Strategic Bidding in a Pool-Based Electricity Market under Load Forecast Uncertainty

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Abstract: This paper proposes a method for determining the price bidding strategies of market participants consisting of Generation Companies (GENCOs) and Distribution Companies (DISCOs) in a day-ahead electricity market, while taking into consideration the load forecast uncertainty and demand response programs. The proposed algorithm tries to find a Pareto optimal point for a risk neutral participant in the market. Because of the complexity of the problem a stochastic method is used. In the proposed method, two approaches are used simultaneously. First approach is Fuzzy Genetic Algorithm for finding the best bidding strategies of market players, and another one is Mont-Carlo Method that models the uncertainty of load in price determining algorithm. It is demonstrated that with considering transmission flow constraints in the problem, load uncertainty can considerably influences the profits of companies and so using the second part of the proposed algorithm will be useful in such situation. It is also illustrated when there are no transmission flow constraints, the effect of load uncertainty can be modeled without using a stochastic model. The algorithm is finally tested on an 8 bus system.

Keywords: Electricity market, Bidding strategy, Game theory, Genetic algorithm, Fuzzy sets, Mont-Carlo simulation.

1 Introduction

In recent decades, the electricity supply industry throughout the world has been moved from centralized and vertically integrated structure to an open market environment. The main objective of these markets is decreasing the cost of electricity through competition [1]. There are different models for an electricity market, and the wholesale market is a common one. In this model no central organization is responsible for the provision of electrical energy. Instead, Distribution Companies (DISCOs) purchase the electrical energy on behalf of their customers directly from Generating Companies (GENCOs). These transactions can take the form of a pool or bilateral transactions [2]. In a pool market the producers and consumers submit their sales and purchasing bids, including a pair of quantity-price to market operator and the market clearing price (MCP) is announced by the market operator. Regardless of the bidding prices from suppliers and customers, all selected suppliers are paid and on the other hand all customers pay according to MCP [3]. In such a market,

each agent tries to establish a suitable price bidding strategy to maximize its profit. In a perfect competition, all participants are price-takers which means no participant can influence the market price unilaterally. Theoretically, in perfect competitive markets, suppliers should bid at, or very close to, their marginal production costs to maximize returns [3]. [4] Presents a model for generation scheduling in a competitive environment. The proposed model takes into account the main purposes of GENCOs which are selling electricity as much as possible and making higher profit. However, electricity markets are more akin to oligopoly than perfect competitive environments. In an imperfect competition, at least one company's action has significant influence on other companies' profits. Therefore, each company in establishing its bidding strategy must consider other companies' actions in the market. In such a competition, game theory is a very useful method for determining bidding strategies of market participants. In [5], competition among pool participants was modeled as a non-cooperative game with incomplete information. [6] Utilizes the game theory to simulate price bidding behaviors of GENCOs and develops Nash equilibrium bidding strategies for GENCOs in electricity markets. However, with the complexity of the problem, the global optimal solution is difficult to be found by this approach. Reference [7]

presents two particle swarm optimization (PSO) algorithms to determine bid prices and quantities under the rules of a competitive power market. A bi-level programming technique is formulated in [8] to develop an optimal bidding strategy for a GENCO in the network constrained electricity markets, and Fuzzy adaptive particle swarm optimization (FAPSO) is applied to obtain the global solution of the proposed problem for single hourly and multi-hourly market clearings while opponents' bidding behavior is modeled with probabilistic estimation. In [9] GA is employed to solve this Bi-level optimization problem, and the formulation is expanded to account for different market participants' risk profiles and it is shown that risk aversion may influence the optimal bidding strategy of an individual. A more detailed review of published works related to the GENCOs' bidding strategies is given in [10]. [11] Investigates the problem of developing optimal bidding strategies of GENCOs considering participants' market power and transmission constraints. However, Demand response programs and the effect of load uncertainty have not been considered in these works.

On the other hand, it is widely recognized that markets will work better when demand response programs are available. [12] Shows that even a small increase in demand elasticity in electricity markets can result in appreciable improvement of the market performance in terms of Lerner index, reducing congestion in the network and mitigating the strategic bidding behavior of the producers. Electric Power Research Institute (EPRI) estimates that demand response has the potential to reduce peak demand in the United States by 45,000 MW [13]. In [14] the reliability-constrained unit commitment problem is formulated in a mixed-integer program format. In this research, In addition to spinning reserve of generating units, interruptible load as a demand response program is also included as a part of operating reserve. Options for demand-responsive resource acquisition encompass a broad range of price-based (e.g. time-varying rates and interruptible tariffs) or incentive-based (e.g., direct load control, demand buy-back, demand bidding, and dispatchable stand-by generation) strategies, for example Direct load Control (DLC) interrupts consumer load by remotely shutting down or cycling consumers' electrical appliances such as air conditioners and water heaters. Consumers usually receive remuneration in the form of a bill reduction in return for participation. A thorough examination of various types of demand response program can be found in [15].

Interrupting Loads (ILs) and Distributed Generations (DGs) are two major resources of demand response programs which DISCOs can exploit to improve their market activities and increase their payoffs. DISCOs can affect MCP and GENCOs' profits by these programs, therefore the effect of these activities must be considered in the market performance. [16] Presents a

model for investigating interactions of GENCOs and DISCOs in a day-ahead pool electricity market. In this structure each GENCO tries to maximize its profit by establishing its generators supply curves, and each DISCO pursues a similar objective through using DGs and ILs. Therefore, DISCOs are not passive in the market and they can influence the MCP. In this research the strategic bidding problem has been formulated as a bi-level optimization problem, which its upper level sub-problem maximizes participants' payoffs and the lower sub-problem solves the independent system operator's (ISO) market clearing problem. On the other hand there are many uncertainties in a power system and the common one is uncertainty of load forecasting. This uncertainty can affect companies' estimation of their profits and consequently it can influence companies' bidding strategies. However the uncertainty of load forecasting has not been considered in [16].

This paper investigates the impact of load forecasting uncertainty in prices bidding strategies of market participants. The general structure of the presented research is similar to [16]. This work tries to obtain an equation between load and market participants' profits, and it is demonstrated that how load uncertainty can influence the profit of a risk neutral participant in an unconstrained power network. It is also shown that if the load is considered as a random variable with a normal probability distribution, mathematical expectation of GENCOs' profits will increase as the standard deviation of random load increases and DISCOs' profit will decrease as the standard deviation of load decreases. On the other hand, if lines flow capacity constraints are considered in the model, the relation between companies' expected profits and the standard deviation of uncertain load greatly depends on the location of companies in the network, and in this case some companies' profit shows high sensitivity to the standard deviation of load. So in such situations, considering load uncertainty is crucial in bidding strategy problem. In this paper, Mont-Carlo method is combined with the method mentioned in [16], for considering load uncertainty in bidding strategy problem.

