Stochastic Assessment of Voltage Sags in Distribution Networks

M. Aliakbar-Golkar* and Y. Raisee-Gahrooyi*

Abstract: This paper compares fault position and Monte Carlo methods as the most common methods in stochastic assessment of voltage sags. To compare their abilities, symmetrical and unsymmetrical faults with different probability distribution of fault positions along the lines are applied in a test system. The voltage sag magnitude in different nodes of test system is calculated. The problem with these two methods is that they require unknown number of iteration in Monte Carlo Method and number of fault position to converge to an acceptable solution. This paper proposes a method based on characteristic behavior of Monte Carlo simulations for determination required number of iteration in Monte Carlo method.

Keywords: Confidence Interval, Distribution Networks, Stochastic Methods, Voltage Sag.

1 Introduction

Voltage sag and interruption are two most serious power quality problems responsible for frequent malfunctions of electrical equipments in industrial and commercial installation [1]. Voltage sag indices are used for simpler description, quantification and comparison of characteristic and performance of different events, sites or systems. The indices are very important part in addressing voltage sag issues, because they often serve as a "specification values" in contracts between the utilities and customers. For example, system average RMS variation frequency index (SARFIx) of single-site, provides a count of all voltage sags for a site which are bellow voltage threshold X and with duration less than 60 seconds and SARFIx system index is calculated as the number of sites experiencing at least one SARFIx single- site index divided by the number of all sites in the system [2].

The calculation of voltage sag indices can be based on actual measurement [3-4] or by using stochastic prediction methods. Without any doubt, the major cause of voltage sags are short circuit faults in power supply system and inside the customer's installation. Stochastic methods (critical distances [5], fault positions, Monte Carlo) have been proposed for the assessment of voltage sag due to faults [6]-[8].

The method of critical distance provides accurate results for the stochastic assessment of voltage sag but it requires high computational effort that becomes even more complex for larger networks. The method of fault position and the Monte Carlo method are simpler and easier to implement [3].

The problem in these two methods is that they require unknown number of fault position and iteration to converge to an acceptable solution and it's very important especially in large networks.

Ref. [8] talks about the minimum required number of iterations assumed by the Monte Carlo method to an acceptable solution but it doesn't mention any practical method.

This paper compares the abilities of fault position and Monte Carlo method by applying symmetrical and unsymmetrical faults with different probability distribution of fault position along the lines to reach balanced and unbalanced voltage sag. Furthermore, this paper presents an appropriate method based on characteristic behavior of Monte Carlo simulations for determination required number of iteration in Monte Carlo method and can be applied to large networks easily.

It is organized as follows; Section 2 presents sample test system, Sections 3 and 4 briefly recalls the principles of fault position and Monte Carlo methods. The effect of different probability distribution of fault positions is described in Section 5. In Section 6 a new method to determine number of iteration in Monte Carlo method is proposed. In Section 7 conclusions are drawn.
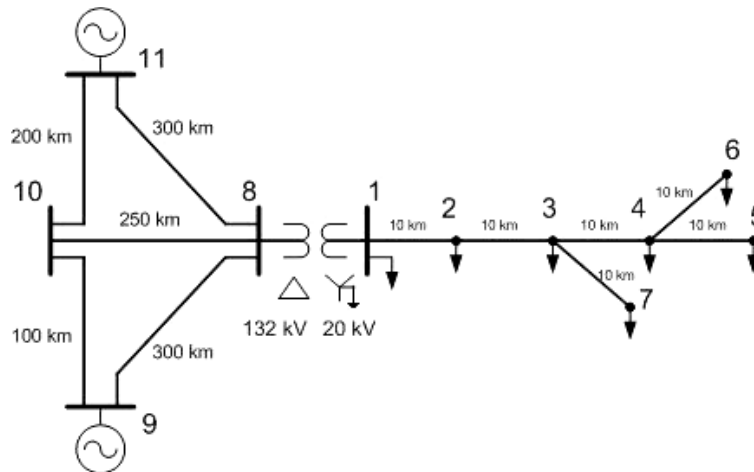
Fig. 1 Sample test system.

2 Test System

The diagram of the test system is shown in Fig. 1. Seven critical customers are connected at seven nodes of the same 20 kV distribution line (60 km total length) through a solidly grounded delta wye transformer. It should be noted that this network represents a typical distribution network with several hundred nodes spread along the main feeder and laterals.

The nodes that supply the most critical customer need to be examined. The equivalent transmission system consists of five 132 kV lines and is relatively of large size (1150 km total length).

3 Fault Position Method

In the method of fault position the voltage in every bus is calculated for various fault positions spread at equal distances for each line. One drawback of the method of fault positions is the lack of clarity in the appropriate number of fault positions to be considered. The more fault positions procedure the more accurate procedure. In the limit situation, infinite number of

faults should be simulated until the results remain unaltered.

