Power Allocation Strategies for MIMO Radar Waveform Design

A. Merline* and S. J. Thiruvengadam**

Abstract: The role of waveform design is central to effective radar resource management for state-of-the art radar systems. The waveform shape employed by any radar system has always been a key factor in determining the performance and application. The design of radar waveform to minimize mean square error (MMSE) in estimating the target impulse response is based on power allocation using waterfilling. This paper shows the effect of various power control strategies in the MMSE performance of the waveform design. We find that the truncated power control strategy exhibits a good MMSE performance. The performance improvement results from the fact that with the truncated power control no power is wasted in poor quality modes.

Keywords: MIMO Radar, Truncated Power Control, Waterfilling, Waveform Design.

1 Introduction

The problem of radar waveform design is of fundamental importance in designing state-of-the-art radar systems. Most radar systems operate by radiating a specific electromagnetic signal into a region and detecting the echo returned from the reflecting targets. The nature of the echo signal provides information about the target, such as range, radial velocity, angular direction, size, shape, and so on [1]. This signal is usually referred to as the radar waveform, and plays a key role in the accuracy, resolution, and ambiguity for radar in performing the above mentioned tasks [2]. Recently there is a great interest in Multiple-Input Multiple-Output (MIMO) radars which uses multiple transmit and receive antennas with large spatial aperture to overcome target fading [3], [4]. MIMO systems compared with Single-Input Single-Output (SISO) systems are used in the mobile communication systems due to their remarkable advantages such as increased frequency spectrum efficiency, quality-improved transmission signals, system coverage area extension, capabilities of supporting higher speed of data transmission and meeting the needs of high capacity and so on. Motivated by these developments, the MIMO concept in communication systems is extended into radar area to solve the problems of receiving signals and target detection. The MIMO radars can be grouped into two classes according to their antenna configurations. One class uses the transmit antennas separated far from each other. Given differences in observing angles on a particular target, the spatial transmit diversity gain can be obtained. According to their receive antennas, this class can be further divided into two kinds of configurations.

The first kind is with the conventional receive array, like the phased array that performs DF (Direction Finding). The second is with the widely-separated antennas, leading to the spatial receive diversity gain. The other class is the configuration whose transmit antennas are closely spaced. Radars in this class usually transmit spatially orthogonal signals to achieve the spatial diversity gain, leading to significantly improved detection performance including virtual aperture extension, spatial coverage extension, beam pattern improvement and increase of the limit on but much less has been done on MIMO waveform design. One important work on waveform design in MIMO context is by Yang and Blum [4] and De Maio and Lops [5]. Yang and Blum have applied MIMO point-to-point communication theory to design radar waveforms by waterfilling the power over the spatial modes of the overall radar scene (channel).

The well-known classical waterfilling solution solves the problem of maximizing the mutual information between the input and the output of a

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channel composed of several subchannels (such as a frequency-selective channel, a time-varying channel, or a set of parallel subchannels arising from the use of multiple antennas at both sides of the link) with a global power constraint at the transmitter [6]. In particular, when the transmitter and receiver are jointly designed for communications through multiple-input multiple-output (MIMO) channels, these types of waterfilling solutions typically appear [7]. Perhaps the most popular of such problems is the minimization of the sum of the mean square errors (MSEs) (equivalently, the trace of the MSE matrix) of the different subchannels existing within a MIMO channel, resulting in a waterfilling solution [8]. For frequency-selective single-input single-output (SISO) channels, similar solutions were obtained already in the 1960s [9].

In this paper, waveform design for MIMO radar for estimation of extended targets modeled using an impulse response is considered. It is assumed that the transmitter has the knowledge of the target's second-order statistics (which could be obtained through some feedback mechanism referred to as covariance feedback) and the transmitter power is constraint. An asymptotic model of the target impulse response is considered as it reduces the required knowledge about the statistical model to just a few samples of the power spectral density (PSD) as it is more suitable in practice. In the remainder of this paper, we first introduce the Signal Model and Optimal Waveform Design. In section 3 the waterfilling algorithms are discussed. In Section 4 the Truncated Power Control strategy is proposed for MIMO Radar. Simulation Results for Waveform Design and the Waterfilling algorithms are given in Section 5. Finally in Section 6, conclusions are presented.