The rest of the paper is organized as follows. In sections 2 and 3, GENCOs' and DISCOs' profit model are formulated and their strategies in the market are presented. Section 4 describes the market clearing model. In section 5 the bidding model is obtained. The relation between load and companies' profits is formulated in 6, the algorithm for solving the bidding model is presented in 7, and a numerical example surveys the algorithm in section 8. We conclude with a brief summary in the last section.

2 GENCOs' Profit Model and Strategies

It is assumed that each GENCO has Ng_n generators whose cost functions are as following:

$$C_{g,i} = C(P_{g,i}) = a_{g,i}P_{g,i}^2 + b_{g,i}P_{g,i} + c_{g,i} \quad , i = 1, 2, \ldots, Ng_n \quad (1)$$

Therefore, generators marginal cost is:

$$MC_i = 2.a_{g,i}.P_{g,i} + b_{g,i} \quad (2)$$

A generator's marginal cost is a linear function of power generated by generator ($P_{g,i}$), and it is the generator's linear bid curve under perfect competition. The variation in price bidding is modeled as the variation of a single parameter k multiplied by the generators' marginal cost. Therefore, a generator's supply function could be modeled as:

$$Bid(P_{g,i}) = \rho_{g,i}(P_{g,i}) = k_{g,i}.(2.a_{g,i}.P_{g,i} + b_{g,i}) \quad (3)$$

Each GENCO submits its supply curve, $P_{g,i}^{min}$ and $P_{g,i}^{max}$ to the ISO, and ISO clears the market using a security constraint economic dispatch and determines each generator's production amount and Local Marginal Prices (LMPs). According to the LMPs, each GENCO's profits formulated as:

$$Rg_n = \sum_{j=1}^{Ng_n} LMP_j.P_{g,j} - \sum_{j=1}^{Ng_n} (a_{g,j}.P_{g,j}^2 + b_{g,j}.P_{g,j} + c_{g,j})$$
$$, n \in Ngco \quad (4)$$

3 DISCOs' Profit Model and Strategies

The objective of a DISCO is to maximize its profit by scheduling its DGs. A DISCO does not bid its DGs into day-ahead market but schedules its DGs according to the estimated LMPs, and also utilizes ILs to restrain high LMPs. It is assumed that the cost function of a DG is as follows [17, 18]:

$$C(P_{dg,i}) = a_{dg,i}.P_{dg,i}^2 + b_{dg,i}.P_{dg,i} \quad , i \in Ndg \quad (5)$$

It should be noted that, DGs with different technologies and cost functions, don't affect the outline of the proposed algorithm and their cost functions can be replaced with Eq. (5). The cost curve of a customer for curtailing its load according to [19] is given as:

$$C(P_{IL,i}) = a_{IL,i}.P_{IL,i}^2 + b_{IL,i}.P_{IL,i} \quad , i \in NIL \quad (6)$$

We assume that if the load of a DISCO is interrupted, the DISCO will be paid according to LMP and load reduction amount, and the DISCO returns all the compensation collected from the interruption of load to the interrupted end customer and does not benefit from the IL compensation. More detail explanations about the mentioned structure can be found in [17], [18]. After clearing the market, each DISCO pays ISO the price of purchased energy. The cost of purchased energy is:

$$C(P_{d,i}) = LMP_i.P_{d,i} \quad , i \in Sn(m) \quad (7)$$

where $P_{d,i}$ is the energy purchased from the market by a DISCO at bus i. Therefore, a DISCO's profit is the difference between the revenue it collects from customers and the cost it pays to purchase energy from market and produce energy by its DGs. The profit of a DISCO can be formulated as follows:

$$Rd_m = \sum_{i \in Sn(m)} \begin{bmatrix} \lambda_c.(P_{d,i}^0 - P_{IL,i}) - C(P_{dg,i}) - \\ LMP_i.(P_{d,i}^0 - P_{IL,i} - P_{dg,i}) \end{bmatrix}, m \in Ndco \quad (8)$$

4 Market Clearing Model

In this market, each GENCO submits its supply curve and each DISCO submits its demand, lower and upper IL limits and cost curve of ILs to ISO. ISO then clears market using a security constraint economic dispatch. The objective in this level is to minimize generation costs and cost of compensating ILs subject to the bids and line flow constraints:

$$\min \sum_{i \in Ng} (k_{g,i}.a_{g,i}.P_{g,i}^2 + k_{g,i}.b_{g,i}.P_{g,i}) + \sum_{i \in Nil} C_{IL}(P_{IL,i}) \quad (9)$$

Subject to:

$$P_{g,i}^{min} \leq P_{g,i} \leq P_{g,i}^{max} \quad , i \in Ng \quad (10)$$

$$-P_{ij}^{max} \leq P_{ij} \leq P_{ij}^{max} \quad , ij \in Nl \quad (11)$$

$$P_{IL,i}^{min} \leq P_{IL,i} \leq P_{IL,i}^{max} \quad , i \in NIL \quad (12)$$

$$(P_{d,i}^0 - P_{IL,i} - P_{dg,i}) - P_{g,i} + \sum_{j \in n(i)} P_{ij} = 0 \quad , i \in n \quad (13)$$

$$\sum_{ij \in Nl(l)} P_{ij}.x_{ij} = 0 \quad , l \in Nlp \quad (14)$$

Constraints in Eqs. (10) and (11) represent generations and transmission lines capacity constraints and Eq. (12) shows IL capacity constraint. Eq. (13) is the load balance constraint on each bus, and Eq. (14) means that the summation of branch voltages in any independent loop must be equal to zero. This constraint is regarded according to DC power flow [20]. By solving this optimization problem, ISO determines $P_{g,i}$, $P_{IL,i}$ and LMP on each bus which is the Lagrangian multiplier of constraint in Eq. (13).

5 Participants' Bidding Model

In a market each agent tries to maximize its profit. The competition assumed in this paper is imperfect which means each agent's strategy influences other agents' profits. If any agent in such competition strives to optimize its own utility or cost unilaterally, it can be

regarded as a non-cooperative game. If each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute Nash equilibrium. Nash equilibrium may, however, be Pareto inefficient [21], and would be the only outcome that agents can credibly achieve in a non-repeated interaction. The rationale of this proceeding is that agents only seek their private benefits, ignoring the benefits or disadvantages of their actions for the opponents and it makes the game to be converged in a point which does not necessarily have the best payoffs for all players [22].