Two cases have been considered:
Case 1: voltage sags in bus 1 to 7 due to faults in distribution system.

At equal distances of each line and for each type of fault 600 faults have been applied. The worth of each type is considered by using the distribution probability of fault type in Fig. 2. Cumulative annual frequency of dips at bus 1 to 7 is shown in Fig. 3.
Case 2: voltage sags for each bus in distribution system due to faults at the transmission system.

The effect of power transformer on voltage sag is considered. To do this, 2000 faults in equal distance of transmission system for each fault type are applied. The worth of each type is considered by Fig. 2. The result is shown in Fig. 4. Because of transmission system configuration, it is clear that bus 8 is influenced by every fault in transmission system. It shows this fact that faults event at 100 km away from the customers will cause sever sags.
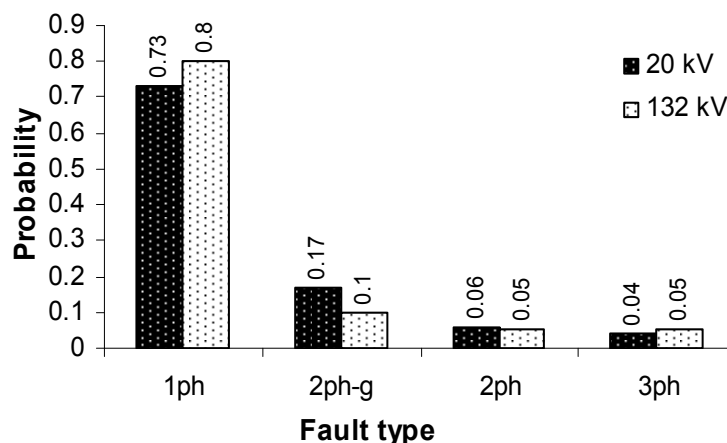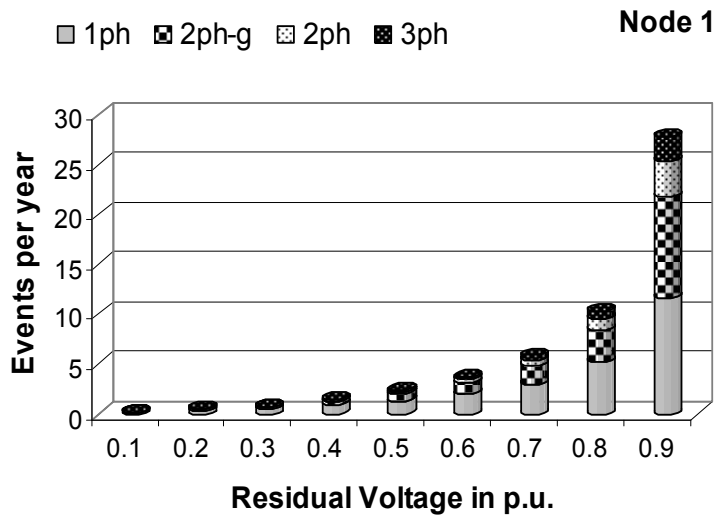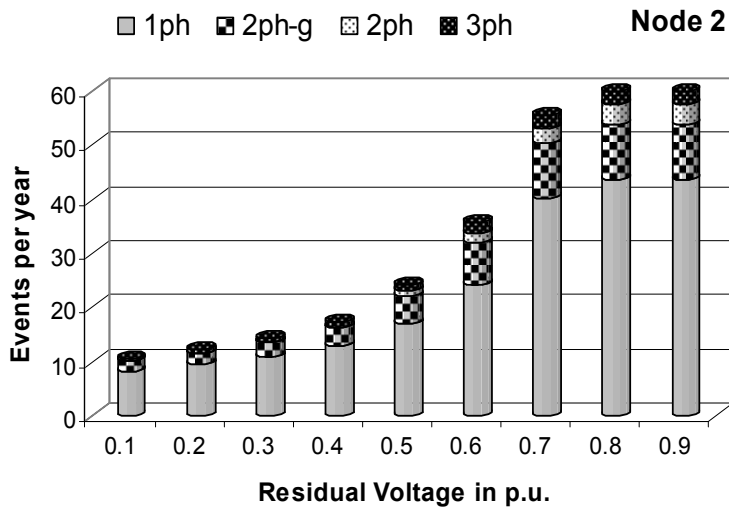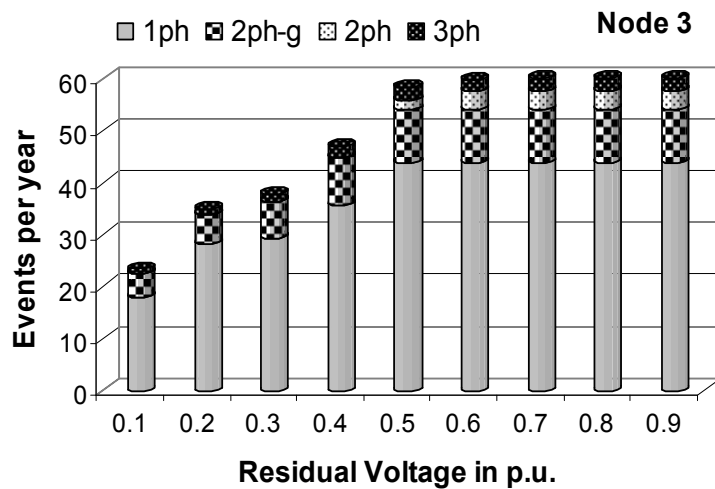

Fig. 2 Probability for each fault type in 20 kV and 132 kV.

