Cooperative Orthogonal Space-Time-Frequency Block Codes over a MIMO-OFDM Frequency Selective Channel

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Abstract: In this paper, a cooperative algorithm to improve the orthogonal space-time-frequency block codes (OSTFBC) in frequency selective channels for $2 \times 1, 2 \times 2, 4 \times 1, 4 \times 2$ MIMO-OFDM systems, is presented. The algorithm of three node, a source node, a relay node and a destination node is formed, and is implemented in two stages. During the first stage, the destination and the relay antennas receive the symbols sent by the source antennas. The destination node and the relay node obtain the decision variables employing time-space-frequency decoding process by the received signals. During the second stage, the relay node transmits decision variables to the destination node. Due to the increasing diversity in the proposed algorithm, decision variables in the destination node are increased to improve system performance. The bit error rate of the proposed algorithm at high SNR is estimated by considering the BPSK modulation. The simulation results show that cooperative orthogonal space-time-frequency block coding, improves system performance and reduces the BER in a frequency selective channel.

Keywords: BER, Cooperative, Frequency-Selective Channel, MIMO-OFDM, Multipath Diversity, OSTFBC, Relay, Spatial Diversity.

1 Introduction

Multi-antenna systems that are called MIMO (multiple-input multiple-output) technology, can eliminate the destructive effects of fading channel and increases system capacity by producing multiple independent channels between the transmitter and receiver. OFDM (orthogonal frequency-division multiplexing) is proposed for high data rate communications, because of its ability to soothe the frequency selective fading channel by splitting the bandwidth channel into several narrow band flat channels. Combining the technologies of MIMO and OFDM together with coding in time domains can lead to high data rate telecommunication with proper error performance and appropriate complexity.

In a MIMO-OFDM system, in addition to spatial diversity, there is another degree of diversity known as multipath or frequency diversity. It is also accomplished that the diversity order of a MIMO-OFDM system with quasi-static channel, is equal to the product of the number of transmit antennas (M_T), the number of receive antennas (M_R) and the number of independent taps (L) for a frequency-selective channel. This order of diversity can be achieved by coding through space, time and frequency domains [1-5].

Space-time-frequency coding is a transmit diversity technique that deploys orthogonal frequency-division multiplexing considering MIMO technology. In [4], a space-time-frequency coding scheme over MIMO-OFDM sub-channels is presented and two code design methods that can guarantee to achieve the maximum diversity order are proposed. The first design method is a repetition coding approach using full diversity space frequency codes, and the second design method is a block coding approach that can guarantee both full symbol rate and full diversity. In [5], the authors produce a relationship between frequency tones and antennas which proposes a grouping technique to reduce the complexity of the code for the MIMO-OFDM systems.

Recently, lot of works have been dedicated to analyzing the performance of Alamouti space-time coding [6-8]. In [9], a non-regenerative dual-hope wireless system based on a distributed Alamouti space time coding is presented and to minimize the possibility of error are derived as scaling function for each relay. A cooperative diversity technique in a MIMO-OFDMA system was considered in [10], in which the base station can detect the transmit signal for each user since the transmission code of cooperative diversity is based on a

Iranian Journal of Electrical & Electronic Engineering, 2016.
Paper received 25 February 2015 and accepted 23 January 2016.
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space time coding. In [11], a cooperative 2×2 Alamouti algorithm in frequency flat channels is presented. In [12], a simple OST transmission scheme is proposed for asynchronous cooperative systems. In [13], a new distributed STFC is presented that employs subcarrier grouping and linear constellation precoding as well as distributed linear dispersion coding. Unlike the existing works, in this paper, a cooperative-OSTFBC algorithm for $2\times 1, 2\times 2, 4\times 1, 4\times 2$, MIMO-OFDM systems are investigated for frequency-selective channels. The achievable diversity order is obtained and the BER are approximated for all four cases.

The rest of the paper is organized as follows. In Section 2, the desired system model is described. In Section 3, a cooperative-OSTFBC algorithm for 2×1 MIMO-OFDM system is obtained. Analytical BER and SER results for orthogonal STFBC in a 2×2 MIMO-OFDM system are attained in Section 4 with achievable diversity order. In section 5 simulation results to obtained BER for 2×1, 2×2, 4×1 and 4×2 systems are presented and compared with the case without the relay node. Finally, conclusions are given in Section 6.

2 System and Channel Model

We assume that a broadband wireless relay network employs MIMO-OFDM system with $M_T\times M_R$ antennas considering N subcarriers to compose of 3 terminals, including a source node S, a relay node R, a destination node D, and every node in the network is subject to a half-duplex constraint, i.e., a node cannot simultaneously transmit and receive as shown in Fig. 1. Both the source node and the relay node are equipped with STFB coding. The channels of S→ R or R→D or S→D are assumed to be the quasi-static frequency-selective Rayleigh fading channels, so that $h_{SR}=[h_{SR}(0),\dots,h_{SR}(L-1)]^T$, $h_{RD}=[h_{RD}(0),\dots,h_{RD}(L-1)]^T$ and $h_{SD}=[h_{SD}(0),\dots,h_{SD}(L-1)]^T$ to denote the channel impulse responses between S → R, R→D and S →D, respectively. They are assumed to be independent zero-mean complex Gaussian vectors, where L denote the number of channel taps. It is also assume that all paths in every channel are independent with each other, also the channels between any two nodes are uncorrelated. The channel impulse response of S→ R between the i[th] transmit and the j[th] receive antennas is given by:

$$h_{i,j,SR}(t,\tau)=\sum_{l=0}^{L-1}h^t_{i,j,SR}(l)\delta(\tau-\tau_l) \qquad (1)$$

