

# Modification of DC Optimal Power Flow, Based on Nodal Approximation of Transmission Losses

M. R. Baghayipour\* and A. Akbari Foroud\*

**Abstract:** The Optimal Power Flow is one of the fundamental problems in power system analyses. Some essential studies in power system operation and planning typically require a large number of repetitive OPF solutions. In these analyses, the convergence speed of the OPF solutions beside their accuracy are two key objects. The full ACOPF is accurate, but takes long solution time. The DCOPF is a simple approximation of OPF that is very fast but is not so accurate. This paper presents a method to improve the accuracy of DCOPF, based on evaluating some nodal shares of transmission losses. Like the previous DCOPF, the Modified DCOPF is derived from a non-iterative DC power flow, and thus its solution requires no long run time. Moreover, it can simply be realized in the form of Lagrange representation, makes it possible to be considered as some constraints in the body of any bi-level optimization problem, with its internal level including the OPF satisfaction. The efficiency of the Modified DCOPF is illustrated through implementing on three test cases (IEEE 30 & 118 Bus test systems and Iran 2006 Transmission Network) and comparing the results (generation levels, line flows, and voltage angles) with the conventional DCOPF and full ACOPF.

**Keywords:** B Matrix Loss formula, DC Power Flow, DC Optimal Power Flow, Nodal Marginal Losses

## 1 Introduction

Optimal Power Flow (OPF) problem indeed establishes the foundation of a wide variety of power system analyses. Noticeably, the use of OPF in power system operation and planning as well as power market related issues are some significant aspects of its usage. For instance, one of the important operation processes that require the OPF solution is the Automatic Generation Control (AGC). In this process, the OPF problem should be solved as accurate and soon as possible, to generate correct and fast decision variables and ensure the power system stability as well as its economic optimality. On the other hand, in the planning purposes, typically a large iteration number of the OPF solutions are required to model the system operation repeatedly for all of the proposed scenarios. Clearly, in these types of problems, each OPF solution should not take long time to avoid the whole problem from being too time consuming.

The ACOPF problem is the full representation of OPF founded on the Full AC Power Flow realization, which due to its nonlinear nature needs to be solved via

some iterative algorithms, such as Newton-Raphson or Gauss-Sidel. Consequently, this causes the ACPF and especially ACOPF problems not only to be time consuming for solution in large scales, but also practically impossible for using in the body of large iterative algorithms, like the planning processes.

The DCOPF problem presents a simple approximation of the main OPF, using the well-known DC Power Flow formula. As it is known, the DCPF formula is a matrix equation that can be solved with no iteration, but with the answers some different from the main ACPF solution, as it ignores the effects of the active and reactive losses as well as reactive power exchanges in the transmission system. Accordingly, the DCOPF solution is fast but not so accurate. However, due to its relatively short run time, the DCOPF approximation currently has a vast usage in the problems founded on the OPF satisfaction, such as planning processes, irrespective of its relatively inaccurate results.

However, due to its wide usage, the conventional DCOPF formulation is expressed and used in many of the previous works, such as [1-7], but only a few of them have presented some mechanisms for reforming it. [8-11] are in such group. Some of those papers have modified the vector of nodal net real power injections by calculating some nodal shares of transmission losses.

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\* The Authors are with the Department of Electrical Engineering, Semnan University, Semnan.

E-mails: [mbpower@gmail.com](mailto:mbpower@gmail.com) and [aakbari@semnan.ac.ir](mailto:aakbari@semnan.ac.ir).

In this way, [8] allocates the loss of each line only to the load buses directly or indirectly connected to it, whereas [9] proposes a mathematical mechanism for allotment of each line loss between its two connected nodes. That paper splits all of the system buses into two generation and demand categories according to the signs of their net injected powers, and calculates the shares of the transmission losses for various generation and load nodes in the system. Those shares are used in that paper to modify the conventional DC Power Flow solution by reforming the vector of nodal net real power injections. Ref. [9] then has developed an iterative algorithm to realize the DCOPF solution modified by considering transmission losses. That method iteratively executes three steps below, starting from a lossless case, until the changes in the nodal net real power injections become under a pre-specified tolerance:

- 1- Solving the well-known Economic Load Dispatch (ELD) problem with loss penalty factors to determine the optimal generation levels in each step.
- 2- Determining the generation and load buses and reforming the vector of the net nodal injections, considering the nodal shares of line losses based on the above-mentioned mechanism.
- 3- Solving the DC Power Flow problem modified by new injections, to update the nodal voltage angles as well as the loss penalty factors in that step. Those factors are used in the first step of the next iteration to result in the updated generation levels according to the ELD (with loss) solution.

However, [8] and [9] have developed some approaches to consider the effect of transmission losses in DCOPF; they both have some essential drawbacks. Since, the approach of [9] is indeed an improved version of that of [8], here only the main weaknesses of [9] is pointed out, as follows:

- 1- It does not respect to the main constraints of the DCOPF problem, namely the constraints of the allowable generation limits and transmission flow limits.
- 2- Due to its inherent complexity, the algorithm of [8] cannot be formulated in the conventional form of optimization problems, i.e. some explicit objective functions and constraints. Consequently, the Lagrange representation for it cannot simply be realized.

Some of the mentioned papers have exploited the DCOPF to calculate the Locational Marginal Prices (LMPs). Since, the conventional DCOPF does not consider the system losses and consequently results in smooth LMPs unlike the main ACOPF, those papers like the predecessors have modified the conventional DCOPF such that it can regard the system losses. For example, [10] and [11] have utilized the marginal Loss Factors as well as the Delivery Factors to form an

iterative DCOPF algorithm. As [10] mentions, its algorithm may be up to 60 times slower than the conventional DCOPF, however it is still faster than the ACOPF. Anyway, their proposed methods have several drawbacks. For instance, they have considered linear generation cost functions and have established the iterative DCOPF as iterations of some LP (Linear Programming) solutions, but the main cost functions are typically in the quadratic or nonlinear forms. Thus those methods may be much more time consuming in actual cases.

In order to obtain a mechanism to improve the previous DCOPF problem such that it considers the effect of the transmission losses, first it is necessary to find such a simple mechanism for nodal allotment of the transmission losses that it can be approximately calculated before solving the DC Power Flow problem. In other words, it should be possible to be formulated independent of the DCPF solution. However, some of the papers that mainly discuss about the Transmission Loss Allocation issue (like [12]) have presented some related criteria, but their formulations are mostly either dependent to the DCPF solution or difficult for use in the DCOPF problem modification.

In this paper, at first a new mechanism for determining the approximate nodal shares of the transmission losses *before the DCPF solution* is presented. This mechanism is based on the well-known B-Loss approximation [1], [13], and its relation with the nodal Marginal Losses through the matrix of B-Coefficients [13]. Then the Modified DCOPF formulation is developed through inserting the nodal loss shares into the body of the conventional DCOPF as some new nodal demands in the real power balance constraint. The Modified DCOPF considers the transmission losses and so, results in much accurate consequences than the conventional DCOPF. However, it is not comparable with the full ACOPF, which considers many decision variables such as transformer taps, it can perfectly be compared with a base ACOPF solution (which its only decision variables are active and reactive power generations as well as bus voltage magnitudes and angles).

The rest of the paper is organized as follows. First, the brief formulation of the conventional DCOPF problem is presented in section 2.1. Then, the mathematical realization of the new nodal loss allotment mechanism and the resulted improved DCOPF problem are developed in sections 2.2 and 2.3, respectively. The Lagrange representation of the Modified DCOPF problem is also presented in section 3. Then, section 4 demonstrates the efficiency of the proposed approach by applying it on the IEEE 30 & 118 bus test system and Iran 2006 Transmission Network and comparing the numerical results with the conventional DCOPF solution as well as the full AC one. Finally, section 5 concludes the paper.

## 2 The Proposed Methodology

### 2.1 Conventional DCOFF Formulation

The conventional DCOFF problem is formulated here, considering the given inelastic demand vector  $\mathbf{d}$  as in Equations (1-4). This problem finds the optimal generation levels as well as nodal voltage angles, with the goal of minimizing the total generation cost of Eq. (1), subject to the nodal power balance constraint of Eq. (2), and allowable upper and lower limits for generation levels and line flows, as formulated in Equations(3) and (4), respectively. The associated Lagrange multipliers are shown in brackets next to the corresponding constraints.

$$\min_{\mathbf{g}, \delta} \{C(\mathbf{g})\} \quad (1)$$

Subject to:

$$\mathbf{g} - \mathbf{d} = \mathbf{P} = \mathbf{B}\delta; (\lambda) \quad (2)$$

$$-\mathbf{g}^L \leq \mathbf{g} \leq \mathbf{g}^U; (\sigma_L, \sigma_U) \quad (3)$$

$$-\mathbf{f}^L \leq \mathbf{H}\delta \leq \mathbf{f}^U; (\gamma_L, \gamma_U) \quad (4)$$

### 2.2 The Well-Known B Matrix Loss Formula

The B Matrix Loss Formula is an explicit mathematical method for calculating the total transmission loss in the power system. This method exploits the quadratic relationship between losses and power flow on each line and expresses the system total loss as a function of the net real power injections, according to Eq. (5) [1], [6].

$$P_{Loss} = \mathbf{P}^T \mathbf{B}_{Loss} \mathbf{P} + \mathbf{B}_{0, Loss}^T \mathbf{P} + B_{00, Loss} \quad (5)$$

In [6] the above formula was simplified to Eq. (6) by ignoring its second and third terms. Then, a simple approximation for calculating the constant  $\mathbf{B}_{Loss}$  matrix has been presented, which its resulted formula is as Eq. (7). Note that the  $\mathbf{B}_{Loss}$  matrix can be approximated using a variety of developed formulas. For example, Eq. (8) proposes another formulation to obtain it. This formula can be derived in a similar way to Eq. (7), but by ignoring the line resistances in the denominators of the  $\mathbf{G}$  matrix diagonal elements.

$$P_{Loss} = \mathbf{P}^T \mathbf{B}_{Loss} \mathbf{P} \quad (6)$$

$$\mathbf{B}_{Loss} = \mathbf{B}^{-1} \mathbf{A}^T \mathbf{G} \mathbf{A} \mathbf{B}^{-1} \quad (7)$$

$$\mathbf{B}_{Loss} = \mathbf{B}^{-1} \mathbf{H}^T \mathbf{R} \mathbf{H} \mathbf{B}^{-1} \quad (8)$$