(a)
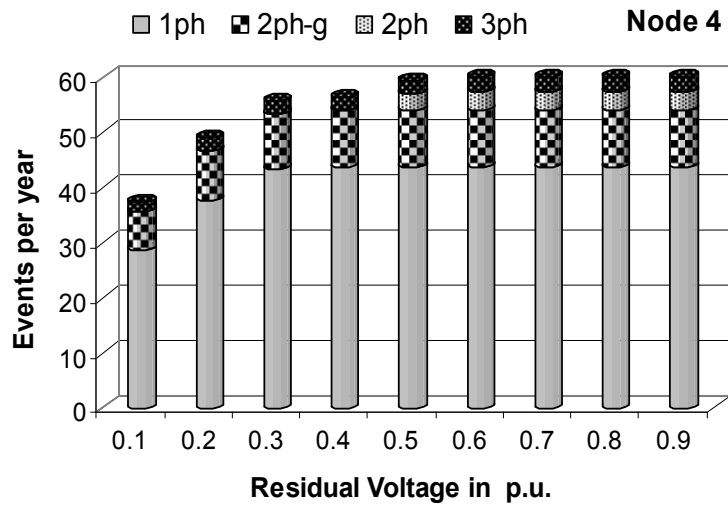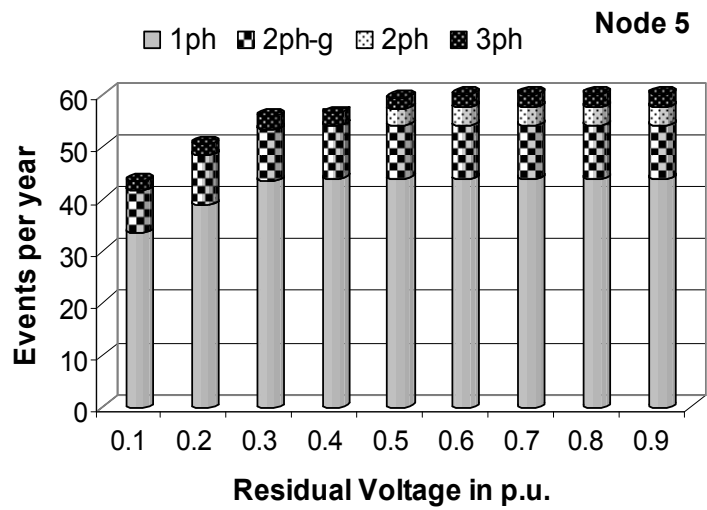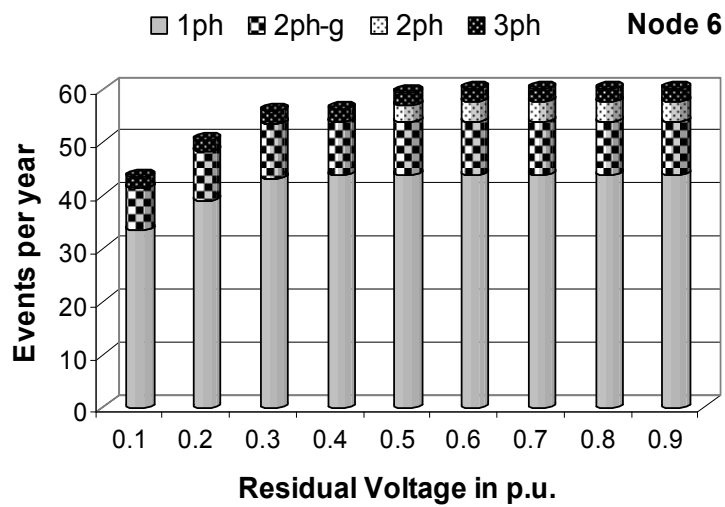