where τ_l is the time delay of the l[th] path, $h^t_{i,j,SR}(l)$ denotes the l[th] path gain from i[th] transmit and the j[th] receive antenna that is a zero mean Gaussian random variable with variance σ_l^2. For normalization, assume that $\sum_{l=0}^{L-1}\sigma_l^2=1$. It is also assumed that all channels between transmitter and receiver antennas have the same Power-Delay Profile (PDP). The channel gain of S→ R between the i[th] transmit and the j[th] receive antennas for the n^{th} subcarrier remains constant during the transmission of one OFDM symbol, and can be written as:

$$H^t_{i,j,SR}(n)=\sum_{l=0}^{L-1}h^t_{i,j,SR}(l)e^{-j2\pi n\Delta f\tau_l} \qquad (2)$$

where $j=\sqrt{-1}$, n represents the nth subcarrier, n =0, 1, 2, …, N-1, $\Delta f=\frac{1}{T_s}$, T_s is the duration of one OFDM symbol and $H^t_{i,j,SR}(n)$ denotes the fading coefficient in the frequency domain that is a zero mean Gaussian random variable with variance 1.

The presented algorithm is implemented in two stages. During the first stage, the destination and the relay antennas receive the symbols sent by the source antennas as shown in Fig. 2. The destination node and the relay node obtain the decision variables employing time-space-frequency decoding process by the received signals. During the second stage, the relay node transmits decision variables to the destination node as shown in Fig. 3. In this paper, it is shown that the number of decision variables in the destination node increases and improves system performance.

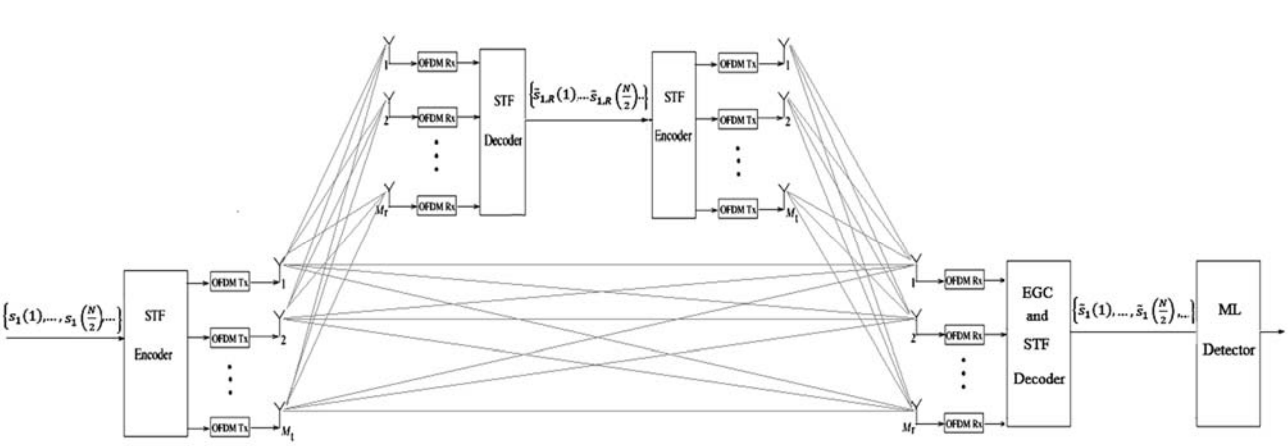

Fig. 1 The proposed cooperative algorithm equivalent model.

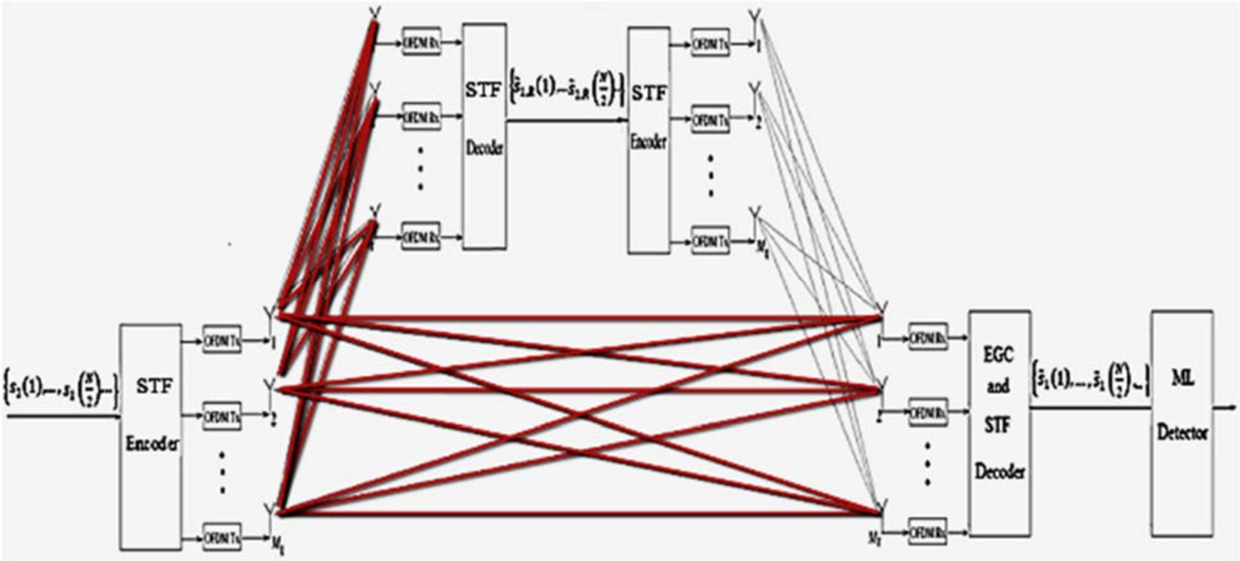

Fig. 2 The first stage.