But in reality a day-ahead market occurs every day, and power market could not be categorized as such games, but could be considered as an infinitely repeated game. In such games, since players have long term relationships the value of future interaction serves as the rewards and penalties to discipline the players' current behavior and in such a case, aligning individual incentives with social goals is essential for efficiency [23]. Regarding this instead of Nash equilibrium we try to find a Pareto optimal point, A Pareto optimal outcome is one such that there is no other outcome where some agent's utility can be increased without decreasing the utility of some other agent. To find this point, each player must maximize its profit, along with other players' profit and tries to find a socially optimal point. So each participant, for determining its bid, has to solve a bi-level optimization problem in which upper level sub-problem maximizes all participants' profits, and in lower level sub-problem market clearing model is considered. These can be formulated as follows:

$$\begin{cases} \max Rd_1 \\ \max Rd_2 \\ \vdots \\ \max Rd_{Ndco} \\ \max Rg_1 \\ \max Rg_2 \\ \vdots \\ \max Rg_{Ngco} \end{cases} \qquad (15)$$

Subject to:

$$K_{g,i}^{min} \le K_{g,i} \le K_{g,i}^{max} \quad , i = 1, \dots, Ng \qquad (16)$$

$$P_{dg,i}^{min} \le P_{dg,i} \le P_{dg,i}^{max} \quad , i = 1, \dots, Ndg \qquad (17)$$

Eqs. (9)-(14) $\qquad (18)$

Relations and parameters in reality however are not that much simple, and there are many uncertainties in them. One of the common uncertainties in power systems is load uncertainty, and it can influence participants' bidding strategy, so each participant has to arrange his bid according to this influence. Therefore, before proposing a method for solving bidding strategy problem, we obtain the effect of load uncertainty on the profit of a risk neutral participant, and then design our algorithm with regard to this effect.

6 Modeling Participants' Profit and Load

To investigate the relation between participants' profit and load, we assume a simple system without any line flow capacity constraint. In this system, there is a GENCO which has a generator with a cost function according to Eq. (1) and submits Eq. (3) as its bid, and also it is assumed that there is a DISCO which has an IL with a cost function according to Eq. (6), and therefore IL's marginal cost is $2.a_{IL}.P_{IL} + b_{IL}$. If it is assumed that IL has a lower and upper limit ($P_{IL\,min} \le P_{IL} \le P_{IL\,max}$), the profits of participants could be derived according to Fig. 1. Curves No. 1 and No. 2 in Fig. 1 are generator's marginal cost curve and bidding curve respectively, L is the amount of load, and $L - P_{IL\,Max}$ is the amount of load after interrupting IL fully and λ_c is DISCO's retail price. If curve No. 2 crosses load in a point between L and $L - P_{IL\,min}$, load could be considered as L or $L - P_{IL\,min}$ depending on the IL's interruption policy. It should be noticed that fixed cost coefficient (C) is not considered in GENCO's profit in Fig. 1 and must finally be subtracted from GENCO's profit.

Generally for considering load uncertainty a normal probability distribution function (PDF) is used. Since the load is a random variable with specific probability distribution, finding participants' expected profit is like oscillating load curve in Fig. 1 to left and right according to load's randomness pattern and averaging GENCO's and DISCO's profit, thus the standard deviation of load's PDF could affect participant's expected profit. Since there is IL in the system, to find this effect the problem has to be analyzed in following states:

1. No load is interrupted
2. Part of IL is interrupting
3. IL is fully interrupted
4. There are multiple suppliers in system

(It is assumed that the lower limit of IL is zero).

6.1 No Load Is Interrupted

In this state, no load is curtailed with respect to GENCO's bid, the condition for being in this state is:

$$k(2.a.L + b) \prec b_{IL} \rightarrow L \prec \frac{b_{IL} - k.b}{2.k.a} \qquad (19)$$

Also, by considering Fig. 1, GENCO's profit could be obtained as follows:

$$Rg = L.(2.k.a.L + b.k) - (a.L^2 + b.L + c)$$
$$= (2.k.a - a).L^2 + (k.b - b).L - c \qquad (20)$$

Fig. 1 Companies profits in a simple market.
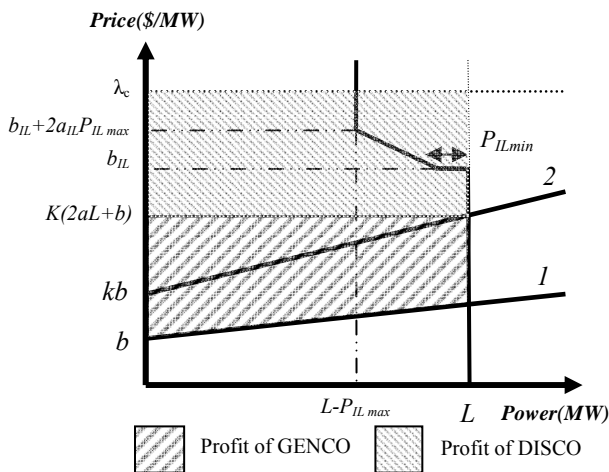

Fig. 2 Part of IL is interrupting.

This equation shows that profit in this state is a quadratic function of load. Thus by taking random sample of load, the expected profit of GENCO will be more than the case in which load is considered as its mean value. For example, we consider a quadratic function $f(x) = \alpha x^2 + \beta x + c$, and we assume that x has a normal probability distribution which x_0 and σ denote its mean value and standard deviation respectively. If we want to calculate expected value of $f(x)$ by taking random samples of x, for every two samples that have ε difference from x_0 we have:

$$\frac{f(x_0+\varepsilon)+f(x_0-\varepsilon)}{2}$$
$$=\frac{\alpha(x_0+\varepsilon)^2+\beta(x_0+\varepsilon)+c}{2}+\frac{\alpha(x_0-\varepsilon)^2+\beta(x_0-\varepsilon)+c}{2} \quad (21)$$
$$=\alpha x_0^2+\beta x_0+c+\alpha\varepsilon^2$$
$$=f(x_0)+\alpha\varepsilon^2$$

Since the differences between the samples and mean value is σ on average, $\alpha\sigma^2$ will be added to the value of $f(x)$ at x_0. Therefore, if the load is considered as a random variable in such system, we can obtain GENCO's expected profit just by obtaining GENCO's profit with the mean value of load and adding $(2ka - a)\sigma^2$ to it, and thus no stochastic sampling is needed. Note that, the expected profit of GENCO increases with increasing the standard deviation of load.