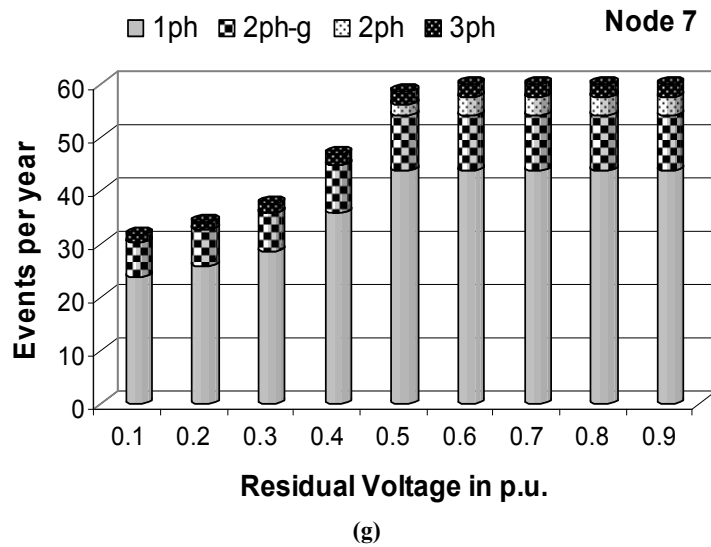**(b)**

(c)

(d)

(e)

(f)

Fig. 3 Cumulative sags frequency at nodes 1-7 (Contribution of balanced and unbalanced sags are shown).
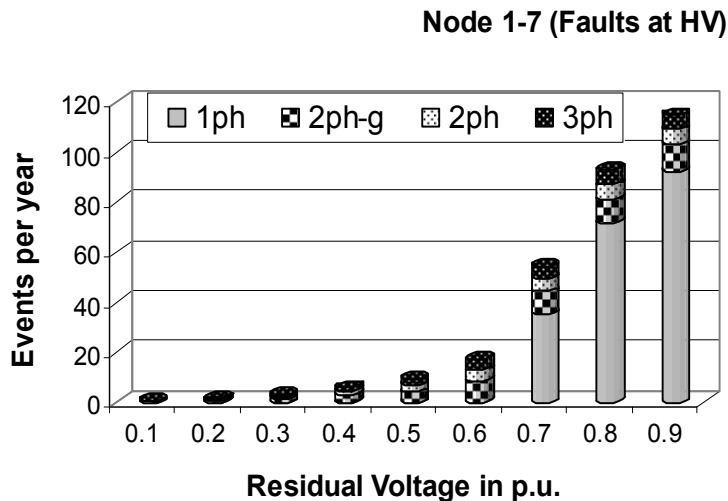

Fig. 4 Cumulative sags frequency at nodes 1-7 due to faults in transmission system.

4 Monte Carlo Method

Monte Carlo simulation is a powerful numerical technique for solving stochastic problems by modeling random variables. The key factor in Monte Carlo simulation is the use of random numbers to model the behavior of stochastic variables involved in the process. Instead of using just the average value to model uncertain variables, the complete distribution function is used number of iterations to describe their behavior. This of course requires more information regarding the historical performance of the system. After an infinite number of iterations, the response of the system studied converges to a solution theoretically

In the procedure of voltage sag assessment with Monte Carlo method assumes that sags are due only to faults caused with the distribution network. Random

variables in our case are mean-time-to-fault, fault position and fault type. Every time the system is run, several quantities are randomly generated prior to calculations. Voltage sag characteristics at the nodes are recorded every run. At the end of simulation time, the SARFIx at the nodes is calculated.

The algorithm implemented can be summarized as follows:

1- Set a simulation time in years.
2- For each element (line or bus) generate a random number and convert it into time-to-fault according to the probability of distribution function of the time-to-fault of the element. Calculate the cumulative time.
3- Select the fault location by generating random number and convert it into fault position according

to the probability distribution function of this parameter.

4- For each fault, generate a random number and convert it into fault type (SLG, LL, LLG and LLL) according to the probability distribution function of the fault type.

5- Calculate the residual voltage at the buses.

6- If cumulative time is less than the simulation time, go to 2.

7- Out put results.

Simulation Parameters:
- The mean-time-to-fault is taken as the reciprocal of the fault rate (1 faults/year-km). This parameter for 60 km distribution radial system is 0.0167 years. This parameter is assumed uniformly distributed. It means that for each year, distribution system has exactly 60 faults.
- The fault resistance is 0 ohm.
- The probability of each type of fault is assumed as Fig. 2.
- The fault location is assumed uniformly distributed along each line.

The test system is run for 50 years and the results are cumulative sag frequency for each node. Fig. 5 compares the results of Monte Carlo and fault position method at node 1. The comparisons are made in the range 0.1-0.9 p.u.

Fig. 5 shows that with above conditions and parameters Monte Carlo and fault position method have the same results. Two main parameters that are important to consider are the probability distribution of fault position and number of iteration that Monte Carlo converges to an acceptable solution.

5 The Effect of Different Probability Distribution of Fault Positions

The effect of probability distribution of fault position in Mont Carlo method is studied in three different cases in distribution network. Each line is 10 km (20 × 0.5 km).