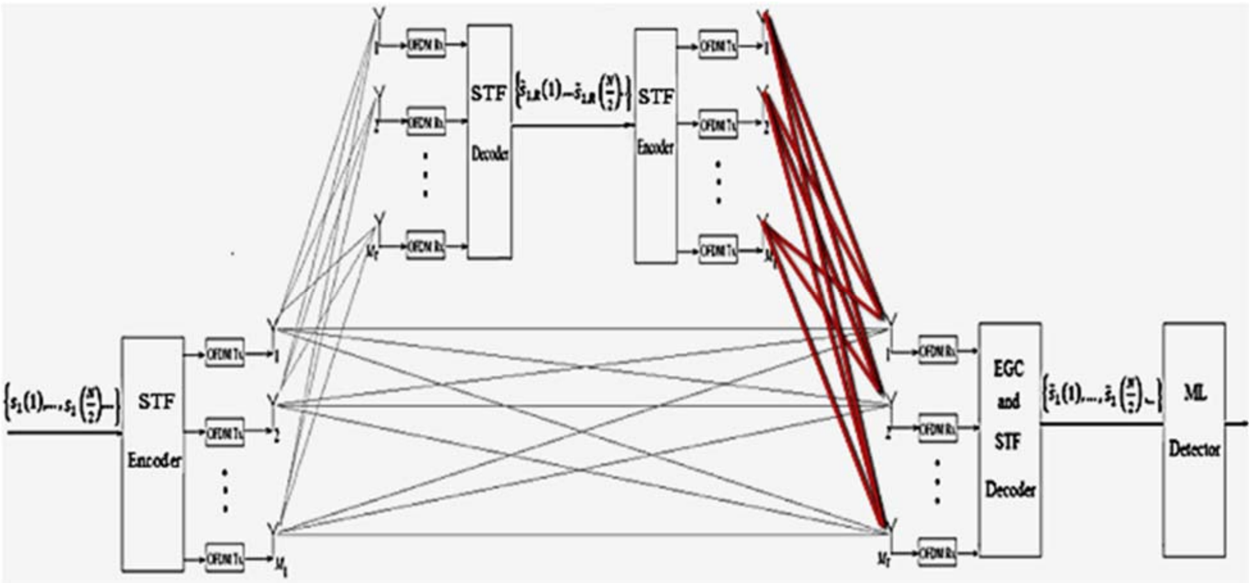

Fig. 3 The second stage.

3 The Proposed Cooperative Algorithm

To achieve the full diversity order of a MIMO-OFDM-2×1 system, which is G_d= 2L, each signal must be sent 2L with repetition space-time-frequency coding scheme [14] by L OFDM subchannels and during the two time periods. At the receiver, the received signal vector at the relay node:

$$Y_{SR} = SH_{SR} + \mathcal{N}_{SR} \tag{3}$$

where S is transmitted signal matrix of size $2N \times 2N$ that is constructed from the STF codeword, H_{SR} is S→ R channel vector of size $2N \times 1$ and $\mathcal{N}_{SR}$ is noise vector of size $2N \times 1$.

In this algorithm, the OSTFBC presented in [14] is used in S→ R, R→D and S→D. The simplest OSTBC (Alamouti code) [15] for two transmit antennas of block size $M_b = 2$ for the OSTFBC proposed in [14], and repeated given below as [15]:

$$S_k = I_{\Gamma\times\Gamma} \otimes \begin{pmatrix} S_1(k) & S_2(k) \\ -S_2^*(k) & S_1^*(k) \end{pmatrix} \tag{4}$$

where $S_{i,k}$ are i.i.d information symbols, k is the index number of independent STBC blocks, and k = 1, 2, …, $\frac{N}{\Gamma}$. Also, $\Gamma = 2^{\lceil \log_2 L \rceil}$ is the number of the times a symbol is repeated. For an N-subcarrier STFBC block, $\frac{N}{\Gamma}$ STBC blocks are assigned to N subcarriers with each of them repeated by Γ times. So a whole encoded STF block code is represented by [16]:

$$S = \mathrm{diag}\left(S_1, S_2, \dots, S_{\frac{N}{\Gamma}}\right)_{M_bN \times M_TN} \quad (5)$$

This repetition scheme across subcarriers is to achieve the full multipath diversity as shown in [17]. Compared with the Alamouti code, the OSTFBC achieves more diversity gain from multi paths at the cost of the lower symbol rate. It is shown in [14] that the above OSTFBC can achieve the full-diversity order $M_T M_R L$.

