefore, $\vec{\omega}_{IL}$ absolutely belongs to LOS definition. By comparing Eqs. (3) and (6), it seems that this two equations are equal, but in reality $\vec{\Omega} \neq \vec{\omega}_{IL}$. By comparing relations, Eqs. (4) and (7), we will obtain

$$\bar{\omega}_{IL} \times \vec{r} = \vec{\Omega} \times \vec{r} \qquad (7)$$

The above relation shows that vectors $\vec{r}$, $\bar{\omega}_{IL}$ and $\vec{\Omega}$ are located in one plane. Since $\vec{\Omega}$ should always be on a plane perpendicular to $\vec{r}$, then the image if $\bar{\omega}_{IL}$ shall always be on this plane. In Fig. 3, locations of LOS angular rate vector and rotation vector are illustrated.

The only condition restricting LOS coordinate system selection is that its X-axis should lie along the $\vec{r}$. With the exception of this point, we can obtain LOS coordinate system from inertial coordinate system by using three Euler rotation angles of ψ_L, $-\theta_L$ and φ_L. Therefore, we will have,

$$\bar{\omega}_{IL}^L = \begin{bmatrix} \dot{\varphi}_L + \dot{\psi}_L S\theta_L \\ -\dot{\theta}_L C\varphi_L + \dot{\psi}_L S\varphi_L C\theta_L \\ \dot{\theta}_L S\varphi_L + \dot{\psi}_L C\varphi_L C\theta_L \end{bmatrix} \qquad (8)$$

(Here we have $S\theta_L = \sin(\theta_L)$ and $C\theta_L = \cos(\theta_L)$, and etc.). We will also demonstrate that selection of LOS coordinate system affects the acceleration relationship format. Here, two methods of selecting LOS coordinate system are illustrated.

In the first method, selection is made such that $\bar{\omega}_{IL}$ is located on a plane perpendicular to $\vec{r}$. As it is clear by Theorem 1 conclusion, in this case the relation $\bar{\omega}_{IL} = \vec{\Omega}$ will be obtained. In Fig. 4 the location of LOS angular rate vector and rotation vector in the LOS coordinate system have been illustrated. Therefore, in this case the LOS angular rate vector will have two components as follows,

$$\bar{\omega}_{IL}^L = \begin{bmatrix} 0 \\ \omega_t \\ \omega \end{bmatrix} \qquad (9)$$

The acceleration command will be as the following [7],

$$\vec{a}_m = N\,\vec{\Omega} \times \vec{v}_m \qquad (10)$$

In fact, this method of applying missile commanded acceleration without considering the afore-mentioned points has been used in [8] by mistake. However, in this reference the $\bar{\omega}_{IL}$ has been assumed to have three components as follows,

$$\bar{\omega}_{IL}^L = \begin{bmatrix} \dot{\psi}_L\,S\theta_L \\ -\dot{\theta}_L \\ \dot{\psi}_L\,C\theta_L \end{bmatrix} = \begin{bmatrix} \dot{\lambda}_x \\ \dot{\lambda}_y \\ \dot{\lambda}_z \end{bmatrix} \qquad (11)$$

As it has been shown in Eq. (9), we should have $\dot{\lambda}_x = 0$. It is worth noting that author of [8] in deriving missile acceleration equations has explained that the seeker is incapable of measuring this parameter and hence disregarded it, but has demonstrated that the value of this parameter, due to method of selecting LOS coordinates system is practically zero. The Eq. (11) has also another problem that is lacking φ_L Euler angle. Although missile and target coordinate systems have been defined, in these methods as depicted in [8], missile acceleration vector will be obtained only by use of two Euler angles as follows,