2 Signal Model and Waveform Design

2.1 Signal Model

MIMO radar that is equipped with M transmitting elements and *N* receiving elements is considered. The duration of the observed signals, in discrete-time and baseband, is denoted by L. We consider the extended target as an FIR (Finite Impulse Response) linear system with order V. Here we assume L> V. The impulse response of the signal transmitted by the m[th] antenna and received by the n[th] antenna by $g_{m,n}(l), l\epsilon[0, V]$. The received waveform at the n[th] receive element can be formulated as

$$y_n(k) = \sum_{m=1}^{M} \sum_{l=0}^{V} g_{m,n}(l) x_m(k-l) + n_n(k) \qquad (1)$$

where $x_m(k)$ is the waveform transmitted from m[th] transmit element and $n_n(k)$ is the additive complex Gaussian noise measured at the n[th] receive element. Put the elements at the same time into a column vector, and using (1), the received signal can be formulated as

$$\mathbf{y}_n = \sum_{m=1}^{M} \mathbf{X}_m \, \mathbf{g}_{m,n} + \mathbf{n}_n \qquad (2)$$

where $\mathbf{X}_m$ is an L x K (K = V +1) Toeplitz matrix which contains the waveforms transmitted from the m[th] transmit element. We further define $\bar{\mathbf{X}} = [\, \mathbf{X}_1, \mathbf{X}_2, \ldots \mathbf{X}_m]$ and $\bar{\mathbf{g}}_n = \left[(g_{1,n})^T, \ldots (g_{m,n})^T\right]$, so equation (2) can be rewritten as

$$\mathbf{y}_n = \bar{\mathbf{X}}\bar{\mathbf{g}}_n + \mathbf{n}_n \qquad (3)$$

Collecting the received waveforms from all the N receive antennas to create $\bar{\mathbf{Y}} = [\mathbf{y}_1{}^T, \ldots . \mathbf{y}_n{}^T]^T$ and defining $\boldsymbol{\chi} = \mathbf{I}_N \otimes \bar{\mathbf{X}}$ we have

$$\bar{y} = \boldsymbol{\chi}\bar{\mathbf{g}} + \bar{\mathbf{n}} \qquad (4)$$

where $\bar{\mathbf{g}} = [\bar{\mathbf{g}}_1{}^T, \ldots \bar{\mathbf{g}}_n{}^T]$ and $\bar{\mathbf{n}} = [\mathbf{n}_1{}^T, \ldots \mathbf{n}_N{}^T]$.

Here it is assumed that the target impulse response vector $\bar{\mathbf{g}}$ is a Gaussian random vector. Its covariance matrix

$$\Sigma_{\bar{g}} = \mathbf{U\Lambda U^H} \qquad (5)$$

where $\mathbf{U}$ is a unitary matrix whose columns are eigenvectors and $\mathbf{\Lambda}$=diag $\{\Lambda_{11}, \ldots, \Lambda_{MNP,MNP}\}$ is a diagonal matrix with each diagonal entry given by a real and nonnegative eigenvalue. The elements of vector $\mathbf{n}$ are assumed to be independently and identically distributed and complex Gaussian, with zero mean and variance σ_n^2.

2.2 Waveform Design

In [4] the methods of waveform design based on maximizing mutual information (MI) and minimizing MMSE under the constraint $\mathrm{tr}\{\boldsymbol{\chi}^H\boldsymbol{\chi}\} \le LNP_o$, where P_o stands for total available power. The problem of waveform design based on MMSE are expressed as

$$\min_{\chi} \mathrm{tr}\left\{\left(\sigma_n^{-2}\boldsymbol{\chi}^H\boldsymbol{\chi} + \Sigma_{\bar{g}}^{-1}\right)^{-1}\right\} \qquad (6)$$

$$\text{s.t.} \ \mathrm{tr}\{\boldsymbol{\chi}^H\boldsymbol{\chi}\} \le LNP_o$$

The optimum waveform is

$$\boldsymbol{\chi} = \boldsymbol{\psi}\left(\mathrm{diag}\left[\left(\mu - \frac{\sigma_n^2}{\Lambda_{11}}\right)^+, \ldots, \left(\mu - \frac{\sigma_n^2}{\Lambda_{MNP,MNP}}\right)^+\right]\right)^{1/2} \mathbf{U^H} \qquad (7)$$

Diag$\{\mathbf{a}\}$ is the diagonal matrix with its diagonal given by the vector $\mathbf{a}$,$(a)^+$ =max [0,a]. The term $\mathbf{U}$ is defined in (5) and it is the eigen value decomposition of the covariance matrix of the target impulse response vector $\bar{\mathbf{g}}$. $\boldsymbol{\psi}$ is an LN X MNP matrix with orthonormal columns, and the scalar constant μ satisfies

$$\sum_{i=1}^{MNP} \left(\mu - \frac{\sigma_n^2}{\Lambda_{11}}\right)^+ = LN\, P_o \qquad (8)$$

The resulting minimum value of MMSE is

$$\mathrm{MMSE} = \sum_{i=1}^{MNP} \frac{\Lambda_{ii}}{(\Lambda_{ii}\sigma_n^{-2}\mu - 1)^+ + 1} \qquad (9)$$

In the equation of the waveform the term with the brackets refer to the waterfilling power allocation. So it is obvious that the waveform design mainly depends on the power allocation.