The vector of the nodal marginal losses, namely, the sensitivity of total transmission loss to a small increment of the net real injected power at each node (and outpoured from the slack bus) can be calculated by differentiating Eq. (6) with respect to the nodal injections. Due to inherent symmetry of the  $\mathbf{B}_{Loss}$  matrix, the above calculation results the vector of nodal marginal losses as in Eq. (9).

$$\mathbf{ML} = 2\mathbf{B}_{Loss} \mathbf{P} \quad (9)$$

It is noticed that calculating the  $\mathbf{B}_{Loss}$  matrix according to Equations (7) or (8) entails the  $\mathbf{B}$  matrix inversion in a similar way to the conventional DC Power Flow calculation. This means that, since the network susceptance matrix ( $\mathbf{B}$ ) is inherently nonsingular with the determinate of zero, its inverse should be calculated through eliminating the row and column of the reference (slack) bus, and then, one row and one column with all elements equal to zero should be appended to the resulted inversed matrix. Consequently, it leads to the value of zero for the marginal loss at the reference node. This point can be explained noting the fact that adding a small injection at the reference node does not change the total transmission loss, as it is withdrawn at the same bus as injected, and thus, it does not flow from the transmission network. Hence, according to Eq. (6), the total transmission loss has no terms of the slack generation. Although this point is so simple and obvious, as will be seen later, indeed it is an essential key in realizing the proposed approach of this paper.

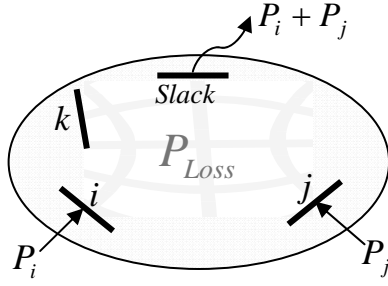
### 2.3 Defining Some Nodal Shares of Transmission Losses Based on the B Matrix Loss Formula

The proposed method of this paper for defining the nodal loss shares, not only is rational and acceptable, but also it is so simple and easy to understand. This method focuses on the simplified “B Matrix Loss Formula” according to Eq. (6) and its relation with the vector of nodal marginal losses as in Eq. (9). By comparing these two equations, it can be observed that total transmission loss can simply be equated to the summation of the products of net real injected powers at various nodes by half of their marginal losses. Hence, the nodal loss shares can be defined according to Eq. (10).

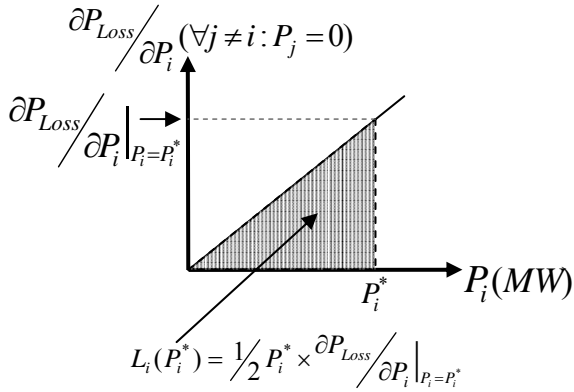
$$\mathbf{L} = \mathbf{P} \cdot \times \mathbf{B}_{Loss} \mathbf{P} = \mathbf{P} \cdot \times \left( \frac{1}{2} \mathbf{ML} \right) \quad (10)$$

In Eq. (10), the operator “ $\cdot \times$ ” denotes the elemental matrix multiply. The vector of nodal loss shares has two principal properties confirming its validity:

- 1- The summation of its all elements yields the approximate total transmission loss (according to the “B Matrix Loss Formula”).
- 2- Each of its elements equals to the product of the net real power injection at its corresponding node and half of its related marginal loss. This approximation is valid because the nodal marginal losses inherently represent the effects of changes in the net real power injections at various nodes on total transmission loss. For more clarity, consider the total transmission loss variation, due to the changes in the net real power injections at two sample nodes,  $i$  and  $j$  (Fig. 1).



**Fig. 1** A sample wide power system with all nodal injections equal to zero, except two buses under study, namely  $i$  and  $j$ . The real injected powers at these two buses (i.e.  $P_i$  &  $P_j$ ) are outpoured from the *Slack*. This case generates the total system real power loss equal to  $P_{Loss}$ .



**Fig. 2** Definition of the nodal loss share  $L_i$  for bus  $i$  (in the case of only one net real injection in the system at bus  $i$  and the others equal to zero), based on the area under the curve of the related nodal marginal loss function for bus  $i$ . Symbols  $P_i$  and  $P_i^*$  respectively denote the variable and present net real power injections at bus  $i$ , and  $P_{Loss}$  is the total system real power loss. The vertical axis represents the marginal loss at bus  $i$  supposing all the nodal injections except bus  $i$  equal to zero, and the horizontal axis is the injected real power at bus  $i$ .

Let to start from the case with all the net injections equal to zero, and consequently no transmission losses. Then, add a net real power injection of  $P_i$  to node  $i$ , outpoured from the slack bus. According to (6), this causes the system total transmission loss to increase by  $B_{ii}P_i^2$ , where  $B_{ii}$  denotes the  $i$ th diagonal element of the  $\mathbf{B}_{Loss}$  matrix. This consequence equals to the product of the  $i$ th net real injected power and half of its related marginal loss. The curve of the nodal marginal loss at bus  $i$  in this case is illustrated in Fig. 2. As seen in this figure, the share of total transmission loss for bus  $i$  in this case equals the area under the curve.

Now consider the case that another net real power injection of  $P_j$  is added at bus  $j$  to the system. Using Eq. (6) it can be proved that total transmission loss in this case will increase to  $B_{ii}P_i^2 + B_{jj}P_j^2 + 2B_{ij}P_iP_j$ , where  $B_{ii}$  and  $B_{jj}$  are the  $i$ th and  $j$ th diagonal elements, and  $B_{ij}$  the  $i$ - $j$  mutual element of the  $\mathbf{B}_{Loss}$  matrix, respectively (the mathematical proof is presented in Eq. 11).

$$P_{Loss} = \mathbf{P}^T \mathbf{B}_{Loss} \mathbf{P} = \begin{bmatrix} 0 & 0 & \dots & | & P_i & | & 0 & \dots & | & P_j & | & 0 & \dots \end{bmatrix}_{1 \times n} \times \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1i} & \dots & B_{1j} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2i} & \dots & B_{2j} & \dots & B_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_{i1} & B_{i2} & \dots & B_{ii} & \dots & B_{ij} & \dots & B_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_{j1} & B_{j2} & \dots & B_{ji} & \dots & B_{jj} & \dots & B_{jn} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_{n1} & B_{n2} & \dots & B_{ni} & \dots & B_{nj} & \dots & B_{nn} \end{bmatrix}_{n \times n} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ P_i \\ 0 \\ \vdots \\ P_j \\ 0 \\ \vdots \end{bmatrix}_{n \times 1}$$

$$= B_{ii}P_i^2 + P_iB_{ij}P_j + P_jB_{ji}P_i + B_{jj}P_j^2$$

$\mathbf{B}_{Loss}$  Matrix Symmetry

$$= B_{ii}P_i^2 + 2B_{ij}P_iP_j + B_{jj}P_j^2 \quad (11)$$

The new statement for total transmission loss contains two additional terms, as compared to the former case. The first,  $B_{ij}P_iP_j$  is a function of only the net real injection at new bus  $j$ , whereas the second  $2B_{ij}P_iP_j$  is indeed a function of both nodal injections. Here, Eq. (10) offers the new total transmission loss expression to be shared between two buses in this way: like the former case, the term  $B_{ii}P_i^2$  is allocated to bus  $i$ , the new term  $B_{jj}P_j^2$  to bus  $j$ , and the term  $2B_{ij}P_iP_j$  is split between two nodes as two equal  $B_{ij}P_iP_j$  shares. Similar analysis can be performed for a new net real injection of  $P_k$  added to another node  $k$ . The new created terms are  $B_{kk}P_k^2$ ,  $2B_{ik}P_iP_k$ , and  $2B_{jk}P_jP_k$ , where symbols  $B_{kk}$ ,  $B_{ik}$ ,  $B_{jk}$  denote the  $k$ th diagonal element, and the mutual  $i$ - $k$  and  $j$ - $k$  elements of the  $\mathbf{B}_{Loss}$  matrix, respectively. According to Eq. (10), the loss share for node  $k$  in this case is  $B_{kk}P_k^2 + B_{ik}P_iP_k + B_{jk}P_jP_k$ , and the remained two  $B_{ik}P_iP_k$  and  $B_{jk}P_jP_k$  terms are added to the previously calculated nodal loss shares for nodes  $i$  and  $j$ , respectively. This analysis can be reiterated for all of the remained buses in the system. In essence, supposing the system has  $n$  nodes (ignoring the slack bus) that only one node has no real power injection, adding a new net real injection to that node increases total transmission loss by one self-nodal term for the node as well as  $n-1$  mutual terms cooperated with the other  $n-1$  nodes. As explained before, according to Eq. (10), each of these mutual terms can be split into two minor terms, namely for loss allotment of the mentioned node and each of the other nodes. This mechanism seems to be rational, as the mutual terms are affected similarly by real injections at each of the cooperated nodes.

#### 2.4 The Modified DCPF Problem Realization

The distinctive property of the ‘‘B Matrix Loss formula’’ is that it does not use the main decision variables of the Power Flow problem (such as voltage angles) for its approximation, as it only requires the system active generation and demand levels. Note that

due to its formulation features, it does not depend on the active generation of the slack bus. This excellent property is exploited here first to develop such a DCPF problem that considers the transmission losses and then to improve the DCOPF problem using it.

The conventional DCPF problem can easily be realized by immediate solution (with no iteration) of the simple linear matrix equation of the nodal power balance constraint, as seen in Eq. (2), supposing that the generation and demand vectors  $\mathbf{g}$  and  $\mathbf{d}$  are provided. It is noticed that the generation of the slack bus is not given to the DCPF problem. This is because, based on the previously mentioned mechanism for inverting the network susceptance matrix ( $\mathbf{B}$ ) the slack generation is indeed not required, and can simply be calculated afterwards.