On the other hand, DISCO's profit function could be obtained as follows:

$$Rd = \lambda_c.L - k.(2.a.L + b).L = -k.2.a.L^2 + L.(\lambda_c - k.b) \quad (22)$$

Since DISCO's profit function is quadratic and coefficient of L^2 is negative, DISCO's expected profit decreases as the standard deviation of load increases.
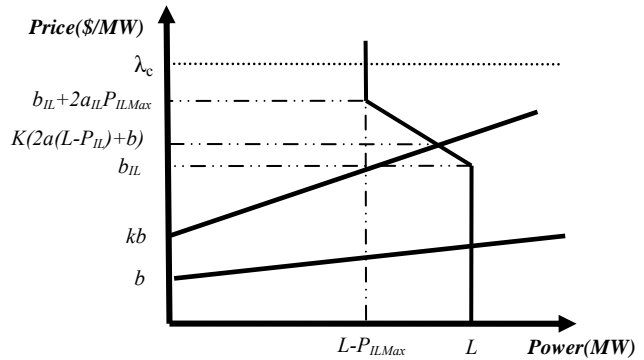
6.2 Part of IL Is Interrupting

In this state, some part of IL with respect to the GENCO's bid is interrupting (Fig. 2), the condition for being in this state is:

$$k.(2.a.L + b) > b_{IL}$$
$$\& \quad (23)$$
$$k.(2.a.(L - P_{ILMax}) + b) < 2.a_{IL}.P_{ILMax} + b_{IL}$$

The amount of interrupted load is:

$$2.a_{IL}.P_{IL} + b_{IL} = k.(2.a.(L - P_{IL}) + b)$$
$$\rightarrow P_{IL} = \frac{2.k.a}{2.a_{IL} + 2.k.a}.L + \frac{k.b - b_{IL}}{2.a_{IL} + 2.k.a} \square \ \alpha.L + \beta \quad (24)$$

Therefore GENCO's profit will be:

$$Rg$$
$$=k.(2.a.(L-P_{IL})+b).(L-P_{IL})-(a.(L-P_{IL})^2+b.(L-P_{IL})+c)$$
$$=(1-\alpha)^2.(2.ka-a).L^2+(1-\alpha).(-2.\beta.(2.ka-a)+ \quad (25)$$
$$(kb-b)).L+...((2.ka-a).\beta^2-(kb-b).\beta-c)$$

where $1-\alpha = \dfrac{2.a_{IL}}{2.a_{IL} + 2.k.a} < 1$.

In this state, profit is also a quadratic function of load. Therefore, the relation between expected profit and load is like state 1, but the coefficient of L^2 is smaller in this state, thus with increasing the standard deviation of load, the expected profit increases but the increment amount is less than state 1. The profit of DISCO in this state is:

$$Rd$$
$$= \lambda_c.L - k.(2.a.(L-P_{IL})+b).(L-P_{IL})$$
$$=-k.2.a.(1-\alpha)^2.L^2+(4.k.a.(1-\alpha).\beta... \quad (26)$$
$$...-k.b.(1-\alpha)+\lambda_c).L+(k.b.\beta-2.k.a.\beta^2)$$

DISCO's profit is also a quadratic function of load and since the absolute value of coefficient of L^2 is negative, with increasing the standard deviation of load

the expected profit of DISCO decreases and the decrement amount is less than that in state 1.

6.3 IL Is Fully Interrupted

The condition for being in this state is:

$$k.(2.a.(L - P_{IL.Max}) + b) > 2.a_{IL}.P_{IL.Max} + b_{IL} \quad (27)$$

This state is like state 1, and the relation between expected profit and load is like that in state 1 and is not obtained here for brevity.

6.4 The Relation with Multiple Suppliers

If there are multiple suppliers in the system, and each supplier submits following equation as its supply function:

$$bid_i = x_i p_i + y_i \quad , i = 1, ..., n \quad (28)$$

ISO clears market by establishing Lagrange equation:

$$C = \sum_{i=1}^{n} (\frac{1}{2}.x_i.p_i^2 + y_i.p_i) + \lambda.(L - \sum_{i=1}^{n} P_i) \quad (29)$$

Using Lagrange multiplier method and some manipulation shown in appendix A, λ is derived as:

$$\lambda = \frac{L}{\sum_{i=1}^{n}(\frac{1}{x_i})} + \frac{\sum_{i=1}^{n}\frac{y_i}{x_i}}{\sum_{i=1}^{n}(\frac{1}{x_i})} \quad (30)$$

This equation represents system's equivalent supply function, which shows the relation between load and market clearing price. With regard to such equivalent supply function, the profit of a supplier, for example GENCO1, is derived:

$$Rg_1 = \lambda p_1 - (a_1 p_1^2 + b_1 p_1 + c_1) \quad (31)$$

$$\lambda = x_1 p_1 + y_1 \rightarrow p_1 = \frac{\lambda - y_1}{x_1} \quad (32)$$

Now we derive the relation between profit and load. Since profit is a quadratic function of load, according to state 1, the coefficient of L^2 will finally affect the expected profit of GENCO, thus by putting Eqs. (30) and (32) into Eq. (31), the coefficient of L^2 will be

$$\frac{L^2}{(\sum_{i=1}^{n}\frac{1}{x_i})^2.x_1} - \frac{a_1.L^2}{(\sum_{i=1}^{n}\frac{1}{x_i})^2.x_1^2} = L^2(\frac{1}{(\sum_{i=1}^{n}\frac{1}{x_i})^2.x_1^2}).(x_1 - a_1) \quad (33)$$

The $(x_1 - a_1)$ is coefficient of L^2 in state1, and $(\frac{1}{(\sum_{i=1}^{n}\frac{1}{x_i})^2.x_1^2})$ is added in the presence of other suppliers; therefore if the standard deviation of load is σ,

according to Eq. (21), the expected profit of jth GENCO could be calculated by putting the mean value of load in GENCO's profit function and adding $(\frac{1}{(\sum_{i=1}^{n}\frac{1}{x_i})^2.x_j^2}).(x_j - a_j).\sigma^2$ to it.

According to above equations in an unconstrained network, participants' profit is a quadratic function of load, and if the load uncertainty is modeled by a normal PDF, the expected profit of a GENCO increases as the standard deviation of load increases and the reverse occurs to DISCO's expected profit.