$$\vec{a}_m^M = \begin{bmatrix} 0 \\ -Nv_m\,\dot{\lambda}_y\,S\theta_m S\psi_m + Nv_m\,\dot{\lambda}_z\,C\theta_m \\ -Nv_m\,\dot{\lambda}_y\,C\psi_m \end{bmatrix} \qquad (12)$$
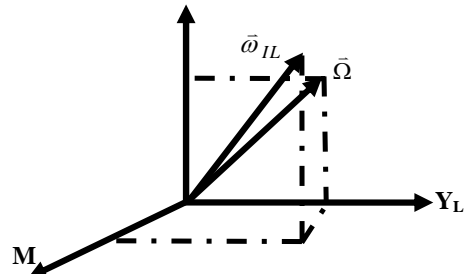
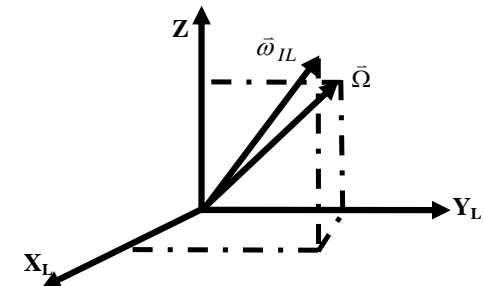**Fig. 3** Illustration of rotation and LOS angular rate vectors in {L}

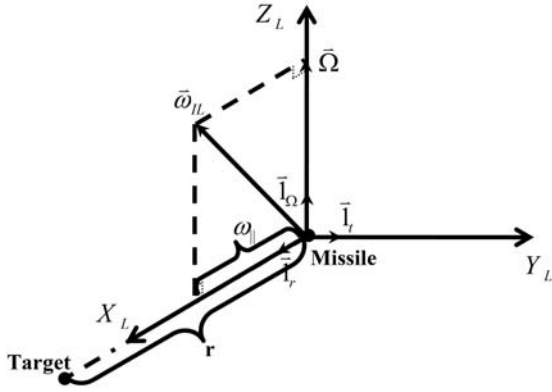**Fig. 4** LOS coordinate system selection in the first method

Fig. 5 LOS coordinate system selection in the second method

In the second method, selection is made such that $\vec{\omega}_{IL}$ will be located in $X_L Z_L$ plane [6]. As it can be seen from Theorem. 1 conclusion, in this case $\vec{\Omega}$ will settle on Z_L axis. In Fig. 5, rotation and LOS angular rate vectors location in the LOS coordinate system has been illustrated. Therefore, LOS angular rate vector has two components in this case as follows,

$$\vec{\omega}_{IL}^{L} = \begin{bmatrix} \omega_{\parallel} \\ 0 \\ \omega \end{bmatrix} \tag{13}$$

The properties of using LOS coordinate system by the second method are explained in [6].

In [6], that is shown that missile command acceleration in missile body coordinate system ({M}) is,

$$\begin{aligned} \vec{a}_m^M &= C_L^M \vec{a}_{mc}^L \\ &= N v_m \omega C \theta_m \vec{j}_M \end{aligned} \tag{14}$$

That is to say "In case of PPNG law in 3D state in terms of the afore-mentioned {L} and {M} definition, it is simply suffices for the missile to have acceleration command component only along the Y-axis of missile body coordinate system ω is measured by the seeker and θ_m can calculated out of seeker gimbals angles.

However, it has been shown that the acceleration command resembles Eq. (12) and it is not as simple as Eq. (14). Hence, second coordinate system will be used.

3 Modification of Guidance Law

Song [9] suggests a modified augmented PNG law, which includes a cross range deviation term to provide initial LOS angle oscillation without sacrificing terminal guidance effectiveness. In [3, 4], the proposed MPNG law for IR homing missiles in which an additional term comprising a cross range compensator [9], is used to provide initial LOS angle oscillations and obtain convergent trajectory and convergent estimates of state variables.

In [6], it can be demonstrated that if the missile commanded acceleration is modified as Eq. (15), then the initial LOS angle oscillation is provided without sacrificing terminal guidance effectiveness,

$$\vec{a}_m^M = (N v_m \omega C \theta_m + \frac{Fr\sigma}{C\psi_m})\vec{j}_M \tag{15}$$

where F is a positive constant, N is navigation constant and σ is the LOS angle such that $\dot{\sigma} = \omega$ and is measured in the Cartesian coordinate system where its X-axis lies along the initial LOS to the target. In other words, we have $\sigma(0) = 0$.