Assuming $L = 2$, signals received by the receiver via the N subcarrier in the relay node will be as follows:

$$\begin{bmatrix} Y_R^1(2n) \\ Y_R^2(2n) \\ Y_R^1(2n+1) \\ Y_R^2(2n+1) \end{bmatrix} = \begin{bmatrix} s_1(k) & s_2(k) & 0 & 0 \\ -s_2^*(k) & s_1^*(k) & 0 & 0 \\ 0 & 0 & s_1(k) & s_2(k) \\ 0 & 0 & -s_2^*(k) & s_1^*(k) \end{bmatrix} \begin{bmatrix} H_{1,1,SR}(2n) \\ H_{2,1,SR}(2n) \\ H_{1,1,SR}(2n+1) \\ H_{2,1,SR}(2n+1) \end{bmatrix} + \begin{bmatrix} N_{SR}^1(2n) \\ N_{SR}^2(2n) \\ N_{SR}^1(2n+1) \\ N_{SR}^2(2n+1) \end{bmatrix} \quad (6)$$

where $k = 1, \dots, \frac{N}{2}$, $n = k-1$, $Y_R^t(\mathcal{n})$ is received signal at the relay node by the $\mathcal{n}^{th}$ subcarrier and t^{th} OFDM block, $t = 1, 2$, $H_{i,j,SR}(\mathcal{n})$ is the channel gain of S→ R between the i^{th} transmit and the j^{th} receive antennas and $N_{SR}^t(\mathcal{n})$ is the additive white Gaussian noise (AWGN) at $\mathcal{n}^{th}$ subcarrier and variance $\sigma_{\mathcal{n}}^2$ is assumed $\sigma_{\mathcal{n}}^2 = 1$. As can be observed, the signals received by each pair of consecutive subcarrier during 2 OFDM block (T=2), will be similar to the MIMO-2×2 system with Alamouti STBC code. So, space time frequency decoding procedure is employed to achieve the following decision variables:

$$\begin{aligned} \tilde{s}_{1,R}(k) = {} & H_{1,1,SR}^*(2n)Y_R^1(2n) + H_{2,1,SR}(2n)Y_R^2(2n)^* \\ & + H_{1,1,SR}^*(2n+1)Y_R^1(2n+1) \\ & + H_{2,1,SR}(2n+1)Y_R^2(2n+1)^* \\ \tilde{s}_{2,R}(k) = {} & H_{2,1,SR}^*(2n)Y_R^1(2n) - H_{1,1,SR}(2n)Y_R^2(2n)^* \\ & + H_{2,1,SR}^*(2n+1)Y_R^1(2n+1) \\ & - H_{1,1,SR}(2n+1)Y_R^2(2n+1)^* \end{aligned} \quad (7)$$

Substituting Eq. (6) into Eq. (7) the following relationship are derived:

$$\begin{aligned} \tilde{s}_{1,R}(k) = {} & s_1(k)\left(\beta_{SR}(k)\right) + H_{1,1,SR}^*(2n)N_{SR}^1(2n) \\ & + H_{2,1,SR}(2n)N_{SR}^2(2n)^* \\ & + H_{1,1,SR}^*(2n+1)N_{SR}^1(2n+1) \\ & + H_{2,1,SR}(2n+1)N_{SR}^2(2n+1)^* \\ \tilde{s}_{2,R}(k) = {} & s_2(k)\left(\beta_{SR}(k)\right) + H_{2,1,SR}^*(2n)N_{SR}^1(2n) \\ & - H_{1,1,SR}(2n)N_{SR}^2(2n)^* \\ & + H_{2,1,SR}^*(2n+1)N_{SR}^1(2n+1) \\ & - H_{1,1,SR}(2n+1)N_{SR}^2(2n+1)^* \end{aligned} \quad (8)$$

$\beta_{SR} = \left\{\beta_{SR}(1) \dots \beta_{SR}(\frac{N}{2})\right\}$ is defined as follows:

$$\begin{aligned} \beta_{SR}(k) = {} & \left|H_{1,1,SR}(2n)\right|^2 + \left|H_{2,1,SR}(2n)\right|^2 \\ & + \left|H_{1,1,SR}(2n+1)\right|^2 \\ & + \left|H_{2,1,SR}(2n+1)\right|^2 \end{aligned} \quad (9)$$

So,

$$\begin{aligned} \mathcal{N}_{1,SR}(k) = {} & H_{1,1,SR}^*(2n)N_{SR}^1(2n) \\ & + H_{2,1,SR}(2n)N_{SR}^2(2n)^* \\ & + H_{1,1,SR}^*(2n+1)N_{SR}^1(2n+1) \\ & + H_{2,1,SR}(2n+1)N_{SR}^2(2n+1)^* \\ \mathcal{N}_{2,SR}(k) = {} & H_{2,1,SR}^*(2n)N_{SR}^1(2n) \\ & - H_{1,1,SR}(2n)N_{SR}^2(2n)^* \\ & + H_{2,1,SR}^*(2n+1)N_{SR}^1(2n+1) \\ & - H_{1,1,SR}(2n+1)N_{SR}^2(2n+1)^* \end{aligned} \quad (10)$$

For simplicity of decision variable in the relay node (8) can be written as follows:

$$\begin{aligned} \tilde{s}_{1.R}(k) &= s_1(k)\beta_{SR}(k) + \mathcal{N}_{1,SR}(k) \\ \tilde{s}_{2.R}(k) &= s_2(k)\beta_{SR}(k) + \mathcal{N}_{2,SR}(k) \end{aligned} \quad (11)$$

Similarly, during the first stage, the decision variables $\left\{\tilde{s}_{1.D}(1), \tilde{s}_{2,D}(1), \dots, \tilde{s}_{1,D}\left(\frac{N}{2}\right), \tilde{s}_{2,D}\left(\frac{N}{2}\right)\right\}$ in the destination node as a result of S →D, the following will be achieved:

$$\begin{aligned} \tilde{s}_{1,D}^1(k) &= s_1(k)\beta_{SD}(k) + \mathcal{N}_{1,SD}(k) \\ \tilde{s}_{2,D}^1(k) &= s_2(k)\beta_{SD}(k) + \mathcal{N}_{2,SD}(k) \end{aligned} \quad (12)$$