The above-mentioned DCPF problem is modified here by inserting the vector of nodal loss shares, defined in Eq. (10), as the vector of new nodal demands. The vector of nodal net real injections ( $\mathbf{P}$ ) in Eq. (10) is considered here as the difference between generation and demand vectors ( $\mathbf{g}-\mathbf{d}$ ), but not  $\mathbf{B}\delta$ , to avoid it to be dependent on the decision variable ( $\delta$ ). Note again that the generation of slack bus is not required in Eq. (10), as all elements of its related row and column in  $\mathbf{B}_{Loss}$  matrix are zero. The resultant power balance constraint (or in other words, the Modified DCPF problem formulation) is seen in Eq. (12).

$$\mathbf{g}-\mathbf{d}-(\mathbf{g}-\mathbf{d})\cdot\times\mathbf{B}_{Loss}(\mathbf{g}-\mathbf{d})=\mathbf{B}\delta\rightarrow$$

$$\delta=\mathbf{B}^{-1}(\mathbf{g}-\mathbf{d}-(\mathbf{g}-\mathbf{d})\cdot\times\mathbf{B}_{Loss}(\mathbf{g}-\mathbf{d})) \quad (12)$$

After calculating the vector  $\delta$  from Eq. (12), the vector of line flows at the lines' sending ends can be obtained using the main DCPF formulation for line flows:  $\mathbf{f}=\mathbf{H}\delta$ . However, due to the line losses, the flows at receiving ends are some different. Using the approximation of  $f_{ij}=G_{ij}(\delta_i-\delta_j)^2$ , for the real power flow ( $f_{ij}$ ) from each line  $ij$ , connected between two nodes  $i$  and  $j$  (where  $G_{ij}$  is the line conductance, and  $\delta_i$  and  $\delta_j$  the voltage angles at nodes  $i$  and  $j$ ), the vectors of line flows at sending and receiving ends can be approximately calculated according to Equations (13) and (14), respectively.

$$\mathbf{f}_{send}=\mathbf{H}\delta=\mathbf{H}\mathbf{B}^{-1}(\mathbf{g}-\mathbf{d}-(\mathbf{g}-\mathbf{d})\cdot\times\mathbf{B}_{Loss}(\mathbf{g}-\mathbf{d})) \quad (13)$$

$$\mathbf{f}_{receive}=\mathbf{f}_{send}-(\mathbf{A}\delta)\cdot\times(\mathbf{G}\mathbf{A}\delta) \quad (14)$$

### 2.5 The Modified DCOPF Problem Formulation

However, the modified formulation of the DCPF problem somewhat improves the accuracy of the resulted line flows, as compared to the previous DCPF problem solution and the full ACPF, its excellent influence is viewed when it is utilized in the DCOPF problem formulation. This is because, in the conventional DCOPF problem, the active generation

levels are determined regardless of the transmission losses, and thus, the resulted generation levels are able just to meet the demands but no losses. This is whereas in the Modified DCOPF problem the transmission losses are considered as well as to the demands. The Modified DCOPF formulation is as in Equations (15-18).

$$\min_{\mathbf{g},\delta}\{C(\mathbf{g})\} \quad (15)$$

Subject to:

$$\mathbf{g}-\mathbf{d}-(\mathbf{g}-\mathbf{d})\cdot\times(\mathbf{B}_{Loss}(\mathbf{g}-\mathbf{d}))=\mathbf{B}\delta;(\lambda) \quad (16)$$

$$-\mathbf{g}^L\leq\mathbf{g}\leq\mathbf{g}^U;(\sigma_L,\sigma_U) \quad (17)$$

$$-\mathbf{f}^L\leq\mathbf{H}\delta\leq\mathbf{f}^U;(\gamma_L,\gamma_U) \quad (18)$$

After solving the above problem for decision variables  $\mathbf{g}$  and  $\delta$ , the vectors of line flows at the sending and receiving ends can easily be calculated, using Equations (13-14). According to the above formulation, the steps of the Modified DCOPF problem solution can be summarized as follows:

- 1- Form the main DCOPF matrices, namely  $\mathbf{B}$ ,  $\mathbf{H}$ , and the network incidence matrix ( $\mathbf{A}$ ). Then calculate the  $\mathbf{B}_{Loss}$  matrix according to Equations (7) or (8) and the method explained in section 2.2 for inverting the network susceptance matrix ( $\mathbf{B}$ ). Note that the formation of matrices  $\mathbf{A}$  and  $\mathbf{H}$  is performed according to hypothetical line flow directions, e.g. from the nodes with lower numbers in the list to the nodes with higher numbers (i.e. 1 to 2, 2 to 3, etc). This assumption is indeed a derivation from the principal KVL and KCL concepts, and implies that the formation of the above two matrices based on various hypothetical line flow directions yields a unique solution for the Modified DCOPF problem.
- 2- Solve the Modified DCOPF problem, formulated in Equations (15-18) and obtain the vectors of nodal generation levels, voltage angles, and subsequently line flows using Equations (13-14).

### 3 The Lagrange Representation of the Modified DCOPF Problem

One of the excellent features of the Modified DCOPF problem developed in this paper is that it can easily be realized in the form of the Lagrange representation through the Karush-Kuhn-Tucker (KKT) optimality conditions. This property helps the developed formulation to be considered as some constraints in the body of any bi-level optimization problem, which its internal level includes the OPF problem satisfaction. In this way, first it is useful to briefly recall the KKT optimality conditions for the conventional DCOPF problem of Equations(1-4), using the dual variables in the parentheses for the constraints of Equations (2-4). The related representation is formulated in the Appendix.

The Lagrange function of the Modified DCOPF problem can be formulated by replacing the second term in Eq. (A.1) for the traditional power balance constraint

by the expression derived from Eq. (12) for the modified loss-considering one. The resulted Lagrange function is according to Eq. (19), with the function symbol “**Diag**” denoting the matrix function that gives the column vectors and converts them to diagonal matrices (Note that for two hypothetical vectors  $\mathbf{x}$  and  $\mathbf{y}$ ,  $\mathbf{x} \cdot \times \mathbf{y} = \mathbf{Diag}(\mathbf{x}) \cdot \mathbf{y}$ ).

$$\begin{aligned} \mathcal{L} = & C(\mathbf{g}) \\ & + \lambda^T (\mathbf{B}\delta - (\mathbf{g} - \mathbf{d}) + \mathbf{Diag}(\mathbf{g} - \mathbf{d})\mathbf{B}_{\text{Loss}}(\mathbf{g} - \mathbf{d})) \\ & + \sigma_U^T (\mathbf{g} - \mathbf{g}^U) + \sigma_L^T (\mathbf{g}^L - \mathbf{g}) + \gamma_U^T (\mathbf{H}\delta - \mathbf{f}^U) \\ & + \gamma_L^T (\mathbf{f}^L - \mathbf{H}\delta) \end{aligned} \quad (19)$$

Similar to the conventional problem, the KKT optimality conditions are derived via differentiating Eq. (19) with respect to the decision variables, mentioned before. It is noticed that the term of the power balance constraint in Eq. (19) is not linear, as it contains the elemental matrix product of two variable vectors, and thus, its differentiation with respect to the generation vector requires an extended mathematical analysis, derived in the Appendix. The resulted formula for differentiating the Lagrange function with respect to the vector of generation levels ( $\mathbf{g}$ ) is expressed as in Eq. (20), where  $\mathbf{I}$  is the identity matrix and  $\mathbf{A}_B$  is a  $n$  by  $n$  matrix defined according to Eq. (21) as a function of  $\mathbf{g}$ .

$$\frac{\partial \mathcal{L}}{\partial \mathbf{g}} = \mathbf{I}\mathbf{C}(\mathbf{g}) + \{\mathbf{A}_B(\mathbf{g}) - \mathbf{I}\}\lambda - \sigma_L + \sigma_U = \mathbf{0} \quad (20)$$

$$\mathbf{A}_B = \mathbf{B}_{\text{Loss}}\mathbf{Diag}(\mathbf{g} - \mathbf{d}) + \mathbf{Diag}[\mathbf{B}_{\text{Loss}}(\mathbf{g} - \mathbf{d})] \quad (21)$$

The KKT optimality condition for the Modified DCOPF problem can be expressed through replacing Eq. (A.2) by Eq. (20), and Eq. (A.4) by Eq. (22). The other equations, namely (A.3) and (A.5-A.13) will be remained unchanged.

$$\mathbf{B}\delta - \mathbf{g} + \mathbf{d} + (\mathbf{g} - \mathbf{d}) \cdot \times (\mathbf{B}_{\text{Loss}}(\mathbf{g} - \mathbf{d})) = \mathbf{0} \quad (22)$$

#### 4 Numerical Analysis

The Modified DCOPF problem developed in this paper as well as the conventional one are applied on two test cases, namely the IEEE 30 and 118 bus test systems and are solved using MATLAB-fmincon general-purpose optimization function. On the other hand, the ACOPF problem solutions for the mentioned test cases are obtained, exploiting the MatPower-runopf program. It is noticed that the modeled ACOPF in this work is a base ACOPF, which only considers the active and reactive generations and bus voltage magnitudes and angles as the decision variables, and any additional variables like transformer taps are neglected. In order to the validity of the proposed method in actual networks be perfectly demonstrated, the mentioned three methods are also implemented on Iran (1385 A.H/2006-2007 A.D) Transmission Network and are solved using GAMS IDE21.6 through its NLP-CONOPT nonlinear programming solver. It is regarded that two first case studies extracted from IEEE CDFs do not include some

data need to run the OPF, such as generation cost functions coefficients, real and reactive power generation limits, voltage limits, and line flow limits (transmission capacities). The third one (Iran network) contains the limits but also without generation cost functions coefficients. Consequently, the additional required data are appended here to the case studies in the way described below:

- 1- The generation cost functions coefficients for each of the first two test cases as well as their allowable active and reactive generation limits are available in Tables 1 and 2. All the lower active generation limits are equal to zero.
- 2- The voltage upper and lower limits for all the cases are assumed equal to 1.06 p.u and 0.94 p.u, respectively (such data are required only for the Full ACOPF problem solution).
- 3- The transmission active capacities (i.e. absolute values of real power flow limits) for IEEE 30 and 118 bus test systems are all assumed equal to 100 MW and 200 MW, respectively. According to the unconstraint case solution, these assumptions result in the bind real power flows for some transmission lines.
- 4- The supposed quadratic coefficients of generation cost function for Iran (1385 A.H/2006-2007 A.D) Transmission Network are displayed in Table 3. The linear coefficients of the function are all assumed equal to 20. The mentioned network has 648 buses, 1029 lines and 273 generators, where the bus No. 606 with the generator No. 231 is slack. For sake of abbreviation, the other data of this network (except the supposed generation cost function coefficients) are not displayed here.