In this paper, for implementation of TAG, the PPNG is modified in combination with the cross term of Eq. (15) and the lead bias term in the following form,

$$\vec{a}_m^M = \left[N v_m (\omega + \omega_b) C \theta_m + \frac{Fr\sigma}{C\psi_m} \right] \vec{j}_M \tag{16}$$

Theorem 2: For modification of the missile acceleration command to intercept the target within a range of **A** meters ahead of it along the target velocity vector, the lead bias modification term must be considered as,

$$\omega_b = K \frac{\dot{r}}{r^2} C \theta_m S \psi_m \tag{17}$$

Proof: suppose that target interception is to be realized within a range of **A** meters ahead of target. Also suppose that target acceleration vector has no component along the target x-axis. Therefore, the target acceleration vector in the target coordinate system ({T}) is explained as follows,

$$\vec{a}_t^T = a_{ty}^T \vec{j}_T + a_{tz}^T \vec{k}_T \tag{18}$$

For changing the interception point within a range of **A** meters ahead of target, it is equally assumed that target covers this range with an additional acceleration of $a_{tx\,LBC}^T$ during flight course. This value can be obtained by the following equation,

$$A = \frac{1}{2} a_{tx\,LBC}^T t_{go}^2 \tag{19}$$

with respect to Eq. (19), we have,

$$a_{tx\,LBC}^T = \frac{2A\dot{r}^2}{r^2} \tag{20}$$

For TAG, it is assumed that target has simply acceleration due to the changed target point and only an acceleration to compensate for this change is applied to the missile. Therefore, for considering TAG, we change the target vector derived from Eq. (20) into the following form

$$\vec{a}_t^T = a_{tx_{LBC}}^T \vec{i}_T \tag{21}$$

Assume that to compensate for target longitudinal acceleration component, missile compensates LOS turning velocity that is the seeker output with a lead bias term ω_b. Our objective is to find this bias value. With respect to PNG law, we know that LOS turning rate assumes a very small value in terminal phase of flight. That is,

$$\omega \approx 0 \tag{22}$$

Hence, from Eq. (22), we will obtain,

$$v_{ty} \approx v_{my} \tag{23}$$

Now from Eq. (23), we will obtain,

$$C\theta_m S\psi_m \approx \rho C\theta_t S\psi_t \tag{24}$$

where $\rho = v_t / v_m$. By assuming $\dot{\omega} \approx 0$, in the terminal phase of flight we have,

$$2\omega_b \dot{r} = a_{ty} - a_{my} \tag{25}$$

Here a_{ty} and a_{my} are respectively components of target and missile acceleration along y-axis due to TAG, which were expressed in LOS coordinate system. By calculating these components and replacing them in Eq. (25), we obtain,

$$2\omega_b \dot{r} \approx a_{tx_{LBC}}^T C\theta_t S\psi_t - Nv_m \omega_b C\theta_m C\psi_m \tag{26}$$

We denote the angle between missile velocity vector and LOS by λ, then we will have,

$$\cos\lambda = C\theta_m C\psi_m \tag{27}$$

In practice, this angle will not be more than 30 degrees. Hence, we obtain,

$$\cos\lambda \approx 1 \tag{28}$$

Also from Eqs. (24) and (27), we can transform Eq. (26) to the following form,

$$(N-2)\rho\omega_b \dot{r} > a_{tx_{LBC}}^T C\theta_m S\psi_m \tag{29}$$

Now from Eq. (20), we can simplify Eq. (29) as,

$$\omega_b > \frac{2A}{(N-2)\rho} \frac{\dot{r}}{r^2} C\theta_m S\psi_m \tag{30}$$

Therefore, we can find a constant like $K < 0$ such that,

$$\omega_b = K \frac{\dot{r}}{r^2} C\theta_m S\psi_m \tag{31}$$

Consequently, MPNG in combination with the lead bias term is realized as follows,

$$\vec{a}_m^M = \left[Nv_m (\omega + \omega_b) C\theta_m \right] \vec{j}_M \tag{32}$$

and the theorem is thus proven. Note that by assuming $\theta_M = 0$, the obtained equations for a two-dimensional case will hold true [3].

For practical implementation, the variables utilized in Eq. (16) will be replaced by the estimated values obtained from a state estimator. Hence, MPNG law is proposed as follows,

$$\vec{a}_m^M = \left[Nv_m (\hat{\omega} + \hat{\omega}_b) C\hat{\theta}_m + \frac{F\hat{r}\hat{\sigma}}{C\hat{\psi}_m} \right] \vec{j}_M \tag{33}$$

4 State Estimation

The choice of coordinate system is crucial for nonlinear filters because the coupling between different coordinates (or states) can seriously degrade their performance. In this paper, a sequential U-D EKF estimator [10] in the MSC has been utilized to estimate the required variables for the implementation of MPNG algorithm expressed in Eq. (33). A major virtue of the MSC approach is that it decouples the relatively accurate states from the downrange states, which prevents covariance matrix ill-conditioning. Thus, the inaccurate range will not appear in the equations to update the covariance matrices, which is an important advantage of MSC [11].