During the second stage, decision variables in the relay node $\left\{\tilde{s}_{1.R}(1), \tilde{s}_{2,R}(1), \dots, \tilde{s}_{1,R}\left(\frac{N}{2}\right), \tilde{s}_{2,R}\left(\frac{N}{2}\right)\right\}$ again through two OFDM block is sent to the destination node as:

$$\begin{aligned} \tilde{s}_{1,D}^2(k) &= \tilde{s}_{1.R}(k)\beta_{RD}(k) + \mathcal{N}_{1,RD}(k) \\ \tilde{s}_{2,D}^2(k) &= \tilde{s}_{2.R}(k)\beta_{RD}(k) + \mathcal{N}_{2,RD}(k) \end{aligned} \quad (13)$$

So, at the end of the second stage, the number of decision variables in the destination node is doubled. The effective decision variables through EGC (Equal Gain Combining) is achieved. So, we have:

$$\begin{aligned} \tilde{s}_1(k) = {} & s_1(k)\left(\beta_{SR}(k)\beta_{RD}(k) + \beta_{SD}(k)\right) \\ & + \mathcal{N}_1(k) \\ \tilde{s}_2(k) = {} & s_2(k)\left(\beta_{SR}(k)\beta_{RD}(k) + \beta_{SD}(k)\right) \\ & + \mathcal{N}_2(k) \end{aligned} \quad (14)$$

and,

$$\begin{aligned} \mathcal{N}_1(k) = {} & \beta_{RD}(k)\mathcal{N}_{1,SR}(k) + \mathcal{N}_{1,RD}(k) \\ & + \mathcal{N}_{1,SD}(k) \\ \mathcal{N}_2(k) = {} & \beta_{RD}(k)\mathcal{N}_{2,SR}(k) + \mathcal{N}_{2,RD}(k) \\ & + \mathcal{N}_{2,SD}(k) \end{aligned} \quad (15)$$

Finally, decision variables $\left\{\tilde{s}_1(1), \tilde{s}_2(1), \dots, \tilde{s}_1\left(\frac{N}{2}\right), \tilde{s}_2\left(\frac{N}{2}\right)\right\}$ are sent to the ML detector for detecting symbols $\left\{s_1(1), s_2(1), \dots, s_1\left(\frac{N}{2}\right), s_2\left(\frac{N}{2}\right)\right\}$.

In fact, the cooperative OSTFBC for 2×2 MIMO-OFDM system would be similar to 2×1 MIMO-OFDM system and The cooperative OSTFBC for 4×1 and 4×2 MIMO-OFDM system, using the following code [18], is similar to the cooperative OSTFBC for 2×1 MIMO-OFDM system.

$$S_k = I_{\Gamma\times\Gamma} \otimes \begin{bmatrix} s_1(k) & s_2(k) & s_3(k) & s_4(k) \\ -s_2(k) & s_1(k) & -s_4(k) & s_3(k) \\ -s_3(k) & s_4(k) & s_1(k) & -s_2(k) \\ -s_4(k) & -s_3(k) & s_2(k) & s_1(k) \\ s_1^*(k) & s_2^*(k) & s_3^*(k) & s_4^*(k) \\ -s_2^*(k) & s_1^*(k) & -s_4^*(k) & s_3^*(k) \\ -s_3^*(k) & s_4^*(k) & s_1^*(k) & -s_2^*(k) \\ -s_4^*(k) & -s_3^*(k) & s_2^*(k) & s_1^*(k) \end{bmatrix} \quad (16)$$

4 The Achievable Diversity Order

The achievable diversity order for the proposed cooperative OSTFBC algorithm is obtained by calculating the BER. For this purpose by considering the BER of a MIMO-OFDM-2×1 system with OSTFBC, the BER of the proposed algorithm is achieved. According to [19], the bit error rate of MIMO-OFDM-2×1 for BPSK modulation can be by:

$$P_{R,b} = \frac{1}{2^{2L}}\left(1-\sqrt{\frac{\bar{\gamma}_b}{2L+\bar{\gamma}_b}}\right)^{2L} \times \sum_{k=0}^{2L-1}\frac{1}{2^k}\binom{2L-1+k}{k}\left(1+\sqrt{\frac{\bar{\gamma}_b}{2L+\bar{\gamma}_b}}\right)^{k} \quad (17)$$

where $\bar{\gamma}_b = \frac{E_b}{N_0}$ is the average noise energy. At high SNRs the above BER can be approximated as:

$$P_{b,R} \approx \binom{4L-1}{2L-1}\left(\frac{L}{2\bar{\gamma}_b}\right)^{2L} \quad (18)$$

It is difficult to accurately calculate the BER in the proposed algorithm. Therefore, an approximate method is given below that can achieve the diversity order. To calculate the BER of the proposed algorithm, it must be noted that, the direction of the desired signal is contained by two paths for signal transmission for the target system. the additional path is $S \to R \to D$ and the actual path is $S \to D$. The first, is the calculation of the BER of the first path that will be achieved through the following conditional probability**:**

$$P_1 = p(e_{D_1}|e_R)p(e_R) + p(e_{D_1}|c_R)p(c_R) \quad (19)$$

where $p(e_{D_1}|e_R)$ is the BER for the destination node when the error has occurred in the relay node and $p(e_R)$ is the BER in the relay node. $p(e_{D_1}|c_R)$ is the BER in the destination node when the error has not occurred in relay node and $p(c_R)$ is the probability that the error has not occurred in the relay node. Note that $p(e_{D_1}|e_R)$ is the probability of correctly received bits in $S \to R \to D$.