Case1:
It is assumed that probability of fault events in middle points of lines is more than the other points. For achievement of this assumption, the fault position is considered normally distributed with a mean value of half of the line and a standard deviation of s in each line as follow relation:

$$f(l) = \frac{1}{s.\sqrt{2\pi}}.e^{\frac{-(l-\mu)^2}{2.s^2}} \tag{1}$$

Fig. 6 shows normal distribution with different amount of s. If fault position is distributed normally like Fig. 7, cumulative sag frequency at bus 1 for different s will be like Fig. 7. and is compared with fault position method.

Case2:
It is assumed that probability of fault events in initial points of lines is more than the other points. To reach this aim, the probability distribution of fault position is assumed to be as follow:

$$f(l) = \frac{2}{s.\sqrt{2\pi}}.e^{\frac{-(l-\bar{l})^2}{2.s^2}} \quad , \bar{l} = 0 \leq l \leq 20^{km} \tag{2}$$

where, s is the spread coefficient from the initial points of the lines.

According to above equation, Fig. 8 shows the probability of each point along the line. If Monte Carlo method is implemented with above probability distribution function with different s, cumulative sag frequency at node 1 will be like Fig. 9. The results are compared with fault position method.

Case 3:
It is assumed that probability of fault events in final points of lines is more than the other points of lines. For achievement of this aim, the probability distribution of fault position is assumed to be as follow:

$$f(l) = \frac{2}{s.\sqrt{2\pi}}.e^{\frac{-(l-\bar{l})^2}{2.s^2}} \quad , 0 \leq l \leq 10^{km} = \bar{l} \tag{3}$$

where, s is the spread coefficient from the initial points of the lines.

According to above equation, Fig. 10 shows the probability of each point along the line. If Monte Carlo method is implemented with above probability distribution function with different s, cumulative sag frequency at node 1 will be like Fig. 11. The results are compared with fault position method.

It's clear from case 1, case 2 and case 3 that fault position method fails in describing the variability of the actual performance of the network and gives only long-term average values. The Monte Carlo approach provides richer results.

6 Number of Iteration in Monte Carlo Method

This section proposes a criterion for determination required number of iterations of Monte Carlo method based on characteristic behavior of Monte Carlo simulations.
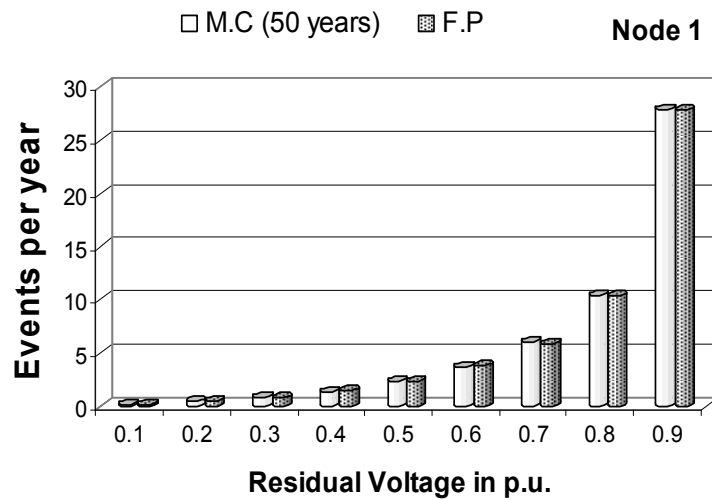
Fig. 5 Comparison between Cumulative sags frequency at node 1 with Monte Carlo and fault position methods when faults are distributed uniformly.
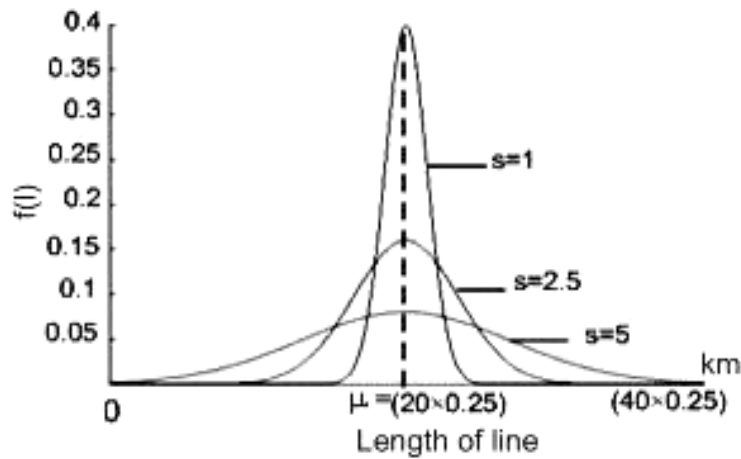