The results for each case study, obtained by any of the mentioned problems are provided in Figs. 3 to 11. Figs. 3, 4 and 5 respectively contain resulted generation levels, voltage angles, and middle line flows (the averages of the sending and receiving flows), versus various generator/bus/line numbers for the modified IEEE 30-Bus test system. The second three figures (i.e. Figs. 6-8) also display similar consequences for the modified IEEE 118-Bus test system, and the third group for Iran (1385 A.H/2006-2007 A.D) Transmission network.

Figs. 3-11 obviously demonstrate the modified DCOPF efficiency. These figures illustrate that the Modified DCOPF results in such accuracy much higher than that of the conventional DCOPF problem, and very close to the Full ACOPF solution, especially for generation levels and line flows. However, for more evidence a criterion is utilized here to compare the result of each DCOPF with ACOPF. This criterion is defined as the vector norm of differences between either of the conventional and modified DCOPF results (in p.u) and the ACOPF corresponding consequences in each case (Eq. 23). Lower values of this criterion imply

lower difference between the tested DCOPF results and the main ACOPF consequences, and thus better answers.

Comparison Criterion

$$= \left\| \begin{array}{l} \text{Modified or Conventional DCOPF Results (p.u)} \\ \text{-ACOPF Results(p.u)} \end{array} \right\| \quad (23)$$

Tables 4-6 contain the related values of the above criterion for different results (generations and middle line flows) of each case study. As seen in these tables, the values of the criterion for the modified DCOPF solution are much lower than the corresponding values of the conventional one. This evidently confirms the modified DCOPF solution accuracy.

On the other hand, the approximate range of the required run times for either of three case studies solved by any of the mentioned problems are also provided in Table 7. According to this table, the Modified DCOPF solution requires considerably less time than the Full ACOPF solution, especially in large scales. For

**Table 1** Generation cost functions coefficients and allowable generation limits for IEEE 30-Bus test system

Bus No.	a	B <sub>g</sub>	g <sup>U</sup>	Qg <sup>U</sup>	Qg <sup>L</sup>
1	0.038	20	360.2	10	0
2	0.010	40	100	50	-40
5	0.01	40	100	40	-40
8	0.01	40	100	40	-10
11	0.01	40	100	24	-6
13	0.01	40	100	24	-6

**Table 2** Generation cost functions coefficients and allowable generation limits for IEEE 118-Bus test system

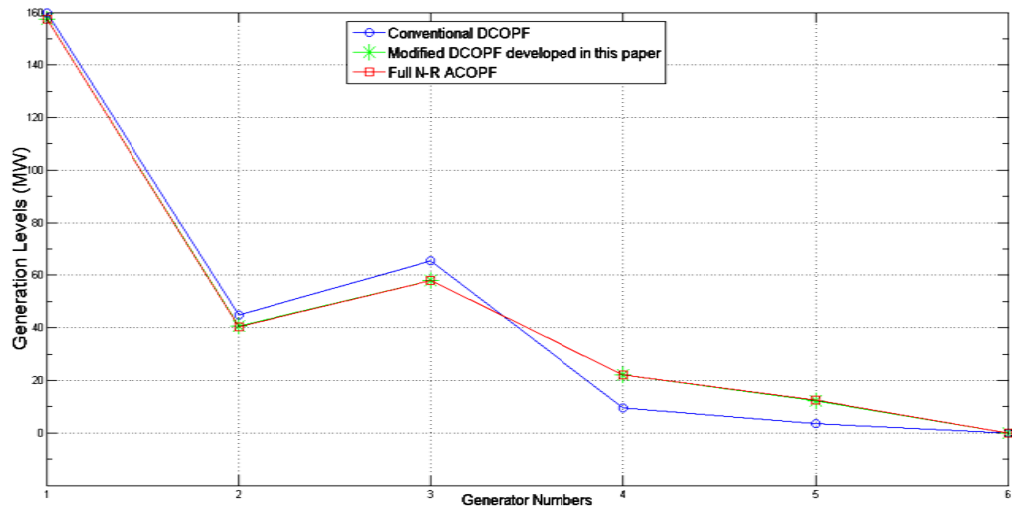
Gen No.	Bus No.	a	b	g <sup>U</sup>	Qg <sup>U</sup>	Qg <sup>L</sup>
1	1	0.010	40	100	15	-5
2	4	0.010	40	100	300	-300
3	6	0.010	40	100	50	-13
4	8	0.010	40	100	300	-300
5	10	0.022	20	550	200	-147
6	12	0.118	20	185	120	-35
7	15	0.010	40	100	30	-10
8	18	0.010	40	100	50	-16
9	19	0.010	40	100	24	-8
10	24	0.010	40	100	300	-300
11	25	0.045	20	320	140	-47
12	26	0.032	20	414	1000	-1000
13	27	0.010	40	100	300	-300

14	31	1.429	20	107	300	-300
15	32	0.010	40	100	42	-14
16	34	0.010	40	100	24	-8
17	36	0.010	40	100	24	-8
18	40	0.010	40	100	300	-300
19	42	0.010	40	100	300	-300
20	46	0.526	20	119	100	-100
21	49	0.049	20	304	210	-85
22	54	0.208	20	148	300	-300
23	55	0.010	40	100	23	-8
24	56	0.010	40	100	15	-8
25	59	0.065	20	255	180	-60
26	61	0.063	20	260	300	-100
27	62	0.010	40	100	20	-20
28	65	0.026	20	491	200	-67
29	66	0.026	20	492	200	-67
30	69	0.019	20	805.2	300	-300
31	70	0.010	40	100	32	-10
32	72	0.010	40	100	100	-100
33	73	0.010	40	100	100	-100
34	74	0.010	40	100	9	-6
35	76	0.010	40	100	23	-8
36	77	0.010	40	100	70	-20
37	80	0.021	20	577	280	-165
38	85	0.010	40	100	23	-8
39	87	2.500	20	104	1000	-100
40	89	0.016	20	707	300	-210
41	90	0.010	40	100	300	-300
42	91	0.010	40	100	100	-100
43	92	0.010	40	100	9	-3
44	99	0.010	40	100	100	-100
45	100	0.040	20	352	155	-50
46	103	0.250	20	140	40	-15
47	104	0.010	40	100	23	-8
48	105	0.010	40	100	23	-8
49	107	0.010	40	100	200	-200
50	110	0.010	40	100	23	-8
51	111	0.278	20	136	1000	-100
52	112	0.010	40	100	1000	-100
53	113	0.010	40	100	200	-100
54	116	0.010	40	100	1000	-1000

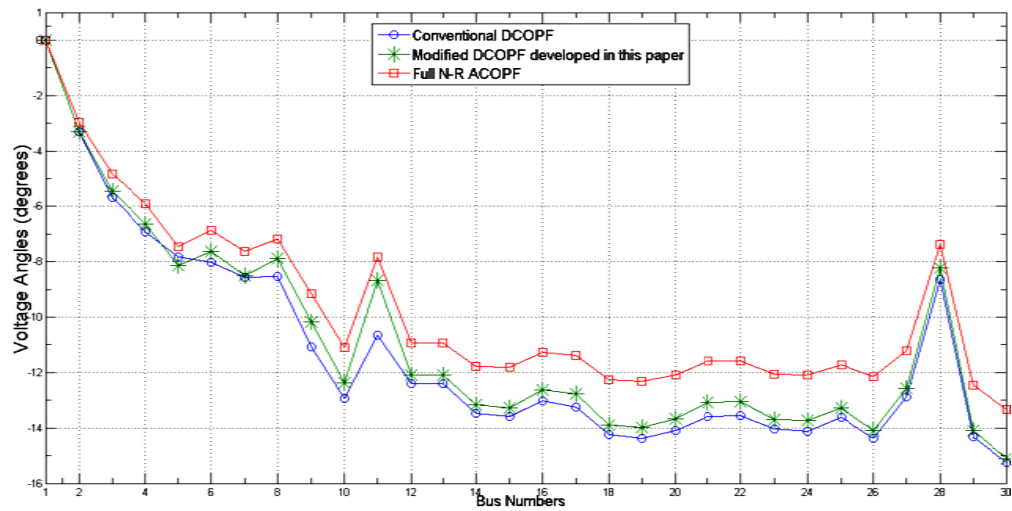
**Table 3** Generation quadratic cost functions coefficients for Iran (1385 A.H/2006-2007 A.D) T Network case study