3…Power Allocation Strategies

In multiple-input multiple-output systems, the channel consists of several sub-channels that share the same transmission media. In general, the subchannels exhibit different characteristics and, for this reason, some sub-channels require more power than others for better system performance. Since the total transmission power is a limited resource, it must be adequately distributed among the sub-channels, a process known as waterfilling. The waterfilling solutions were originally formulated with linear pre-coders and de-coders [10] and [11]. Several optimized waterfilling solutions have been proposed [12], based on the channel matrix eigenvalues. For the case of MIMO Radar based on the stastistical characteristics of the target impulse response, it is possible to estimate the solutions for the waterfilling matrix. In water-filling, more power is allocated to "better" subchannels with higher signal-to-noise ratio.

3.1 Waterfilling for Maximum Mutual Information on a Parallel Gaussian Channel

The waterfilling solution given in the waveform equation maximizes the mutual information [13]. The waterfilling matrix is

$$|\phi_{ii}|^2 = diag\left[\left(\mu - \frac{\sigma_n^2}{\Lambda_{11}}\right)^+, \dots, \left(\mu - \frac{\sigma_n^2}{\Lambda_{MNP,MNP}}\right)^+\right] \quad (10)$$

This water-filling solution allocates the transmitted power to those frequency bins where the PSD of **g** indicates the presence of significant scattering of the extended target; while, for those frequency bins where the target PSD is small as compared with the background noise (large values of σ_n^2/v_{ii}), less transmitted power is allocated. This optimum solution is reasonable and makes intuitive and logical sense.

3.2 Zero Forcing Solution

The solution for each diagonal element ϕ_{ii} in the power allocation matrix $\boldsymbol{\phi}$ can be generalized as

$$|\phi_{ii}|^2 = \alpha\Lambda_{ii}^{-1} \quad (11)$$

subject to the same total power constraint given in (6) the solution for the waterfilling matrix is,

$$|\phi_{ii}|^2 = \frac{\boldsymbol{LNP_O\Lambda^{-1}}_{ii}}{\sum_{i=1}^{MNP}\Lambda_{ii}^{-1}} \quad (12)$$

3.3 Constant Power Control Strategy

This constant power control strategy is introduced as modified constant power policy [14] for narrowband fading channels maintains a constant SNR at the receiver only when the fade level is above an optimized threshold level. It is observed that although the modified policy is relatively simple, it performs very close to the optimal policy. According to this power allocation strategy beyond a cutoff point v_0 all subchannels are allotted the same power.

$$S_k = \begin{cases} S_0, & v_k \geq v_0 \\ 0, & v_k < v_0 \end{cases} \quad (13)$$

where S_k is the available power and S_0 is the power shared equally among the subchannels that have PSD of g beyond the cutoff point. The critical task in waterfilling should be to ensure that the subchannels with low PSD values should be allocated the correct amount of power. A constant power allocation strategy where the transmitter allocates zero power to those subchannels that would receive zero power in exact water-filling, but allocates constant power to subchannels that would receive positive power in exact water-filling is often close to the optimal.

4…Truncated Power Control

The truncated power control strategy is same as original waterfilling except that it does not allocate any power to frequency bins where the target PSD is small compared with background noise. In [15] it is observed that the truncated channel inversion technique has the same spectral efficiency as the variable- rate and variable-power MQAM modulation scheme in high speed transmission over fading channels. The truncated channel inversion technique which is a suboptimal adaptive technique [16] has very simple encoder and decoder designs, but they exhibit a capacity penalty which can be large in severe fading.

In Code Division Multiple Access (CDMA) communication system truncated power control exhibits both a power and capacity gain [17]. It is found that truncated power control is most effective for channels with large power fluctuations or with large background noise. In [18] it is shown that truncated power control can be effectively applied to improve that tradeoff between frame transmission delay and residual errors after retransmissions without increasing the frame transmission delay through the radio link layer and without increasing the energy consumption in Transmission Control Protocol/Internet Protocol (TCP/IP) over wireless links.