$$p(e_{D_1}|e_R) = p(c_{D_1}) = 1 - p(e_{D_1}) = 1 - P_{D_1,b} \quad (20)$$

$$p(e_R) = P_{R,b} \quad (21)$$

$$p(e_{D_1}|c_R) = P_{D_1,b} \quad (22)$$

$$p(c_R) = 1 - P_{R,b} \quad (23)$$

$$P_{D_1,b} = P_{R,b} \quad (24)$$

So,

$$P_1 = (1 - P_{D_1,b})P_{R,b} + P_{D_1,b}(1 - P_{R,b}) \quad (25)$$

And the BER in $S \to D$ is:

$$P_2 = P_{D_2,b} = P_{R,b} \quad (26)$$

At the receiver, the signals received from the two paths, are added together. Thus, at high SNR and considering the BPSK modulation, the approximate BER is calculated considering the error is occurred in both directions ($S \to R \to D$ and $S \to D$) as:

$$\begin{aligned} P_{2\times1} &\approx P_1 \times P_2 = \\ &2\left(1-\binom{4L-1}{2L-1}\left(\frac{2\bar{\gamma}_b}{L}\right)^{-2L}\right)\binom{4L-1}{2L-1}\left(\frac{2\bar{\gamma}_b}{L}\right)^{-2L} \\ &\times\binom{4L-1}{2L-1}\left(\frac{2\bar{\gamma}_b}{L}\right)^{-2L} = \\ &2\left(1-\binom{4L-1}{2L-1}\left(\frac{2\bar{\gamma}_b}{L}\right)^{-2L}\right)\binom{4L-1}{2L-1}^2\left(\frac{2\bar{\gamma}_b}{L}\right)^{-4L} \end{aligned} \quad (27)$$

The relation (26) concludes that the achievable diversity order, in the proposed algorithm, is 4L.

Similarly, the BER for cooperative-OSTFBC $2 \times 2, 4 \times 1$ and 4×2 MIMO-OFDM at high SNR for BPSK modulation is approximated as:

$$P_{2\times2} \approx 2\left(1-\binom{8L-1}{4L-1}\left(\frac{2\bar{\gamma}_b}{L}\right)^{-4L}\right)\binom{8L-1}{4L-1}^2\left(\frac{2\bar{\gamma}_b}{L}\right)^{-8L} \quad (28)$$

$$P_{4\times1} \approx 2 \times \left(1-\binom{8L-1}{4L-1}\left(\frac{2\bar{\gamma}_b}{L}\right)^{-4L}\right)\binom{8L-1}{4L-1}^2\left(\frac{2\bar{\gamma}_b}{L}\right)^{-8L} \quad (29)$$

$$P_{4\times2} \approx 2 \times \left(1-\binom{16L-1}{8L-1}\left(\frac{2\bar{\gamma}_b}{L}\right)^{-8L}\right)\binom{16L-1}{8L-1}^2\left(\frac{2\bar{\gamma}_b}{L}\right)^{-16L} \quad (30)$$

The above equations concludes that the achievable diversity order in the proposed algorithm for cooperative-OSTFBC $2 \times 2, 4 \times 1$ and 4×2 MIMO-OFDM are 8L, 8L and 16L, respectively.

According to [19], the average SER of MIMO-OFDM-2×1 for M-PSK modulation will be the following relation:

$$P_{e(\mathrm{M-PSK})} = \left(1+\frac{\bar{\gamma}}{2L}sin^2\frac{\pi}{M}\right)^{-2L} \times f(L,\bar{\gamma},M) \tag{31}$$

where:

$$f(L,\bar{\gamma},M) = \left\{\frac{\left(2L-\frac{1}{2}\right)!}{2\sqrt{\pi}(2L)!}\,{}_2F_1\left(2L\,,\frac{1}{2}\,;2L+1\,;\frac{1}{1+\frac{\bar{\gamma}}{2L}sin^2\frac{\pi}{M}}\right) + \frac{cos\frac{\pi}{M}}{\pi}F_1\left(\frac{1}{2}\,,2L\,,\frac{1}{2} - 2L\,;\frac{3}{2}\,;\frac{cos^2\frac{\pi}{M}}{1+\frac{\bar{\gamma}}{2L}sin^2\frac{\pi}{M}}\,,cos^2\frac{\pi}{M}\right)\right\} \tag{32}$$

${}_2F_1(a,b\,;c\,;x\,)$ is the gauss hypergeometric function and $F_1(a,b_1,b_2\,;c\,;x,y)$ is Appell hypergeometric. Similarly, the average SER for cooperative-OSTFBC $2\times1-$ MIMO-OFDM at high SNR for M-PSK modulation is approximated as:

$$\mathrm{P}_{2\times1,\mathrm{M-PSK}} \approx 2\left(1-\left(1+\frac{\bar{\gamma}}{2L}sin^2\frac{\pi}{M}\right)^{-2L} \times f(L,\bar{\gamma},M)\right) \times f(L,\bar{\gamma},M)^2\left(1+\frac{\bar{\gamma}}{2L}sin^2\frac{\pi}{M}\right)^{-4L} \tag{33}$$

The relation (33) concludes that the achievable diversity order, for M-PSK modulation in the proposed algorithm, is 4L.