The state vector in MSC is defined as,

$$\begin{aligned}
\vec{x} &= (\sigma, \omega, \theta_m, \psi_m, \frac{1}{r}, \frac{\dot{r}}{r}, a_{tx}^L, a_{ty}^L, a_{tz}^L) \\
&= (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9)
\end{aligned} \tag{34}$$

The continuous system dynamics are represented as follows,

$$\dot{\vec{x}} = \vec{f}\left(\vec{x}, \vec{w}, \vec{a}_m^M \right) \tag{35}$$

The state space equations in scalar form can be put in the following form [6],

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -2x_2 x_6 + x_5(x_8 - a_{my}^M \cos x_4) \\
\dot{x}_3 &= -\frac{x_5 x_9}{x_2} \sin x_3 \\
\dot{x}_4 &= -x_2 + \frac{x_5 x_9}{x_2} \tan x_3 \cos x_4 + \frac{a_{my}^M}{v_m \cos x_3} \\
\dot{x}_5 &= -x_5 x_6 \\
\dot{x}_6 &= x_2^2 - x_6^2 + x_5(x_7 + a_{my}^M \sin x_4) \\
\dot{x}_7 &= w_1 \\
\dot{x}_8 &= w_2 \\
\dot{x}_9 &= w_3
\end{aligned} \tag{36}$$

where a_{my}^M is the component of the missile acceleration along the y-axis in $\{M\}$ coordinate system, and (w_1, w_2, w_3) are Gaussian process noises with the following characteristics:

$$\bar{w} = (w_1, w_2, w_3)$$
$$E(w_1) = 0, E(w_2) = 0, E(w_3) = 0 \tag{37}$$
$$E\left(\bar{w}(\zeta_2), \bar{w}(\zeta_1)\right) = \sigma_T^2 \delta(\zeta_2 - \zeta_1) I_{3\times3}$$

where σ_T^2 is the variance of the process noise. We also have,

$$P_{i|i} = (I - K_i H_i) M_{i|i-1} (I - K_i H_i)^T + K_i R_i K_i^T \tag{38}$$

where K_i the Kalman gain matrix is defined as,

$$K_i = M_{i|i-1} H_i^T (H_i M_{i|i-1} H_i^T + R_i)^{-1} \tag{39}$$

In the above, the measurement matrix H_i and the measurement noise covariance R_i can be defined from the following measurement equation,

$$Z = H \bar{x} + \bar{v} \tag{40}$$

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{41}$$

Here, it has been assumed that the first measurement is the LOS rotation rate measured by the seeker, and the second measurement is the parameter $\dfrac{\dot{r}}{r} = -\dfrac{1}{t_{go}}$, which is accessible [6]. Besides, the measurement noise $\bar{v}_i$ is assumed to be additive, white and Gaussian with the following characteristics,

$$E(\bar{v}_i) = \bar{0} \quad , \quad R_i = E(\bar{v}_i \bar{v}_i^T) = \begin{bmatrix} \sigma_\omega^2 & 0 \\ 0 & \sigma_{\frac{\dot{r}}{r}}^2 \end{bmatrix} \tag{42}$$

Finally, after the i-th measurement, the updating of the state estimate in MSC is given by,

$$\hat{\bar{x}}_{i|i} = \hat{\bar{x}}_{i|i-1} + K_i (Z_i - H_i \hat{\bar{x}}_{i|i}) \tag{43}$$

