An Efficient Algorithm to Design Nearly Perfect-Reconstruction Two-Channel Quadrature Mirror Filter Banks

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Abstract: In this paper, a novel technique for the design of two-channel Quadrature Mirror Filter (QMF) banks with linear phase in frequency domain is presented. To satisfy the exact reconstruction condition of the filter bank, low-pass prototype filter response in pass-band, transition band and stop band is optimized using unconstrained indirect update optimization method. The objective function is formulated as a weighted sum of pass-band error and stop-band residual energy of low-pass prototype filter, and the square error of the distortion transfer function of the QMF bank at the quadrature frequency. The performance of the proposed algorithm is evaluated in terms of Peak Reconstruction Error (PRE), mean square error in pass-band and stop-band regions and stop-band edge attenuation. Design examples are included to illustrate the performance of the proposed algorithm and the quality of the filter banks that can be designed.

Keywords: Nearly Perfect-Reconstruction, Optimization, Phase Distortion, Two-Channel Filter Banks.

1 Introduction

Two-channel Quadrature Mirror Filter (QMF) bank is a multi-rate digital filter structure that consists of two decimators in the signal analysis section and two interpolators in the signal synthesis section. These filter banks were first used in sub-band coding, where the signal is split into two or more sub-bands in the frequency domain, so that each sub-band signal can be processed in an independent manner [1]. QMF banks find applications in various areas, such as automated methods for scoring tissue microarray spots [2], image coding [3], multicarrier modulation systems [4], two-dimensional short-time spectral analysis [5], antenna systems [6], advances in sampling theory [7], biomedical engineering [8], wideband beam forming for sonar [9] and to solve co-existence problem of wireless communication systems [10, 11]. Related previous work on the design of Nearly Perfect-Reconstruction (NPR) two-channel QMF banks [12−27] can be classified into a number of different approaches.

Least-squares [12-14] and Weighted Least-Squares (WLS) [15-17] methods had been applied to carry out the design task. In [13, 14], eigenvalue-eigenvector approach was proposed to find the optimum prototype filter coefficients in time domain. Chen and Lee [15] presented a WLS method based on linearization of the objective function to obtain the optimal filter tap-weights. This approach requires intensive matrix inversions, therefore, not suitable for real-time applications. Various iterative methods [18-23] and genetic algorithm based techniques [24-27] have been proposed for the design of two-channel QMF banks. Authors in [19] have developed an efficient technique by considering filter responses in pass-band, stop-band and also in transition band regions for the design of QMF bank. Recently, [24] presented an improved Particle Swarm Optimization (PSO) method for designing linear phase QMF banks and Ghosh *et al.* [25] proposed an approach based on adaptive-differential-evolution algorithm for the design of NPR two-channel QMF banks. Fig. 1 shows the analysis and synthesis sections of a two-channel QMF bank. The discrete input signal $x(n)$ is divided into two sub-band signals having equal band width, using the low-pass and high-pass analysis filters $H_1(z)$ and $H_2(z)$, respectively. The sub-band signals are decimated by a factor of two to achieve signal compression or to reduce processing complexity. The outputs of the synthesis filters are combined to obtain the reconstructed signal $\hat{x}(n)$. The reconstructed

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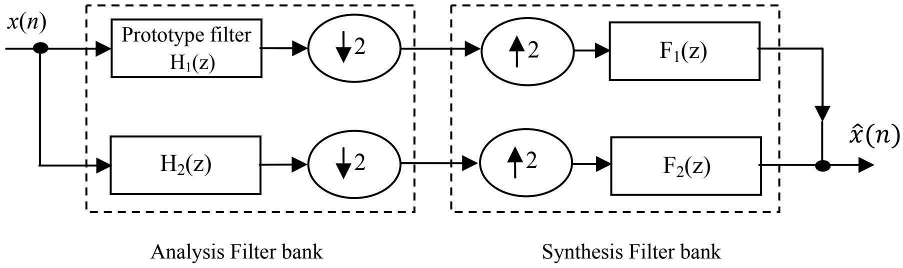

Fig. 1 The 2-channel quadrature mirror filter bank.

signal $\hat{x}(n)$ suffers from Aliasing Distortion (ALD), Phase Distortion (PHD), and Amplitude Distortion (AMD) due to the fact that the analysis and synthesis filters are not ideal [28]. Therefore, the main emphasis of most of the researchers while designing the prototype filter for two-channel QMF bank has been on the elimination or minimization of these three errors to obtain a Perfect Reconstruction (PR) or NPR system [12-17, 28].