Bus No.	a	Bus No.	a	Bus No.	a	Bus No.	a	Bus No.	a	Bus No.	a	Bus No.	a
4	0.1000	415	0.0551	454	1.0000	493	0.1250	532	2.0000	571	0.1111	610	0.1538
5	0.1000	416	0.0551	455	1.0000	494	0.1250	533	2.0000	572	1.0000	611	0.1538
12	0.2857	417	0.0551	456	1.0000	495	0.1031	534	2.0000	573	0.1250	612	0.1538
14	0.2857	418	0.0551	457	1.0000	496	0.1031	535	2.0000	574	0.1250	613	0.1538
66	0.6667	419	0.1905	458	1.0000	497	0.1031	536	0.0714	575	0.1250	614	0.0427
67	0.6667	420	0.1739	459	0.0909	498	0.0826	537	0.0714	576	0.1250	615	0.0427
68	0.6667	421	0.1563	460	0.0909	499	0.0826	538	0.0714	577	0.1111	616	0.0427
69	0.6667	422	0.1563	461	0.2500	500	0.0826	539	0.0714	578	0.1111	617	0.0427
384	0.0290	423	0.1563	462	0.2500	501	0.0826	540	0.0847	579	0.1111	618	0.0427
385	0.0290	424	0.0333	463	0.0952	502	0.0826	541	0.0847	580	0.1111	619	0.0427
386	0.1111	425	0.0333	464	0.0952	503	0.0826	542	0.0847	581	0.1111	620	0.0444
387	0.1111	426	0.0333	465	0.0833	504	0.0826	543	0.0847	582	0.1639	621	0.0444
388	0.1000	427	0.0364	466	0.0274	505	0.0826	544	0.0847	583	0.1639	622	0.0444
389	0.0909	428	0.0364	467	0.0274	506	0.0826	545	0.0847	584	0.1639	623	0.0444
390	0.0909	429	0.0364	468	0.0274	507	0.0826	546	0.1724	585	0.1639	624	0.0444
391	0.0909	430	0.0364	469	0.0274	508	0.0826	547	0.1724	586	0.1639	625	0.1111
392	0.0909	431	0.0847	470	0.4000	509	0.0826	548	0.1724	587	0.1639	626	0.1111
393	0.5000	432	0.0847	471	0.4000	510	0.0444	549	0.1724	588	0.1639	627	0.1111
394	0.5000	433	0.0847	472	0.1250	511	0.0444	550	0.5882	589	0.1639	628	0.1111
395	0.0345	434	0.0847	473	0.1250	512	0.0444	551	0.5882	590	0.0909	629	0.1000
396	0.0345	435	0.0847	474	0.1250	513	0.0444	552	0.5882	591	0.0909	630	0.0847
397	0.0339	436	0.0847	475	0.1250	514	0.2000	553	0.5882	592	0.6667	631	0.0847
398	0.0339	437	0.1000	476	0.1250	515	0.2000	554	0.5882	593	0.6667	632	0.0847
399	0.0339	438	0.1000	477	0.1250	516	0.2000	555	0.5882	594	0.6667	633	0.0847
400	0.0339	439	0.1000	478	0.1111	517	0.1111	556	0.6667	595	0.6667	634	0.1124
401	0.0444	440	0.1000	479	0.1111	518	0.1111	557	0.6667	596	0.6667	635	0.1124
402	0.0444	441	0.1000	480	0.1111	519	0.1111	558	0.6667	597	0.0361	636	0.1299
403	0.0444	442	0.1000	481	0.0444	520	0.1111	559	0.6667	598	0.0361	637	0.1299
404	0.0444	443	0.1000	482	0.0444	521	0.1111	560	0.6667	599	0.0361	638	0.1299
405	0.0345	444	0.1000	483	0.0444	522	0.1111	561	0.1250	600	0.0361	639	0.1111
406	0.0345	445	0.0877	484	0.0444	523	0.0323	562	0.1250	601	0.0361	640	0.1111
407	0.1000	446	0.0877	485	0.0444	524	0.0741	563	0.1250	602	0.0361	641	0.1111
408	0.3333	447	0.0877	486	0.0444	525	0.0741	564	0.1250	603	0.0877	642	0.1111
409	0.3333	448	0.0877	487	0.0444	526	0.0741	565	0.1250	604	0.0877	643	0.1111
410	0.1000	449	0.0877	488	0.0444	527	0.0741	566	0.1250	605	0.0877	644	0.1111
411	0.0551	450	0.0877	489	0.1250	528	0.0741	567	0.1000	606	0.0940	645	0.0111
412	0.0551	451	0.0714	490	0.1250	529	0.0741	568	0.1000	607	0.1538	646	0.2000
413	0.0551	452	0.0714	491	0.1250	530	0.6667	569	0.1000	608	0.1538	647	0.0926
414	0.0551	453	0.0714	492	0.1250	531	0.6667	570	0.1111	609	0.1538	648	0.1000

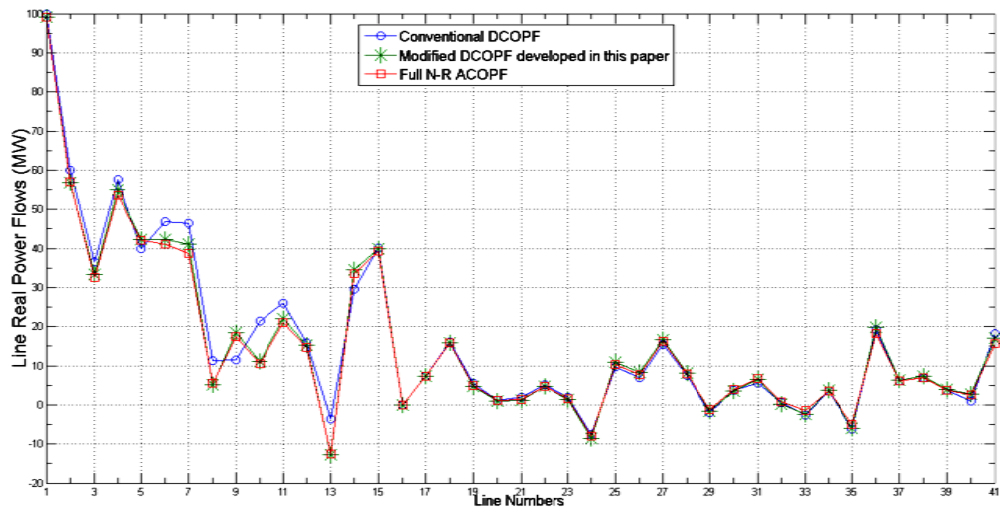




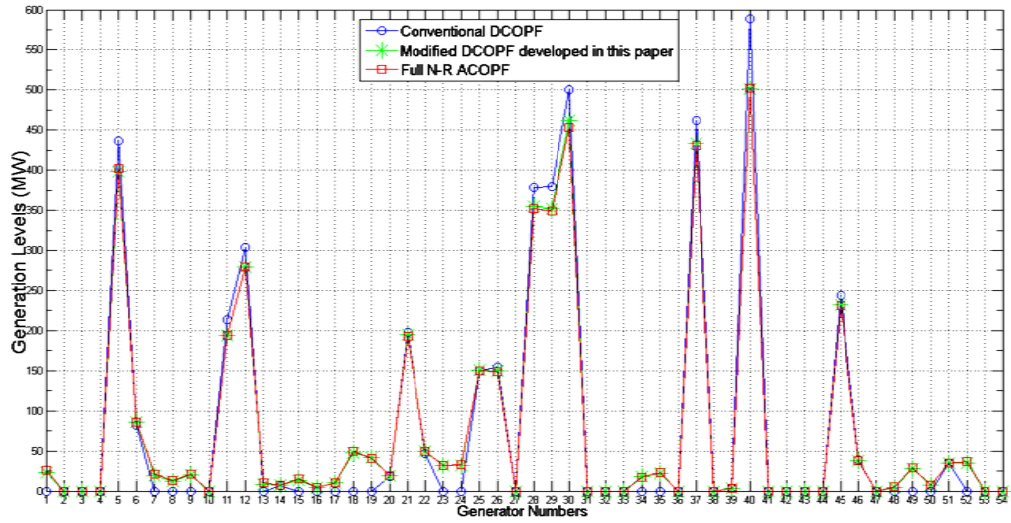
**Fig. 3** Calculated generation levels versus various generator numbers in the modified IEEE 30-Bus test system, using any of three methods: Conventional DCOPF, Modified DCOPF of this paper, and Full N-R ACOPF



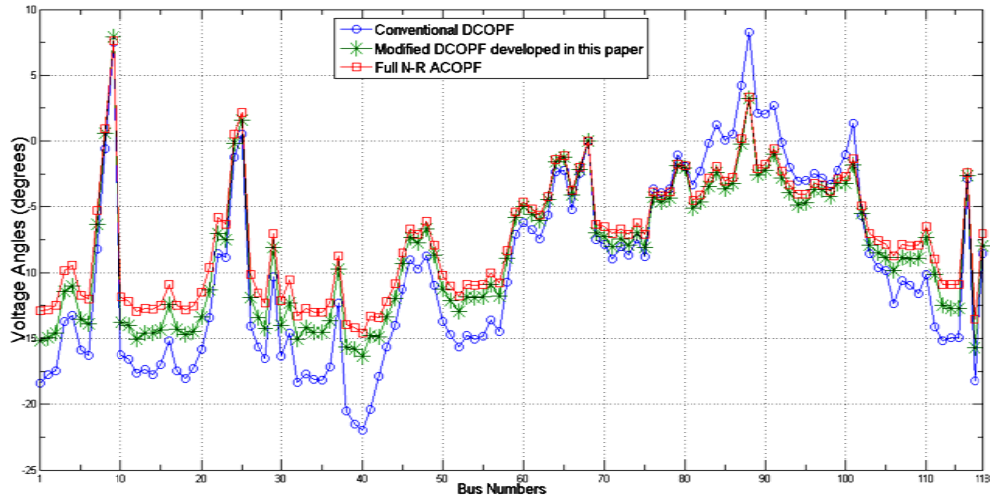
**Fig. 4** Calculated voltage angles versus various bus numbers in the modified IEEE 30-Bus test system, using any of three methods: Conventional DCOPF, Modified DCOPF of this paper, and Full N-R ACOPF