According to [19], the average SER of MIMO-OFDM-2×1 for M-QAM modulation will be the following relation:

$$P_{e(\mathrm{M-QAM})} = \left(1\frac{3\bar{\gamma}}{4L(M-1)}\right)^{-2L} \times g(L,\bar{\gamma},M) - \left(1+\frac{3\bar{\gamma}}{2L(M-1)}\right)^{-2L} \times h(L,\bar{\gamma},M) \tag{34}$$

where:

$$g(L,\bar{\gamma},M) = 2\left(1-\frac{1}{\sqrt{M}}\right) \times \frac{\left(2L-\frac{1}{2}\right)!}{\sqrt{\pi}(2L)!} \times {}_2F_1\left(4L\,,\frac{1}{2}\,;4L+1\,;\frac{1}{1+\frac{3\bar{\gamma}}{4L(M-1)}}\right) \tag{35}$$

and

$$h(L,\bar{\gamma},M) = 2\left(1-\frac{1}{\sqrt{M}}\right)^2 \times \frac{1}{\pi(4L+1)} \times F_1\left(1\,,2L\,,1\,;\;2L+\frac{3}{2}\,;\;\frac{1+\frac{3\bar{\gamma}}{4L(M-1)}}{1+\frac{3\bar{\gamma}}{2L(M-1)}}\,,\frac{1}{2}\right) \tag{36}$$

The average SER for cooperative-OSTFBC $2\times1-$ MIMO-OFDM at high SNR for M-QAM modulation is approximated as:

$$P_{e(\mathrm{M-QAM})} \approx 2\left(1-\left(1\frac{3\bar{\gamma}}{4L(M-1)}\right)^{-2L} \times g(L,\bar{\gamma},M) - \left(1+\frac{3\bar{\gamma}}{2L(M-1)}\right)^{-2L} \times h(L,\bar{\gamma},M)\right)\left(\left(1\frac{3\bar{\gamma}}{4L(M-1)}\right)^{-2L} \times g(L,\bar{\gamma},M) - \left(1+\frac{3\bar{\gamma}}{2L(M-1)}\right)^{-2L} \times h(L,\bar{\gamma},M)\right)^2 \tag{37}$$

It has been seen that for *M*-QAM schemes, diversity order of $G_d=4L$ can be achieved.

5 Simulation Results

In this section, the proposed algorithm is simulated for 2×1, 2×2, 4×1 and 4×2 MIMO-OFDM systems for the uniform channel models with $\mathrm{L}=2{,}3$. In $\mathrm{S}\rightarrow\mathrm{R}$, $\mathrm{R}\rightarrow\mathrm{D}$ and $\mathrm{S}\rightarrow\mathrm{D}$, the simulated MIMO-OFDM system for $\mathrm{N}=256$ subcarriers in 1 MHz bandwidth. Assume that the average symbol power per transmit antenna is $\mathrm{E_s}=\frac{1}{\mathrm{M_T L}}$ and the noise variance is $\frac{1}{\mathrm{SNR}}$. We assume that the perfect channel state information is known at the receiver. The simulations carried out for two uniform channel models. For $\mathrm{L}=2$, 2-ray channel model with time delays $\tau_1=0$ μsec and $\tau_1=1$ μsec and normalized path gains $\sigma_l^2=\frac{1}{2}$ for $l=1,2$ is assumed. For $\mathrm{L}=3$, 3-ray uniform channel model with time delays $\tau_1=0$ μsec,$\tau_1=1$ μsec, $\tau_2=2$ μsec and $\sigma_l^2=\frac{1}{3}$ for $l=1,2,3$. The simulation results for 2×1, 2×2, 4×1 and 4×2 systems compared with orthogonal STFBC in [14] for 2×1, 2×2, 4×1 and 4×2 MIMO-OFDM systems.

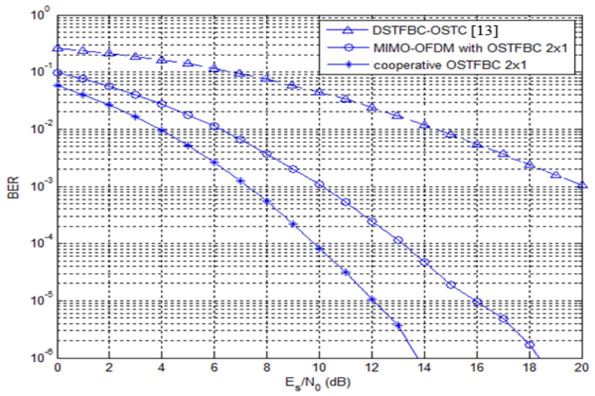

Fig. 4 BER vs. SNR of O-STFBC [14] and cooperative-OSTFBC for $\mathrm{M_T}=2$, $\mathrm{M_R}=1$, $\mathrm{L}=2$ and spectral efficiency of 1 bit/sec/Hz.

Fig. 4 shows the BER performance of orthogonal STFBC [14], DSTFBC-OSTC with two nodes relay [13], and proposed cooperative-orthogonal STFBC with BPSK modulation for $M_T = 2$, $M_R = 1$, $L = 2$. It is evident from the plots in Fig. 4 that the proposed algorithm can reduce the BER, and the system performance is improved by as much as 4dB compared with the orthogonal STFBC [14].

Figs. 5, 6 and 7 depict the BER performance of the orthogonal STFBC [14] and proposed cooperative-orthogonal STFBC with BPSK over uniform channel model for $L = 2$ for 2×2, 4×1 and 4×2 systems, respectively. As it is observed, the OSTFBC [14] can achieve diversity order of $8, 8, 16$ for $L = 2$ but proposed cooperative-OSTFBC can achieve diversity order of 16, 16, 32 for 2×2, 4×1 and 4×2 systems, respectively. It is evident from the figures that the proposed algorithm can reduces the BER and improves system performance.