By setting the synthesis filters cleverly in terms of the analysis filters, aliasing can be cancelled completely and PHD has been eliminated by using linear phase FIR filters [12-17]. The overall transfer function of such an ALD and PHD free system turns out to be a function of the filter tap weights of the low-pass analysis filter only that is known as low-pass prototype filter. In QMF banks, the high-pass and low-pass analysis filters are related to each other by the mirror-image symmetry condition: $H_2(z)=H_1(-z)$, around the quadrature frequency $\pi/2$. Due to this symmetry constraint, the AMD can only be minimized in this case by optimizing the filter tap weights of the low-pass prototype filter [28].

This paper proposes a novel algorithm to design the two-channel QMF bank without using any matrix inversion which generally affects the performance and effectiveness of some of optimization methods. The error measure to be minimized is formulated as a weighted sum of pass-band error and stop-band residual energy of low-pass prototype filter and the square error of the distortion transfer function of the QMF bank at the quadrature frequency $\omega=\pi/2$. Unconstrained variable metric method [29] is used to minimize the error measure by optimizing the filter tap weights of the low-pass prototype filter.

The paper is organized as follows. Section 2 briefly describes the principle of two-channel QMF bank and then formulation of the design problem in frequency domain. Section 3 presents proposed algorithm for minimization of the objective function. In section 4, we discuss the design results of the proposed QMF bank and comparison with different methods. Finally, some concluding remarks are drawn in section 5.

2 The Design Problem

The expression for the overall system function, or distortion transfer function of the alias free two-channel QMF bank can be written as [15-23]

$$T(z) = \tfrac{1}{2}[H_1^2(z) - H_1^2(-z)] \tag{1}$$

where, for alias cancellation, synthesis filters are defined as given below:

$$F_1(z) = H_2(-z) \text{ and } F_2(z) = -H_1(-z) \tag{2}$$

To obtain the perfect reconstruction QMF bank, the overall transfer function $T(z)$ must be a pure delay, i. e.,

$$T(z) = \tfrac{1}{2}[H_1^2(z) - H_1^2(-z)] = cz^{-n_0} \quad \text{or}$$

$$\hat{x}(n) = cx(n - n_0) \tag{3}$$

Equation (3) clearly shows that if the prototype filter $H_1(z)$ is selected to be a linear phase FIR, then $T(z)$ also become linear phase FIR and phase distortion is eliminated. To assure the linear phase FIR constraint, impulse response $h_1[n]$ of the low pass prototype filter $H_1(z)$ should be symmetric $h_1[n]=h_1[N-1-n]$, $0\leq n\leq N-1$, N is the filter length [1]. With this selection, the corresponding frequency response is given by [30].

$$H_1(e^{j\omega}) = A(\omega)e^{-j\omega(N-1)/2} \tag{4}$$

where $A(\omega) = \pm |H_1(e^{j\omega})|$ is the amplitude function. For real impulse response, the magnitude response $|H_1(e^{j\omega})|$ is an even function of ω, hence, by substituting Eq. (4) into Eq. (1), the overall frequency response of the two-channel QMF bank becomes

$$T(e^{j\omega}) = \frac{1}{2}(e^{-j\omega(N-1)})[|H_1(e^{j\omega})|^2 - (-1)^{(N-1)}|H_1(e^{j(\pi-\omega)})|^2] \tag{5}$$