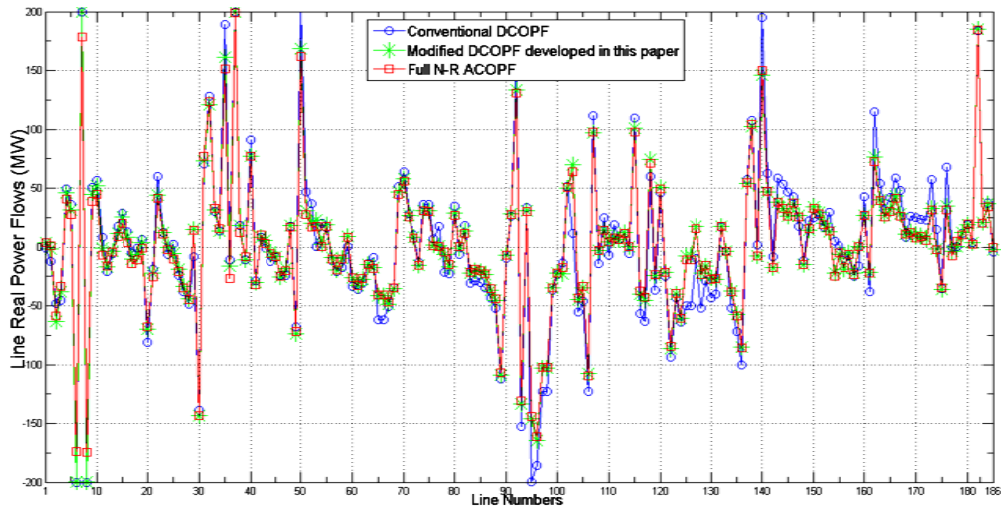
**Fig. 5** Calculated middle line flows versus various line numbers in the modified IEEE 30-Bus test system, using any of three methods: Conventional DCOPF, Modified DCOPF of this paper, and Full N-R ACOPF



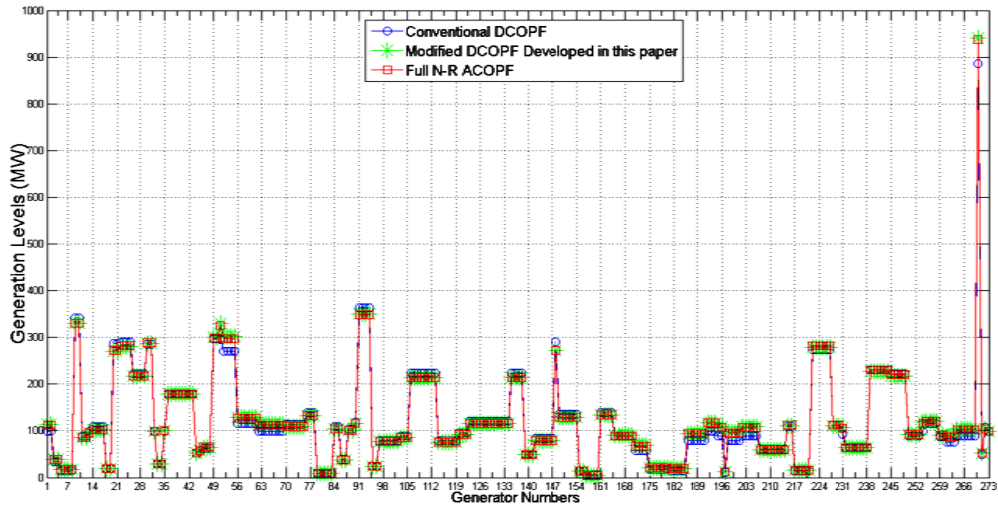
**Fig. 6** Calculated generation levels versus various generator numbers in the modified IEEE 118-Bus test system, using any of three methods: Conventional DCOPF, Modified DCOPF of this paper, and Full N-R ACOPF



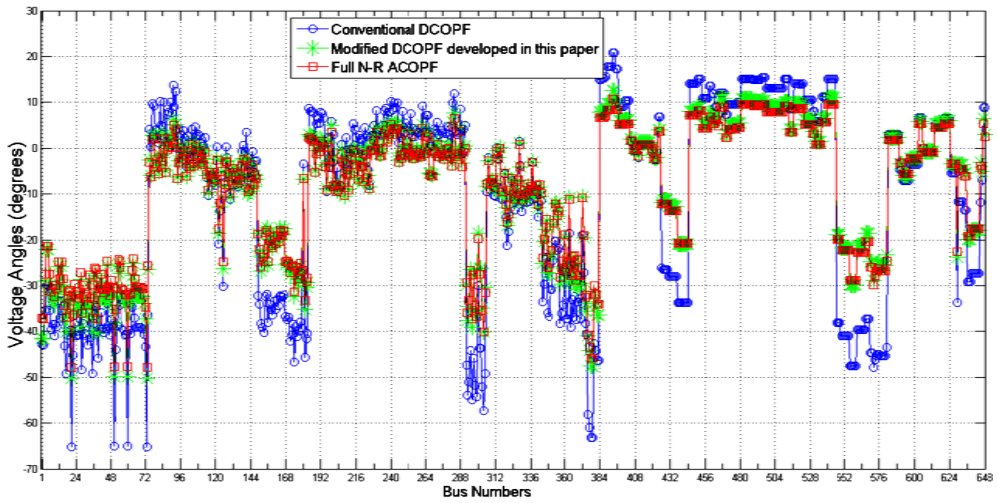
**Fig. 7** Calculated voltage angles versus various bus numbers in the modified IEEE 118-Bus test system, using any of three methods: Conventional DCOPF, Modified DCOPF of this paper, and Full N-R ACOPF



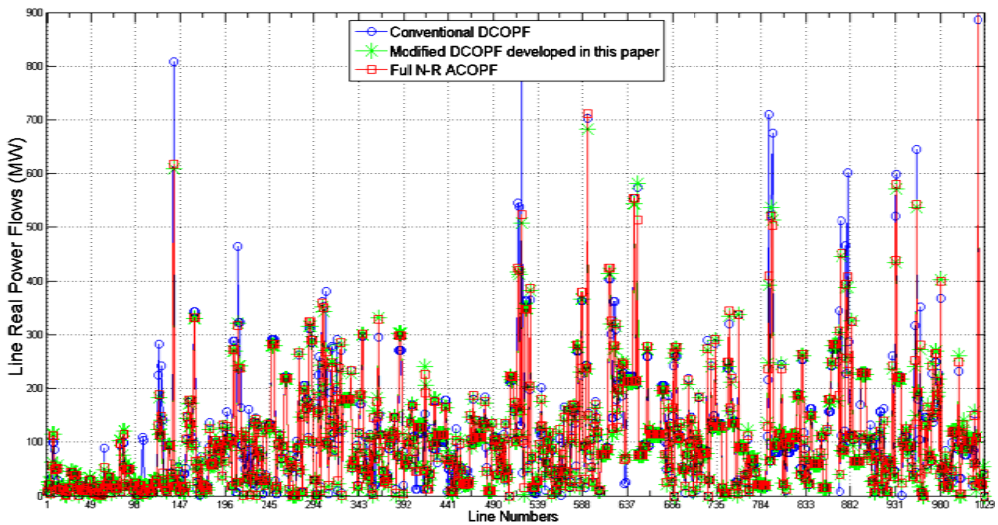
**Fig. 8** Calculated middle line flows versus various line numbers in the modified IEEE 118-Bus test system, using any of three methods: Conventional DCOPF, Modified DCOPF developed in this paper, and Full N-R ACOPF



**Fig. 9** Calculated generation levels versus various generator numbers in Iran (1385 A.H/2006-2007 A.D) Transmission network, using any of three methods: Conventional DCOPF, Modified DCOPF of this paper, and Full N-R ACOPF



**Fig. 10** Calculated voltage angles versus various bus numbers in Iran (1385 A.H/2006-2007 A.D) Transmission network, using any of three methods: Conventional DCOPF, Modified DCOPF of this paper, and Full N-R ACOPF



**Fig. 11** Calculated middle line flows versus various line numbers in Iran (1385 A.H/2006-2007 A.D) Transmission network, using any of three methods: Conventional DCOPF, Modified DCOPF developed in this paper, and Full N-R ACOPF

**Table 4** The values of the vector norms of the differences between either of two DCOPF solutions and the main ACOPF, (defined in Eq. 23) for the Modified IEEE 30-Bus test system

Analized Results \ Different OPFs	Conventional DCOPF	Modified DCOPF
Real Generations (p.u)	0.6076	0.0923
Middle Line Real Flows (p.u)= $\frac{1}{2} \left( \begin{matrix} \text{Sending Real Flows -} \\ \text{Receiving Real Flows} \end{matrix} \right)$	0.5837	0.1428

**Table 5** The values of the vector norms of the differences between either of two DCOPF solutions and the main ACOPF, (defined in Eq. 23) for the Modified IEEE 118-Bus test system

Analized Results \ Different OPFs	Conventional DCOPF	Modified DCOPF
Real Generations (p.u)	1.9662	1.2218
Middle Line Real Flows (p.u)= $\frac{1}{2} \left( \begin{matrix} \text{Sending Real Flows -} \\ \text{Receiving Real Flows} \end{matrix} \right)$	2.2528	1.2087

**Table 6** The values of the vector norms of the differences between either of two DCOPF solutions and the main ACOPF, (defined in Eq. 23) for Iran (1385 A.H/2006-2007 A.D) Transmission Network

Analized Results \ Different OPFs	Conventional DCOPF	Modified DCOPF
Real Generations (p.u)	1.5139	0.2508
Middle Line Real Flows (p.u)= $\frac{1}{2} \left( \begin{matrix} \text{Sending Real Flows -} \\ \text{Receiving Real Flows} \end{matrix} \right)$	8.4789	1.4433

**Table 7** Required run times in Seconds for the applied case studies, using either of three mentioned methods

Different Case studies \ OPFs	Conventional DCOPF	Modified DCOPF	N-R ACOPF
Modified IEEE 30-Bus test system	~ 0.2-0.5 Solver: Matlab (fmincon)	~ 0.2-0.5 Solver: Matlab (fmincon)	~ 0.5-0.8 Solver: Matpower (runopf)
Modified IEEE 118-Bus test system	~ 140 Solver: Matlab (fmincon)	~ 170-180 Solver: Matlab (fmincon)	~ 900-950 Solver: Matpower (runopf)
Iran 2006 T Network	~0.5 Solver: GAMS IDE NLP- CONOPT	~4.5 Solver: GAMS IDE NLP- CONOPT	~300 Solver: GAMS IDE NLP- CONOPT

example, the required run time of the Modified DCOPF solution for the modified IEEE 118-Bus system is less than one fifth of its Full ACOPF solution. However, it is obvious that the required run time of the Modified DCOPF is a bit higher than the conventional one, as its formulation has more complexity, but the excellent accuracy of the Modified DCOPF results, observable in Figs. 3-11 and Tables 4-6, justifies its validity and usefulness.