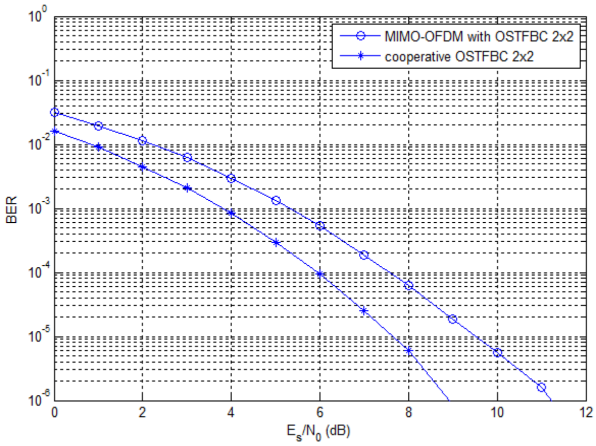

Fig. 5 BER vs. SNR of O-STFBC [14] and cooperative-OSTFBC for $M_T = 2$, $M_R = 2$, $L = 2$ and spectral efficiency of 1 bit/sec/Hz.

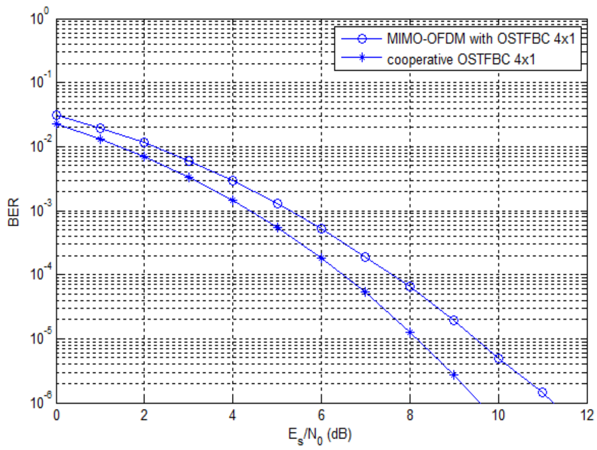

Fig. 6 BER vs. SNR of O-STFBC [14] and cooperative-OSTFBC for $M_T = 4$, $M_R = 1$,$L = 2$ and spectral efficiency of 1 bit/sec/Hz.

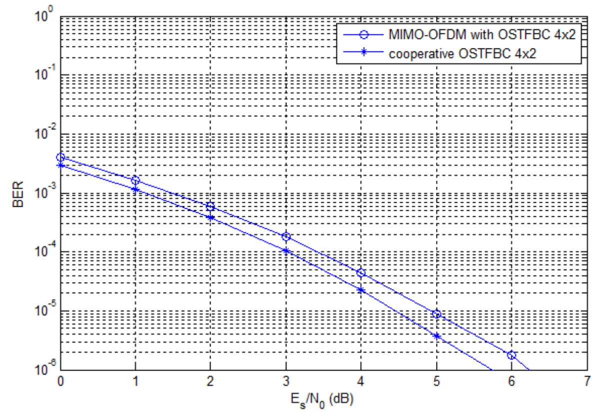

Fig. 7 BER vs. SNR of O-STFBC [14] and cooperative-OSTFBC for $M_T = 4$, $M_R = 2$,$L = 2$ and spectral efficiency of 1 bit/sec/Hz.

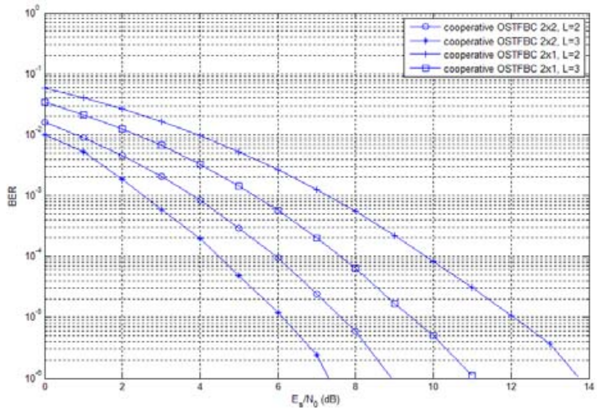

Fig. 8 BER vs. SNR of the cooperative-OSTFBC for $L = 2$ and L=3, $M_T = 2$.

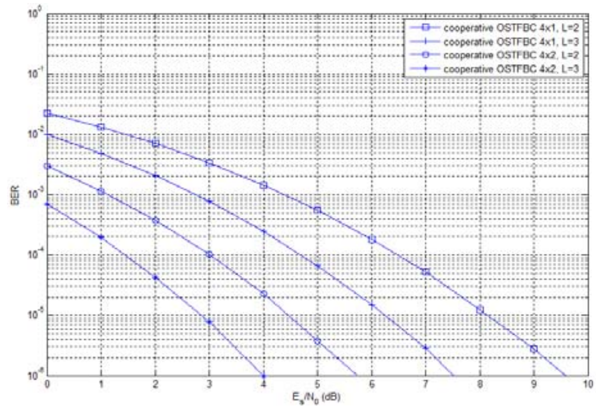

Fig. 9 BER vs. SNR of the cooperative-OSTFBC for $L = 2$ and L=3, $M_T = 4$.

Figs. 8 and 9 depict the BER performance of the proposed cooperative-orthogonal STFBC with BPSK over uniform channel model for $L = 2$ and $L = 3$. It is evident from the figures that the proposed cooperative-orthogonal STFBC with $L = 3$ reduces the bit error rate and improves system performance.

6 Conclusion

In this paper, a cooperative algorithm is suggested for MIMO-OFDM systems with 2×1, 2×2, 4×1 and 4×2 antennas equipped with OSTFBC technique. We have shown that proposed algorithm achieves the full diversity order of $2M_TM_RL$. Simulation results are presented to show the performance improvement and the proposed algorithm can reduce the bit error rate by adding a relay node.

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