For odd filter length, above equation gives $T(e^{j\omega})= 0$ at $\omega = \pi/2$, this entails severe amplitude distortion around quadrature frequency. Therefore, N must be chosen even to avoid this distortion and from Eq. (5), Exact Reconstruction (ER) condition can be written as

$$|T(\omega)| = |A(\omega)|^2 + |A(\pi-\omega)|^2 = c\,, \quad \text{for all } \omega. \tag{6}$$

In this case after eliminating ALD and PHD, we can only minimize amplitude distortion rather than completely eliminated due to mirror image symmetry constraint [30]. If the prototype filter $H_1(z)$ characteristics are assumed ideal in pass band and stop band regions then the exact reconstruction condition is automatically satisfied in the range of frequencies $0<\omega<\omega_p$ and $\omega_s<\omega<\pi$ [1], where, ω_p and ω_s are pass band and stop band edge frequencies, respectively. The main difficulty comes in transition band region ($\omega_p<\omega<\omega_s$), therefore the AMD must be controlled in this region. Hence, the aim is to optimize the coefficients of $H_1(z)$, such that the exact reconstruction condition nearly satisfied.

To design the low-pass analysis filter $H_1(z)$, we propose to minimize the following error function 'ϕ' based on the method presented in [19].

$$\phi = \alpha_1.\phi_p + \alpha_2.\phi_s + \alpha_3.\phi_t \tag{7}$$

where ϕ_p and ϕ_s are the measure of pass-band error and stop-band residual energy of filter $H_1(z)$, ϕ_t is the square error of $T(z)$ at $\pi/2$, in transition band, and α_1, α_2 and α_3 are real constants.

The ER condition of Eq. (6) at $\omega = \pi/2$, can be reduced to [19].

$$A\left(\frac{\pi}{2}\right) = (1/2)^{1/2} A(0) \tag{8}$$

where $A(0)$ and $A(\pi/2)$ are the amplitude responses of prototype filter at zero frequency and quadrature frequency, respectively. Consequently, the square error ϕ_t is given by:

$$\phi_t = [A\left(\frac{\pi}{2}\right) - (1/2)^{1/2} A(0)]^2 \tag{9}$$

ϕ_p and ϕ_s may be taken as:

$$\phi_p = \frac{1}{\pi}\int_0^{\omega_p} \left[A(0) - A(\omega)\right]^2 d\omega \tag{10}$$

$$\phi_s = \frac{1}{\pi}\int_{\omega_s}^{\pi} \left[A(\omega)\right]^2 d\omega \tag{11}$$

3 Computation of Optimum Filter Tap Weights

For real symmetric impulse response with even N, the corresponding frequency response of low-pass prototype filter $H_1(e^{j\omega})$ is given by [1].

$$H_1(e^{j\omega}) = \left[\sum_{n=0}^{(N/2-1)} 2h_1(n)\cos\omega\left(\frac{(N-1)}{2} - n\right)\right].e^{-j\omega(N-1)/2} \tag{12}$$

$$H_1(e^{j\omega}) = A(\omega)\, e^{-j\omega(N-1)/2} \tag{13}$$

where $A(\omega)$ is the amplitude function and given by:

$$A(\omega) = 2\mathbf{h}_1^{\mathbf{T}}\mathbf{c}(\omega) = \mathbf{b}^{\mathbf{T}}\mathbf{c}(\omega) \tag{14}$$

The vector $\mathbf{c}(\omega)$ and vector $\mathbf{b}^{\mathrm{T}} = 2\mathbf{h}_1^{\mathbf{T}}$ are given by:

$$\mathbf{c}(\omega) = [\cos\omega((N-1)/2)\ \cos\omega((N-1)/2) - 1) \ldots \cos(\omega/2)]^{\mathrm{T}}, \mathbf{b} = [b_0\, b_1 b_2 \ldots . b_{(N/2-1)}]^{\mathrm{T}} \tag{15}$$

At $\omega = 0$, the amplitude function is calculated as

$$A(0) = \mathbf{b}^{\mathbf{T}}\mathbf{c}(0) = \mathbf{b}^{\mathbf{T}}\mathbf{1}, \tag{16}$$

where **1** is the vector of all 1's.