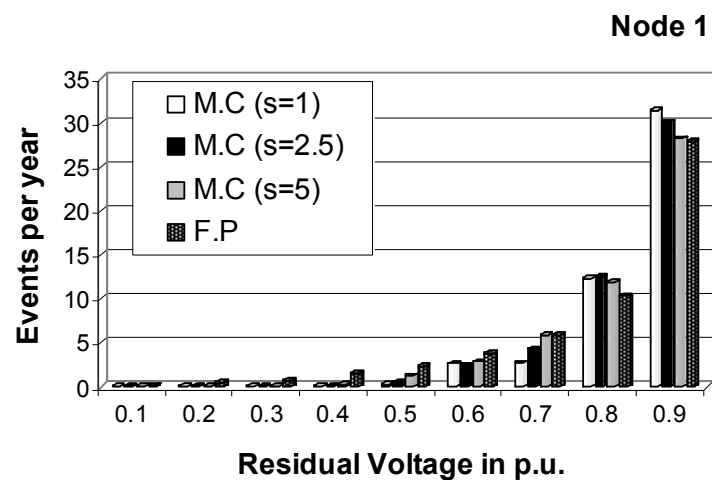**Fig. 6** Normal distribution of faults along the lines in distribution network with s = 1, 2.5, 5.

Node 1

Fig. 7 Comparison between Cumulative sags frequency at node 1 with Monte Carlo and fault position method when faults are distributed like Fig. 6.

Fig. 8 Distribution of faults when the probability in initial points of line is more and spread coefficient is s = 1, 2.5, 5.

Node 1

Fig. 9 Comparison between Cumulative sags frequency at node 1 with Monte Carlo and fault position method when faults are distributed like Fig. 8.

Fig. 10 Distribution of faults when the probability in final points of line is more and spread coefficient is s = 1, 2.5, 5.

Node 1

Fig. 11 Comparison between Cumulative sags frequency at node 1 with Monte Carlo and fault position method when faults are distributed like Fig. 10.
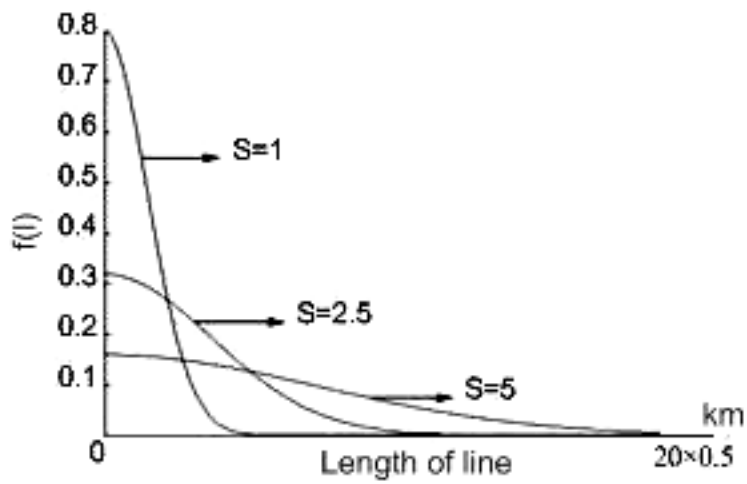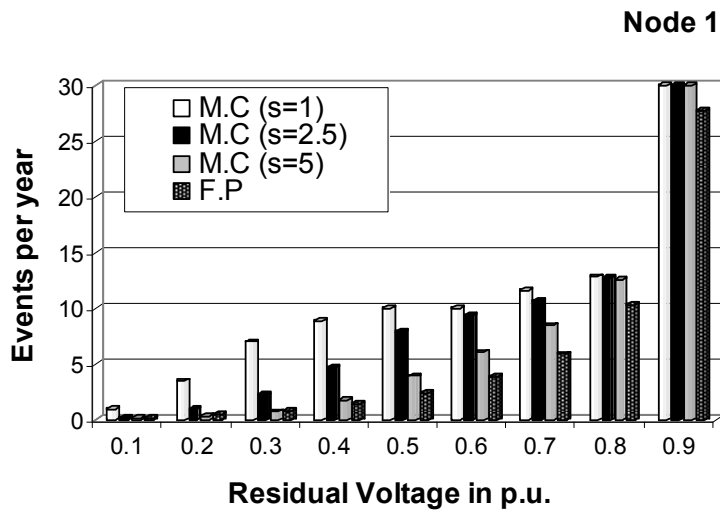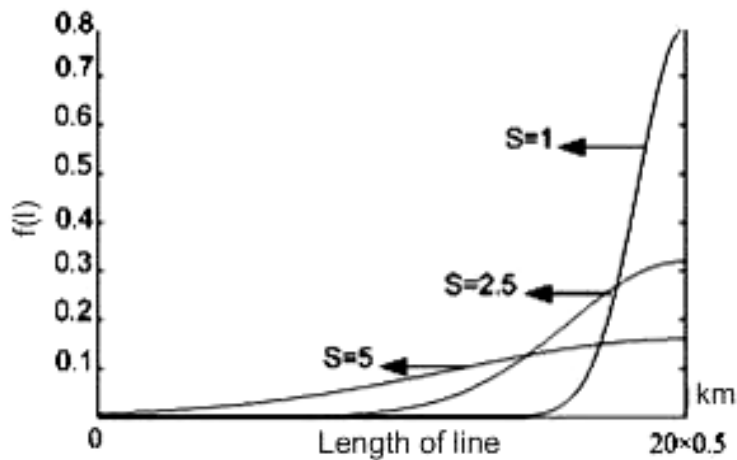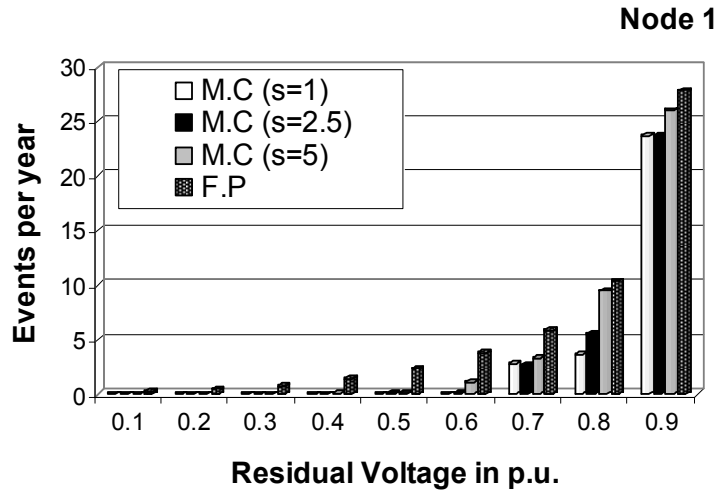
6.1 Characteristic Behavior of Monte Carlo