5 Simulation

In this section, the proposed MNPG law in Eq. (33) has been used in a Monte Carlo simulation for practical applications. The missile-target geometry depicted in Fig. 6 with various initial intercept conditions is utilized to set up engagement scenarios. The initial values for error covariance are adopted as follows [6],

$$P_{0_{1,1}} = \sigma_\sigma^2 \; , \; \sigma_\sigma = 0.026$$
$$P_{0_{2,2}} = \sigma_\omega^2 \; , \; \sigma_\omega = 0.03$$
$$P_{0_{3,3}} = \sigma_{\theta_m}^2 \; , \; \sigma_{\theta_m} = 0.087$$
$$P_{0_{4,4}} = \sigma_{\psi_m}^2 \; , \; \sigma_{\psi_m} = 0.087$$
$$P_{0_{5,5}} = \sigma_r^2 \hat{x}_{0_5}^4 \; , \; \sigma_r = 3000$$
$$P_{0_{6,6}} = \left[\sigma_{\dot{r}}^2 \hat{x}_{0_5}\right]^2 - 2C_{r\dot{r}} \sigma_r \sigma_{\dot{r}} \hat{x}_{0_5}^2 \hat{x}_{0_6} \tag{44}$$
$$\qquad + \left[\hat{x}_{0_5} \hat{x}_{0_6} \sigma_r\right]^2, \; \sigma_{\dot{r}} = 300, C_{r\dot{r}} = 0.5$$
$$P_{0_{5,6}} = -C_{r\dot{r}} \sigma_r \sigma_{\dot{r}} \hat{x}_{0_5}^3 + \hat{x}_{0_5}^3 \hat{x}_{0_6} \sigma_r^2$$
$$P_{0_{6,5}} = P_{0_{5,6}}$$
$$P_{0_{7,7}} = P_{0_{8,8}} = P_{0_{9,9}} = \sigma_T^2 \; , \; \sigma_T = 40$$

The results of 50 runs of Monte Carlo simulation are presented in Figs. 7-14. The error in the figures represents the RMS-type values. The values of parameters are presented in Table. 1.

Table 1 Parameter values

Parameter	Value	Description
T	0.01 Sec	Sampling interval
v_m	450 m/s	Missile velocity
v_t	250 m/s	Target velocity
N	3.5	The effective navigation constant
F	4	The guidance constant in first cross term
K	-1	The guidance constant in second cross term
$\sigma(0)$	0 deg	The initial LOS angle
A	8 m	Distance between hot point and target fuseledge

The simulation has been performed for four engagement scenarios.

Launch scenario 1: The initial estimates relative to the LOS coordinate system are as the following,

$$r(0) = 2800\,m$$
$$\theta_t(0) = 20 \text{ deg} \;, \; \psi_t(0) = 135 \text{ deg}$$
$$\theta_m(0) = 10 \text{ deg} \;, \; \psi_m(0) = 0 \text{ deg} \tag{45}$$
$$a_{tx}(0) = 0 \;, a_{ty}(0) = 0 \;, a_{tz}(0) = 0$$

and the initial estimates of filter are,

$$\hat{r}(0) = 5000\,m \;, \hat{\dot{r}}(0) = -450\,m/s$$
$$\hat{a}_{tx}(0) = 0 \;, \hat{a}_{ty}(0) = 0, \hat{a}_{tz}(0) = 0 \tag{46}$$

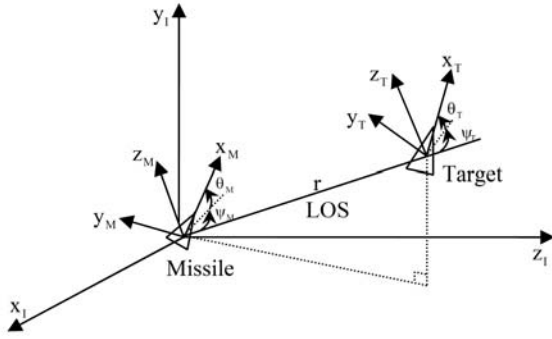Miss-distance values in cases of with and without estimate are as follows,

Fig. 6 The target-missile engagement geometry

$$M.D_{True} = 0.06\,m$$
$$M.D._{Estimated} = 0.4\,m \tag{47}$$

Launch scenario 2: The initial estimates relative to the LOS coordinate system are as follows,

$$r(0) = 4500\,m$$
$$\theta_t(0) = 20 \text{ deg} \ , \ \psi_t(0) = 154 \text{ deg}$$
$$\theta_m(0) = 10 \text{ deg} \ , \ \psi_m(0) = 0 \text{ deg} \tag{48}$$
$$a_{tx}(0) = 0 \ , a_{ty}(0) = 0 \ , a_{tz}(0) = 0$$

and the initial estimates of filter are,

$$\hat{r}(0) = 5000\,m \ , \hat{\dot{r}}(0) = -450\,m/s$$
$$\hat{a}_{tx}(0) = 0 \ , \hat{a}_{ty}(0) = 0 , \hat{a}_{tz}(0) = 0 \tag{49}$$