3.1 Expressions for ϕ_t, ϕ_p and ϕ_s

By using Eqs. (9), (14) and (16), ϕ_t can be expressed as:

$$\phi_t = [\mathbf{b}^{\mathbf{T}}\mathbf{c}(\pi/2) - (1/2)^{1/2}\,\mathbf{b}^{\mathbf{T}}\mathbf{1}]^2 = [\mathbf{b}^{\mathbf{T}}\mathbf{q} - A_1]^2 \tag{17}$$

where vector $\mathbf{q}$ is equal to vector $\mathbf{c}(\omega)$, when it is evaluated at $\omega = \pi/2$ and $A_1 = 0.707\, A(0)$.

Similarly pass band error ϕ_p can be realized.

$$\phi_p = \mathbf{b}^{\mathbf{T}}\mathbf{W}\mathbf{b} \tag{18}$$

where $\mathbf{W}$ is a real, symmetric and positive definite matrix, given by

$$\mathbf{W} = \int_0^{\omega_p} [\mathbf{c}(0) - \mathbf{c}(\omega)][\mathbf{c}(0) - \mathbf{c}(\omega)]^{\mathrm{T}} \frac{d\omega}{\pi} \tag{19}$$

Stop band error ϕ_s is given as:

$$\phi_s = \mathbf{b}^{\mathbf{T}}\mathbf{Z}\mathbf{b} \tag{20}$$

where $\mathbf{Z}$ is a real, symmetric and positive definite matrix, calculated as:

$$\mathbf{Z} = \int_{\omega_s}^{\pi} \mathbf{c}(\omega)\mathbf{c}^{\mathrm{T}}(\omega)\frac{d\omega}{\pi} \tag{21}$$

3.2 Minimization of the Objective Function

The error function or objective function 'ϕ'can be realized by substituting Eqs. (17), (18) and (20) into Eq. (7).

$$\phi = \alpha_1\mathbf{b}^{\mathbf{T}}\mathbf{W}\mathbf{b} + \alpha_2\mathbf{b}^{\mathbf{T}}\mathbf{Z}\mathbf{b} + \alpha_3[\mathbf{b}^{\mathbf{T}}\mathbf{q} - A_1]^2$$

$$= \alpha_1\mathbf{b}^{\mathbf{T}}\mathbf{W}\mathbf{b} + \alpha_2\mathbf{b}^{\mathbf{T}}\mathbf{Z}\mathbf{b} + \alpha_3[\mathbf{b}^{\mathbf{T}}\mathbf{Y}\,\mathbf{b} - 2A_1\mathbf{b}^{\mathbf{T}}\mathbf{q} + A_1{}^2] \tag{22}$$

$$\phi = \mathbf{b}^{\mathbf{T}}\mathbf{U}\mathbf{b} + \alpha_3[-2A_1\mathbf{b}^{\mathbf{T}}\mathbf{q} + A_1{}^2] \tag{23}$$

where matrix **U** and **Y** are given by**:**

$$\mathbf{U} = \alpha_1\mathbf{W} + \alpha_2\mathbf{Z} + \alpha_3\mathbf{Y} \quad \text{and} \quad \mathbf{Y} = \mathbf{q}\mathbf{q}^{\mathbf{T}} \tag{24}$$

The error function given by Eq. (22) is a quadratic function and matrix $\mathbf{U}$ is a symmetric and positive definite matrix, therefore, ϕ can be minimized by unconstrained variable metric method [29]. In this method the inverse of Hessian matrix is approximated using Broyden-Fletcher-Goldfarb-Shanno formula. This method has self-correcting properties and exhibits super-linear convergences near the optimal point therefore, it is suitable for QMF design problem. If $\mathbf{b_i}$ is the approximation of the minimum point at the ith stage of iteration and λ_i is the optimal step length in the search direction, then the improved approximation can be calculated as:

$$\mathbf{b}_{i+1} = \mathbf{b}_i - \lambda_i[\mathbf{H}_i]\nabla\phi_i = \mathbf{b}_i + \lambda_i\mathbf{s}_i \tag{25}$$