7 Problem Solving Algorithm

As mentioned before each participant for determining its best strategy has to solve Eqs. (15)-(18). On the other hand, participants could not predict the amount of load precisely, and they use a normal probability distribution for modeling load uncertainty. Therefore, in modeling the problem, the parameter $P_{d,i}^0$ in Eq. (13) is uncertain and just its probability distribution is available. Since the problem is complicated, the analytical methods could not solve it. Therefore, stochastic methods like Genetic Algorithm could be helpful. In this paper fuzzy satisfying method and Genetic Algorithm are used simultaneously to solve bi-level optimization problem and Mont-Carlo method is also used to model load uncertainty. Since there are several objectives in the problem, fuzzy satisfying method is useful. In fuzzy set theory, each object x in a fuzzy set X is given a membership function denoted by $\mu(x)$, which is corresponding to the characteristic function of the crisp set whose values range between zero and one. In fuzzy sets the closer the value $\mu(x)$ to 1, the more x belongs to X. The x in this problem is participants' profit, and therefore for every participant a minimum and maximum profit has to be obtained. After finding a maximum and minimum profit for each participant, the membership function of each objective could be defined as:

$$\mu(R_i) = \begin{cases} 0 & R_i \leq R_i^{min} \\ \frac{R_i^{max} - R_i}{R_i^{max} - R_i^{min}} & R_i^{min} \leq R_i \leq R_i^{max} \\ 1 & R_i \geq R_i^{max} \end{cases} \quad (34)$$

Note that maximizing all participants' profits in Eq. (15) does not mean that all participants will finally profit well, but rather if a participant has not the ability to influence market clearance point, other participants should not regard the profit of such participant in their optimization problem, or they can put maximum profit of such participant in Eq. (34) so low that it does not impose any constraint on their optimization.

Before calculating objective functions that are GENCO's and DISCO's profit, and their membership

values, $P_{g,i}$ s, $P_{IL,i}$ s and LMPs must be calculated. These parameters are obtained in lower level sub-problem. On the other hand, in market clearing model, according to Eqs. (9)-(14), $K_{g,i}$ and $P_{dg,i}$ are inputs, thus for every combination of $K_{g,i}$ s and $P_{dg,i}$ s there is a set of profits, and by putting these profits into their membership function fuzzy set of profits is obtained. In other words, for every combination of inputs we will have a set of fuzzy profits in output. To compare the value of two fuzzy sets of profits, the most attention is given to the minimum value of each set. This can be formulated as follows:

$$\text{Maximize} \left[\min_{i=1,...,N} (\mu(R_i)) + \rho \sum_{i=1}^{N} \mu(R_i) \right] \quad (35)$$

To circumvent the necessity to perform the Pareto optimality test, the term $\rho . \sum_{i=1}^{N} \mu(R_i)$ is added, where ρ is a sufficiently small positive number [24]. For finding participants' best strategy to maximize Eq. (35) Genetic algorithm is applied. Each chromosome in this method is $\text{Chromosome} = [Kg_1,...,Kg_n, Pdg_1,...,Pdg_m]$. The fitness function in Genetic Algorithm is Eq. (35) that determine more desirable chromosome.

To model load uncertainty, Mont-Carlo method is combined with the above mentioned method. For this purpose a normal probability distribution is considered for each load. For each chromosome that enters the OPF, sufficient samples are taken from these loads and the profits of companies is calculated with respect to the sampled loads, and the average of the profits is taken to determine chromosome's fitness.

Executing the algorithm involves following steps:
1. For every objective establish a membership function according to objective's maximum and minimum value like Eq. (34).
2. Assign a normal probability distribution for load at each bus.
3. Create a Genetic algorithm whose chromosomes consist of Ng+Ndg genes, and each gene represents either $k_{g,i}$ or $P_{dg,i}$.
4. Initialize GA population and specify the number of generations allowed (the number of chromosomes in each generation is denoted by *Popsize*)
5. Set GA generation counter $t_{gen} = 1$.
6. Set the population counter $t_{pop} = 1$.
7. To calculate the fitness of each chromosome , do the following for chromosome t_{pop}:

   a) Specify the number of Mont-Carlo trials denoted by T_{Mont}.
   b) Set MC trial counter $t_{mont} = 1$.
   c) Take sample of each random load.
   d) Solve market clearing problem Eqs. (9)-(14).

e) Save $P_{g,i}$ s, $P_{IL,i}$ s and LMPs derived from (d), and with them calculate GENCOs' and DISCOs' profit.
f) $t_{mont} = t_{mont} + 1$. If $t_{mont} \le T_{Mont}$ go to 7.c
g) Calculate the average profit of each t_{pop}:

$$R_{t_{pop}} = \frac{1}{T_{Mont}} \left(\sum_{j=1}^{T_{Mont}} R_j \right) \quad (36)$$

h) $t_{pop} = t_{pop} + 1$. If $t_{pop} \le$ Popsize go to 7.
8. Put expected profits derived in 7.g into Eq. (34) to derive fuzzy value of each expected profit.
9. By putting fuzzy values derived in step 8 into Eq. (35) a number will be derived for every chromosome t_{pop} that determines chromosome's fitness.
10. $t_{gen} = t_{gen} + 1$. If $t_{gen} \le T_{gen}$, go to11, or else go to step 12.
11. Perform standard GA operators such as crossover, mutation and etc and then go to step 6.
12. Save the best chromosome derived in step 9 as participants' best strategies.

The flowchart of this algorithm is shown in Fig. 3. It must be noted that participants are considered risk neutral which means expected profit, or in fact, their average profit in large trials is their deciding factor. Eq. (36) could be changed depending on participant's risk sensitivity.

8 Numerical Results
8.1 Part A
To test what was derived in section 6, we consider a system with 3 suppliers whose parameters are shown in Table 1. By considering load as a random variable and using Mont-Carlo method, the expected profit of GENCOs are derived which are shown in Table 2. For example the first row of this table shows that if load is 300 MW how much is the expected profit of GENCOs, and the last row shows the expected profit of GENCOs when the load is a random variable and its mean value and standard deviation are 300 MW and 45 MW respectively. It should be noted that the expected profit of GENCOs increase as the standard deviation of load increases. The expected profit of GENCOs could also be calculated by using what derived in Eqs. (21) and (33). For example when the standard deviation of load is 36 MW, the expected profit of Gen1 could be calculated according to Eq. (21):