The amount of data generated by Monte Carlo simulation is huge. Only a very small sample of the results is presented here. Fig. 12 shows a characteristic behavior of Monte Carlo simulations for 40 years when faults are distributed uniformly. The system average rms variation frequency index at node 1 estimated via Monte Carlo simulation is shown in Fig. 12.

The SARFI-0.9 resulting for each simulation and the average SARFI-0.9 calculated over the cumulative number of simulations are shown. Although the SARFI-0.9 corresponding to each year is volatile, the average is rather stable.

The outcomes of the Monte Carlo approach are average values calculated over different sample sizes. Therefore, the outcome of the Monte Carlo method distributes normally and the error in the estimate can be calculated building a confident interval for the actual mean value.

In order to evaluate how accurate the last average value in Fig. 12 is, the sample standard deviation s is used to build a confidence interval for the expected SARFI-0.9. The standard deviation s correspond to the n = 40 simulations in Fig. 12 is about 1.58 and the average SARFI-0.9 calculated over the 40 previous values is 27.15. The 95% confidence interval results:

$$(\text{SARFI} - 0.9 = \mu) \in \left[\bar{X} - \frac{1.96.s}{\sqrt{n}}, \bar{X} + \frac{1.96.s}{\sqrt{n}} \right] \quad (4)$$

$$(\text{SARFI} - 0.9 = \mu) \in \left[27.64, 26.65 \right] \quad (5)$$

where 1.96 is the critical value corresponding to a 95% of confidence of the normal distributed variable.

It means that we are 95% sure that unknown population value of Monte Carlo simulation has captured within the interval. Equation shows that the error can be reducing by increasing the number of simulations. If the number of samples is less than 30, t-distributed must be used.

6.2 Sample Size (Number of Iteration) Determination in Monte Carlo Method

In general and in order to estimate mean of population to a bound d with (1-alpha) %100 confidence, the required sample size found as follows:

$$n = \left[\frac{z_{\alpha/2}\sigma}{d} \right]^2 \quad (6)$$

The value of sigma in a population is unknown. It can be estimated by the standard deviation s from a prior sample. The value of n must be rounded upward to ensure that the sample size will be sufficient to achieve the specified reliability. d is known as margin of error and is a measure of the width of the confidence interval. $z_{\alpha/2}$ depends on alpha and is 1.96 for $\alpha = \%5$.

In the population of Monte Carlo simulation results, the value of sigma is unknown. A prior sample is selected as guide sample. To do this, in the presented algorithm in Monte Carlo method (Section 4), output results (SARFI-x) are calculated for different simulation times. The set of out put results is Monte Carlo guide sample. The sigma is estimated with standard deviation of guide sample.

By replacing the estimated sigma in Eq. 6 and with $z_{\alpha/2} = 1.96$, the decreasing function of n (number of iteration) in term of d (margin of error) is obtained.

In the left side of bending area in the diagram of decreasing function of n (number of iteration) in term of d (margin of error), Fig. 13, a few changes in d lead to great changes in number of iteration (n). Therefore, in the Monte Carlo method it's not at all worthwhile to apply many calculations to get only a bit improvement in d (margin of error) and it's advisable to consider the n (number of iteration) in bending point as required number of simulation time in Monte Carlo method.

6.3 Case Study

In order to become 0.95% sure that the estimation of SARFI-0.9 in bus 1 with characteristic behavior of Fig.