where $\nabla\phi_i$ is the gradient of the objective function ϕ and $\mathbf{s}_i$ is the search direction, when evaluated at the design vector $\mathbf{b}_i$, both are given by:

$$\nabla\phi_i = 2\mathbf{Ub} + \alpha_3[-2A_1\mathbf{q}] \tag{26}$$

and

$$\mathbf{s}_i = -[\mathbf{H}_i]\nabla\phi_i\ , \tag{27}$$

and matrix $[\mathbf{H}_i]$ is the estimate of inverse of Hessian matrix. Initially, the matrix $[\mathbf{H}_i]$ is taken as the identity matrix $[\mathbf{I}]$ and up-dation of this matrix is done using Broyden-Fletcher-Goldfarb-Shanno formula [29].

$$[\mathbf{H}_{i+1}] = [\mathbf{H}_i] + \left[1 + \frac{\mathbf{g}_i^T[\mathbf{H}_i]\mathbf{g}_i}{\mathbf{d}_i^T\mathbf{g}_i}\right]\frac{\mathbf{d}_i\mathbf{d}_i^T}{\mathbf{d}_i^T\mathbf{g}_i} - \frac{\mathbf{d}_i\mathbf{g}_i^T[\mathbf{H}_i]}{\mathbf{d}_i^T\mathbf{g}_i} - \frac{[\mathbf{H}_i]\mathbf{g}_i\mathbf{d}_i^T}{\mathbf{d}_i^T\mathbf{g}_i} \tag{28}$$

where

$$\mathbf{g}_i = \nabla\phi_{i+1} - \nabla\phi_i \tag{29}$$

and

$$\mathbf{d}_i = \mathbf{b}_{i+1} - \mathbf{b}_i = -\lambda_i[\mathbf{H}_i]\,\nabla\phi_i \tag{30}$$

In the direction of $\mathbf{s}_i$, the optimum step length λ_i can be obtained by equating the derivative of error function $(\mathbf{b}_i + \lambda_i\mathbf{s}_i)$ with respect to λ, to zero. The derivative $d\phi/d\lambda = 0$, gives following term:

$$\lambda_i = \{\mathbf{b}_i^T\mathbf{U}\mathbf{s}_i - \alpha_3 A_1\mathbf{q}^T\mathbf{s}_i\}/\{\mathbf{s}_i^T\mathbf{U}\mathbf{s}_i\} \tag{31}$$

Initial values of filter coefficients $h_1(n)$, are chosen as the same which was taken in [14], [23] to satisfy unit energy constraint on the filter coefficients. The step-by-step description of the design algorithm that minimizes the error function is as shown in Table 1.

4 Case Study

A MATLAB program has been written which implements the design procedure for prototype low-pass filter described in the previous section and tested on a desktop computer equipped with an Intel Dual core CPU @ 2.10 GHz with 1 GB RAM. This section presents two design examples to examine the effectiveness of the proposed algorithm. The performance of the algorithm is evaluated in terms of five significant quantities:

(i) Mean square error in the pass band (ϕ_p),

(ii) Stop band error (ϕ_s),

(iii) Stop-band first lobe attenuation (A_L),

(iv) Stop-band edge attenuation (A_s)= $-20\log_{10}(H_1(\omega_s))$

(v) Measure of reconstruction error (e) in dB = $\max_\omega|10\log|T(e^{j\omega})|| - \min_\omega|10\log|T(e^{j\omega})||$.

In both the examples stop-band first lobe attenuation (A_L) has been obtained from respective zoomed attenuation curves. It has been observed that the algorithm works regardless of the values of the constants α_1, α_2 and α_3. However, we have used trial and error method to select the values of these constants, in order to obtain the best possible solution.

Example 1: For $N = 42$, $\omega_s = 0.6\pi$, $\omega_p = 0.4\pi$, $\alpha_1 = 0.9$, $\alpha_2 = 0.15$ and $\alpha_3 = 1$, the filter coefficients obtained (for $0 \le n \le N/2-1$) are listed in Table 2.