## 5 Conclusion

In this paper, a new mechanism to improve the accuracy of the conventional DCOPF is presented. This method modifies the principal power balance constraint in the DCOPF problem through determining some nodal shares of transmission losses and adding them to the system as some new nodal loads. The validity of this method is proved through a mathematical discussion about how it allocates the nodal losses shares to several net real injections on two or more buses. The numerical results also confirm the efficiency and accuracy of the Modified DCOPF developed in this paper, through comparing with the results of the conventional DCOPF as well as the ACOPF. It is noticed that the main results of the Modified DCOPF are only sets of generation levels, voltage angles and line real power flows, and thus are not fully comparable with the full ACOPF, which considers many decision variables such as transformer taps. However, the Modified DCOPF solution can perfectly be compared with a base ACOPF solution, which its only decision variables are the active and reactive power generations as well as bus voltage magnitudes and angles. According to the results, the required run time of the Modified DCOPF is much lower than the ACOPF solution time and only a little more than the conventional DCOPF solution time, but instead it produces much more accurate consequences. As a result, its solution for generation levels, voltage angles, and line flows is very similar to that of the ACOPF solution, but with a considerably less run time.

Another strong point of this method is its simplicity, as it can easily be realized in the form of Lagrange representation through Karash-Kuhn-Tucker (KKT) optimality conditions. This excellent feature makes it possible to be considered in the body of any bi-level optimization problems, exploiting the OPF satisfaction as their internal (follower) sub-problem.

## Appendix

### A.1 The Lagrange Representation of the Conventional DCOPF Problem

The Lagrange function of the conventional DCOPF problem is expressed as in Eq. (A.1).

$$\mathcal{L} = C(\mathbf{g}) + \lambda^T(\mathbf{B}\delta - \mathbf{g} + \mathbf{d}) + \sigma_U^T(\mathbf{g} - \mathbf{g}^U) + \sigma_L^T(\mathbf{g}^L - \mathbf{g}) + \gamma_U^T(\mathbf{H}\delta - \mathbf{f}^U) + \gamma_L^T(\mathbf{f}^L - \mathbf{H}\delta) \quad (\text{A.1})$$

Consequently, the KKT optimality conditions can be obtained through differentiating Eq. (A.1) with respect

to the decision variables  $\mathbf{g}$ ,  $\delta$ ,  $\lambda$ ,  $\sigma_L$ ,  $\sigma_U$ ,  $\gamma_L$ ,  $\gamma_U$ , as in Equations (A.2-A.13).

$$\mathbf{I}\mathbf{C}(\mathbf{g}) - \lambda - \sigma_L + \sigma_U = \mathbf{0} \quad (\text{A.2})$$

$$\mathbf{B}\lambda + \mathbf{H}^T(\gamma_U - \gamma_L) = \mathbf{0} \quad (\text{A.3})$$

$$\mathbf{B}\delta - \mathbf{g} + \mathbf{d} = \mathbf{0} \quad (\text{A.4})$$

$$\mathbf{g}^L - \mathbf{g} \leq \mathbf{0} \quad (\text{A.5})$$

$$\mathbf{g} - \mathbf{g}^U \leq \mathbf{0} \quad (\text{A.6})$$

$$\mathbf{f}^L - \mathbf{H}\delta \leq \mathbf{0} \quad (\text{A.7})$$

$$\mathbf{H}\delta - \mathbf{f}^U \leq \mathbf{0} \quad (\text{A.8})$$

$$\sigma_L^T(\mathbf{g}^L - \mathbf{g}) = \mathbf{0} \quad (\text{A.9})$$

$$\sigma_U^T(\mathbf{g} - \mathbf{g}^U) = \mathbf{0} \quad (\text{A.10})$$

$$\gamma_L^T(\mathbf{f}^L - \mathbf{H}\delta) = \mathbf{0} \quad (\text{A.11})$$

$$\gamma_U^T(\mathbf{H}\delta - \mathbf{f}^U) = \mathbf{0} \quad (\text{A.12})$$

$$\gamma_L, \gamma_U, \sigma_U, \sigma_L \geq \mathbf{0} \quad (\text{A.13})$$

## A.2 The Mathematical Proof of Eq. (21)

According to Eq. (19), the second term in Eq. (20) can be calculated as in Eqns. (A.14) to (A.16).

$$\{\mathbf{A}_B(\mathbf{g}) - \mathbf{I}\}\lambda = \frac{\partial}{\partial \mathbf{g}} \left\{ \lambda^T \begin{pmatrix} \mathbf{B}\delta - (\mathbf{g} - \mathbf{d}) \\ + \text{Diag}(\mathbf{g} - \mathbf{d})\mathbf{B}_{\text{Loss}}(\mathbf{g} - \mathbf{d}) \end{pmatrix} \right\} \quad (\text{A.14})$$

$$= \frac{\partial}{\partial \mathbf{g}} \left\{ \begin{matrix} -\lambda^T \mathbf{g} \\ + \lambda^T \text{Diag}(\mathbf{g} - \mathbf{d})\mathbf{B}_{\text{Loss}}(\mathbf{g} - \mathbf{d}) \end{matrix} \right\} \quad (\text{A.15})$$

$$= -\lambda + \frac{\partial}{\partial \mathbf{g}} \{ \lambda^T \text{Diag}(\mathbf{g} - \mathbf{d})\mathbf{B}_{\text{Loss}}(\mathbf{g} - \mathbf{d}) \} \quad (\text{A.16})$$

Comparing the left side of Eq. (A.14) with the right side of Eq. (A.16) yields:

$$\mathbf{A}_B(\mathbf{g})\lambda = \frac{\partial}{\partial \mathbf{g}} \{ \lambda^T \text{Diag}(\mathbf{g} - \mathbf{d})\mathbf{B}_{\text{Loss}}(\mathbf{g} - \mathbf{d}) \} \quad (\text{A.17})$$

The right side of Eq. (A.17) can be extended as below:

$$\mathbf{A}_B(\mathbf{g})\lambda = \frac{\partial}{\partial \mathbf{g}} \left\{ \begin{matrix} [\lambda_1 \ \lambda_2 \ \dots \ \lambda_n] \times \\ \begin{bmatrix} g_1 - d_1 & 0 & \dots & 0 \\ 0 & g_2 - d_2 & 0 & \dots & 0 \\ \vdots & 0 & \ddots & & 0 \\ 0 & \dots & 0 & g_n - d_n \end{bmatrix} \\ \times \\ \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} & B_{n2} & \dots & B_{nn} \end{bmatrix} \\ \begin{bmatrix} g_1 - d_1 \\ g_2 - d_2 \\ \vdots \\ g_n - d_n \end{bmatrix} \end{matrix} \right\} = \begin{bmatrix} \frac{\partial}{\partial g_1} \\ \frac{\partial}{\partial g_2} \\ \vdots \\ \frac{\partial}{\partial g_n} \end{bmatrix} \{\Psi\} \quad (\text{A.18})$$

where,  $\lambda_i, g_i$  and  $d_i$  denote the elements of  $\lambda$ ,  $\mathbf{g}$  and  $\mathbf{d}$  matrices, respectively, and  $B_{ij}$  elements represent the elements of the  $\mathbf{B}_{\text{Loss}}$  matrix. Symbol  $\Psi$  is a scalar expression, formulated as in Eq. (A.19).

$$\Psi = [\lambda_1 \ \lambda_2 \ \dots \ \lambda_n] \times \begin{bmatrix} B_{11}(g_1 - d_1)^2 + B_{12}(g_1 - d_1)(g_2 - d_2) + \dots \\ \quad + B_{1n}(g_1 - d_1)(g_n - d_n) \\ B_{21}(g_2 - d_2)(g_1 - d_1) + B_{22}(g_2 - d_2)^2 + \dots \\ \quad + B_{2n}(g_2 - d_2)(g_n - d_n) \\ \vdots \\ B_{n1}(g_n - d_n)(g_1 - d_1) + B_{n2}(g_n - d_n)(g_2 - d_2) + \dots \\ \quad + B_{nn}(g_n - d_n)^2 \end{bmatrix} \\ = \lambda_1 \left( B_{11}(g_1 - d_1)^2 + B_{12}(g_1 - d_1)(g_2 - d_2) + \dots \right) \\ \quad + B_{1n}(g_1 - d_1)(g_n - d_n) \\ + \lambda_2 \left( B_{21}(g_2 - d_2)(g_1 - d_1) + B_{22}(g_2 - d_2)^2 + \dots \right) \\ \quad + B_{2n}(g_2 - d_2)(g_n - d_n) \\ + \dots \\ + \lambda_n \left( B_{n1}(g_n - d_n)(g_1 - d_1) \right. \\ \quad \left. + B_{n2}(g_n - d_n)(g_2 - d_2) + \dots \right) \\ \quad + B_{nn}(g_n - d_n)^2 \quad (\text{A.19})$$

According to Eq. (A.19), the elements in the right side of Eq. (A.18) are formulated as below:

$$\frac{\partial \Psi}{\partial g_1} = \lambda_1 (2B_{11}(g_1 - d_1) + B_{12}(g_2 - d_2) + \dots \\ \quad + B_{1n}(g_n - d_n)) + \lambda_2 B_{21}(g_2 - d_2) \\ \quad + \dots + \lambda_n B_{n1}(g_n - d_n) \\ = \left( \sum_{i=1}^n B_{1i}(g_i - d_i) \right) \lambda_1 \\ \quad + \sum_{i=1}^n (B_{i1}(g_i - d_i)) \lambda_i \quad (\text{A.20})$$

Similar expressions can be restated for the other partial derivatives in Eq. (A.18).

$$\frac{\partial \Psi}{\partial g_2} = \left( \sum_{i=1}^n B_{2i}(g_i - d_i) \right) \lambda_2 \\ \quad + \sum_{i=1}^n (B_{i2}(g_i - d_i)) \lambda_i \quad (\text{A.21})$$

$$\frac{\partial \Psi}{\partial g_n} = \left( \sum_{i=1}^n B_{ni}(g_i - d_i) \right) \lambda_n \\ \quad + \sum_{i=1}^n (B_{in}(g_i - d_i)) \lambda_i \quad (\text{A.22})$$

In the matrix form, Equations (A.20) to (A.22) can be rearranged as Eq. (A.23).