With truncated power control for MIMO radar, transmit power is allocated in proportion to the quality of particular mode beyond a cut-off point. The transmit power is adjusted such that,

$$\phi_{ii} = \begin{cases} \left(\mu - \frac{\sigma_n^2}{v_{ii}}\right)^+, & v_{ii} \geq v_0 \\ 0, & v_{ii} < v_0 \end{cases} \quad (14)$$

The average transmit power, S_T is given by,

$$S_T = E[\phi_{ii}] \quad (15)$$

The parameter ϕ_{ii} depends upon the target impulse response. The target impulse response can be modeled as Log-normal distribution. The log-normal distribution of cross section σ results when a Gaussian random variable x with mean a and variance b^2 is converted through the transformation σ = exp (x). The log-normal random cross section is defined by,

$$f(\sigma) = \frac{1}{\sigma\sqrt{2\pi b^2}}\exp\frac{(\ln(\sigma)-a^2)}{2b^2}u(\sigma) \quad (16)$$

$$F(\sigma) = \left\{1 - Q\frac{\ln(\sigma)-a}{b}\right\}u(\sigma) \quad (17)$$

where, f(σ) is the probability density function of σ, F(σ) is the probability distribution function of σ, and Q(.) is the Q-function.

5 Simulation Results

5.1 Radar with Single Transmit/Receive Antenna (Asymptotic)

The special case of radar with a single transmit antenna and receive antenna is studied. The antenna index m or n could be omitted for the ensuing analysis and assume L is asymptotically large. An extended target with a finite-energy, complex Gaussian impulse response g with memory V=19 or K=20 is considered. The waveform becomes,

$$\boldsymbol{\chi} = \overline{\mathbf{X}} \tag{18}$$

$$\mathbf{X} = \boldsymbol{\psi}\left(\operatorname{diag}\left[\left(\eta - \frac{\sigma_n^2}{\mathbf{v}_{11}}\right)^{+}, .., \left(\eta - \frac{\sigma_n^2}{\mathbf{v}_{\mathbf{MM}}}\right)^{+}\right]\right)^{1/2} \mathbf{F}_{\mathbf{M}}^{\mathbf{H}} \tag{19}$$

It is assumed that the samples of PSD of g are available and take on the values, ν_{ii} (i=1,2,….20) are available.

Fig. 1 illustrates the samples of the PSD of g, target impulse response. In all simulation it is assumed that noise is complex and i.i.d. Gaussian distributed with σ_n^2 normalized to 1.

It is known that Toeplitz matrices can be approximated by their associated circulant matrices, which can be the result of the diagonalization of discrete Fourier transform (DFT) matrix [19]. Here we assume the vector $\mathbf{g}$ is wide sense stationary and Matrix $\sum_g$ has a Toeplitz structure, so the asymptotic simplification can be employed. The following simulations are all carried out in the asymptotic approach.

5.2 Waterfilling for Maximum Mutual Information on a Parallel Gaussian Channel

This water-filling solution allocates the transmitted power to those frequency bins where the PSD of **g** indicates the presence of significant scattering of the extended target, while, for those frequency bins where the target PSD is small as compared with the background noise (large values of σ_n^2 /V_{ii}), less transmitted power is allocated. If one wishes to measure a target, one should concentrate the power where there is the least uncertainty about the target in order to overcome the noise. On the other hand, it is inefficient to focus the power where the noise hides the target image.

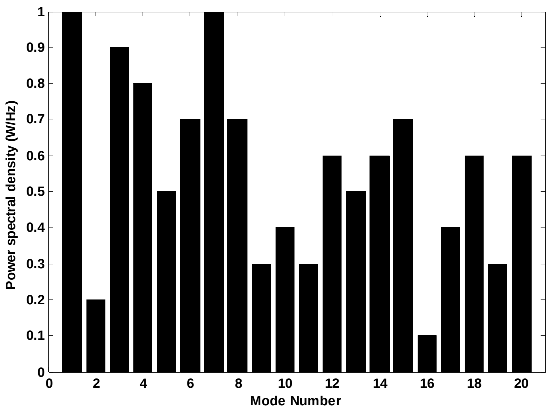

Fig. 1 Samples of the PSD of g.

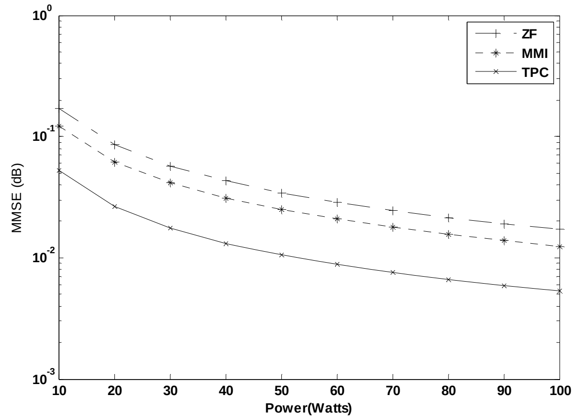

Fig. 2 MMSE as a function of L and P_o for various power allocation schemes.