All frequencies $[0, \pi]$ are normalized to [0, 1]. The corresponding normalized magnitude plots of analysis filters $H_1(z)$, $H_2(z)$ and distortion function $T(z)$ are shown in Fig. 2(a). The reconstruction error (in dB) of QMF bank is plotted in Fig. 2(b). The significant quantities obtained are PRE (e) in dB = 0.0088, (E_p)= 3.622×10^{-9}, (E_s)= 3.70×10^{-7}, (A_s) = 44.69 dB and (A_L) = 54.63 dB.

Table 1 Algorithm for designing the prototype filter.

	Steps of Design Algorithm
1:	Select filter length (N), ω_s and ω_p.
2:	Start with an initial design vector $\mathbf{h_1} = [h(0)\ h(1)\ h(2) \ldots\ldots h((N/2)-1)]$, $\mathbf{h_1}$ is zero except $h((N/2)-1) = \frac{1}{\sqrt{2}}$, and assume initial values of α_1, α_2 and α_3.
3:	Compute $\mathbf{b}_i = 2\mathbf{h_1}$ and set the iteration number, $i = 1$.
4:	Find the objective function ϕ_i, by using Eq. (22), at the design vector $\mathbf{b}_i$.
5:	Compute $\nabla\phi_i$ and the search direction $\mathbf{s}_i$, by using Eqs. (26) and (27), at $\mathbf{b}_i$. Determine the optimum step length λ_i, by using Eq. (31).
6:	Compute the new approximation $$\mathbf{b}_{i+1} = \mathbf{b}_i - \lambda_i[\mathbf{H}_i]\nabla\phi_i = \mathbf{b}_i + \lambda_i\mathbf{s}_i$$
7:	Find ϕ_{i+1} and $\nabla\phi_{i+1}$ at the design vector $\mathbf{b}_{i+1}$. Also compute matrix $[\mathbf{H}_{i+1}]$ using Eq. (28). If $\phi_{i+1} \ge \phi_i$, choose the optimum point as $\mathbf{b}_i$, stop the procedure and go to step (9). If $\phi_{i+1} < \phi_i$, set $\phi_i = \phi_{i+1}$, and $\mathbf{b}_i = \mathbf{b}_{i+1}$.
8:	Set the new iteration number as $i = i + 1$, and go to step (4).
9:	Compute $\mathbf{h_1} = (1/2)\mathbf{b}_i$, as the optimum solution and stop the procedure.

Table 2 Optimized filter tap weights in example 1.

Filter Taps (N = 42)	
$h_1(0)$= 0.00003002780	$h_1(1)$= 0.00056684555
$h_1(2)$ = -0.00088975255	$h_1(3)$ = -0.00106662003
$h_1(4)$ = 0.00263112814	$h_1(5)$= 0.00143944439
$h_1(6)$ = -0.00576110160	$h_1(7)$ = -0.00129397138
$h_1(8)$ = 0.01075131240	$h_1(9)$ = -0.00003089173
$h_1(10)$= -0.01818470103	$h_1(11)$ =0.00349951307
$h_1(12)$ = 0.02886533479	$h_1(13)$ = -0.01076105979
$h_1(14)$ = -0.04451022334	$h_1(15)$ = 0.02518183956
$h_1(16)$= 0.07003271098	$h_1(17)$ = -0.05705282394
$h_1(18)$ = -0.12723852401	$h_1(19)$ = 0.1702847829
$h_1(20)$ = 0.59424041885	