$R_{Gen1} \cong 3626.4 + 0.0147(36^2) = 3645.5$

This is approximately equal to the expected profit of Gen1 in Table 2. Therefore, the expected profit could be calculated just by using above method, and thus no stochastic sampling is needed in such case.
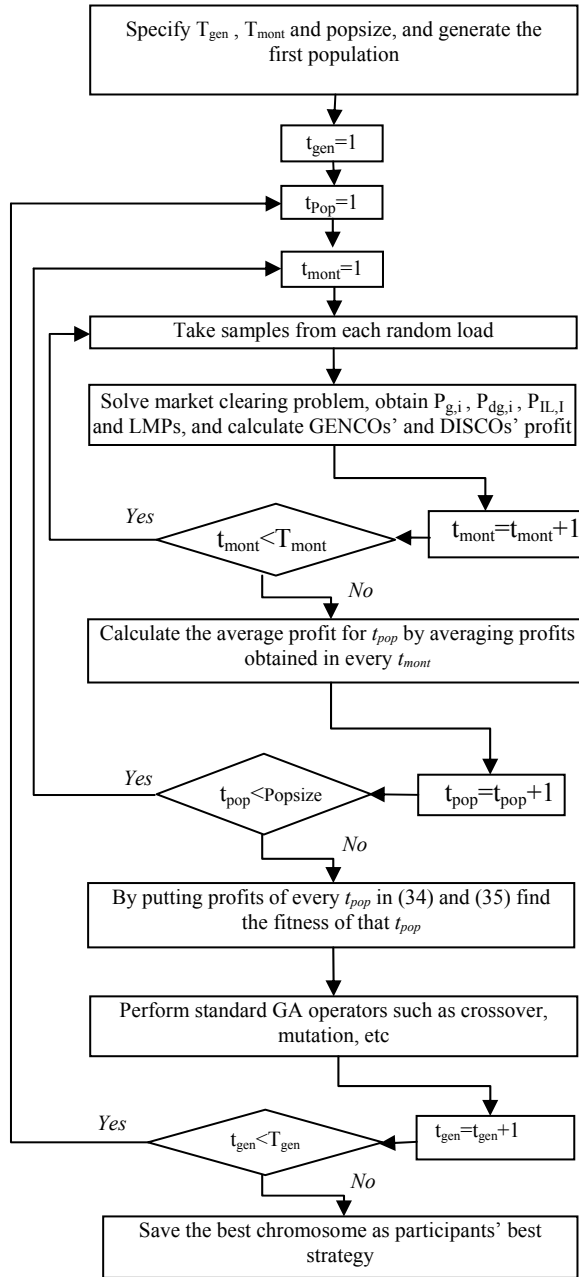
Fig. 3 Flowchart of proposed method.

8.2 Part B

The shown system in Fig. 4 is used to examine the proposed algorithm. The information of participants and transmission lines are shown in Tables (B-1)-(B-4) in Appendix B. The following assumptions are made in this study:

- The simulation is running from GENCO3 viewpoint.
- Price that DISCOs charge their end customers for energy is $\lambda_c = 95 \ \$/MWh$.
- $P_{IL,Max}$ is one fifteenth of peak load
- Since with the available methods the error of load forecast could be reduced to 3% [25], the

standard deviation of random load is assumed to be 3.5%.

- The maximum and minimum profit of participants are assumed according to Table 3.

By using the method described in previous section the following results are obtained.

The best strategy for participants is according to Table 4. With respect to these strategies and load uncertainty the probability distribution of GENCO3's profit is shown in Fig. 5. Since participants are risk neutral they decide on their expected profit.

Table 1 Generators Parameters.

	a_g ($/MW²h)	b_g ($/MWh)	P_{max} (MW)	bid function	Coef. of L^2 in (33)
Gen1	0.1	30	140	0.2P+60	0.0147
Gen2	0.09	21.5	160	0.23P+55	0.0156
Gen3	0.12	22.2	150	0.27p+50	0.0121

Table 2 Generators Profits with respect to increasing in standard deviation of load.

	Profit of Gen1 ($)	Profit of Gen2 ($)	Profit of Gen3 ($)
N(300,0)	3626.4	4879.6	4601.9
N(300,9)	3628	4881.5	4603.4
N(300,18)	3632.9	4886.6	4607.4
N(300,27)	3635.3	4889	4609.1
N(300,36)	3648.6	4903.4	4620.5
N(300,45)	3655	4910.4	4625.7

Table 3 Maximum and Minimum Profits.

	DISCO 1	DISCO 2	DISCO 3	GENCO 1	GENCO 2	GENCO 3
Min Profit ($)	-2500	-3000	-1000	-1500	-1500	-1500
Max Profit ($)	4000	4500	2000	7500	7000	9500

Table 4 Participants' best strategies.

K_{G2}	K_{G4}	K_{G5}	K_{G6}	K_{G7}	K_{G8}	P_{DG1} (MW)	P_{DG2} (MW)	P_{DG3} (MW)
1.905	2.656	3.434	3.711	2.1	4.01	8.07	5.015	4.033

Table 5 Participants' expected profits.

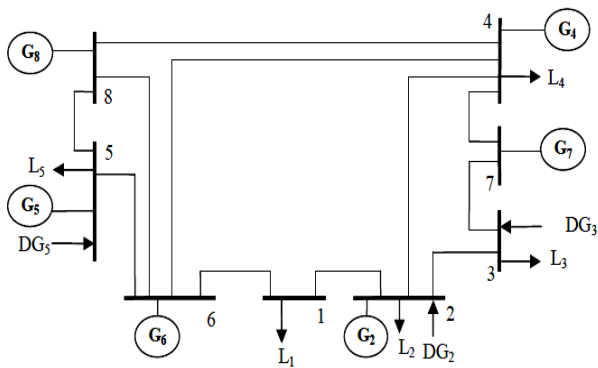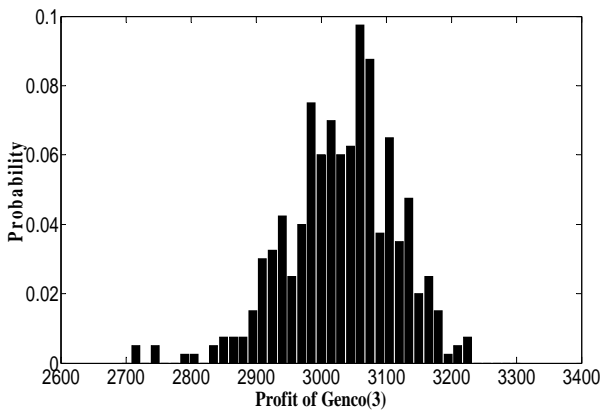|	DISCO 1	DISCO 2	DISCO 3	GENCO 1	GENCO 2	GENCO 3
Profit ($)	207.2	129.2	239.8	2212	2309	3031

Fig. 4 Sample system.

Fig. 5 Probability distribution of GENCO3's Profit.

The expected profit of participants with respect to these strategies is shown in Table 5, and also the average amount of LMPs, lines flow, interrupted loads and power generated by generators are shown in Tables (6)-(8) respectively.

Now if the amount of standard deviation of load is more or less than the value that is assumed in the problem, how it will affect the participant's profits. In other words, if participants don't pay attention to the uncertainty of load forecasting and estimate the load

with a crisp value, how the error could affect their profits?