Miss-distance values in cases of with and without estimate are as follows,

$$M.D_{True} = 0.01\,m$$
$$M.D._{Estimated} = 0.8\,m \tag{50}$$

Launch scenario 3: The initial estimates relative to the LOS coordinate system are as follows,

$$r(0) = 8200\,m$$
$$\theta_t(0) = 20 \text{ deg} \ , \ \psi_t(0) = 166 \text{ deg}$$
$$\theta_m(0) = 10 \text{ deg} \ , \ \psi_m(0) = 0 \text{ deg} \tag{51}$$
$$a_{tx}(0) = 0 \ , a_{ty}(0) = 0 \ , a_{tz}(0) = 0$$

and the initial estimate of the filter are,

$$\hat{r}(0) = 5000\,m \ , \hat{\dot{r}}(0) = -450\,m/s$$
$$\hat{a}_{tx}(0) = 0 \ , \hat{a}_{ty}(0) = 0 , \hat{a}_{tz}(0) = 0 \tag{52}$$

Miss-distance values in cases of with and without estimate are as follows,

$$M.D_{True} = 0.1\,m$$
$$M.D._{Estimated} = 1.9\,m \tag{53}$$

Launch scenario 4: The initial estimates relative to the LOS coordinate system are as follows,

$$r(0) = 8000\,m$$
$$\theta_t(0) = 20 \text{ deg} \ , \ \psi_t(0) = 172 \text{ deg}$$
$$\theta_m(0) = 10 \text{ deg} \ , \ \psi_m(0) = 0 \text{ deg} \tag{54}$$
$$a_{tx}(0) = 0 \ , a_{ty}(0) = 0 \ , a_{tz}(0) = 0$$

and the initial estimate of the filter are,

$$\hat{r}(0) = 5000\,m \ , \hat{\dot{r}}(0) = -450\,m/s$$
$$\hat{a}_{tx}(0) = 0 \ , \hat{a}_{ty}(0) = 0 , \hat{a}_{tz}(0) = 0 \tag{55}$$

Miss-distance values in cases of with and without estimate are as follows,

$$M.D_{True} = 0.3\,m$$
$$M.D._{Estimated} = 1.8\,m \tag{56}$$

In the performed simulations, a maximum of 40g acceleration limit has been considered for the missile. Figs. 7, 9, 11 and 13 indicate missile acceleration command for all scenarios. These figures show an oscillatory motion due to dual acceleration command that result in provision of observability in the missile homing phase. The required $\hat{\dot{\sigma}}_{LBC}$ values for implementation of terminal phase of guidance, using the proposed modified guidance law have been shown in Figs. 8, 10, 12 and 14. As it can be seen from these figures, the value of lead bias term in terminal phase of flight is considerable. Regarding these figures, performance improvement in the terminal phase is obvious.
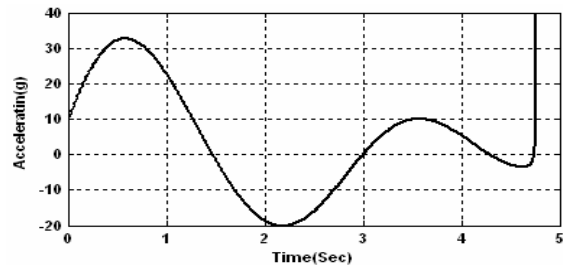

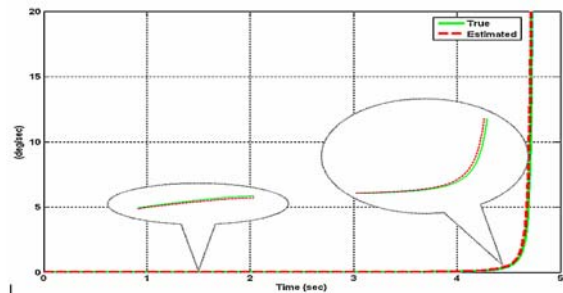**Fig. 7** Required missile acceleration in scenario 1