12, isn't more than d, a guide sample of SARFI-0.9 with length of 8 (for different simulation times) has been selected. The standard deviation of prior sample has been considered as standard deviation of population. Standard deviation of prior sample is s = 0.557.

By replacing the standard deviation of this sample in Eq. 6, sample size is expressed as a function of d (margin of error).

The diagram of sample size (n) as a function of d is shown in Fig. 13. It is clear that with a little decrease in error at A zone, the sample size (n) will have considerable growth.

The estimation error difference in two Monte Carlo simulations with n = 49 and n = 81 (year) is only 0.04. In other words, it is advantage to choose sample size with n = 49 year a point in bending area of this diagram.

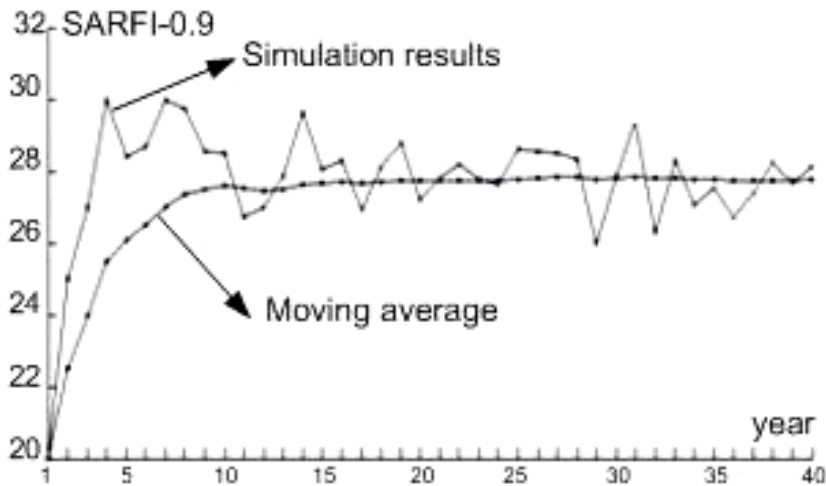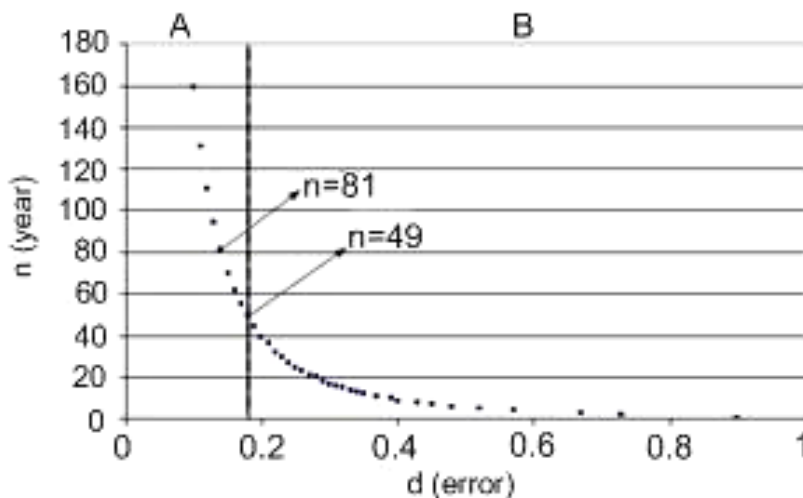**Fig. 12** SARFI-0.9 for node 1 with 40 Monte Carlo simulations.

Fig. 13 Sample sizes (n) in term of margin of error (d).

7 Conclusion

This paper compares two most common methods in stochastic assessment of voltage sags (Fault Position and Monte Carlo methods). It is shown that if fault positions distributed uniformly along the lines, their results are similar. However, if fault positions along the line don't distributed uniformly and the probability of fault positions in different points of each line be various, their results aren't same. This conclusion is clear because in fault position method faults spread at equal distances for each line. Therefore, Monte Carlo method is more flexible than fault position method. Fault position provides mean value for stochastic assessment. Moreover, in these two methods number of fault position and iteration are unknown and simulation should be continuing till the results remain unaltered, this procedure requires a lot of computational efforts that are very time consuming in large networks. Since the behavior of results in Monte Carlo method is normal, a method based on characteristic behavior of Monte Carlo simulations has proposed to estimate iteration in Monte Carlo method. In this method, by forming a guide sample of results of Monte Carlo method and deriving standard deviation of it, number of irritation can be determined properly.

Appendix

In table I impedances of lines and transformers are presented.

Table I Impedance of Lines (Ω/km) and transformer (Ω).

-	+ Sequence	- Sequence	0 Sequence
Distribution	1.2+0.4i	0.22+0.37i	0.37+1.5i
Subtransmission	0.1+0.4i	0.1+0.4i	0.5+0.4i
transformer	0.02i	0.02i	0.005i

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