In section 6 the relation between participants' profit and load was obtained, and it is said that with a fixed bid and uncertain load, as the standard deviation of load increases the expected profit of GENCOs increases and DISCOs' expected profit decreases. To illustrate this influence in this system, we increase the standard deviation of loads, and by considering that participants' strategies are according to Table 4, we obtain the expected profit of participants. The results are shown in Fig. 6.

As shown in Fig. 6, as the standard deviation of load is increased the expected profit of DISCO1 and DISCO2 are decreased, GENCO2's expected profit is increased, and the expected profit of DISCO3, GENCO1 and GENCO3 are approximately stayed constant. This trend in profits is inconsistent with what concluded in section 6. This difference originates from the fact that what derived in section 6 was in an ideal network, but in this network we have congested lines that makes the relation between standard deviation of load and expected profits to be dependent on participants' location in the network. In above network the line 11 has been congested, and as a consequence in most sampled loads in Mont-Carlo simulation, we have LMPs instead of MCP in the system. In the left side of this line, at the buses 5, 6 and 8, LMPs are low, and in the right side of this line, at the buses 1, 2, 3, 4 and 7, LMPs are high, and the reason is that supplying an additional megawatt of load at left nodes of the line decreases the congestion of line11 and the reverse occurs in the right side, and with considering this matter, the peculiar behavior in profits can be explained. DISCO1's loads are located at buses 1 and 2, thus with increase in standard deviation of load, there are some high amount of loads among sampled loads that makes LMPs in these two buses increase significantly, and hence DISCO1's expected profit decreases.

Table 6 Average amount of LMPs.

Bus 1 ($/MWh)	Bus 2 ($/MWh)	Bus 3 ($/MWh)	Bus 4 ($/MWh)	Bus 5 ($/MWh)	Bus 6 ($/MWh)	Bus 7 ($/MWh)	Bus 8 ($/MWh)
101.7	100.1	99.2	97.32	95.17	94.96	98.1	95.3

Table 7 Average amount of Lines flow.

Line 1 (MW)	Line 2 (MW)	Line 3 (MW)	Line 4 (MW)	Line 5 (MW)	Line 6 (MW)	Line 7 (MW)	Line 8 (MW)	Line 9 (MW)	Line 10 (MW)	Line 11 (MW)
-15.57	9.48	-1.12	-15.25	8.74	-7.36	-8.94	7.29	-0.86	2.585	14.1ᵛ

Table 8 Average amount of generation and interrupted loads.

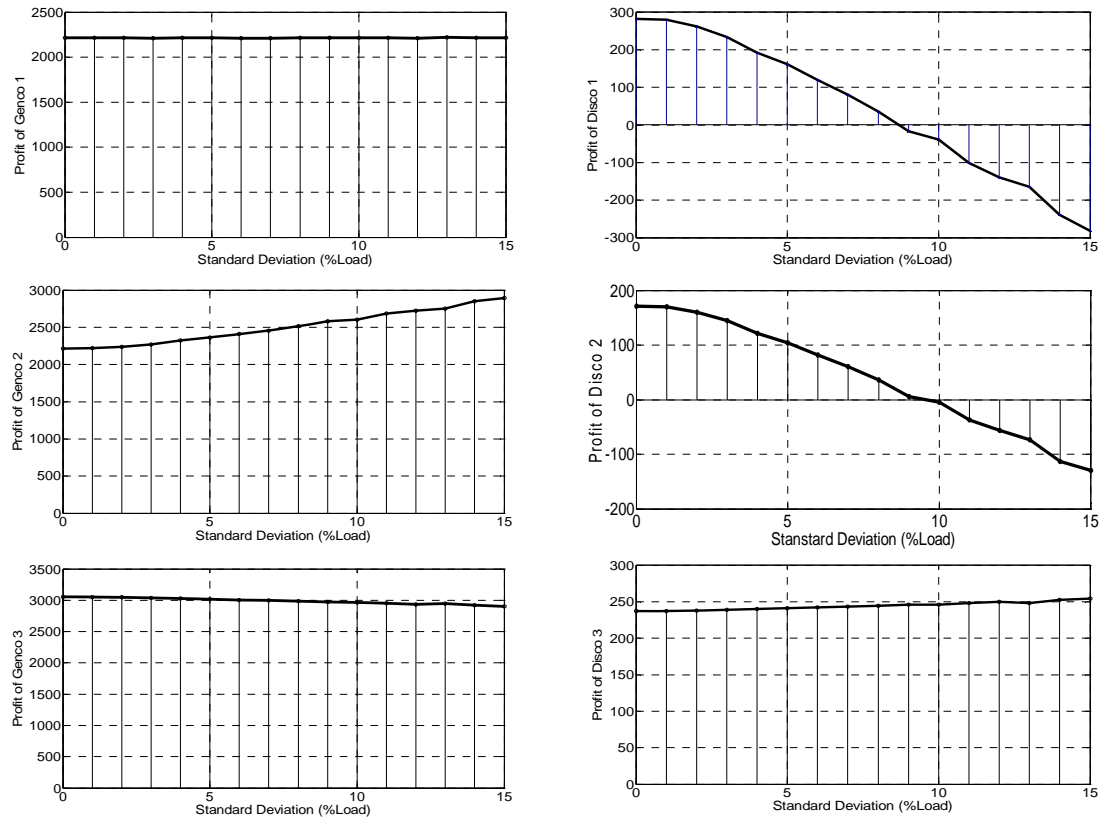| P_{G2} (MW)	P_{G4} (MW)	P_{G5} (MW)	P_{G6} (MW)	P_{G7} (MW)	P_{G8} (MW)	P_{IL1} (MW)	P_{IL2} (MW)	P_{IL3} (MW)	P_{IL4} (MW)	P_{IL5} (MW)
38.81	5.86	29.14	11.69	24	15.37	5.25	4.05	5.25	5.25	5.25

Fig. 6 GENCOs' and DISCOs' expected profit with respect to various amounts of standard deviation of load.

DISCO2's loads are located at nodes 3 and 4, and according to the same reasons its profit decreases with increasing standard deviation. DISCO3's load is located at node 3, and because supplying an additional megawatt of load in this node does not considerably influence flow of line11, DISCO3's expected profit is remained constant. Generators of GENCO1 are at nodes 6 and 7.