$$\begin{aligned}
\begin{bmatrix} \partial\Psi/\partial g_1 \\ \partial\Psi/\partial g_2 \\ \vdots \\ \partial\Psi/\partial g_n \end{bmatrix} &= \begin{bmatrix} [B_{11} \ B_{12} \ \dots \ B_{1n}] \begin{bmatrix} g_1 - d_1 \\ g_2 - d_2 \\ \vdots \\ g_n - d_n \end{bmatrix} & 0 & \dots & 0 \\ 0 & [B_{21} \ B_{22} \ \dots \ B_{2n}] \begin{bmatrix} g_1 - d_1 \\ g_2 - d_2 \\ \vdots \\ g_n - d_n \end{bmatrix} & 0 & \dots & 0 \\ \vdots & 0 & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & [B_{n1} \ B_{n2} \ \dots \ B_{nn}] \begin{bmatrix} g_1 - d_1 \\ g_2 - d_2 \\ \vdots \\ g_n - d_n \end{bmatrix} & \vdots \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} \\
+ \begin{bmatrix} B_{11} & B_{21} & \dots & B_{n1} \\ B_{12} & B_{22} & \dots & B_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ B_{1n} & B_{2n} & \dots & B_{nn} \end{bmatrix} \begin{bmatrix} g_1 - d_1 & 0 & \dots & 0 \\ 0 & g_2 - d_2 & 0 & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & g_n - d_n \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} & \quad (A.23)
\end{aligned}$$

From Eqns. (A.18) and (A.23):

$$\mathbf{A}_{B(g)\lambda} = \left\{ \begin{array}{l} \mathbf{Diag}[\mathbf{B}_{Loss}(\mathbf{g} - \mathbf{d})] \\ + \mathbf{B}_{Loss}^T \mathbf{Diag}(\mathbf{g} - \mathbf{d}) \end{array} \right\} \boldsymbol{\lambda} \quad (A.24)$$

Finally, considering the inherent symmetry of  $\mathbf{B}_{Loss}$  matrix, Eq. (21) is proved as in Eq. (A.25).

$$\mathbf{A}_{B(g)} = \mathbf{B}_{Loss} \mathbf{Diag}(\mathbf{g} - \mathbf{d}) + \mathbf{Diag}[\mathbf{B}_{Loss}(\mathbf{g} - \mathbf{d})] \Delta \quad (A.25)$$

### A.3 Nomenclature

Here the symbols that are more practical across the paper are listed. Additional symbols have been defined throughout the text as needed. The symbols are categorized into two classes: Scalars are remarked with italic fonts and not bolded (such as  $x$ ), and vectors and matrices are bolded with no italic fonts (like  $\mathbf{x}$ ).

$n$ : Number of the system buses.

$m$ : Number of the system transmission lines.

**Diag**: A matrix function that gives the column vectors and converts them to diagonal matrices (for two hypothetical vectors  $\mathbf{x}$  and  $\mathbf{y}$ ,  $\mathbf{x} \cdot \times \mathbf{y} = \mathbf{Diag}(\mathbf{x}) \cdot \mathbf{y}$ ).

$\mathbf{g}$ : Vector of active nodal generation levels ( $n \times 1$ , MW).

$C(\mathbf{g})$ : Scalar total generation cost in the system as a function of generation levels;  $C(\mathbf{g}) = \mathbf{a}^T \mathbf{g} + 0.5 \mathbf{g}^T \mathbf{Diag}(\mathbf{b}) \mathbf{g}$ , with  $\mathbf{a}$  being the vector of linear cost parameters and  $\mathbf{b}$  the vector of quadratic cost parameters (\$/hour).

**IC**( $\mathbf{g}$ ): Vector of generation incremental costs;

**IC**( $\mathbf{g}$ ) =  $\mathbf{a} + \mathbf{Diag}(\mathbf{b}) \mathbf{g}$  ( $n \times 1$ , \$/MWh).

$\mathbf{d}$ : Vector of active demand levels ( $n \times 1$ , MW).

$\boldsymbol{\delta}$ : Vector of voltage phase angles ( $n \times 1$ , rad).

**A**: Network incidence matrix ( $m \times n$ , dimensionless).

**R**, **G**: Diagonal square matrices of transmission line resistances and conductances, respectively ( $n \times n$ ,  $\Omega$ ,  $\Omega^{-1}$ ).

**B**: Network susceptance matrix ( $n \times n$ ,  $\Omega^{-1}$ ).

**B<sub>Loss</sub>**: A constant square matrix, contains the quadratic coefficients for transmission loss calculation ( $n \times n$ , dimensionless).

**B<sub>0, Loss</sub>**: A constant vector, contains the linear coefficients for transmission loss calculation ( $n \times 1$ , dimensionless).

$B_{00, Loss}$ : A constant scalar coefficient, denotes the fixed part of total transmission loss (dimensionless).

**ML**: Vector of nodal marginal losses ( $n \times 1$ , MW).

**L**: Vector of nodal shares of transmission active losses ( $n \times 1$ , MW).

$P_{Loss}$ : Scalar value of total transmission loss (MW).

**f**: Vector of apparent line flows. In DCOPTF this vector is supposed as active line flows. ( $m \times 1$ , MW).

$\mathbf{g}^L, \mathbf{g}^U$ : Vectors of lower and upper active generation limits ( $n \times 1$ , MW).

$\mathbf{Qg}^L, \mathbf{Qg}^U$ : Vectors of lower and upper reactive generation limits ( $n \times 1$ , MW).

$\mathbf{f}^L, \mathbf{f}^U$ : Vectors of lower and upper apparent line flow limits. In DCOPTF these vectors are supposed as active line flow limits. ( $n \times 1$ , MW).

**H**: Matrix relating voltage angles to line flows under DC power flow;  $\mathbf{f} = \mathbf{H} \boldsymbol{\delta}$  ( $m \times n$ ,  $\Omega^{-1}$ ).

**P**: Vector of nodal net real power injections ( $n \times 1$ , MW).

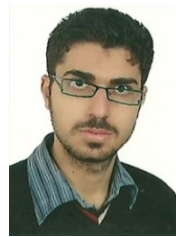
$\boldsymbol{\lambda}$ : Vector of Lagrange multiplier associated with real power balance constraint under DC power flow ( $n \times 1$ , \$/MWh).

$\boldsymbol{\sigma}_L, \boldsymbol{\sigma}_U$ : Vectors of Lagrange multipliers associated with generation lower and upper limits, respectively ( $n \times 1$ , \$/MWh).

$\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_U$ : Vectors of Lagrange multipliers associated with lower and upper line flow limits, respectively ( $m \times 1$ , \$/MWh).

## References

- [1] Wood A. J. and Wollenberg B. F., *Power generation, operation, and control*, Second Edition, New York: Wiley-IEEE; 1996.
- [2] Gil H. A., Galiana F. D., and da Silva E. L., "Nodal price control: a mechanism for transmission network cost allocation", *IEEE Trans. Power Syst.*, Vol. 21, No. 1, pp. 3-10, Feb. 2006.
- [3] Ghayeni M. and Ghazi R., "Transmission cost allocation in restructured power systems based on nodal pricing approach by controlling the marginal prices", *Iranian journal of Electrical & Electronic Engineering*, Vol. 6, No. 2, pp. 93-102, Jun. 2010.
- [4] Ghayeni M. and Ghazi R., "Multi area transmission cost allocation in large power systems using the nodal pricing control approach", *Iranian journal of Electrical & Electronic Engineering*, Vol. 6, No. 4, pp. 238-247, Dec. 2010.
- [5] Abdi H., Parsa Moghaddam M., and Javidi M. H., "A probabilistic approach to transmission expansion planning in deregulated power systems under uncertainties", *Iranian journal of Electrical & Electronic Engineering*, Vol. 1, No. 3, pp. 43-52, Jul. 2005.
- [6] Badri A., Jadid S., and Parsa Moghaddam M., "Impact of participants' market power and transmission constraints on GenCos' nash equilibrium point", *Iranian journal of Electrical & Electronic Engineering*, Vol. 3, No. 1 & 2, pp. 1-9, Jan. 2007.
- [7] Sarkar V. and Khaparde S. A., "DCOPF-based marginal loss pricing with enhanced power flow accuracy by using matrix loss distribution", *IEEE Trans. Power Syst.*, Vol. 24, No. 3, pp. 1435-1445, Aug. 2009.
- [8] Yamashiro S., "Optimization of power flow by DC method", *The Journal of the IEEJ B*, Vol. 97-B, No. 11, pp. 679-684, 1977.
- [9] Li X., Yamashiro S., Wu L., Liu Z. and Ouyang M., "Generation scheduling in deregulated power market taking into account transmission loss allocation", *IET Gen. Trans. Dist.*, Vol. 4, No. 7, pp. 883-892, Jan. 2010.
- [10] Li F. and Bo R., "DCOPF-based LMP simulation: algorithm, comparison with ACOPF, and sensitivity", *IEEE Trans. Power Syst.*, Vol. 22, No. 4, pp. 1475-1485, Nov. 2007.
- [11] Litvinov E., Zheng T., Rosenwald G. and Shamsollahi P., "Marginal loss modeling in LMP calculation", *IEEE Trans. Power Syst.*, Vol. 19, No. 2, pp. 880-888, May. 2004.
- [12] Conejo A. J., Galiana F. D. and Kockar I., "Z-Bus loss allocation", *IEEE Trans. Power Syst.*, Vol. 16, No. 1, pp. 105-110, Feb. 2001.
- [13] Cardell J. B., "Marginal loss pricing for hours with transmission congestion", *IEEE Trans. Power Syst.*, Vol. 22, No. 4, pp. 1466-1474, Nov. 2007.



**Mohammad Reza Baghayipour** was born in Yazd, Iran, in 1985. He received both the B.Sc. and M.Sc. degrees in electrical engineering from Semnan University, Semnan, Iran. His research interests include several different issues concerning the restructured power systems, such as transmission cost and loss allocation and planning.



**Asghar Akbari Foroud** was born in Hamadan, Iran, in 1972. He received B.Sc. degree from Tehran University and M.Sc. and PhD degrees from Tarbiat-modares University, Tehran, Iran. He is now with Semnan University. His research interests include power system dynamics & operation and restructuring.