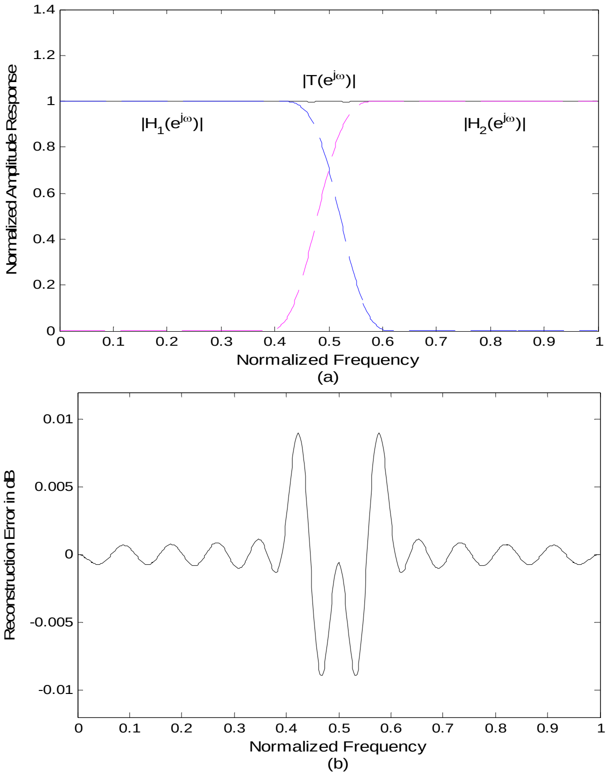

Fig. 2 (a) Magnitude response of analysis filters and overall filter bank for $N = 42$, (b) Reconstruction error in dB.

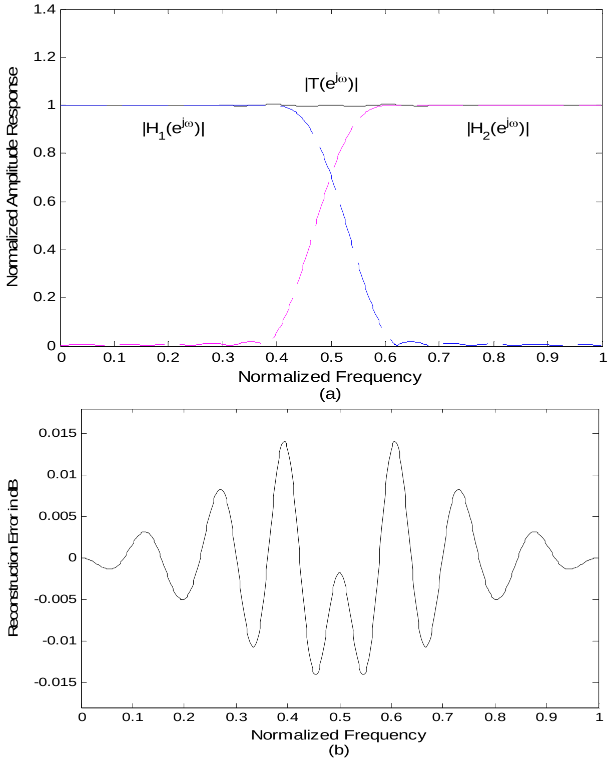

Fig. 3 (a) Magnitude response of analysis filters and overall filter bank for $N = 24$, (b) Reconstruction error in dB.

Example* 2:* For filter length $N = 24$, $\omega_s= 0.6\pi$, $\omega_p= 0.4\pi$, $\alpha_1 =0.7$, $\alpha_2 =0.1$ and $\alpha_3 = 1$, the filter coefficients obtained (for $0 \le n \le N/2-1$) are listed in Table 3.

Normalized magnitude plots of analysis filters $H_1(z)$, $H_2(z)$ and distortion function $T(z)$ are shown in Fig. 3(a). The reconstruction error (in dB) of QMF bank is plotted in Fig. 3(b). The significant quantities obtained are PRE (e) in dB = 0.0139, $(E_p)= 1.162\times10^{-8}$, $(E_s)= 7.48\times10^{-5}$, $(A_s) = 25.06$ dB and $(A_L) = 34.85$ dB.

Table 3 Optimized filter tap weights in example 2.