Therefore load uncertainty could be classified into two types, if there is no constraint in the system, section 6 demonstrated that how the profits of companies will change. In such systems since the effect of load uncertainty on expected profits is determinable, participants can have a good estimation of their expected profits without using any stochastic algorithm, and if they want to use section 7's algorithm to find their best strategy they can omit Mont-Carlo part in that algorithm. If there is line capacity constraint in the system, the relation between standard deviation of load and profits considerably depends on participants' location in the network. For instance, in the above example the expected profit of DISCO3 was not affected by Standard deviation of load, but DISCO1's expected profit was sensitive to the standard deviation of load because of its location in the network. Therefore, according to such bidding, DISCO3 could rely more on its expected profit, but DISCO1could not be confident enough and if he miscalculates the amount of load, its profit will change because of its location in the network, and finally the proposed algorithm for bidding is better to be used in such system.

9..Conclusion

Since in an imperfect competition each participant's action has influence on market clearing price, participants have to consider other participants' actions when maximizing their profits. In this paper a method for participating in such a competition in a pool market was proposed. In this method, because power transaction in a power market occurs frequently, strategy for finding a socially optimum point was presented and since load uncertainty can affect participants' expected profit we modeled this effect into their bidding strategy. Therefore the relation between participants' profit and load in an unconstrained network was derived and it was shown that when the standard deviation of random load increases, the expected profit of GENCOs and DISCOs increase and decrease respectively, and it was derived that the effect of load uncertainty in the expected profit of a risk neutral participant is determinable. It was also shown that, if there is line capacity constraint in the network, the relation between participants' expected profit and standard deviation of uncertain load has a different pattern, which differs with respect to participants' location in the network. Therefore, in such system participants have to pay more attention in the effect of load in their bidding strategy, and using the proposed algorithm is helpful in such situations.

Appendices

Appendix A

ISO clears market by establishing Eq. (29) and by using Lagrange multiplier according to following:

$$\frac{\partial c}{\partial P_i} = 0 \rightarrow x_i P_i + y_i - \lambda = 0 \rightarrow P_i = \frac{\lambda - y_i}{x_i} \qquad (A\text{-}1)$$

$$\frac{\partial c}{\partial \lambda} = 0 \rightarrow L - \sum_{i=1}^{n} P_i = 0 \rightarrow L = \sum_{i=1}^{n} P_i \qquad (A\text{-}2)$$

By putting Eq. (A-1) into Eq. (A-2) we will have:

$$L = \sum_{i=1}^{n} P_i \rightarrow L = \sum_{i=1}^{n} \frac{\lambda - y_i}{x_i}$$

$$\rightarrow L = \sum_{i=1}^{n} \frac{\lambda}{x_i} - \sum_{i=1}^{n} \frac{y_i}{x_i}$$

$$\rightarrow L = \lambda . \sum_{i=1}^{n} \frac{1}{x_i} - \sum_{i=1}^{n} \frac{y_i}{x_i} \qquad (A\text{-}3)$$

$$\rightarrow \lambda = \frac{L}{\sum_{i=1}^{n} \frac{1}{x_i}} + \frac{\sum_{i=1}^{n} \frac{y_i}{x_i}}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

Appendix B

Table B-1 GENCOs' parameters.

GENCO No.	Gen No.	P_{min}	P_{Max}	a_g ($/MW²h)	b_g ($/MWh)	c_g ($)
2	G2	0	40	0.08	45.62	17.64
2	G4	0	58	0.11	35.35	31.6
3	G5	0	40	0.09	22.47	49.75
1	G6	0	50	0.095	23.37	89.62
1	G7	0	24	0.085	33.47	24.05
3	G8	0	60	0.078	21.39	79.78

Table B-2 DISCOs' parameters.

DISCO No.	Load No.	Bus No.	Load (MW)	a_{IL} ($/MW²h)	b_{IL} ($/MWh)
1	L1	1	N(35,1.23)*	1	50.25
1	L2	2	N(27,0.95)	1	52.26
2	L3	3	N(35,1.23)	1	54.27
2	L4	4	N(35,1.23)	1	55.61
3	L5	5	N(35,1.23)	1	63.65
*N(35,1.23) = normal distribution with mean value 35 and standard deviation 1.23					

Table B-3 DGs' parameters.

DISCO No.	DG NO.	Bus No.	P_{min}	P_{Max}	a_{dg} ($/MW²h)	b_{dg} ($/MWh)
1	DG2	2	0	10	0.09	34
2	DG3	3	0	9	0.09	34
3	DG5	5	0	8	0.09	34

Table B-4 Lines' parameters.

Line No.	From Bus	To Bus	X(p.u)	Limits (MW)
1	1	2	0.011	30
2	2	3	0.018	30
3	2	4	0.03	20
4	3	7	0.022	40
5	7	4	0.015	30
6	4	6	0.03	30
7	4	8	0.03	40
8	8	6	0.0065	40
9	8	5	0.02	20
10	5	6	0.025	38
11	6	1	0.03	14.2

Appendix C

Nl	Set of all branches
Nl(l)	Set of branches in independent loop l
Ndco	Set of all DISCOs
Ngco	Set of all GENCOs
Sn(m)	Set of buses for DISCO m
Ng_n	Set of generators for GENCO n
Ndg	Set of all DGs
NIL	Set of all ILs
Nlp	Set of independent loops
n	Set of all buses
n(i)	Set of buses connected to bus i
$P_{d,i}^0$	Total demand of a DISCO
P_{ij}^{max}	Maximum flow limit on line ij
x_{ij}	Reactance of line ij
λ_c	DISCO's retail energy price
$k_{g,i}$	GENCO's strategic parameter
$P_{g,i}$	Generation of a generator
$P_{dg,i}$	Generation of a DG
$P_{IL,i}$	IL granted to a DISCO
P_{ij}	Power flow of line ij
Rd_i	Profit of DISCO i
Rg_i	Profit of GENCO i
$\mu(R_i)$	Profit's membership function
$C(P_{g,i})$	Cost function of a GENCO
$C(P_{dg,i})$	Cost function of a DG
$C(P_{IL,i})$	Cost function of DISCO's IL
$P_{g,i}^{min}, P_{g,i}^{max}$	Generator's lower and upper limits
$P_{dg,i}^{min}, P_{dg,i}^{max}$	DG's lower and upper limits

$P_{IL,i}^{min}, P_{IL,i}^{max}$ IL's lower and upper limits

$a_{dg,i}, b_{dg,i}$ Generation cost coefficients of a DG

$a_{IL,i}, b_{IL,i}$ IL cost coefficients of a DISCO

$a_{g,i}, b_{g,i}, c_{g,i}$ Cost coefficients of a generator

Acknowledgment

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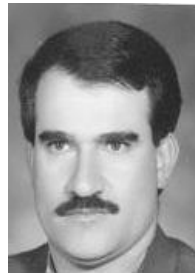
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