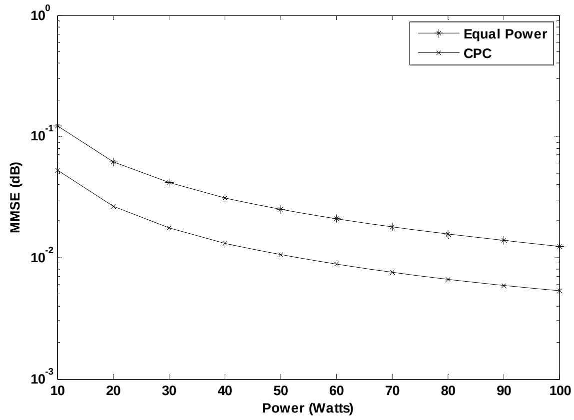

Fig. 3 MMSE as a function of L and P_o for various power allocation schemes.

In Fig. 2, the value of MMSE for different values of P_o and L=320 is shown for the power allocation schemes such as Maximum Mutual Information (MMI), Zero Forcing (ZF) and Truncated Power Control (TPC) for a single transmit and receive antenna case. It is observed that the performance of MMI type of power allocation is optimal. The performance of ZF type is inferior to MMI and that of TPC is superior to MMI. As discussed in the previous section TPC is a threshold based power allocation strategy.

In Fig. 3, the power allocation schemes are compared for Equal Power and Constant Power Control (CPC) scheme. It is observed that the MMSE performance is better for CPC scheme which is also a threshold based power allocation strategy.

It is observed from Fig. 4 that the performance of waterfilling and equal power allocation schemes is very close to each other**.** Waterfilling is of little help only at low SNR. At very low Power values it is observed in Fig. 4 that the performance of MMI type of waterfilling is superior to Equal power allocation.

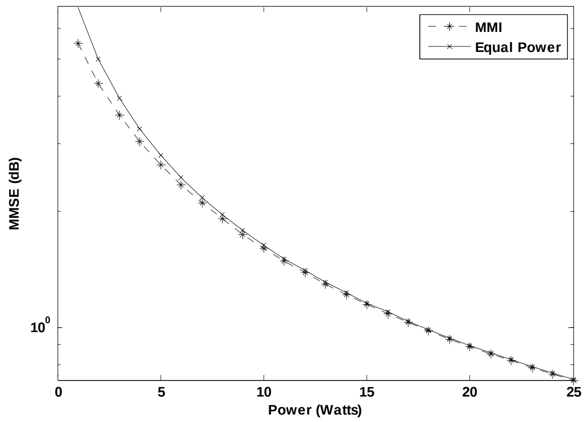

Fig. 4 MMSE as a function of L and P_o for low power values.

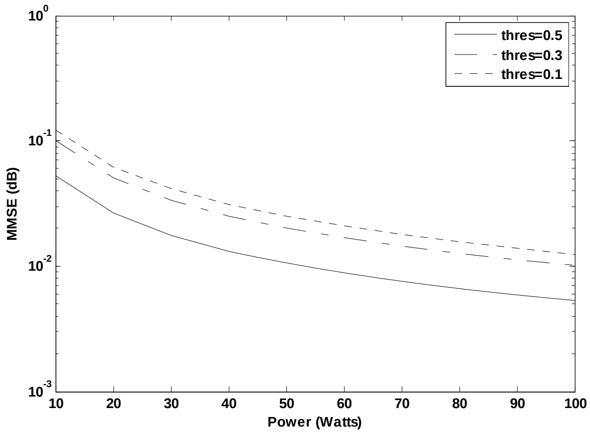

Fig. 5 MMSE as a function of L and P_o for different Truncation Thresholds.

Figure 5 shows the MMSE performance for various truncation thresholds. As the threshold is increased the MMSE performance improves.

6 Conclusion

For the design of optimum radar waveform for MIMO radar, the second order statistics of the extended target impulse response which contains important information regarding the target characteristics is exploited in this paper. It is observed that the optimum solution employs waterfilling, which allocates the transmitter power in proportion to the quality of the particular mode. Low complexity sub optimal power allocation methods are also discussed. The performance of the truncated power allocation strategy is compared with the conventional power allocation methods. It is observed that the performance of truncated power allocation strategy is better than all the conventional power allocation methods.

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