Filter Taps (N = 24)	
$h_1(0)$= 0.00447423300	$h_1(1)$ = -0.00914936838
$h_1(2)$ = -0.00294793823	$h_1(3)$ = 0.02070572773
$h_1(4)$ = -0.00236718296	$h_1(5)$ = -0.03861209528
$h_1(6)$= 0.01581765598	$h_1(7)$= 0.06758584810
$h_1(8)$= -0.04819461467	$h_1(9)$= -0.12906666302
$h_1(10)$= 0.16402285083	$h_1(11)$= 0.60339846987

4.1 Discussion of Results

Table 4 presents a comparison of the proposed method results with other existing methods [19, 20, 21, 25, 26] results (with similar design specifications of two-channel QMF bank i. e., $\omega_s= 0.6\pi$ and $\omega_p= 0.4\pi$) for $N = 24$. The proposed method gives improved performance than all the other methods in terms of PRE and (ϕ_p).

The percentage reduction in peak reconstruction error with respect to other existing methods [25, 19, 20, 26, 21] is 53.66%, 44.62%, 31.81%, 26.84%, and 18.71%, respectively, calculated using Eq. (32). Similarly the percentage reduction in mean square error in pass-band (ϕ_p) is 98.73%, 87.43%, 92.79%, 93.71%, and 85.24%, respectively, compared to all considered algorithms.

$$Percentage\ Reduction\ in\ PRE = \frac{PRE_{other\ method} - PRE_{proposed\ method}}{PRE_{other\ method}} \times 100\ \% \qquad (32)$$

The proposed method also gives improved performance than the methods of [19], [25] and [26] in terms of better stop- band error (ϕ_s) and stop-band edge attenuation (A_s). However algorithm presented in [21] obtains best results for ϕ_s and A_s. Table 5 shows that the proposed method is better than considered existing method in terms of computational complexity because, it does not involve any matrix inversion and determination of eigenvectors in each iteration. The proposed method requires less computation time to design the low-pass prototype filter in comparison of other existing methods. Consequently, the overall performance of the proposed method is better than other existing methods and filter bank designed by this method can be used for real time applications.

Table 4 Comparison of the proposed algorithm with other existing optimization algorithms based on the significant quantities for filter length $N = 24$.

Methods	ϕ_s	ϕ_p	PRE (dB)	A_s (dB)	Phase Response
Upender *et al*. [26]	7.99×10^{-5}	1.845×10^{-7}	0.019	22.78	Linear
Kumar *et al*. [21]	1.74×10^{-5}	7.86×10^{-8}	0.0171	28.31	Linear
Algorithm in [20]	7.49×10^{-5}	1.61×10^{-7}	0.0202	26.15	Linear
Sahu *et al*. [19]	8.49×10^{-5}	9.23×10^{-8}	0.0251	23.03	Linear
Ghosh *et al*. [25] (MJADE_pBX algorithm)	1.30×10^{-4}	9.14×10^{-7}	0.030	20.11	Linear
Ghosh *et al*. [25] (JADE algorithm)	3.19×10^{-4}	4.10×10^{-6}	0.050	20.25	Linear
Proposed method	7.48×10^{-5}	1.16×10^{-8}	0.0139	25.06	Linear

Table 5 Comparison of the proposed algorithm with some other existing optimization algorithms based on computational complexities.

Technique	Any matrix inversion in each iteration	Selection of initial $h_1(n)$	Evaluation of eigenvectors in each iteration	CPU time (sec.)
Proposed method	No	Assumed	No	0.094
Kumar *et al*. [21]	Yes	Assumed	No	0.11
Upender *et al*. [26]	No	Assumed	No	4.48
Sahu *et al*. [19]	Yes	Assumed	No	0.66
Chen & Lee [15]	Yes	To be determined	No	----
Ghosh *et al*. [25]	No	Assumed	No	----
Jain & Crochiere [14]	Yes	Assumed	Yes	1.68

5 Conclusion

This paper has presented an efficient algorithm for designing two-channel QMF banks with linear phase. The QMF design problem is basically a nonlinear and multidimensional optimization problem. The proposed algorithm exhibits superlinear convergence near the optimal point therefore, it is suitable for QMF design problem. Error function for the design problem is minimized by optimizing the prototype filter coefficients. The proposed algorithm has been tested on two design examples by writing a MATLAB program. The Design results clearly show that this technique gives improved performance in terms of peak reconstruction error as compared with several state-of-art existing techniques and it is also efficient for QMF banks with larger filter taps.

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