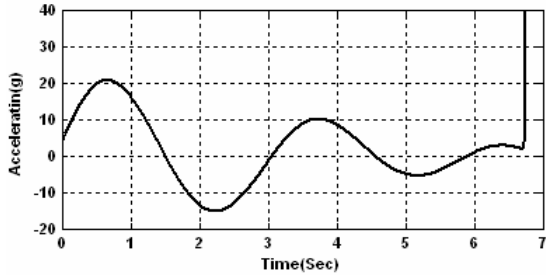**Fig. 8** True and estimated lead bias term in scenario 1

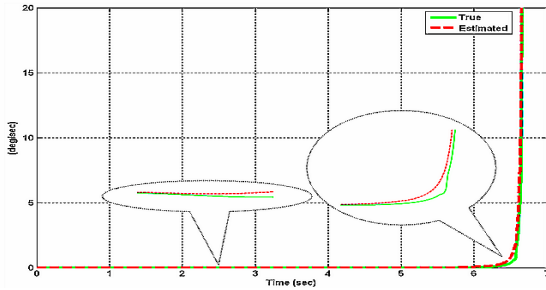**Fig. 9** Required missile acceleration in scenario 2
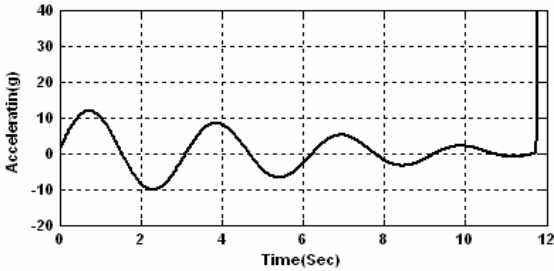

Fig. 10 True and estimated lead bias term in scenario 2

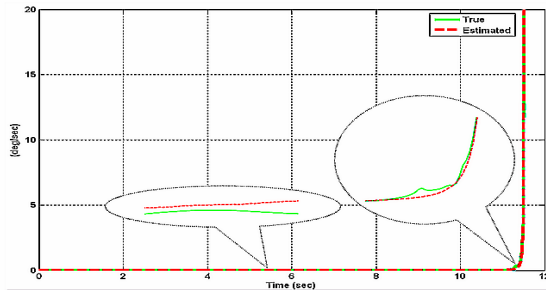**Fig. 11** Required missile acceleration in scenario 3

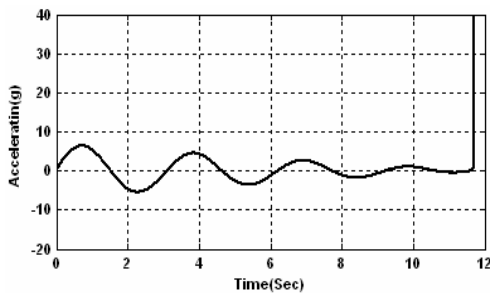**Fig.12** True and estimated lead bias term in scenario 3

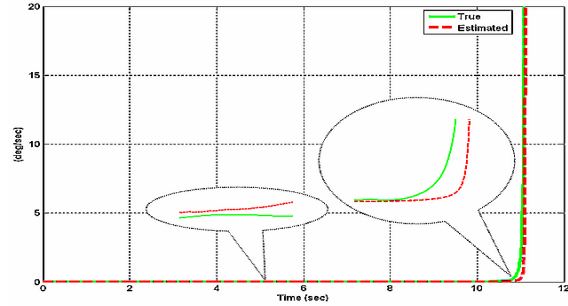**Fig. 13** Required missile acceleration in scenario 4

Fig. 14 True and estimated lead bias term in scenario 4

6 Conclusion

In this paper TAG for passive homing missiles is explained. Modeling has been performed by using LOS coordinates with a particular definition. To obtain convergent estimates of those state variables (involved particularly unavailable from IR trackers), a sequential U-D EKF in MSC combined with a MPNG law are proposed. The proposed guidance law is the traditional PPNG law with two additional terms, which has been calculated in 3-dimensional case. First term includes a cross range compensator and the second is lead bias term. Simulation results indicate that the lead bias term considered for MPNG properly provides terminal conditions for missile-target homing interception.

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